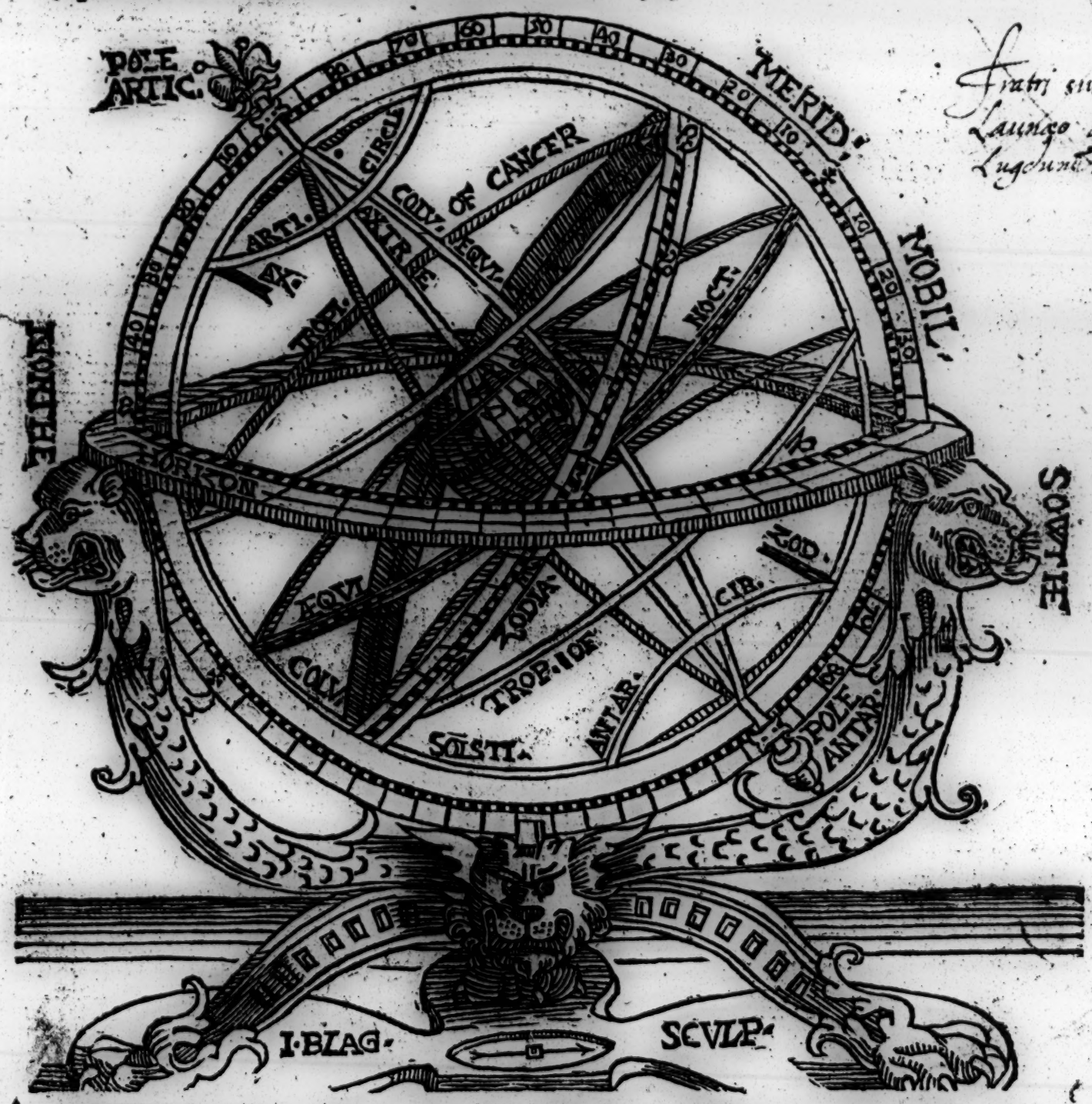
Harvard 528. n. 20
1-9

THE MATHEMATICAL IEWEL,

Shewing the making, and moſt excellent vſe of a ſinguler *Inſtrument ſo called: in that it performeth with wonderfull* dexteritie, whatſoeuer is to be done, either by Quadrant, Ship, Circle, Cylinder, Ring, Dyall, Horoſcope, Aſtrolabe, Sphere, Globe, or any ſuch like heretofore deuiſed: yea or by moſt Tables commonly extant: and that generally to all places from Pole to Pole.

The vſe of which Iewel, is ſo aboundant and ample, that it lea- deth any man practiſing thereon, the direct pathway (from the firſt ſteppe to the laſt) through the whole Artes of Aſtronomy, Coſmography, Geography, Topography, Nauigation, Longitudes of Regions, Dyalling, Sphericall triangles, Setting figures, and briefely of whatſoeuer concerneth the Globe or Sphere: with great and incredible ſpeede, plaineneſſe, facillitie, and pleaſure:

The moſt part newly founde out by the Author, Compiled and publiſhed for the furtherance, as well of Gentlemen and others deſirous of ſpeculatiue knowledge, and priuate practiſe: as alſo for the furniſhing of ſuch worthy mindes, Nauigators, and traueylers, that pretend long voyages or new diſcoueries: By *Iohn Blagraue* of *Reading* Gentleman and well willer to the Mathematickes, who hath cut all the prints or pictures of the whole worke with his owne hands. 1585.



Fratri sue Nathanaely Lauingo donum mittit Lugdunensis Bat. Nordouicus P. Lauinius.

Imprinted at London by Walter Venge, dwelling in Fleetelane ouer againſt the Maiden head.

MARGARITA MATHEMATICA.

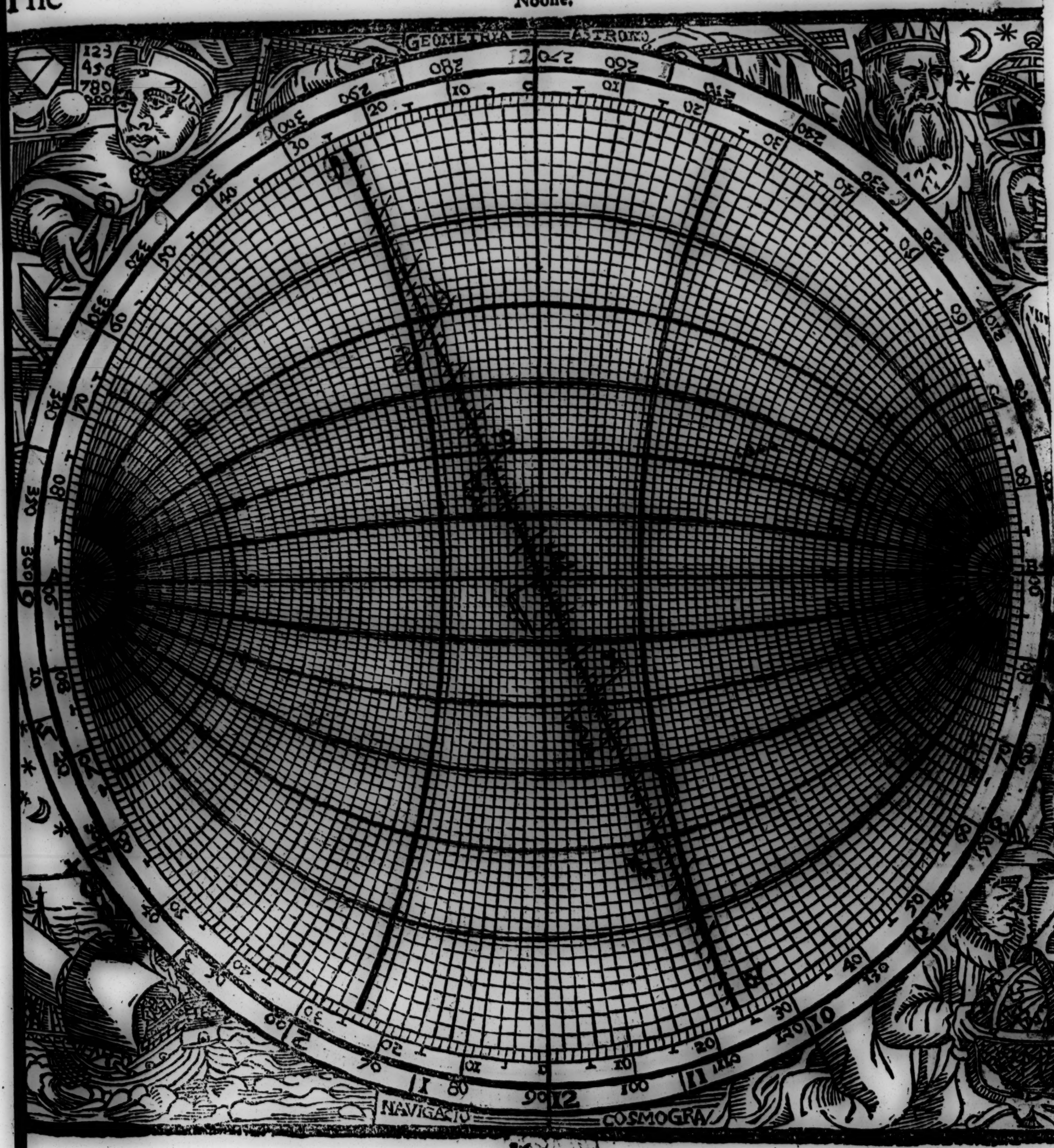
Astronomis nobilissimum, Geometris iucundissimum, Nauigantibus prestan
tissimum, Cosmographis commodissimum, Philosophis, Medicis, & aliquid sublimè affectantibus gratissimum, Tyronibusq; facillimum.

The handle is heere to be set on.

Meridies.
Noone.

The

Mater

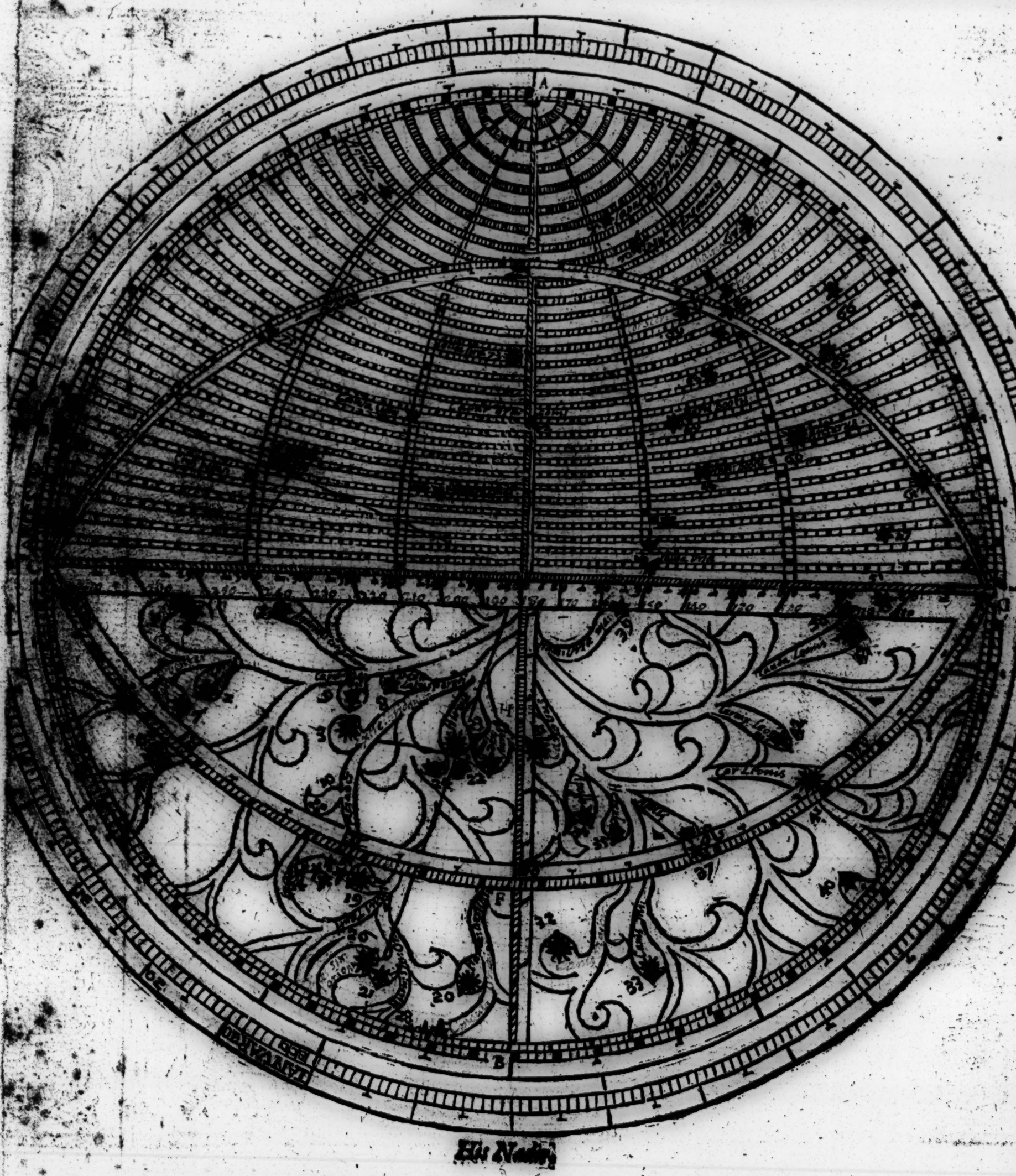


Media nox.
North.

Per *Ioannem Blagrauum* Readingensem, conditum, editum, & Sculptum.

1584.

The Reete.



To the Right Honorable Sir VVilliam Cycill Knight

Baron of Burleigh, Lord high Treſurer of Englande, Maſter of her Maieſties Wards and Liueries, Knight of the moſt honorable order of the Garter, and of her heighneſſe moſt honorable Priuie Counſel.

BEing (Right Honorable) no leſſe then the reſt of my deere friends, full fraught with the feruent and dueefull zeale, wherein the whole progenie of vs are, & ſhalbe euerlaſtingly [illegible] vnto your good L. diuerſe wiſe: namely, for the ſyncere Iuſtice and pittie, or rather piety, ſo charitably extended in our late extreame & moſt iniurious vexation. Sounding in equitie the depth of the monſtrous and deteſtable driftes and deuiſes of thoſe notorious, lewde & wicked practiſers againſt vs, who ſought *Quaq; in-turia*, to ſtrippe vs out of whatſoeuer we poſſeſſed, without any colour almoſt, of matter or cauſe: but onely preſuming and truſting to the packing and corruption of the time, and of our innocencie and inſufficiencie to withſtand their aſſaultes. It were a treble tragedie to declare euery circumſtance: and needeleſſe, the matter being wholly handled ſo ſundrie times before your L. that your L ordſhip no doubt remembreth it: and of vs, who haue all their trecherie in records exemplified, can neuer be forgotten: How wickedly they ſtale away (as they thought) all our euidences, and thereupon by what ſiniſter meanes they wrought her Maieſtie a title: howe cunningly or rather cozeningly they conueyed it againe from her Heighneſſe for three ſcore and ten pounds of money, which coſt my friends almoſt 3000.li. neere 40. yeares paſt: How they alſo altered the tenure from *Capite* to *Socage*, and what vile corruption, notable packing, craftie confederacies, wicked ſubornations, vnconſcionable wreſtings, ſubtill ſhiftings, indirect proſecutions, and vnchriſtianlike dealings they vſed and practiſed, againſt vs poore ſoules euerie way vnable, had not the right of the cauſe found grace before your H. chiefly and other honorable perſonages & good Iudges and Iuſtices of this land, euen to the glory & praiſe of Almightie God in the eye of the world. The continuall remembrance whereof neuer ceaſing: yea rather dayly encreaſing, hath pricked me forth, perhaps (as it may be thought) ſomwhat ouer boldly, al things in equall ballance conſidered, (though of two extremities, I haue rather ſorted to hazard the ridiculous diſdaine of preſumption, then the moſt heinous reproch of an vngratefull minde:) euen to preſent your Honour heere at this time with a *Iewel*, not wrought of Minerals, or ſet with ſtately ſtones, or brought home from beyond the ſeas by ſundry our countrie men, in their venterous and worthy voyages lately performed. But a *Mathematicall Iewel*, of no ſmall vertue and efficacie, to furniſh the willing wits of this our age for the like enterpriſes. Containing in ſum, *A reduction of the Arts Mathematicke tending thereunto, and to diuerſe other good vſes, from that deepe difficultie, wherewith hitherto they haue bene ſequeſtred and cloſed vp as it were in ſeuerall, onely to the learneſt ſort: vnto an eaſie, methodious, plaine, and practique diſcipline, lying wide open vnto euerie ingenious practiſer*, whenſe I preſume, many ſingular inuentions, and notable commodities in time ſhall enſue and ſpring, yea a number yet vnthought of, euen from the common ſort of handie craftes men and trauailers. The which making it no better then it ſhall I hope proue, I moſt humblie craue of your H. to accept at his hands, who thinketh himſelfe in no leſſe good caſe then the happieſt, in that it hath pleaſed God to grant him but this poore meanes, whereby he may attaine to ſhew ſome leaſt ſparke of the vnquenchable fire of his loyal and dutifull zeale towards your H. Humblie ingrating that by your L. good fauour and protection, this little maſſe may be garded and freed from detraction, and the rather borne withall in any eſcape or want, of that others now can or hereafter by gather heereof may ſupplie better, or enlarge. And thus, as moſt bounden in duetie, reddie in ſeruice, and dayly in prayer, vnto Almightie God to be your Honours good [illegible] and fortreſſe both in this life and the life to come, reſt moſt humble at your H. commaund.

Iohn Blagraue.

The Author to the curteous Reader.

IT is ſtrange (gentle Reader) to ſpeake of the breuitie, methode, plainneſſe and facilitie, whereby this *Iewel* performeth all and that with the vttermoſt, which I haue in my title auouched, yea & to be admired, that one onely & ſo ſingle an inſtrument ſhould performe ſo infinite a number of concluſions, more then I thinke I ſhall euer haue time to write, and thoſe no baſe ſtuffe inuented to fill vp a volume as in other bookes are many thruſt in to ſmall purpoſe. Wherefore be not ſparing of a little paines at the firſt entrance to vnderſtand the particularities of your *Iewel* though it ſeeme a little harde, and aſſure your ſelfe being once entred, that neuer inſtrument was more eaſie and pleaſant: the 2. booke eſpecially the ſecond part will, I preſume, ſufficiently inſtruct you. I haue beſides here following ſet an Index or Table directing to the expoſition of any harde terme chancing in the way. Neuer giue ouer at the firſt reading though any thing ſeeme hard rather aske a little helpe. And if yee deſire to be excellent perfite in your inſtrument, abridge my whole worke, and you ſhall finde it will ſtand you in more ſteede then 20. times reading, I haue alwayes done ſo with any booke I liked of. I craue nothing for my part but that ye will lightly paſſe ouer euery eſcape of mine and the Printers, which I doubt wil fal out in ouer many places (the coppy in truth being brought vnto him very defuſe and enterlined) though none I hope ſo great to alter the ſenſe of the matter, which I could not helpe, by reaſon I haue bene wonderfully letted as is too too wel knowne to the world, for theſe 6. or 7. yeares togither in the behalfe of my friends, who were monſtrouſly troubled for no leſſe then their whole liuings & poſſeſſions, wherein my part alſo ſomwhat lay, by certaine promooting mates or to terme them more aptly concealed knaues, who after that ſome of them had (as they thought) filcht away all our euidences, though by the great prouidence of God the chiefeſt were ſtrangly preſerued, then was there no ſubtill, indirect or corrupt dealing to be imagined but they practiſed it againſt vs. For, ioyning with a famous lewde pettifogger they got by his meanes commiſſions out, and offices found vpon ſtelth at pleaſure to entitle the Queene thereto as concealed lands, whereby they had it vnder the broad ſeale before we heard any inckling of it, and came vpon vs with a writ of Intruſion, to further which action, and others after that: they fell to ſuch packing, ſhifting, ſuborning wreſting ſeducing & ſuch diabolical practiſes & that ſo hardly followed it, that the world was a great whiles *Aequidubium* of our ſucceſſe. It is a world to ſpeak of al their wicked drifts, which bicauſe it would be but a glaſſe, wherein vile people of like diſpoſition might ſee the whole commō welth of villanous trechery, I here omit, togither with their names: and the rather I do it, for that in conſcience I thinke they would reioyce and glory to bee in print though for egregious knauerie, euen like that lewd fellow that ſet *Dianaes* temple on fire to get him a laſting name. And yet one of their auncеſtors in the booke of monuments is in print alredie looking through a Pillorie which I cannot helpe. Wherfore (curteous Reader if any thing be amiſſe blame thoſe caterpillers of the common welth, thoſe vnconſcionable hunters of mens titles, thoſe wicked *Hamons*, being ſuch, of whom it may rightly be ſaid, *Qui nunc in terra, viuunt hic quaſi viuerra, Cuniculos vellens pauperes de limine pellens*, who hindred my trauailes, & brought me dayly out of courſe with my determinations: ſo that commonly at the ende of euery Terme I was with *Penelope* newly to begin againe my webbe. And for one two yeares togither could ſcarce intende to looke on booke, by meanes whereof my minde alwayes being troubled, and partly by meanes of my young yeares, then wanting iudgement when I wrote the firſt coppy of ſome part of this work, I know I haue harſhly written ſome things therin, wel neare vnto the middle of the third booke, which to this day being aboue 7. yeares betweene, leiſure would neuer allow my riper yeares fully to tranlace to my minde, and therfore you muſt vſe a little the more induſtrie to vnderſtand it. For ſince our 6. yeares turmoile in law ended, I haue bene ſo greatly called away with other needfull affaires, and ſo much applied for meaſuring of land, plateing ſeruеying and ſuch like, that time hath bene ſhorter with me then before. Beſides that I was forced to cut al the prints my ſelfe to my great paines and let of time, that had it not beene for the importunacie of ſundrie Gent. my friends who perceiued the ſinguler commoditie of the inſtrument, that were deſirous to haue me publiſh it with ſpeede, leaſt time ſhould by ſome meanes haue ſet it quite aſide, I would either haue had ſufficient time to haue fined it, and new written the coppie, or ſtaide it ſtill in my ſtudie, but being as it is, I muſt requeſt euery man to take in good part ſome part of my ill hap, promiſing to make amends with a little more diligence in my next ſixe bookes of the *Iewel*, which I purpoſe with as much ſpeede as I can after theſe to publiſh. And thus wiſhing my ſelfe preſent at euery doubt with my hartie good will farewell this 20. of Ianuarie 1584.

The

The Authour in his owne defence.

IF poore man with his paine, May pick vp many a pounde,
If ſtreames with often rayne, May quell the greedie grounde,
If ruſticke ſoyle may yeelde, Both hearbes and flowres faire,
If tylth the hungrie field, May maintaine and repaire:
Then why ſhoulde *Momus* howte, Or cauill at my quill,
To ſee hym now ſend out, That he hath haachte at will?
Or why ſhoulde painfull penne, that long hath toyld in hope,
By meanes of ſpitefull men Be turned all aſlope.
As though the *Muſes* all were niggardes now of late,
And would not let the ſmall To enter at their gate:
Or ſtoupt to none but ſtates, Or lou'd to lodge in Towers,
Diſdaining countrie cates Or courſe contriued bowres.
No no, you neuer knew, That ere they were ſo ſtrange,
And I dare vouche for true They neuer will ſo chaunge.
VVhy? did not they attende *Diogenes* his tubbe?
And meekly to him bend Yea vnder many a ſhrubbe?
And were they not at hand VVith *Arion* on the Seas?
VVith *Orpheus* on the lande Amid the wildernesſe?
Let no man nowe then doubt But haply I may ſhow
Such ſtuffe as late came out, And did in *Parnaſſe* grow.
Though that my name be not Among the learned rolde.
Let not that bee a blot Ere that my tale be tolde.
Yet *Zoylus* ſeemes to ſay, VVhy? what? and whence is he?
A childe but yeſterday, And now to ſcale the skie?
VVhere gathered he his skill? VVhat tutor tolde him in?
The Vniuerſities denill That ere he dwelt therein.
And London laughes to thinke She ſcarce doth knowe hys face,
How commes he then to linke VVith *Vranes* worthy grace.
My aunſweare ſhall bee ſhort, My paine this peece hath pend,
God lent it to my lot, And hee ſhall mee defende.
The Pſalmes ſay ſucklinges young His glory ſhall diſcloſe
VVhich warrants me among My wrangling wrabbiſh foes.

I. B.

The Authours dumpe.

VVHo coucheth cloſe in countrie coaſt, the ruſticke *Nymphes* among,
In *Codrus* caue obſcured aye, a farre from any throng,
VVill ſhrinke to come amid the rouge, that braue in bewties lappes,
That ſit and ſwigge their bellies full, from queene *Minerues* pappes.
Lo I the ſame, though lingred long, at laſt am come to light,
But ſtande amazde, a newe come gueſt, I ſeeme in eache mans ſight.
No wight I ſee putteth forth his pen, my needles prayſe to paint
But euen alone a countrie drone I ſtand in this conſtraint.
Eche paltrie pamphlet in theſe dayes hath praiſers many here,
VVhich mountaines promiſe to the worlde, where mole hilles ſcarce appeere.
If then ſuch praiſes prooue to mockes, and come before their time:
I reake the leſſe, I looke the ende ſhall prayſe this worke of mine.
To thee dame *Vrane* I referre the lot of my ſucceſſe,
Giue me therefore an hapfull houre, then I ſhall care the leſſe.
Graunt *Iupiter* be Lorde thereof, and *Mercurie* aſcendent,
Let *Saturne* ſtande aloofe the whyle, and *Mars* in detriment.
The *Moneſſepe* aſpected well his *Lords* in exaltation,
And Lady *Luna* haſting to ſome bliſſefull application.
My *Radix* with the *fortune partes* to agree thereto take care,
And I thy fame to ſling abroade henceforth will neuer ſpare.

I. B.

¶ A Table of the Contents of all the Chapters *contained in this worke.*

The first booke of this Mathematicall *Iewell* containeth very needefull Geometricall principles which may be called the rules of the compasse, and also after that the principles and rudiments of *Astronomie* and *Cosmographie*.

OF certaine definitions Geometricall.

CONCLVSIONS

1 TO erect a perpendiculer vppon any point of a groundline assigned, and thereby to make a right angle.
2 To make a right angle another way, and easier.
3 To deuide any streight line or arche of a circle into two equall parts very speedily,
4 From a point proposed to draw a perpendiculer to any ground line.
5 From a point proposed to draw a perpendiculer to any grounde line another way where the groundline lacketh length.
6 From the extreeme or end of a ground line to draw a perpēdiculer very easily wher the same ground line cannot be extended.
7 To doe the same more readily another way with once opening of your compasse.
8 From a point proposed to drawe a line parallel to any line proposed.
9 To make a triangle of any three lines proposed.
10 To make a square of any side giuen,
11 To make an angle equall to any angle proposed.
12 To finde the very point where the touch line toucheth any circle.
13 To deuide speedily and exactly any line into many equall partes.
14 To prepare a triangle too bee alwaies readie for deuiding of a line exactly into as many euen parts as shalbe required.
15 Another speedie way to deuide a line into many equall partes.
16 The same a third way and readiest.
17 Any three prickes howsoeuer not being in a streight line how to describe a circle that shall cut them al three.
18 How to finde the center of any circle of arche proposed.
19 To make an angle equal to any angle assigned otherwise then in the 11. conclusion.
20 To deuide easily a very small circle into a number of equall diuisions.

Heere followe the principles and rudiments of Astronomie and Cosmographie, with the vnderstanding of euery circle of the spheare, and their vocables

CHAP. 1.

VVHat Cosmographie, Geographie, and Topographie are.
2 Of the placing of speeres and the diuision of the worlde.
3 VVhat a spheare or globe his axtree and poles are.
4 Of the fiue chiefe great circles of the spheare.
5 Of the foure chiefe lesser circles of the spheare.
6 Of the Zodiacke and his diuision into degrees and signes, and their caracters.
7 A farther discourse of the circles of the spheare.
8 Of the circles of the spheare chaungeably with euery horizon.
9 Of the motions of the spheare, and all the tenne speeres.
10 In what time euery speere and planet finisheth his course, and of the caracters of the planets.
11 Of Zones, Climates, Parallels and Meridianes.
12 Of the longitudes and latitudes of places.
13 Of the Theorickes of the planets.

The second booke of the Mathematicall Iewell, *sheweth the making of the* Iewell, *and also the definition and explanation of this* Iewell, *and of euery circle and part thereof*

CHAP. 1.

A Necessary discourse which *Gemma Frisius* vsed as a preamble to his booke *De Astrolabio Catholico*, by what reason the Globe is brought into a flat or plaine forme.
2 VVhat materiall this *Iewell* were best to bee made of.
3 The meridianes and parallels drawen how to finish the *Mater*.
4 Howe to finde the centers of the Meridianes three seuerall wayes.
5 Howe to finde the centers of the parallels artficially.
6 How to make a *limbe* to the *mater* and the *reete* both out of one peece with all their lineaments,
7 How the zodiacke of this *reete* may be deuided diuers other wayes.
8 How to place the fixed starres in the *reete*.
9 Howe to finish and cut out the *reete* of this *Iewell*.
10 How to make the *label* for the *mater* and the rule to the backside with sightes.
11 How to furnish the back of the *Iewel* with the Theorick and other necessaries.
12 How to set together all this worke of the *Iewel*.

Howe

Heere followeth the inuention, definition and explanation of the Iewell and of euery circle part and thing therein contained.

13 VVhat moued the Authour to his inuention of this *Iewell* and whence he obtained it.
14 Of the definition of the *Iewel* and his principall partes.
15 Of the diuision or two seuerall distinctions of this *Iewell*.
16 Of the dispositions of the circles both in the *mater*, *reet*, & *limb*, for the first & chiefest distinction of this Instrument.
17 The circles seruing the first & chiefest distinction knowen by the last chap. how to place them to the lati. of your countrie, where it shall rest fixed to all conclusions a few excepted euen in maner of the globe or speare with a farther exposition of the lineaments of the *Iewell* so placed.
18 Of the dispositions of the circles both in *mater*, *reete*, & *limb* for the 2. distinction of the *Iewel*.
19 A farther declaration cõcerning the 2. part of this *Iewel* for the Zodiack, generall hori.. & fixed starres
20 How to choose out which of the 90. horizons serueth to euery countrie.
21 How the Zodiacke & fixed starres rise and set on the horizon now founde.

The third booke of the Mathematicall Iewel, containeth al the propositions & conclusions of all former writers, with diuers new additions.

CHAP. 1.

HOw by the *Iewel* to take the altitude of the Sun, Moone, or any planet or starre aboue the horizon.
2 How to get or rather examine the Suns greatest declination and distance of the Tropickes and therby also to learne the latitude of any countrey.
3 How by the Theorick on the backe of the *Iewell* to get the sunnes place or deg. in the Zodiack any day.
4 How to finde the sunnes declination for euery day in the yeare, and also the declination of euery part of the Ecliptick.
5 How to finde the latitude or poles eleuation any day proposed in any place or countrey whatsoeuer.
6 How to know what it is a clocke at any time the sunne shining and in any region in the world.
7 The latitude of any place knowen how to gette the declination of the Sunne, or any Starre, Planet or Comet by obseruation.
8 How the latitude or poles eleuation may be had in any region by the starres that neuer set.
9 The lat. knowen how to finde what day of the moneth it is though it were lost as they say by the Sun shining at noone.
10 To find the right ascension for any portion of the ecliptik, and what degree of the equinoctiall doth coascend or rise togethers with euery degree of the ecliptick in a right spheare.
11 The same more compendiously by the *reete*.
12 VVhat point of the ecliptick it is as well in the first quadrant as in the rest also that hath the greatest difference betweene the arch of the ecliptick, & the arche of the Equinoctiall coascending with him.
13 Of the difference of ascensions. 14. The same very plainely by the *reete*.
15 To know the oblique ascensions. 16 Of diseensions both right and oblique.
17 The longi. & lati. of any starres proposed, to know their right or oblique ascens. and how much they decline from the equinoctiall, and how to place starres in the *mater* of the *Iewel*.
18 The declination of fixed starres proposed with their latitude which neuer altereth, how to finde their longitude in the Zodiacke.
19 The difference of ascension of any part of the Zodiacke, or fixed starre proposed or the oblique ascension to know what the latitude of the countrie is.
20 To find the amplitude of the rising & setting of the Sunne or starres.
21 The amplitude of the rising of any knowen starre giuen, or of the Sun with his place in the Zodiacke to know the latitude of the countrie agreeable.
22 To know the rising and setting of the Sun & the semidiurnal arch of the Sun & starres.
23 The quantitie of the longest day of any countrie proposed, how to get the eleuation of the pole there and of the distinction of the climates and parallels.
24 To know what houre of ♑ day or night, the Moone or any planet or star riseth or setteth on the horiz.
25 To know what starres neuer rise or set to euery region, and which are verticall.
26 Of the Cosmicall, Eliacall, and Acronicall, rising and setting of the starres.
27 How to know what it is a clocke in the night time by the starres.
28 Of vnequall or planetarie houres.
29 How by the *Iewell* to know the meridian or greatest altitude of the Sunne and starres.
30 How to find the degree of *medium cæli* or culmination, called midheauen at any time proposed.
31 To knowe how much any degree or point of the heauen is distaunt from the meridian at any time by degrees of the equinoctiall.
32 How by the *Iewell* to know the height of the Sun aboue the hori. euery houre & time of the yere sitting within dores, and thereby to make tables to any lati. for making of particuler dials, as cilinders, ringes, quadrants and such like.
33 How to know the height of any star aboue the horizon sitting close within dores, & thereby to learne to know the starres in the skie.
34 How to find the meridian line, and the foure quarters of the world called *cardines*, diuers wayes, and specially by the Astrolabe.
35 The meridian found, to finde the azimuth of the Sunne or starres by sight without the houre.
36 The verticall circle of the Sunne or anye starre the degree of the Sunne with their altitude aboue the horizon giuen, how to know the latitude, and what it is a clocke.

37 The latitude, the Sunnes azimuth, and almicantare giuen to finde his place in the Zodiacke.
38 To know what houre the Sunne or any ſtarre cometh to any azimuth propoſed.
39 A comet, a planet, or any vnknowen ſtarre being ſeene in the ſkie, how to get his longitude and latitude in the Zodiacke, and how by the right aſcenſion and declination of any ſtarre to get his longitude & latitude.
40 How to knowe the quantitie of the angles which the ecliptick maketh with the meridian at any moment.
41 To do the ſame another way.
42 Howe to finde the Horoſcope called the aſcendent at any time appointed.
43 How to finde the Horoſcope or aſcendent another way by the *mater*.
44 Of the 12. houſes, and what a circle of poſition is.
45 To finde the circle of poſition according to *Ioh. Regiomontanus* and *Campanus* of any Point propoſed, and how much the pole is eleuated aboue any ſuch circle.
46 Of the other eight houſes according to *Regiomontanus*, and how to ſet a figure.
47 Of the other eight houſes according to *Campanus* and *Gaculus*.
48 To know in what houſe any ſtarre is at any inſtant, and to place the chiefe ſtarres in the 12. houſes.
49 Of directions, what direction is, and by what meanes it is perfourmed.
50 Of conuerſe or euerſe direction or dimiſion.
51 How farre or into what part of the Zodiacke dimiſion or direction ſhall come any yeere.
52 To find what deg. of the ecliptick occupieth any circle of poſition at any time.
53 To know the quantitie of the angle of the inclination of the Ecliptick too the horiz on at any degree of the ecliptick.
54 How to finde the altitude of the Sun or any point of the ecliptick aboue the horiz on at any time otherwiſe then hath yet been ſhewed.
55 To finde what angles the circle of altitude maketh with the ecliptick at any point thereof aſsigned.
56 How to finde the *parallaxis* or diuerſitie of ſight of the Moone in the circle of altitude.
57 How to finde the *Parallaxes* of the Moone or any other planet or ſtarre, in longitude and latitude of the ecliptick.
58 How to finde when the eclipſe of the Sunne ſhall bee.
59 Another maner to finde out the *Parallaxes* in longitude and latitude aſwell in the Sun, as the Moone.
60 To finde the latitude of the Moone.
61 How much of the Sunne ſhall be darkened.
62 How long the eclipſe of the Sunne ſhall endure.
63 Of the eclipſe of the Moone, the greatneſſe and duration of the ſame.
64 How to get the *parallaxis* of the Moone out of the heauens at any houre propoſed the latitude knowen with the longitude and latitude of the Moone.
65 To knowe the greateſt or horizontall *Parallaxis* of the Moone when ſhee is to be ſeene.
66 The longitude and latitude of the two ſtarres giuen him to know the diſtance between them.
67 How the diſtance of any two ſtarres vnknowen may be had.
68 To knowe howe long the tayle of a Comet is.
69 Another way to ſearch the diſtances of ſtarres one from another.
70 The longitude and latitude of two ſtarres giuen to get the angle of ſtaſion of one from another.
71 To knowe whether three ſtarres or cities be in one great circle,
72 That the tayle of a comet extendeth it ſelfe directly from the Sun, and how you may trie it.
73 To know what point of the heauen is in the merid. of any place at any inſtant propoſed, and what is is a cloke in any other countrie.
74 How to direct a ſhip by the ſtarres in any voyage.
75 To make an horizontall and South wall diall by the *Iewel*.
76 To finde the declination of any wall from the South, ane fitting a diall therto.
77 How to know the oblique aſcenſ. of any deg. of the Zodiack, or of any ſtar, the difference of aſcenſion not regarded.
78 The oblique aſcention giuen to finde the deg. of the Zodiacke coaſcending.
79 Of the recourſe & reuolutions of the yeeres of the world, neceſſary in natiuities & directions.
80 Of progreſſions and diuiſions as they be called.

The fourth booke of the Mathematicall Iewell *containing diuers other Aſtronomicall propoſitions and vſes of the* Iewell *of the authours owne inuention.*

CHAP. 1.

AN aduertiſement of the authour concerning neceſſary inſtruments.
2 How to make an inſtrument of 3. ſtreight rules better then any compaſſes to deſcribe any arch with, bee his center neuer ſo far of.
3 The vſe of the former inſtrument in drawyng of any arch.
4 Of a certaine ſecrete in diuiding the meridianes of great Inſtruments.
5 Another ſecrete founde by the Authour in deuiding the parallels of great Inſtruments.
6 Of drawing and diuiding the ecliptick line on the *mater*.
7 By the verticall circle or azimuth of the Sun or ſtarres taken to know what it is a clocke very certainly and exactly.
8 By the lati. of any planet & his diſtance from any knowen fixed ſtar had, how to get his exact lõgitude.
9 How at all times you may behold on the *mater* of your *Iewel*, the whole circumference of the ecliptik according to his cituation aboue the horiz. Which bringeth with it pleaſant vſes as followeth.

10 By

10 By the Eclipticke circle conſtituted as in the laſt chap. how to get the altitude and azimuth of the Sunne or any point of the Eclipticke, and therby alſo of the other planets ſo they haue no latitude.
11 By the ſame conſtitution of the Eclipticke circle how to finde his *nonageſ grad*, with the altitude therof, being the quantity of the angle made betweene the Eclipticke and the horiſon.
12 How by the ſame conſtitution of the Eclipticke to ſee all the circles of poſition liuely on the *Iewell* and what deg. of the Zodiacke is in any of them.
13 How by the ſame conſtitution of the Eclipticke circle to ſet a figure very ſenſibly and apparantly.
14 How to ſet a Figure with more pleaſure and eaſe, the laſt chap. wel vnderſtood, ſo that you may behold your whole 12. houſes in their beyng altogether on your *Iewel*.
15. How by the 11. chap. to get the altitude of the *nonageſ. grad.* more readily then there is expreſſed.
16 How to place the Eclipticke aboue the *Finiter* after another ſort then is ſhewed in the 9. chap. and in the ſame conſtitution to find out euery needful circle wherby moſt excellent concluſions are to be performed.
17 How by the conſtitution of the 16. chap. to get both the altitude and aſimuth of the Sun or any point of the Eclipticke or of any other planet or ſtarre how much ſoeuer they ſhalbe in latitude.
18 How by the conſtitution of the Eclipticke as in the 16. chap. to get the longit. and latitude of any ſtarre or planet, that is to be ſeene in the skie: and therby alſo his right aſcenſion and declination.
19 The latitude and azimuth of the Sun or any planet or ſtarre being taken, howe by the conſtitution of the 16. chap. to get their exacte longitude.
20 How to get the true and exact azimuth of any comet, planet, or ſtarre vnknowne.
21 The moone ſhining on any Sun diall, to know in the night what it is a clocke very ſpeedily by helpe of the *Iewel*.
22 A ſpeedy way to know what it is a clocke by the ſtarres, without taking their altitude.
23 The vſe of the 3. tables comprehended in the 2. Figure of the 2. booke 13. chap.

The 5. booke of the Mathematical Iewell, *teaching moſt plainly to performe on the* Iewell *the whole art of ſpherical triangles, and may be called the key of the knowledge of the ſpheare.*

CHAP. 1.

OF the expoſition of a Sinicall quadrant, and al things thereto appertayning.
2 Of certaine definitions needful to the better vnderſtanding of this woorke.
3 Of certain Theoremes needful to be premiſed for the better vnderſtanding of this my preſent work of ſphericall triangles.
4 If of a right angled ſpherical triangle two ſides ſhalbe knowne, how to get the quantity of the third ſide, and the other 2. angles.
5 The declination of the Sun, moone, or any other planet or ſtarre from the Equinoctiall giuen together with the latitude of the countrey, how by the 4. chap. to get the amplitude the difference of aſcenſion, the oblique aſcenſion and the diurnall arch of any of them.
6 To performe the 4. chap. on the *Iewel* by another maner of working.
7 To performe the ſame by the *Iewell* after a third manner.
8 The quantity of the longeſt day in any countrey giuen, to know the latit, & in what climat the ſame is.
9 If one ſide, of a right angled ſpherical triangle with one of the two angles not right bee giuen, howe to fynd the other two ſides and angle.
10 If you would make an horiſontall or vertical dial to any countrey, how by this laſt chap. to fynd what number of deg. euery houres ſpace containeth.
11 If the 3. ſides of any ſpherical triangle whatſoeuer ſhalbe giuen, to fynd the quantity of euery of hys 3. angles.
12 The declination of the Sunne giuen at anye time together with her altitude aboue the horiſon how by meane of the 11. chap. to get what houre and minute it is, and in what azimuth the Sun then is.
13 If of a ſpherical triangle 2. ſides containing any one knowne angle ſhalbe giuen how to fynd the other ſide and angles.
14 How by the helpe of the 13. chap. to know the height of the Sun at any houre and min. propoſed in the whole yeere, and alſo in what aſimuth he is.
15 *Ioan. Regiomon. lib. 1. cap. 51. & lib. 4. cap. 29.* propoſeth and ſaieth that the knowing of two ſides of a triangle not right angled with one angle ſubtended by one of thoſe ſides, cánot ſerue to get the other ſide & angles.
16 The aſimuth of the Sunne giuen or taken at any time by the inſtrument with his declination, howe by helpe of the 15. chap. to know what height the Sun is of, and what it is a clocke.
17 If of any ſphericall triangle not right angled, 2. angles ſhalbe giuen with the ſide lying betweene thoſe angles or els the ſide ſubtending one of them, how to get the other two ſides and 3. angle.
18 How by help of the 17. chap. to make any declining diall, that is to ſay a diall to any wall ſhouldring or bending from the ful beholding of the South.
19 How to let downe a perpendiculer arch from any angle of a knowne ſpherical triangle vnto his baſe or ſubtending ſide, and to know the quantity of the ſame perpendiculer and of the partes wherinto hee deuideth the baſe.
20 To doe the ſame after another maner of working, yea though one of the other 2. angles, and one of the other 2. ſides of the triangle were vnknowne.
21 If 2. great circles of the ſpheare croſſe one another, making a knowne angle howe to get the perpendiculer arch falling from any point aſſigned of the one circle vnto the circumference of the other, together with the arch of the ſame circle betweene the perpendiculer and the croſsing.
22 How by helpe of the 21. chap. to know both the height of the cocke to anye declining diall, and howe much

much he muſt be placed byas from the perpendiculer which I cal his deflexion.
23 In a right angled ſpherical triangle where one of the angles ſhal chaunce to be but certaine minutes as it oft happeneth, how to get the ſide ſubtending that angle preciſely notwithſtanding.
24 If the 3. angles of a right angled ſpherical triangle ſhal be giuen how to get the quantity of the 3. ſides vnknowne.
25 If by the croſſe ſtaffe or other inſtrument you ſhall take the true diſtance of any planet, comet, or ſtarre vnknowne from two known ſtarres, how by the helpe of this my worke of ſpherical triangles, to get the longitude and latitude in the Zodiacke of the ſame planet, comet, or ſtarre, and alſo his declination and right aſcenſion: and what you had neede to do in all ſpherical queſtions.

The 6. booke of the Mathematical Iewell, *ſhewing the Theoricall reaſon, ground, and making of all dialles.*

CHAP. 1.

Of certaine preambles to be noted in vnderſtanding this worke following.
2 Of the dyſtinction of dials into 2. kinds, Inſtrumentall and ſpherical.
3 Of the definition and deriuation of dials, and the ground of them all.
4 VVhat the cocke or Gnomon of euery diall is, & by what reaſon they giue the true ſhade to the diall.
5 Of the diuiſion of horizons, and eſpecially into 3. ſorts.
6 Of the diuiſion of al dials into 3. ſorts, & why they make ſhew of greater diuerſity then there is.
7 How to make the firſt kind of dial to the Equinoctial horizon, that is, where one of the poles is Zenith.
8 How to make the ſecond kind of dial to a right horizon, that is, where the Equinoctial cutteth the Zenith, together with the ground and reaſon of all dialles to any right horizon.
9 How to make the third kind of diall to an oblique horizon, which they commonly cal an horizon dial, & that very eaſily.
10 How to make a dial to a perpendiculer wal beholding the ſouth, called of ſome authors *Horologium verticale*.
11 To make a dial to a ful North wall.
12 How to make a dial to an Eaſt wal.
13 How to make a diall to a VVeſt wall.
14 Of declining walles and what circles of the ſpheare they repreſent.
15 How to know the ſcituation of any wall, and to find how much he declineth.
16 How by the declination of any wall known to finde his eleuation with his angle of deflexion.
17 To performe the laſt chap. a ſecond way by the 6. chap. of ſpherical triangles.
18 To performe the ſame a third way.
19 How to make a dial to any declining wall reſpecting the South.
20 How to make a diall to any declining wal reſpecting the North.
21 How to order dialles that decline very farre Eaſt and VVeſt.
22 Of walles reclining and inclining, and of the diuerſity of them.
23 How to take the reclination or inclinatian of any wal or flat, together with his declination if any be.
24 How to make a dial to a South or North reclining or inclining wal, banke or flat.
25 How to make a diall to an Eaſt and VVeſt wall, banke, or flat, reclining or inclining.
26 How to find the angles of the poles eleuation, dialles deflexion, and meridians aſcenſion to any reclyning or inclining wall declining.
27 How to perform the laſt chap. by ſpherical triangles very eaſily & with great pleaſure and profite.
28 The angles of reclination, declination, eleuation, deflexion, and aſcenſion had as before, howe to make the diall to a reclining wal, banke or flat declining.
29 Of inclining dialles declining.
30 How to know in what countrey any declining, reclining or inclining dial woulde, or of right ſhoulde ſerue as an horizon dial, & alſo to make him ſhew the houres of the ſame place together with yours.
31 How to place a flat repreſenting the horizon of any countrey, city or place in the world both according to the longitude and latitude of the ſame.
32 To performe the ſame by ſpherical triangles.
33 To deſcribe al maner of declining dials, & alſo the Eaſt and VVeſt reclining and inclining more at pleaſure then yet hath bin ſhewed, without foreknowing the angles of eleuation and deflexion.
34 To reduce al reclining and inclining flats declining vnto Eaſt & VVeſt reclining & inclining flattes to ſome one latitude or other, and thereby moſt eaſily to make dials.
35 To doe the ſame ſomewhat eaſier.
36 To performe the 31. chap. more eaſily, or at the leaſt more pleaſingly.
37 A moſt eaſie and briefe way to reduce al reclining & inclining flats declining vnto Eaſt and VVeſt, reclining flats to a new latitude otherwiſe then in the 34. & 35. chap. by helpe of ſpherical triangles.
38 A brief note how Eaſt & VVeſt reclining or inclining dialles are to be made 2. ſeueral wayes.
39 How to make a dial to any Eaſt or VVeſt, reclining or inclining flatte by helpe of ſpherical triangles moſt eaſily.
40 Moſt excellently and eaſily to make al manner of reclining flats declining by the *Iewel* and help of ſpherical triangles.
41 To know in what longitude and latitude, our horizon, or anye other ſhall repreſent any declining wall propoſed, or els any circle of poſition propoſed.

An Index or Table directing any man vnto ſuch places in this booke where al the termes hereafter following, are defined and expounded. The firſt number ſignifiyng the booke, the ſecond the chapter.

A

Angle right, ſharp or blũt, 1.0.
Arch. 1.0.& 5.1.
Axtree of the world, 1.3.
Articke circle, 1.5.
Antarticke circle, 1.5.
Azimuthes, 1.8.& 2.18.
Almicantares, 1.8.& 2.18
Apogeon, 1.13.
Aux, Idem.
Abſis. Idem.
Equinoctial of the ſphere. 1.4.
Aetheriall parte of the worlde. 1.2.
Aequans. 1.13.
Aequinoctium. 1.4.
Aſtrolabe. 2.16.
Equinoctial of the *Mater.* 2.19
Axtree. Idem.
Apex. 2.21.
Altitude. 3.1.
Equinoctials height. 3.7.
Aſcenſion. 3.10.
Amplitude. 3.20.
Acronical riſing & ſetting of ſtarres. 3.26.
Angle of the earth. 3.30.
Aſcendent. 3.42.
Apheta. 3.49.

B

Baſe line. 6.23
Baſe line perpẽdiculer. Idẽ

C

Circle. 1.0.
Center. Idem.
Chorde. 1.0.& 5.1.
Circumference. 1.0.
Coſmography. 1.1.
Corography. 1.1.
Colures. 1.4.
Caret. of the ſignes. 1.6.
Circles of ſtation. 1.8.
Circles of poſition. 1.8.& 3.44.
Caracters of the planets. 1.10.
Caput draconis, 1.10.
Cauda draconis. 1.10.
Climates. 1.11.& 3.33.
Culmination. 3.30.& 3.31.
Cardines. 3.34.
Conjunctio viſa & vera. 3.62.
Comet. 3.72.
Complement. 5.1.
Coſmical riſing. 3.26.

D

Diameter. 1.0.
Degr. of the Zodiack. 1.6.
Difference of longit. 1.12
Deferentes Apogeon or *Excentricum*, 1.13.
Deferens Epicyclum. Idem.
Direct. Idem.
Deuiding boord. 2.2.
Degree. 3.1.
Declination of the Sun or ſtarres. 3.7.& 3.4.
Differẽce of aſcenſiõ. 3.13
Diurnal arch. 3.22.
Direction. 3.49.
Dimiſion. 3.49.
Direction conuerſe. 3.50.
Diameters of the earth, ſũ and moone. 3.63.
Diuiſions. 3.80.
Diall. 6.2.
Declining walles. 6.14.
Declinatiõ of a wal. 6.14.
Deflexion. 6.16.& 6.27.

E

Elements. 1.2.
Elemental part. Idem.
Ecliptick. 1.6.
Excentricitye of the planets. 1.13.
Ecliptick of the *mater*. 2.19
Eclipſe of the ſun. 3.63.
Eclipſe of the moone. Idẽ.
Eleuation of a wal. 6.16.

F

Flat. 1.0.
Firſt mouer. 1.9.
Finitor. 2.18.
Finitor to the latitude. 2.19

G

General horizons. 2.6.
Ground line. 1.0.
Great circle of the ſphere. 5.2.
Great circles, axetree and poles Idem.
Geography. 1.1.
Globe. 1.3.

H

Horizon. 1.4.
Houre lines. 1.7.
Hori. *rectus & obliquus*. 2.19
Houre lines of the *Mater*. 2.19.
Heliacall riſing & ſetting of ſtarres. 3.26.
Horoſcope. 3.42.
Houſes of a figure, 3.46.
Horizontal parallax. 3.56
Horizons poles. 6.1.
Horizon *rectus, Equinoctialis, obliquus*. 6.5.

I

Imum Cœli. 3.30.
Including ſide. 5.2.
Inclining walles. 6.22.
Inclination. 6.23.

L

Line. 1.0.
Lines parallel, Idem.
Lines ſpirall, Idem.
Long ſquare. Idem.
Loſeinge. Idẽ.
Likeiame. Idẽ.
Latitude of planets. 1.10.
Longit. of regions. 1.12.
Latitude of regions. Idem
Longitudo longior. 1.13.
Longitudo propior. Idem.
Longitudo media, Idem.
Limbe of the *Iewel*. 2.16.
Latit. of the moone. 3.60
Latitudo viſa & vera. 3.62.
Label. 2.16.

M

Meridian circle. 1.4.
Minutes. 1.6.
Meridians. 1.7.& 1.11.
Motion of the firſte mouer. 1.9.
Medius motus. 1.13.
Mater. 2.16.
Meridi. of the *mater*. 2.19.
Meridian altit. 2.1& 3.29
Medium cœli. 3.30.
Meridian line. 3.34.
Meridians aſcenſion. 6.26
Motus viſus & verus. 3.62.
Mediatio cœli. 3.11.

N

Nadir. 1.8.
North horizons. 2.21.
North ſignes. 2.21.
Nonageſ. gradus eclip. 3.53.

O

Opticke. 2.1.
Oblique aſcenſion. 3.10.
Oblique aſcending ſignes. 3.13.
Oblique horizons. 6.5.
Oppoſitum angis. 1.13.

P

Parallel lines. 1.0.
Point. 1.0.
Perpendiculer. 1.
Poles of the world. 1.3.
Parallels of declinatiõ. 1.7
Poles of the horizon. 1.8.
Proper motion of the ſpheares. 1.9.
Parallelles of Coſmography. 1.11. & 3.23.
Perigeon. 1.13.
Playne ſpheare. 2.16.
Parallelles of the *mater*. 2.19.
Pole circles of the *mater*. 2.19.
Planetary houres. 3.28.
Palallaxis of altit. 3.56.
Parallax. of longitude and latitude. 3.57.
Progreſſions. 3.80.
Perpendiculer arch. 5.2.
Primum mobile. 1.9.
Pole Zenith. 6.1
Pole Nadir. 6.1.

Q

Quadrant. 5.1.
Quadrant Sinical. Idem.
Quadrantes baſe and ſide. Idem.

R

Right aſcenſion. 1.7. and 3.7.
Retrograde. 1.13.
Reete. Idem.
Rule. 2.16. and 2.18.
Right horizon. 3.7.
Recourſe. 3.79.
Reuolutions. Idem.
Right Sine. 5.1.
Reclining walles. 6.22.
Reclination. 6.23.

S

Spiral lines. 1.0
Semidiameter. 1.0.
Square. Idem.
Speeres of planets. 1.2
Spheare. 1.3.
Sommer Solſtice. 1.5.
Solſtitium hybernum. Idem.
Signes of the ☌odiack. 1.6.
Seconds. Idem.
Speere. 1.9.
Stacionary. 1.13.
South horizons. 2.21.
South ſignes. 2.21.
Suns greateſt declina. 3,2.
Signes right aſcend. 3.13.
Semidiurnal arch. 3.22.
Syne. 5.1.
Syne totall. Idem.
Sinus rectus & verſus. Idem.
Supplement of a Sine. Idẽ.
Spherical angle. 5.2.
Spherical triangle. Idem.
Subtending ſide. Idem.

T

Touchline. 1.0.
Topography. 1.1.
Triangle. 1.0.
Tropicks of ☋ ♈. 1.5.
Thirds. 1.6
Theoricke. 1.13.& 2.13. 16.
Tropic. of the *mater*. 2.19.

V & VV.

VVorld. 1,2.
VVinter Solſtice. 1.5.
Vertical point. 1.8.
Verus motus. 1.13.

Z

Zodiacke. 1.4.
Zodiacks bredth. 1.6.
Zenith. 1.8.
Zones. 1.11.
Zenith poynt of the *Reete*. 2.18.
Zenith line of the *Reete*. 2. 19.

Of certaine ſupplies for the better finiſhing of the figure of the Iewell *ioyned to this booke, and firſt for the* Mater *and his* Limbe.

YOu ſee that in the vndermoſt halfe A D B of the vttermoſt ſpace of the *Maters Limbe*, there are certayne great blacke numbers ſeruing indeede to the houres of the night, the very like muſt you ſet in the other halfe A C B in redde inke to ſerue the day houres, then in the middlemoſt ſpace of the *Limb*, which you ſee numbred from the Equinoctial C D ending at either pole in 90. I would haue you ſet the like numbers in redde ynke beginning from either pole and ending at the Equinoct. C D with 90. So much for the *Limbe*.

Now for the *Mater* I would haue you with a pen or penſil, & very good ynke or colour, make euery 3. meridian counted on ech way from the axetree line ſo much bigger. that they might eaſily be ſeen from the reſt, which is done for eaſe of memory & ſpeedy nũbring them. But on this condition, that thoſe aboue the axtree line towards C be done with red ynk or colour, the other with black. So much for the *mater*, the reſt you may learn in the 2. book 3. chap. but that I would haue the Figures of the houre circles ſeruing to the day, and the caracters of the South ſignes to be red, the reſt blacke: and if you doe guild the tropicks, pole circles, axetree, Eclipticke and Equinoct. line, then would the ſpheare ſhew in kinde.

Now for the Reete.

FIrſt read ouer the 2. booke 6. 8. & 9. chap. then would I haue the ſouth half of the zodiack caractered and nũbred with red ynke, & likewiſe the ſouth ſtarres a litle ſhadowed, & their names written in red, the reſt al in black. As for cutting out of the *reete* it is taught in the ſaid 9. chap. or rather the 4. book 1. chap.

Of the Theoricke or Calender.

TO ſaue the making of a ſeueral print, I haue placed the circles & diuiſions of the Theoricke on the out ſide of the *reete*, and are to be cut off from the *reete* in the circle A B C D deſcribed, as you ſee through the middeſt of the innermoſt graduation onely touching the extreames of the zodiack C and D. And therin write the monethes and dayes of the whole yeere as I haue begunne with Ianuary, and the names of the ſignes as I haue begun with Aries: alſo the ſaints daies or names as in the 2. booke is ſhewed, which done clap it on the backe of the *Iewel*, ſo that the centre agree thereto, and within it you may place what deuiſe lyketh you beſt.

Laſtly, whereſoeuer I haue committed any fault by ſlip of knife for want of handineſſe, beeing that theſe were the firſt that euer I cut, I muſt requeſt you to amend it with your pen, and aſſure your ſelfe if you handle this paper inſtrument wel in the paſting that he retch not. and take care in the cutting out of the branches & barres, he wil ſerue you a long time to better vſe then one of mettall, beeing charily kepte, if you get very fine paſtboord made of purpoſe, and then ſpred your paſte very fine thereon, & quickly laying on this picture & clappe it ſtreight into a preſſe before it bee thorowe wette with the paſte, ſo maye you keepe it from retching. I would wiſh you to make experience by ſome other peece of paper of like bignes firſt before you vẽture your picture.

Faultes eſcaped.

PAge 2. line 42. for D N reade K, pag. 2. line 52. for D E B reade D E, pag. 10. line 34. for my Iewel reade on my Iewel, pag. 11. line 23. for D E read nothing. p. 13. l. 4. for vz. M A. read in A. p. 13. l. 5. for M A read in A. p. 13. l. 9. for from read for, p. 16. l. 7. for 108. read 180. p. 20. l. 24. for firſt read fiſt. p. 21. l. 30. for one of the Limb read on the Limbe, p. 24. l. 30. for line leuel read line of leuel. pag. 27. l. 47. for 60. out of 60 1/3. read then taking 38 1/3. out of 60 1/3. pa. 29. l. 53. in the centre read the centre. pag. 32. l. 21. for oblique aſcenſ. read oblique deſcenſion, p. 35. l. 4. for and a halfe the reade and halfe the: p. 35. l. 44. for layd the deg. r. lay the deg. p. 38. l. 8. for diſtãce from r. diſtãt from: p. 38. l. 19. for but the Finit. r. but moue the Finit. p. 44. l. 27. for point found. r. point: p. 45. lin. 15. for of the 12. houſes, r. of the 12. houſe. p. 49. l. 14. for 7 1/2. of the clock r. 7. of the cloake: p. 49. l. 26 for vpwards 20. deg. r. vpwards 90. deg. p. 49. l. 38. for of the limb on the r. on the lymb of the: p. 51. l. 9. for which luminaries r. which the luminaries: p. 51. l. 51. for I would needs. r. example. &c. p. 51. l. 59. for his muſt r. this muſt: p. 52. for T H G of the ☉ r. T G of the ☉: p. 53. l. 32. for 53. almoſt r. 35. almoſt: p. 54. l. 36. for Sunne r. ſumme: p. 58. l. 4. for with the diſt. read which is the diſt: p. 58. l. 8. for M Y r. in Y: p. 69. l. 23. for thence from the r. thence on the: p. 72. l. 40. for ſhewing hers. ſheweth her: p. 72. for the alt. tit. and azim. r. the latitude and azim p. 76. l. 40. for in euery 90. deg. r. in euery parte 90. deg. p. 77. l. 34. croſſing M A r. croſſing in A: p. 79. l. 1. for G D or M r. G D or V M: p. 81. l. 6. for C B knowne r. C B vnknowne: p. 87. l. 22. for to be A K. rea. B A K. p. 87. l. 29. for in A. from r. in A A. from: p. 87. l. 52. of the ſaid r. or of the ſaid: p. 8. l. 24. for and the 17 1/3. r. and the pole 17 2/3. p. 93. l. 56. for yet knowne r. yea knowing: pag. 104. l. 49. for Iewell or meridian r. Iewell our meridian. p. 107. l. 33. for A plumbe for the flat reade A. plumbe from the flatte: pag. 109. lyne 19. for this complement reade his complement pag. 111. li. 18. for place r. plat: p. 111. l. 22. for A K C r. A K G: pa. 111. lin. 24. 55. deg. r. 56. deg. pag. 112. for deliniation of inclining dials reclining read declination of inclining dials declining. pag. 119. lin. 29. for Balmenus read Balbinus.

Gentle Reader, take a litle paynes to amend theſe ſmal faults which I wold aduertiſe you to do before you read the booke. Though the ſence of the matter and the figures explaning the meaning would, I know, haue cauſed the learned to eſpie theſe faults eaſily: yet for eaſe of the yong learner, I haue taken this paines to ſhew them downe: note that in numbring the lines for theſe faults I haue neuer accounted the title of any chap.

Note alſo that a little before the impreſſion I left 2. chapters out at the beginning of the 2. book, & therfore you ſhall find in the reſt of the bookes, the 2. book 20. chap. auouched when it is meant now the 18. and the 21. when it is the 19. & ſo of the reſt, which I though good to warne you of.

The first booke of the Mathemati-

call Iewel: set foorth by Iohn Blagraue of Reading, Gentleman,

Conteineth very needfull Geometricall principles, which may be called

the rules of the compasse: it containeth also after that, the principles and Rudiments of Astronomie and Cosmographie, set downe as the first footesteppe for the vnlearned, which I aduertise the learned to passe ouer.

Of certaine Deffinitions Geometricall.

KNOWING how needfull & speedefull it is as well to the making of this *Iewel*, as also in dyalling and all other Mechanicall working of Mathematical deuises to haue a handines & dexteritie in the vse of the rule and compasse: for contriuing and dispoling of angles, lineaments, and inuentions to euerie purpose and conclusion. Therefore for those that haue not beene elsewhere studied in Geometrie, I could not choose but set downe these principles and conclusions following, that the sequeale of my booke might be the better vnderstood and practised.

1 A *point* is a pricke indiuisible or rather imagined, or that hath neither length nor bredth.

2 A *line* is a length without bredth whose extreames are two points.

3 A *Superficies* or flat is the vppermost face of any thing, a bredth without thickneſſe and bounded with a line or lines.

4 An *angle* is the touching of two lines not lying in one streight line.

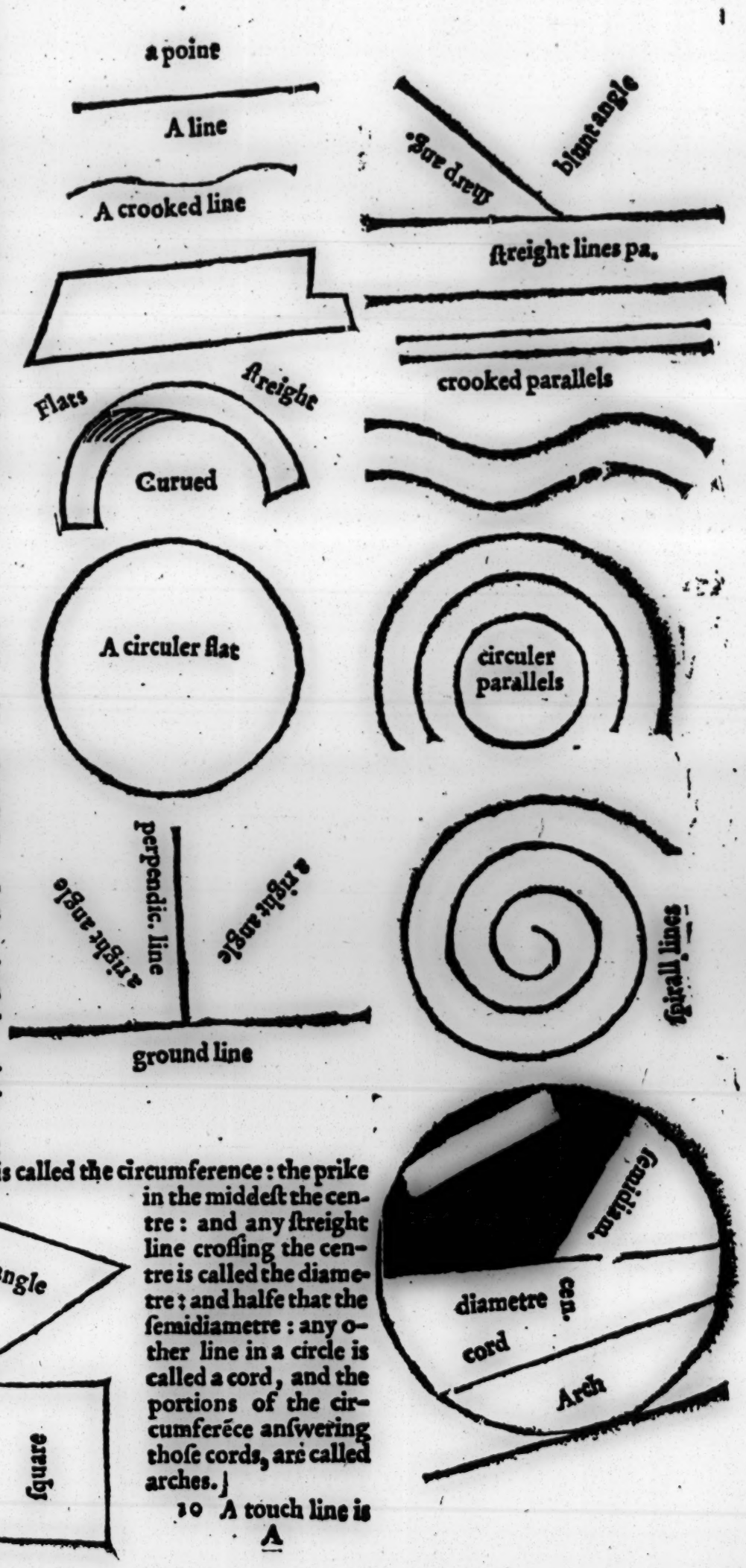
5 A *perpendiculer* or plum line, is a line lighting on another line, making therewith two equal angles, on ech side one: which are called right angles. And the line whereon the perpendiculer lighteth is called the ground line.

6 An *angle* lesse then a right angle is called a sharp angle and that which is greater then a right angle, is called a blunt angle.

7 Those lines are called *parallell* lines whether they be streight or crooked which are drawne ech where of like width: and being drawne foorth infinitely will neuer meete.

8 A *spirall* line is a line representing manie circuler paralels and yet is but one onely line of some called a wormeline.

9 A *circle* is made by the reuolution of a streight line, his one end remaining fixed: of circles the ring is called the circumference: the prike in the middest the centre: and any streight line crossing the centre is called the diametre: and halfe that the semidiametre: any other line in a circle is called a cord, and the portions of the circumferēce answering those cords, are called arches.



10 A touch line is

when a line doth onely touch a circle.

11 A tryangle is a flat of three sides.

12 A square is a flat of foure equall sides, and foure right angles.

13 A long square is a flatte of foure right angles the sides vnequall.

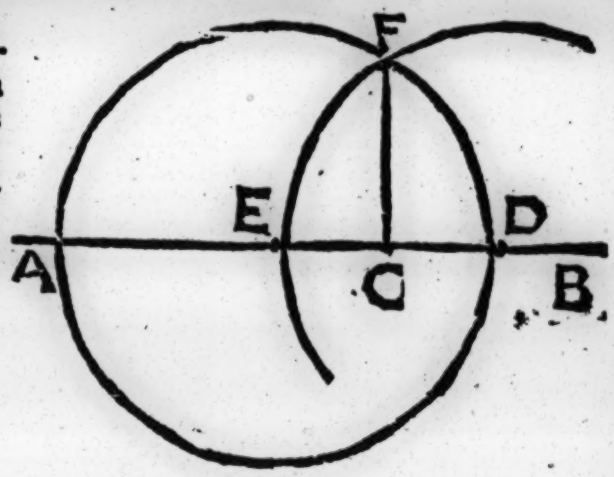
14 A loseing is a flatte of foure equall sides whose opposite angles be also equall.

Diuerse other superficiall figures there are as well regulare as irregulare vppon whose definitions it were superfluous for me to stande, since I entend to entreate of Geomatrie no farther then is requisite to the vnderstanding of this worke of the *Iewel*.

The first conclusion.

How to erect a perpendiculer vppon any point of a groundline assigned: and thereby to make a right angle.

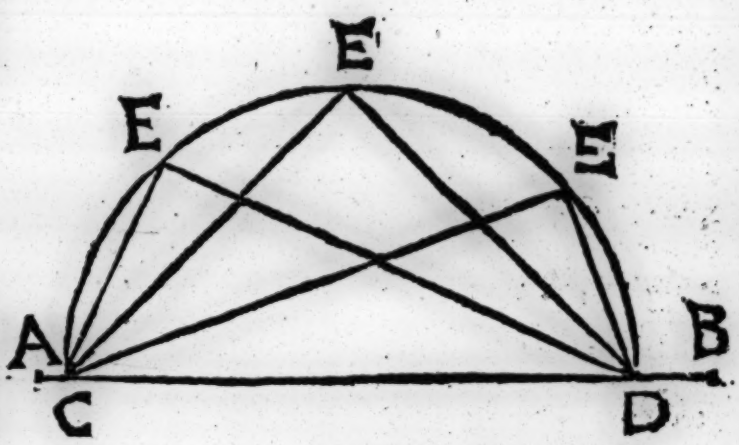
ech: probl: 4. FOr example I would erect a perpendiculer on the point C. of the groundline A B, which to do I open my compasse to a reasonable scantlet at a venture and make therewith two notes vz. D and E in the groundline A B. ech of a like distance from C. which done my compasse I open to a bigger widenes at a venture, and setting one foote in D. I make a circle or draw an arch aboue the line A B ouer the point C. likewise the compasse vnstirred and one foote placed in E. there draw another circle or arch crossing the first aboue C vz. in the point F. lastly drawing the line F C you haue your desire vz. F C perpendiculer to A B. and the angles F C B and F C A both right angles by the fift definition.



2. Conclusion.

To make a right angle another way and easier.

ech: prob. 9. 17. YOu shall draw a groundline videlizet A B then opening the compasse to any reasonable scantlet make thereon a semicircle vz. C E D. Now take this for a generall rule, and note it well, for it will stand you in great steade to many purposes, that any two lines meeting in the circumference of a circle issuing from the ends of his diametre do make a right angle, as in this figure any of the angles marked with E are right angles: it is demonstrated in Euclide lib. 3. propos. 30.



3 Conclusion.

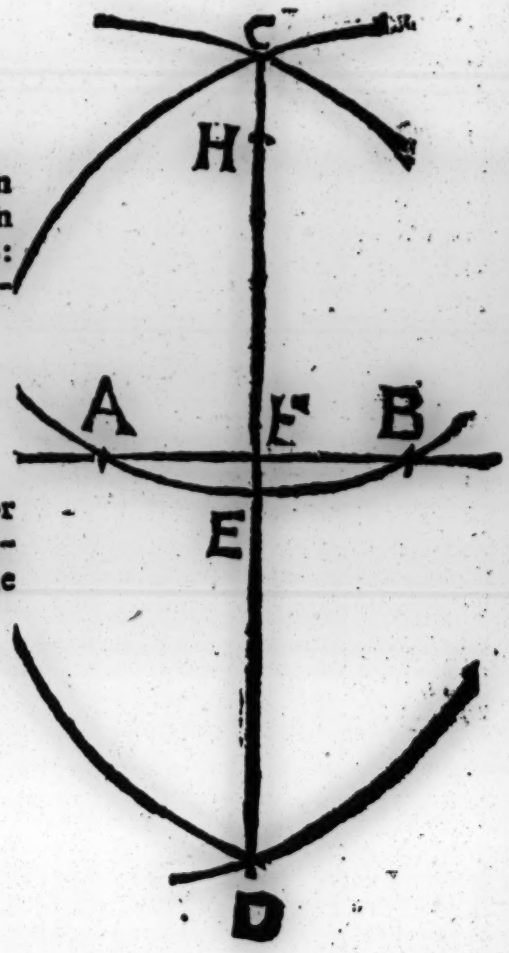
To deuide any streight line or arch of a circle into two equall parts verie speedilie.

ech: pr: 7. LEt there be proposed the line A B or the arch A E B to be equally diuided; vpon the extreames of either of these vz. A and B. I pitch my compasse opened to some ordinarie scantlet, and on those two centres A and B. I make two circles or arches crossing at C aboue A B and D below, then drawing the line C D, it shall equally diuide both the line A B in the point F, and the arch A E B in E.

4 Conclusion.

From a point proposed to draw a perpendiculer to any groundline.

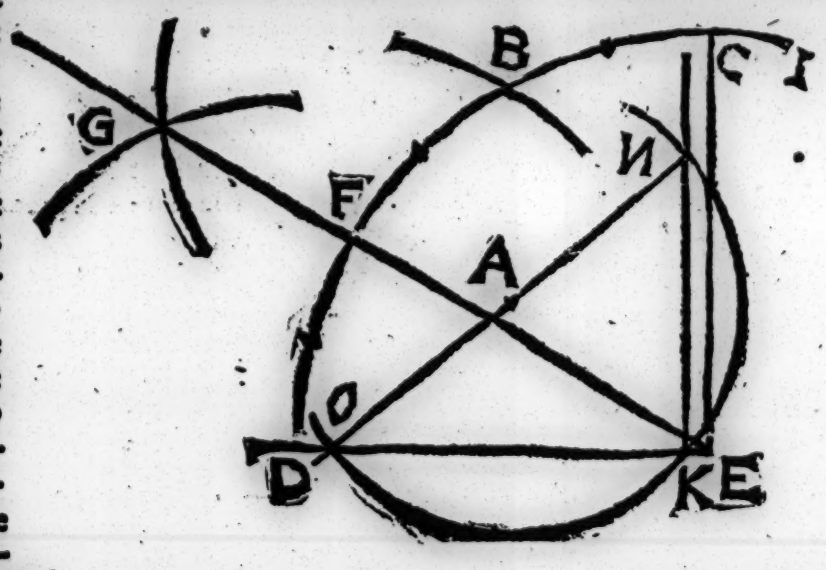
ech: prob. 5. LEt H be the point proposed and A B the groundline, set one foote of your compasse in H and extend the other somwhat beyond the groundline vz. to E. and therewith on the centre H make the arch A E B, cutting the groundline A B in the points A and B: then making A and B centres, you shall by the 3. Conclusion deuide the line A B into equall parts in F: lastly drawing the line H F the same is your perpendiculer desired.



5 Conclusion.

From a point proposed to draw a perpendiculer to any groundline a redier way where the groundline lacketh length.

ech: prob. 6. FOr example let there be proposed the plaine D E C whether it be a boorde, stone or plate, &c. and that from N being a point therein proposed, I would draw a perpendiculer to the groundline D E. Now because this perpendiculer will light so neere the point E being the extreame of this plaine, that I cannot worke it by the third conclusion, you shall therefore from ℔ N draw a streight line crossing the groundline D E. by as any where at auenture somwhat towards D vz. in O, and on the middest of N O. vz. on the centre A, describe a semicircle towards E according to the semidiametre A N or A O which shal cut the groundline D E ℔ in K, thē drawing the line N K it muste needs be your perpendiculer desired by the 2. conclusion.



6 Conclus.

6 Concluſion.

From the extreame or end of a groundline to draw a perpendiculer verie eaſily where the ſame groundline cannot be extended.

IT happeneth often in making of quadrants and other geometricall operations that we are driuen euen from a very angle to erect a perpendiculer, either to make or trie the ſquare angle truely, as for example in this quadrant D E C. From K being the extreame of K D. I would erect a perpendiculer, here you ſee no roome left to worke by the firſt concluſion: Wherefore you ſhall open your compaſſe to any reaſonable ſcantlet whatſoeuer, and ſetting one foote in K extend the other within your figure where you thinke good vz. to A. and on the centre A according to the ſemediametre A K deſcribe a circle or a porcion bigger then a ſemicircle, & where this circle cutteth D K vz. at O thenſe lay a rule on A. and draw the line O A extending it ſelfe infinitely beyond A and it ſhall cut the ſaid circle on the otherſide in N. Laſtly draw the line N K and of neceſsitie it muſt be the perpendiculer deſired by the 2. concluſion.

7 Concluſion.

To do the ſame more redily another way with once opening of your compaſſe.

ADmit that vpon the extreame ot point E of the groundline D E. I would erect a perpendiculer which to do I open my compaſſe to any reaſonable ſcantlet and therewith on the centre E. I deſcribe an arch ſomwhat bigger then a quadrant vz. H I. cutting the groundline A E in D. then on the centre D with the quantity D E. I make an arch croſsing H I. in B, & on the centers D and B. the compaſſe ſtyll vnſtyrred, I drawe two other arches croſsing without the arche H I in G. then I draw the line G E which deuideth the arch D B in halfe, as well as the 3. concluſion vz. in F. Laſtly on the centre F (the compaſſe remaining at the firſt width vz. the width D E) I draw an arch croſsing the arch H I. beyond B in the point C. and ſo drawing a line from E to C the ſame is your perpendiculer deſired.

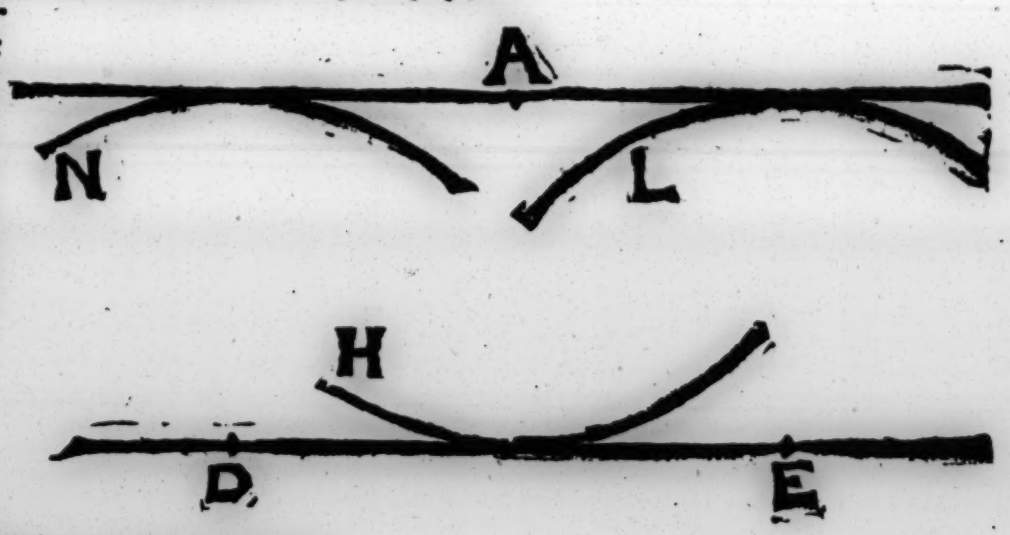
Note that the arch D F B C. is the iuſt quadrant of a circle deuided into three equal ſpaces by the points F and B. eche ſpace being 30. degrees ſuch as the quadrant D C is 90. euerie of which three ſpaces you may deuide in halfe as you deuided the arch D B. then will there be ſixe ſpaces ech 15. deg. thus may you with eaſe graduate a quadrant.

You may performe this concluſion another way thus. Erect a perpendiculer in ſome place of the groundline A E by the firſt concluſion vz. K L vnto which from the point E draw a line parallel by the 8. concluſion the ſame ſhall be the line C E your deſired perpendiculer. Thus you ſee how many wayes the Geometer can contriue one thing neither ſhall any draught ſo croſſe him but he can inuent ſome one way or other to performe his deſire. Theſe three laſt concluſions, eſpecially the firſt two of them I neuer read them in any booke, but in contriuing of deuiſes I haue beene driuen to caſt my wits to ſeeke them out, ſome will ſay to what purpoſe are ſo many wayes, one is as good as twentie: he is deceiued greatly, for at a time he ſhall finde that ſome one of theſe ſhall ſerue his torne when all the reſt ſhall do no good, as by examples I could here ſhew if it were materiall. Alſo there is a way by three quantities vz. 3. 4. and 5. making a triangle of them by the 9. concluſion the angle ſubtended by 5. ſhall be a right angle and diuerſe ſuch like. Rath: prob.

8 Concluſion.

From a point propoſed to draw a line parallell to any line propoſed.

TO draw a line parallel to the line D E propoſed which ſhall cut the propoſed point A. ſet one foote of your compaſſe in A and open the other to ſuch width that it make an arch onely touching the line D E vz. the arch H. Then keeping the compaſſe at that width on two ſeuerall points taken at auenture in the line D E vz. D and E. make two ſeueral arches vz. L and N. then drawing a line which may be a toucheline to both thoſe arches the ſame ſhall cut the propoſed point A. and be parallel to the propoſed line D E as you heere ſee.

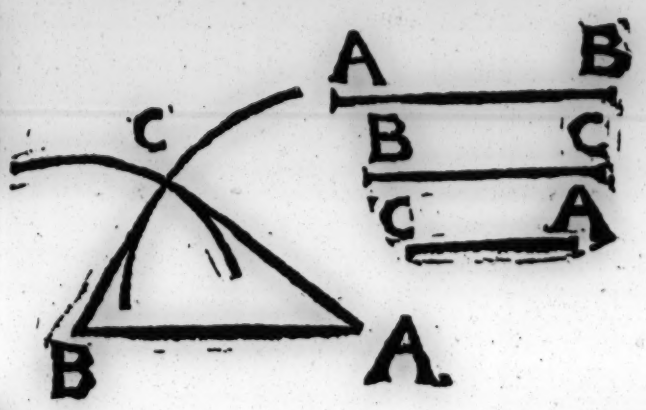


Rath: prob. 2

9 Concluſion.

To make a triangle of any three lines propoſed.

LEt A B. B C. and C A. be the three lines propoſed, firſt draw a line equall to any one of them, as to A B. then taking the quantitie of B C in your compaſſe, therewith vpon the point B of this ſecond line draw an arch, laſtly, take C A in your compaſſe, and making the other end A centre deſcribe another arch croſsing the firſt & where they croſſe ſet C. then drawing lines from C to A & B your triangle is made whereby you may gather, that except two of the lines propoſed be longer then the third, there can be no triangle made of them.

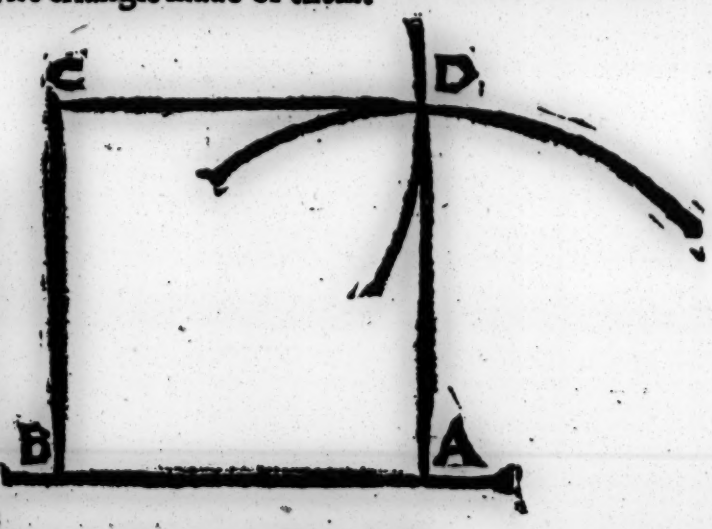


Rath. prob. 4

10 Concluſion.

To make a ſquare of any ſide giuen.

LEt there be propoſed the ſide of a ſquare vz. A B. vpon the point B erect a perpẽdiculer equal to A B. by the 1. or 6. cõcluſion vz. B C. then take in your compaſſe the quãtitie A B or B C and making A & C centres, therwith make two Arches croſsing in D. laſtly drawing the lines D C and D A. Fiet.



Rath. prob. 5

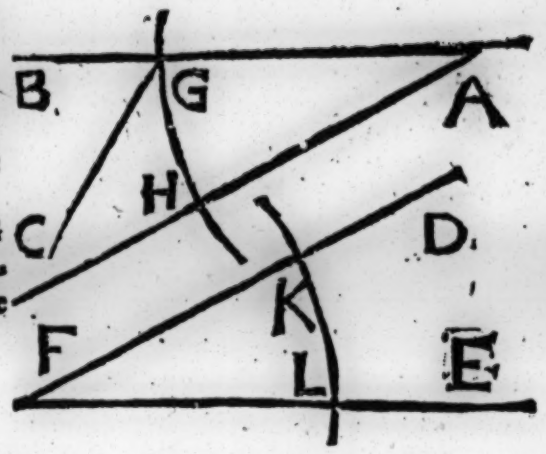
11 Concluſion.

To make an angle equall to any angle propoſed.

Eth. prob. 8.

LEt there be propoſed the angle B A C. now opening my compaſſe at auenture & making A the centre, I draw an arch cutting the two lines containing the angle propoſed in the points G and H. Then my compaſſe remaining at the ſame width, I draw the like arch vpon the centre F wherein I ſet the ſpace K L equall to G H. laſtly drawing the lines D K F. and E L F. I haue the angle D F E. equall to the angle B A C. propoſed.

The ſame is to be done after another ſort: you may by drawing a third line as G C make the angle propoſed a triangle and then if by the 9 concluſion you make another triangle equall thereto, the ſame ſhall haue equall angles ech to his match to the other.

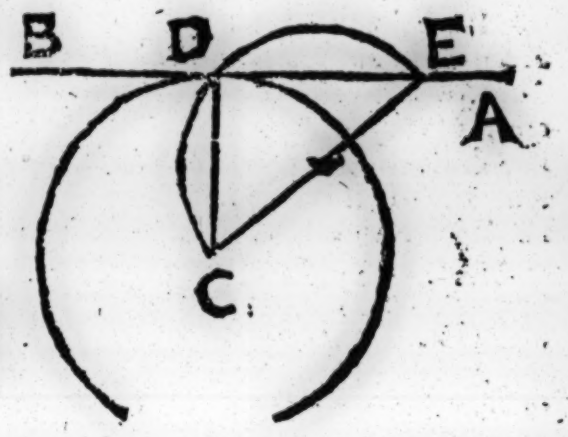


12 Concluſion.

To find the very point where a touchline toucheth any circle.

Eth. prob. 31.

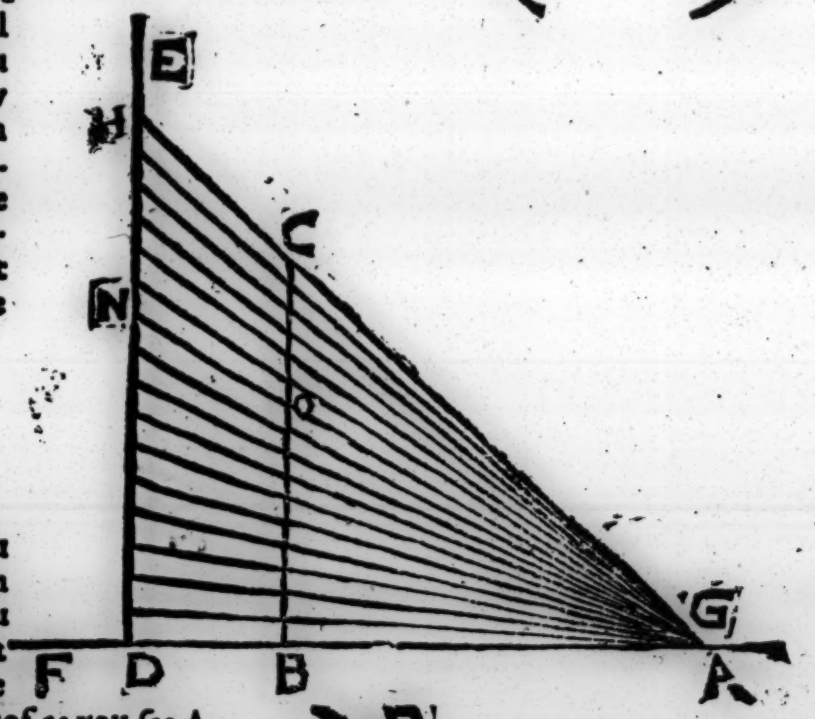
DRaw a line from any point of the touchline A B. vnto the centre of the circle vz. E C. and thereon make a ſemicircle vz. E D C. which ſhall cut the touchline and the circle in the very touch point vz. D. marke therefore the manifold vſe of the 2. concluſion.



13 Concluſion.

To diuide ſpeedily and exactly any line into many equall parts.

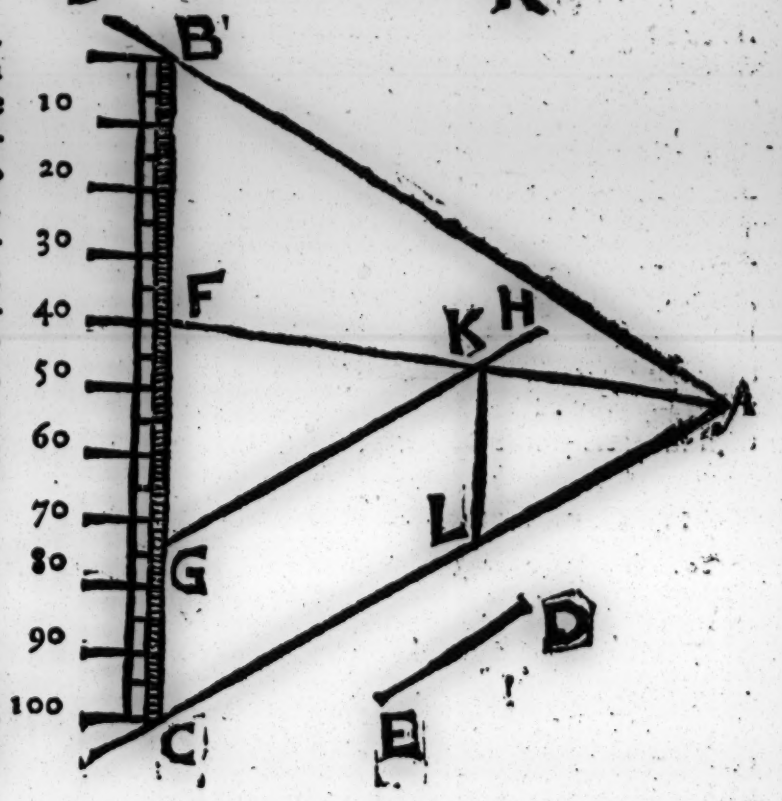
LEt there be a line propoſed vz. B C. to be deuided into 16. equall parts which to do, erect him on the groundline A F. perpendiculer or inclining all is one: Then draw another line vz. D E. parallel to B C. cutting A F. in D. this done open your compaſſe to a conuenient ſmall ſcantlet & therewith beginning from D. marke out 16. equall diuiſions in the line D E. ending in H. to which you may ſet numbers, then laying a rule on H and C. draw foorth the line E C. till it cut the groundline A F. in the point G. then laying a ruler on the point G. and euery of the 16. notes in the line D E. draw ſo manie ſtreight lines iſſuing from G. vz. 16. and they ſhall deuide the line B C. propoſed into 16. equall parts. But if you would haue B C. into 40. 60. 100. or more partes then ſhould you ſet D E. farther off.



14 Concluſion.

To prepare a triangle to be alwayes readie for deuiding of lines exactly into as many euen parts as shalbe required.

FIrſt make you a ſcale of as many equall parts as you thinke will beſt fit ſuch purpoſes as you pretend, an 100. equall parts ſerued me wel, and will ſerue you to deuide a line into any number of equall parts not exceeding 100. Then by the 9. concluſ. make a triangle of three equall ſides ſo that this ſcale be the baſe thereof as you ſee A B C. which I did make on a faire boord couered with parchment then hauing alwayes a rule with a centrchole towards the one end to ride vppon the point A. where muſt be a fine pinne or ſpaniſh needle erected fit to the hole: ſo haue you a verie neceſſarie inſtrument made to diuide lines yea of ſmall quantitie into as many equal parts as you liſt. as for example, in menſurations & plattings we vſe ſmal meaſures proportionable to the great, which ſome haue called the arte of litle foote: for which purpoſe I haue oftentimes diuided an ynch in 60. parts ſomtimes more ſomtimes leſſe as ſerued beſt my purpoſe, which I wil ſhew you heere to do though your ſcale be an hundred: let D E. be the quantitie of an ynch to be ſubdiuided into 60. parts. Now therefore I draw an obſcure line from A. to 60. parts of the ſcale vz. A F. then I appoint the quantitie C G. in the ſcale equall to D E. and from the point G. do I draw another obſcure line parallel to A C. by the 8. concluſion vz. G H. which cutteth the firſt obſcure line A F. in the point K. then drawing the line K L. by the ſaid 8. concluſion parallel alſo to B C. the ſame ſhalbe equall to D E. and ſtand readily placed in your triangle to be diuided into 60. equall parts like as in the 13. concluſion by laying the rule on A. & euery of the ſaid 60 parts. It were heere far beyond my preſent determination to ſhewe the infinite commodities of theſe two laſt figures as wel in ſhrinking and ſtretching of platformes and for manifold proportions Geometricall, as alſo in proſpectiue.



15 Concluſion.

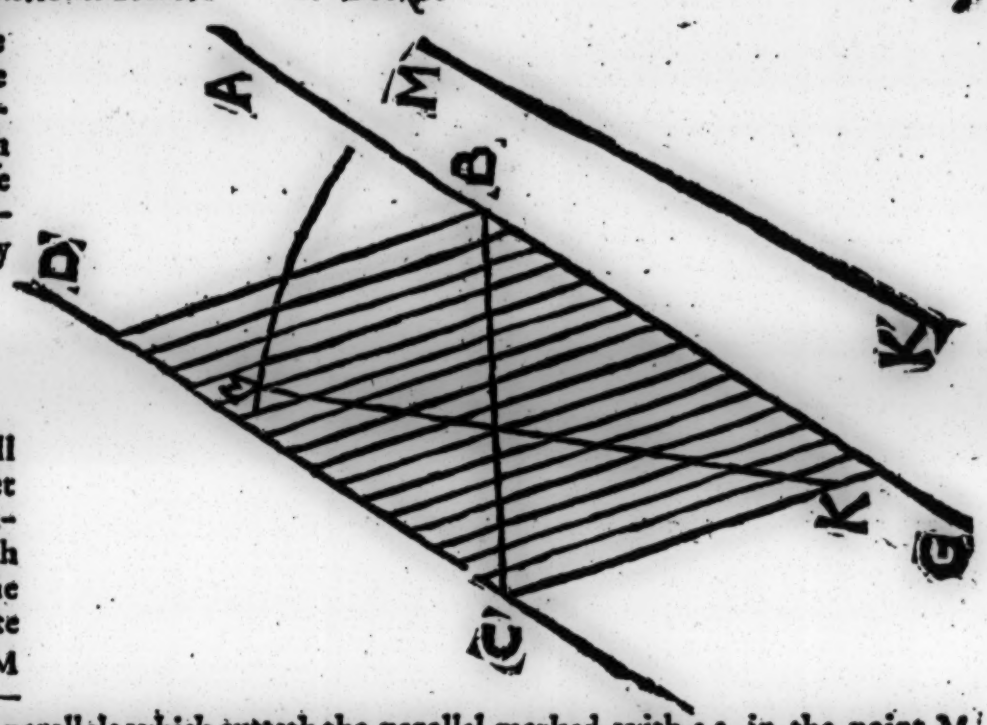
Another ſpeedie way to diuide a line into many equall parts.

FOr example I will deuide the line B C. mentioned in the 13. concluſion into 16. equall parts another way, which to do ~~wa Ird~~ two long parallels vpon the extreames of B C. vz. A B G and D C E. then opening my compaſſe to a reaſonable

drawe

ſcantlet

ſcantlet I marke out 16. equall parts in ech of thoſe parallels beginning from the extreames of B C. in the one vpwards, vz. from C to D. in the other downewards, vz. from B. to G. and ſo number them, which done I drawe 17. narrow parallels according to thoſe diuiſions croſsing the line B C. in 15. places and do diuide the line B C. into 16. equall partes as you may better ſee by this figure then by many words.



Rath. prob.

16 Concluſion.

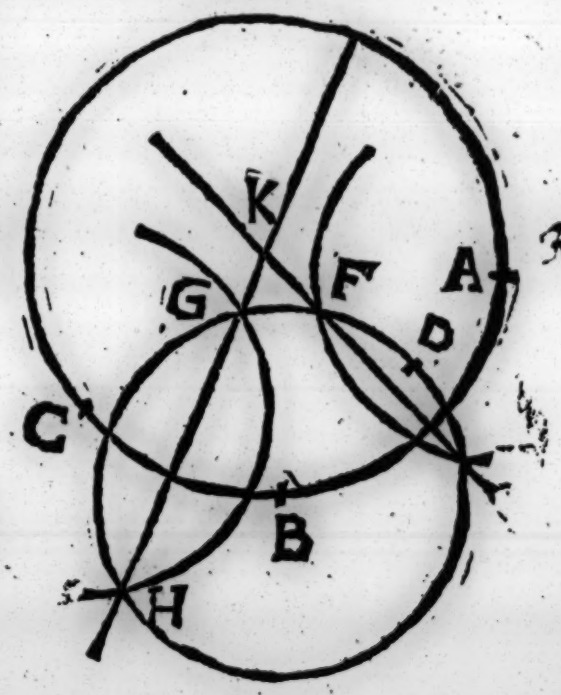
The ſame a third way and redieſt.

FOr this you muſt alwayes haue a table drawne full of parallel lines being all of equall ſpaces and ſet numbers vnto them, as for example by theſe 16. parallels reddie drawne, I wil diuide this line K M. which is longer then B C. into 12. equall parts, I take the quantitie thereof in my compaſſe & ſetting one foote in the loweſt parallel C G ——— K ——— M vz. at K. with the other I make an arch amongſt the parallels which cutteth the parallel marked with 12. in the point M. then drawing the line K M. croſſe the parallels, the ſame is equall to this line K L. propoſed and diuided into 12. equal parts: and if you would haue K M. diuided into but three parts, recon euery 4. parallels for one, you haue your deſire.

17 Concluſion.

Any three prickes ſet downe howſoeuer, not being in a ſtreight line: how to deſcribe a circle that ſhall cutte them all three.

LEt A B C. be the three prickes propoſed you ſhall ſet the one foote of your compaſſe in the middlemoſt pricke vz B. and extende the other towards the farthermoſt of the other two, ſomwhat more then halfe way be ſure as you ſee here B D. and therewith on the centre B. deſcribe a circle, then the compaſſe vnſtirred deſcribe two other circles on the centres A and C. that ech of them may cutte the former circle D. in two places vz. in E. and F. and in G. and H. laſtly drawing the lines E F. and G H. where they croſſe at K. there is the centre of the circle deſired, for ſetting one foote of your compaſſe in K. the other extended to any one of the three pricks A B. or C. and therewith deſcribing a circle on the centre K. it ſhall cut the reſt.



Rath. prob.

18 Concluſion.

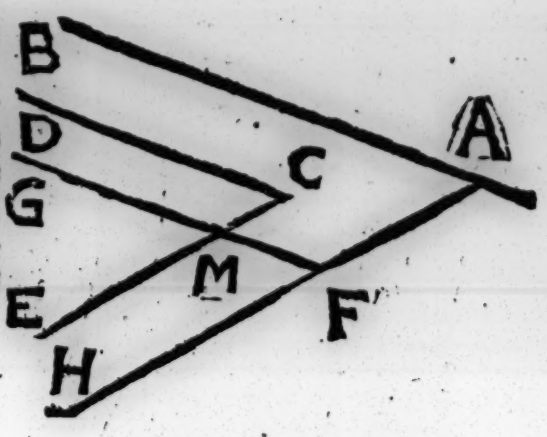
How to finde the centre of any circle or arch propoſed.

IF any circle or arch be propoſed make in the circumference thereof three prickes at auenture as heere you ſee A B. and C. and then by the 17. concluſion finding the centre of theſe three prickes, the ſame ſhalbe alſo the centre of your circle or arch ſought for.

19 Concluſion.

To make an angle equall to any angle aſsigned otherwiſe then in the 11. concluſ.

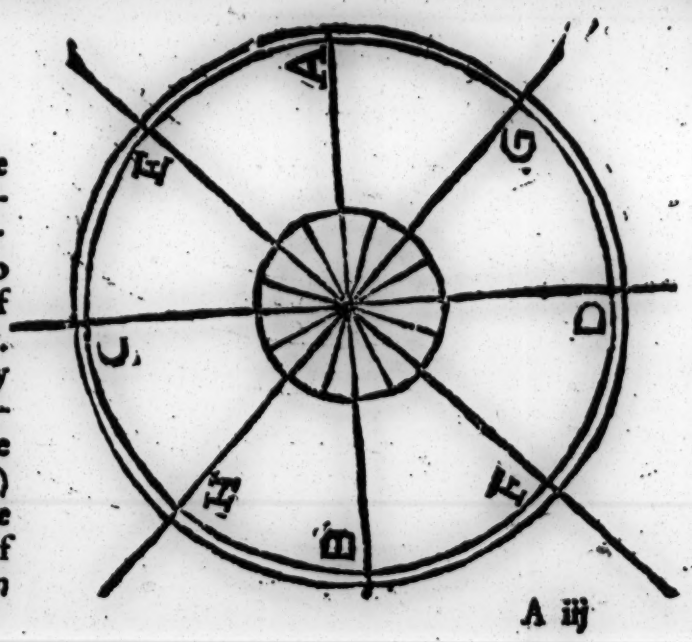
LEt B A H. be the angle aſsigned, then ſhall you draw the line C D. parallel to A B. and C E. parallel to A H. by the 8. concluſion and ſo ſhall you haue where the lines C D. and C E. do croſſe the angle D C E. equall to the angle B A H. aſsigned, as by the 11. concluſion you may make triall. Hereby you ſhal ground this generall rule which will ſtand you diuerſe times in ſteede if you marke it, that all angles whoſe containing ſides are parallel ech to his mach are equall ech to other, as here the angle G M E. is alſo equall to B A H. becauſe M G. is parallel to A B. and M E. to A H. and ſo is G F H. equall to B A H. the like is ſhewed by the line L K. of the 7. concluſion in making of a right angle. This dexteritie in contriuing angles is the ground of all menſuration and platting.



20 Concluſion.

To diuide eaſily a very ſmall circle into a number of equall diuiſions.

YOu ſhall about your ſmall circle deſcribe a very great circle concentricke therto, which great circle you may at pleaſure diuide into what partes you liſt with a paire of compaſſes and helpe of the 3. and 7. concluſion, which done you ſhal (by a rule laide from the common centre of theſe circles to euery part one after an other of your great circles) diuide thereby the ſmal into the like parts, not vnlike to the diuiding of the ſtreight line in the 13. concluſion



wherefore

wherefore if you haue a large circle of two foote diametre alwayes reddie diuided into 360. equall parts, and another into $365\frac{1}{4}$ equall parts, the ſame ſhall ſtand you in great ſtead to diuide any circle of the *Iewel* or any other aſtronomicall inſtrument, be they neuer ſo ſmall.

Heere follow the principles and rudiments of Aſtronomy and Coſmographie with the vnderſtanding of euerie circle of the Spheare and their vocables.

What Coſmographie Geographie and Topographie are.

Chapter 1.

COſmographie is as much to ſay, as the deſcription of the world: as well his Aethereall part, as Elementall, and in this differeth from Geographie, bicauſe it diſtinguiſheth the earth by the celeſtiall circles only: and not by Hilles, Riuers and ſuch like.

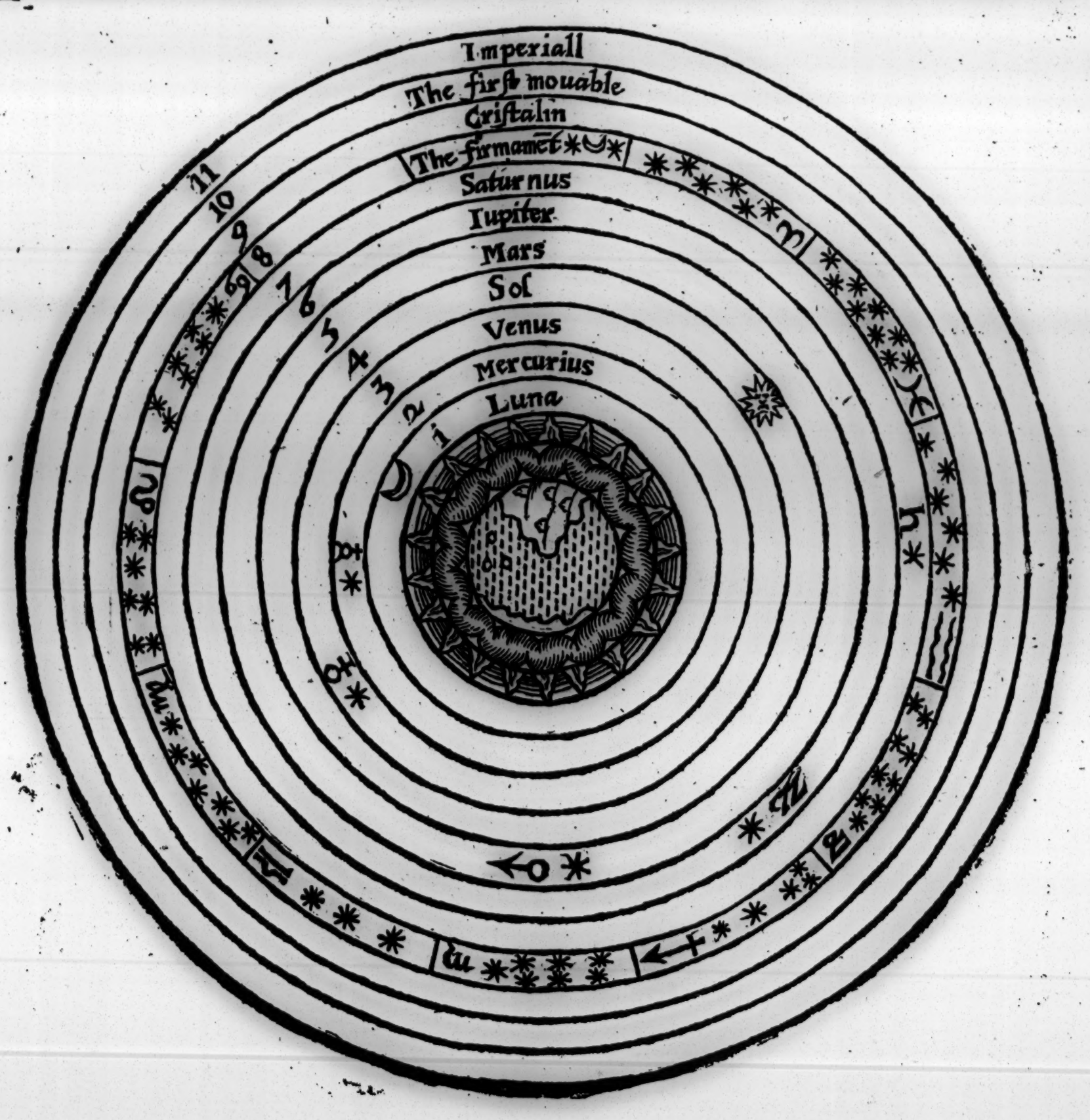
Geographie is a certeine forme and imitation of the picture of the earth, and of his chiefeſt and knowen parts, and differeth from Coſmography, becauſe it diſtinguiſheth the earth by hilles, ryuers, ſeas, and other notable matters, not reſpecting the circles of the ſpheare.

Topographie called alſo Corographie, is the deſcribing of any particular place without relation to the whole, not leauing out the ſmalleſt contents thereof, as ports, townes, ſmall ryuers, buildings, houſes, towers, walles, &c.

Of the placing of the ſpheares and the diuiſion of the world.

Chapter 2.

THe world is diuided into two parts: vz. Elementall, and Aethereall. The Elementall is ſubiect to dayly alteration, and conteineth the 4. elements, the earth, water, ayre, and fire. The Aetheriall part, called of the Philoſophers *Quinta*



Essintia, doth compasse the elementall part with his Concauitie, and is an inuariable substance alwayes abiding: and containeth tenne speares one compasing in another spherically, as you see in this figure of those tenne speeres, the speere of the Moone, is placed next vs, compasing the speere of the fire. And next it, the speere of Mercury, and so forwards in order of Venus, the Sunne, Mars, Iuppiter, Saturne: Next these immediatly is the starrie firmament, then the Christallin heauen, called the ninth speere: lastly the 10. speere called *primum Mobile* doth comprehend them all: whatsoeuer is beyond is the habitacle of God and his elect: for their motions looke the ninth cap. This I say after the opinion of old writers though Copernicus hath ascribed another order.

What a sphere or Globe his axtree and poles are.

Chapter 3.

A Sphere or Globe, is a sollide bodie, contained in one Superficies or flatte: in the middest whereof there is a point called his centre, from which all lines drawne to the vttermost Superficies are equall and called his Semidimetientes or Semidiameters.

The axtree of a Sphere, is his Diametient about which he is mooued: or more plainely, it is a line imagined, passing through his centre to his circumference, about which his motion is performed.

The poles of the world, are the two extreames of the axetree, being likewise two points imagined in the heauens: the one is called the North pole or pole Articke, and is alwayes (to vs that dwell on this side of the equinoctiall) aboue the horizon. The other called the South pole or pole Antarticke, and is alwayes to vs vnder the horizon as much as the other is aboue.

Of the six chiefe great circles of the Sphere.

Chapter 4.

THe *Horizon* called also in latin *Fenitor* is euery where the circle separating the vppermost halfe world or Hemisphere from the lowermost, or more plainely it is a great circle of the Sphere whereon from the verticall point directly ouerhead a plum line let downe standeth perpendiculer from his centre.

The Meridian circle of any place, passeth by the saide verticall point directly ouer head, and the Poles of the worlde: to which when the Sunne commeth, it maketh noone aboue the earth, and midnight vnder the earth.

The Aequinoctiall is a circle crosing square the meridian, and the axtree line, lying iust betweene the poles, in euery part equally distant from them both: and when the sunne commeth thereto which is twise euery yeare it causeth the day and the night to be equall in length through the whole world, and is called Aequinoctium.

The Zodiacke, diuideth the Aequinoctiall in the middest at Aries and Libra: and lieth byas vnto him $23\frac{1}{2}$. degrees, declining on ech side towards the poles: and hath in bredth 16. degrees after some writers: and after others but 12. but all the other circles of the Sphere haue no bredth.

The two Colures are two cyrcles crosing one another square at the poles the one passing by ♈ and ♎ the other by ♋ and ♑ limitting the two Aequinoctiall and the two Solstitiall points of the Zodiacke.

Of the fower chiefe lesser circles of the Sphere.

Chapter 5.

THe Tropicke of Cancer is a circle parallel to the Aequinoctiall distant from him Northwards $23\frac{1}{2}$ degrees, his vse is onely to bound the Eclipticke greatest declination Northwards: to which when the sunne commeth, the day is at the longest in our Horizon, which dwell North from the Aequinoctiall, and the night at the shortest, and there the sunne beginneth to go backe againe, and there said to be in his sommer solstice.

The Tropicke of Capricone is the like circle bounding the ecliptick Southwards: and when the sunne commeth there it maketh the shortest day and longest night, and there begins to go backe againe, I say backe againe in respect of the parallels and lengthening of the day, but in the Zodiacke he goeth on still direct and forwards euery day one degree, as hereafter shal more manifestly be vnderstood, and there also is the sunne said to be in our winter solstice.

The Articke and Antarticke circles commonly called the pole circles, are likewise parallel to the equinoctial and in euery part as farre distant from the poles as the Eclipticke declineth from the equinoctiall vz. $23\frac{1}{2}$ degrees, and are made by the reuolution of the ecliptickes poles: and therefore may well be called pole circles.

Of the Zodiacke and his diuision into degrees and signes and their carecters.

Chapter 6.

WHat the Zodiacke is, I haue shewed in the 4. Chapter, he hath in bredth 12. or 16. degrees as there is said: the line or circle diuiding this bredth in the middest quite round, leauing 6. or 8. degrees that is to say halfe the Zodiackes bredth on ech side, is called the Eclipt ~~li~~circle, and is commonly vsed in writing for the zodiack, this Eclipticke is diuided into 360. degrees as the equinoctiall is, but then those are parted into 12. vz. euery part being 30. degrees. By this meanes the whole bredth of the zodiacke is diuided into 12. signes, but after another sense vz. in respect of the longitudes of fixed starres, Comets and such like: the whole heauens are diuided into 12. signes by circles issuing from the poles of the zodiacke by euery 30. degrees of the Eclipticke, as hereafter shall be more manifest. The first signe of the zodiacke is called Aries and beginneth in the vernall intersection of the ecliptick with the equinoctiall: the rest proceede in order, as followeth: contrarie to the motion of the first moouer, that is from the West towards the East: and these be their carecters whereby they are written and knowne. ♈ Aries ♉ Taurus ♊ Gemini ♋ Cancer ♌ Leo ♍ Virgo ♎ Libra ♏ Scorpio ♐ Sagittarius ♑ Capricornus ♒ Aquarius and ♓ Pisces. The ecliptick, the equinoctical & euerie other great circle of the Spheare is said to contain 360. degrees euery deg. 60. minutes, euerie minute 60. seconds, euery second 60. thirds, & so forwards to the tenthes.

The materiall figure of the circles of the Sphere before mentioned, you shall haue in the title page at the beginning.

A farther discourse of the circles of the Sphere.

Chapter 7.

THis figure heere being the picture of the material Sphere made for commoditie in vsyng doth not altogither agree with the Sphere celestiall. For heere you see the Horizon is set fixed on two pillers, the Sphere with his axtree and poles appointed

pointed moueable, to bend downe, or to be lift vp. to and from this fixed Horizon, according to theleuation of the pole aboue the ſame. Where in trueth, the axtree and poles of the world, and Sphere celeſtiall, are fixed for euer in one ſelfe ſame place: and neuer moue, but all the whole Spheare and Spheares mooue about them. And it is the Horizon that altereth to them continually in euery countrie, which is ſpecially perceiued Northwards or Southwards. For why? a man cannot go Northwards or Southwards 60. miles but the Horizon altereth a degree of the poles eleuation, either higher or lower, as in the 12. Chapter following is more manifeſt. And wheras you ſee in this figure one meridian Mobile: which in the vſe of the material Sphere ſerueth the ſteede of al the Meridians and parallels commodiouſly as it is well knowne to him that vnderſtandeth the vſe of the material Sphere, but aſſure your ſelfe the number of the Meridians are infinite, agreeable to euery ſmalleſt diuerſitie of ſituation. For euery ſeueral countrie, Eaſt or Weſt, from other but 30. miles, hath a ſeuerall Meridian, diſtant halfe a degree from the other: and ſo round. And al the Meridians meete and croſſe at the two poles as the two Colures do, which are Meridians alſo to ſome country. But ſoft, my penne hath runne a little too large of theſe Meridians: for you can ſcarſe ſay that the Sphere hath any Meridian, becauſe it neuer ceaſeth turning: the Meridian circle alwayes cutteth the North & South being deriued of the latin word *Meridies*, in Engliſh the South. There is no one point of the Sphere tarrieth a minute in the South, but turneth continually rounde once euery day: wherefore the Meridians are onely appropriate to the Globe of the earth: which is fixed immoueable, and beginne from the Iles of Canaria, as in the 12. Chapter. Theſe Meridians are likewiſe appointed and imagined fixed in the heauens: and then are called *Circuli horarii*, hower circles. But thoſe ſelfeſame circles imagined & appointed in the Spheare turning dayly about with the Spheare, & all the Speeres celeſtial as is ſaid, are their circles of right aſcention, called in latin, *circuli aſcentionis rectæ*. and begin from the vernal interſection of thecliptike with the equinoctial: that is, from the beginning of Aries, proceeding towards ♋ and ſo round, and they ſerue to as great vſe, in limitting the obliquitie of the zodiack, and euery point of the heauens vniformely: as theMeridians ſerue in diſtinguiſhment of places on the earth. So that you ſee, thoſe circles, which the common ſort call Meridians, are of three ſorts: the firſt fixed as it were on the globe of the earth, and there called moſt aptly Meridians. The ſecond fixed in the heauens and there called hower circles, although they may there aptly ynough be called Meridians alſo but for diſtinction ſake: the third continually moueable with the celeſtiall Speeres: and there called circles of right aſcention: and all theſe haue one ſelfeſame axtree which by the figure of the *Iewel* you may partly perceiue.

Primus 3^x { 1 2 3

Where as alſo there are in the figure of the materiall Sphere, before ſet to the 6. Chapter, ſpecially appointed but fiue parallel circles: as namely thequinoctiall, the two tropicks, and pole circles, I may compare it to a mappe of the world wherein they vſe to ſet downe but the chiefe cities, leauing out all ſmall townes and villages. For you muſt know, that the number of parallels are likewiſe as infinite as the Meridians to be thwact full from thequinoctiall to ech pole, and euery one ſeruing to as great vſe as any of thoſe fiue: as hereafter wilbe ſhewed in my bookes following. But yet in the Sphere, they are not aptly called parallels, but rather *circuli* or *parallelli declinationis*, that is circles or parallels of declination, for that their office is to ſhew the declination of the ſunne, Moone, or any ſtarre, planet, or point of the heauens from thequinoctiall, and are numbred from him towards either pole, there ending at 90. ſo that looke what parallel cutteth at any time the place of the ſunne, Moone, or any ſtarre, the ſame counted from thequinoctiall is called his declination: As for example, either tropicke is but the $23\frac{1}{2}$ parallel and the ſunne being there is ſaide to decline $23\frac{1}{2}$ degrees, either pole circle is but the $66\frac{1}{2}$ parallel, and ſo of the reſt, and euen as the ſunne glides euery day in the zodiacke one deg. forwards, ſo doth he thereby get euery day a new parallel whereby the dayes are lengthened and ſhortned, all which premiſes you may alſo perceiue by the figure of the *Iewel*.

Of other circles of the Sphere, changeable with euery horizon, as Azimuthes, Almicantares and ſuch like.

Chapter 8.

THe Meridians hower lines, and parallels mentioned in the laſt chapter, do alwayes keepe one ſelfeſame ſtate, courſe, and place, but there are yet other circles moueable with euery conſtitution of the Horizon: of which ſort are almicantares, Azimuthes, circles of poſition, circles of ſtation, generall horizons and ſuch like. But all theſe are not vnlike to the Meridians and parallels in forme and faſhion, ſauing they are applied to other purpoſes in their ſundrie vocations, as hereafter will appeare. What the Horizon is hath beene ſhewed in the 4. Chapter, but this much farther is to be noted, that whither the horizon be taken on the ſuperficies of the earth, or through the centre, there riſeth no difference in the riſing or ſetting of the ſtarres and planets, or in any other ſphericall practiſing, except it be in the moone, or ſome comets which are very neere vnto the earth, which is one moſt ſtrong reaſon that the earth is but as a point or pricke in reſpect of the higher ſpheres and firmament: as for example, the ſunne being in the Aequinoctium, is manifeſtly found to tarry 12. houres aboue the earth, and 12. houres vnder, whereas if the earth were not as a pricke in reſpect of the ſphere of the ſunne, he muſt needs ſhine at that time leſſe then 12. houres aboue the earth, by ſo much time as twiſe the ſemidiametre of the earth (that is at riſing and ſetting) ſhould hinder him, which by experience is found nothing. Other proofes Geometricall there are, heere impertinent to my intention, and of the moſt would be little regarded: for hardly will they beleeue that a ſtarre (a planet ſtarre I meane) ſhould be 100. times as bigge as the whole earth: and much leſſe that euery ordinary ſtarre ſhould be ſo bigge as the whole earth. Therefore againe to my purpoſe: in euery horizon the point directly ouer heade, called in latine *punctus verticalis* is commonly called the Zenith; and the point oppoſite thereto, that is to ſay, below the earth vnderfoote as it were, called *Nadir*: or to define them more artificially, theſe two points vz. the Zenith, and *Nadir* are the poles of the horizon, being 90. degrees diſtant from euery part thereof, euen as is ſaid in the 4. the poles of the world are from thequinoctiall.

The Azimuthes called in latin *Circuli verticales*, verticall circles, *quia omnes tranſeunt per verticem*, are great circles of the Sphere, croſsing all at the Zenith deuiding the Horizon into 360. degrees, as the Meridians do thequinoctiall from the poles of the world, and are numbred from theaſt point of the horizon, yet after ſome, the foure quarters are numbred by 90. ending in the north and ſouth points.

The Almicantarathes, called in latin *circuli altitudinum*, circles of alitude, are the very like to the parallels of the Sphere beginning from the Horizon, thenſe growing leſſe and leſſe to nothing at the Zenith, as the parallels do from thequinoctiall to either pole. Circles of ſtation are the very azimuthes as ſhall appeare in the third booke more at large. Circles of poſition are the like, but croſsing at the two ends of the meridian line of the Horizon, as in the third booke. General Horizons be not much vnlike to them, as hereafter ſhalbe ſhewed, and ſo of the reſt: all which will be wonderfull plaine on my *Iewel*, which hath certaine circles on the *Reete* performing the vſes of them all, onely by altering their location, much like a player that playeth many parts with onely chaunging his voice and habite.

Of

Of the motions of the Speere and all the tenne Spheres.

Chapter 9.

I Make this diſtinction betweene the words Sphere, and Speere: by the Sphere, I meane the materiall Sphere with all his appurtenances: or more plainely, the whole frame of the world, as wel his celeſtial as terreſtriall parts, in latin called *Sphera*. By this word Speere I meane the ſphere of any ſeuerall planet, ſtarre or motion, in latin called *orbis*: and hath both a conuex and a concaue ſuperficies: yet euery *orbis* is *Sphera*, but euery *Sphera* is not *orbis*. This noted, you ſhall vnderſtand that the 10. Speere called *primum Mobile*, the firſt moouer, doeth continually whirle about the worlde, vppon the axtrees and poles of the world, euery 24. houres once round, without ceaſing, being ſo appointed by God from the beginning: and all the heauens, and all the nine Speeres are as it were perforce rapt and caryed about therewith euery 24. houres, from the Eaſt to the Weſt, and ſo to Eaſt againe, by the onely violence of the ſame tenth ſphere, called *raptum primum Mobilis*. Now the moſt part, and eſpecially the vnlearned, ſeeing this dayly motion in the ſunne, do thinke that he goeth it of himſelfe: whereas both he and all the other planets, ſtarres, and Speeres go a quite contrarie courſe (but a farre ſlower paſe) and are caried volens, nolens (as it were) this way: much like a man, who being in a long barge or gally, in a very ſwift tide or ſtreame, may walke a ſoftly paſe, as farre as the barge will giue him leaue, againſt the ſtreame and tide, but the ſtreame and tide being very ſwifte and eſpecially if the winde helpe withall, may happen to carrie him a mile backe, before hee can walke the length of the barge forwards: or like as a Spider, that ſhould creepe about a wheele contrary to that way which the wheele is driuen: Euen ſo the ſunne of his owne motion, goeth euery day in the zodiacke, one degree forwards (that is) from the Weſt towards the Eaſt, according to the ſequele of the ſignes: yet, becauſe his courſe is as it were againſt ſtreame, winde, and tide (that is to ſay) againſt the motion of the tenth Speere, he is caryed quite contrarie to his owne motion, by the *raptum primi Mobilis*, from Eaſt towards Weſt round about the world euery 24. howers which is done 360. times ere he can performe his owne courſe, that is in a yeare, euen as a man may driue a wheele 360. times about, before the ſpider can get once about the contrary way. This courſe of the ſunne in the zodiacke is the cauſe of the diuerſitie of the times of the yeare, of ſommer, winter, ſpring, and harueſt, long dayes and ſhort. In the very like manner the courſe of all the other planets and ſtarres in their Speeres, is quite contrary to the dayly motion of the firſt mooucr, and euery of their Speeres hath a ſeuerall motion, one ſlower then other, the higheſt Speeres and neareſt to the *primum mobile* haue ſloweſt motions (as in the next Chapter is ſhewed) and the farther off, neere to the earth, the ſwifter. And ſome Philoſophers holde opinion, that were it not that the violent ſwiftneſſe of the *primum mobile*, were ſomwhat qualified and holden backe as it were, by the ſlow motions of the highermoſt Speeres, as the 9. the 8. the Speeres of Saturne, Iupiter, and Mars, and ſo with the helpe of the reſt, that nothing could liue vnder it, but the violence thereof would deſtroy euery thing.

In what time euery Speere and planet finiſheth his courſe and of the carrecters of the planets.

Chapter 10.

THe Criſtalline or ninth ſpeere his motion is almoſt vnſenſible, requiring 49000. yeares to performe his courſe once about, contrarie to the firſt mooucr, according to Alphonſus: but by Ptolome 36000. yeares. The 8. ſpeere, being the ſtarry firmament, performeth his motion contrarie to the *primum mobile* in 7000. yeares once about the zodiacke.

The 7. ſpeere, which carieth the ſtarre or planet called Saturne (for all the fixed ſtarres are in the 8. ſpeere, the other ſpeeres haue but ech one ſtarre) accompliſheth his courſe once about the zodiacke in 29. yeares 162. dayes. and 12. houres.

The ſixt ſpeere of Iupiter, in 11. yeares 113. dayes and 20. houres.

The fift Speere of Mars in one yeare 321. dayes and 23. houres almoſt.

The 4. Speere of the ſunne, in 365. dayes and ſixe houres almoſt.

The thirde Speere of Venus in 584. dayes and $\frac{1}{64}$ of a day.

The 2. of Mercury in 365. dayes and 6. houres.

The 1. of the Moone in 29. $\frac{1}{2}$ dayes.

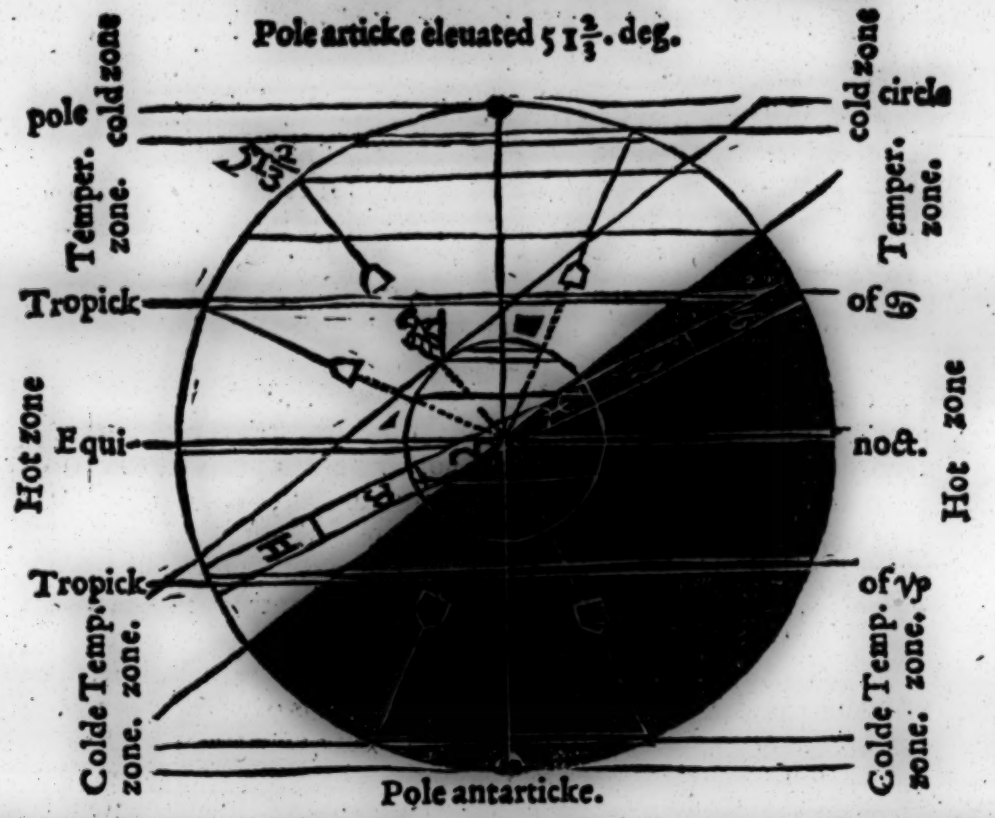
Theſe are the carrecters of the planets ♄ Saturne, ♃ Iupiter, ♂ Mars, ☉ Son, ♀ Venus, ☿ Mercury, ☾ the Moone, ☊ caput Draconis, ☋ cauda Draconis, ☌ coniunction, ☍ opoſition, □ quartile, △ tryne, ✶ ſextile.

There is no place heere to proceede any farther about the theoricall motions of the planets, yet if it pleaſe God to giue me leaſure a little to recouer that time which hath beene hitherto bereft me by meanes of thoſe horrible and vnconſcionable hunters of mens titles with concealements: I meane before I end this booke to ſet the Vranicall inſtruments or theorikes of the ſunne and moone, with apt explanations of them, becauſe they will be the greateſt helpe that may be, to get the longitudes of Regions. In the meane ſeaſon know thus much, that as I haue ſhewed in the laſt chapter, all the ſeuen planets by their owne motions contrarie to the 10. Speere, do continually go about vnder the zodiacke lying one within another within the zodiacke like a number of hoopes which a cooper carieth at his backe, and do neuer ſo much as peepe without the zodiacke on any ſide, but walke vnder it as vnder a bower or canapie. The ſunne he neuer goeth one iote a wry, but alwayes keepes in the ecliptick line: All the other planets their ſpeeres lie byas croſſe the ecliptick, which cauſeth them to go reeling like a dronken man, ſometimes on one ſide of the ecliptick & ſometimes on the other, which reeling is cauſe of their latitude, but they neuer go from vnder ſome part of the zodiack: they neuer reele aboue halfe the bredth thereof: for to that end the zodiacke hath his bredth appointed him to keepe them within compaſſe, as to them that vnderſtand the theoricks is wel knowne. Further the two points where the ſphere of any planet doth cut the ecliptick, are called Caput and Cauda Draconis, as in the ſecond Booke 5. 8. Chapt. you may reede: marke theſe two laſt Chapters well, and reade them againe, I haue ſet them downe as plainely as I can becauſe theſe things in Aſtronomy I could hardly conceiue when I firſt began.

Of Zones, Climats, parallels and meridians.

Chapter 11.

COſmographers minding to limite out the earth, finding by moſt aſſured proofe (aſwell by the eclipſe of the moone, as other philoſophical approbations) that the earth togither with the water maketh a round globe, & is placed in the midſt & centre of the heauens, did therfore imagine & appoint thereon the like circles to the circles of the ſphere, in euery reſpect Symmetrially agreeing vnto them as this figure ſheweth, where I haue made the Globe of the earth farre within the ſphere, ſo that what proportion the circles of the ſphere beare one to another, the ſame do the like circles made on the globe of the earth beare one to another, inſomuch that looke what parallel circle any place is ſcituate in on the earth: the ſame circle in the ſphere doth touch the zenith of the ſame place. For you muſt note that all plum lines round about the world do hang to the



centre of the earth,according to the maxim in philoſophie, *Omne graue tendit ad centrum:* As for example,to them that dwel in the tropick circle on the earth,the tropick circle of the ſpeere toucheth their zenith,ſo that when the ſun commeth to that tropick it ſtandeth that day at noone iuſt ouer their heads. In like manner becauſe heere at reading the latitude is 51 2/3 I ſay that we dwell in the 51 2/3 parallel of the earth and that the 51 2/3 parallel of the heauens(alwayes reconing them from thequinoctial) doth touch our zenith as in this figure you ſee : Now becauſe the ſunne neuer goeth farther then the tropicks on any ſide of thequinoctiall which are but the 23 1/2 parallels. I ſubduct 23 1/2 out of 51 2/3 there reſteth 28 1/6 therefore I ſay that the ſunne when ſhe is at the neareſt to vs commeth not at noone to our zenith by 28 1/6 degrees. But theſe circles being drawne on the globe of the earth,haue contrarie names and deffinitions from thoſe of the ſphere, and are by Coſmographers appointed to diſtinguiſh the earth as followeth: They haue appointed fiue zones in the heauens, of which two lie betweene the pole circles and the poles being horible cold and ſcarſe habitable vnder them : a third lieth betweene the two tropicks and is extreme hotte, and ſcarſe habitable becauſe of heate : the other two are very temperate and habitable, and lye betweene the pole circles and the tropickes and in one of them dwell we. Theſe zones on the globe of the earth are called plagues.

The parallels in Coſmographie are onely thoſe paralels of the ſphere that are ſo farre diſtant from other, that paſsing by the zenith of any horizon they alter the length of the day a quarter of an houre , and thequinoctiall it ſelfe is accounted for the firſt parallel they are vſual 21.on ech ſide of thequinoctial. The climates are ſuch of the parallels as do alter the length of the day halfe an houre,beginning from the 4.of the Coſmographicall parallels,and are 9.in number on ech ſide of thequinoctiall,and haue ſeueral names of ſuch places of the world as they paſſe ouer : as Dia Meroes, Dia Syenes Dia Alexandrias Dia Rhodon Dia Rhomes Dia Ponton Dia Boriſthenes Dia Ripheos Dia Daniam:the other on the Southſide of thequinoctial are called Ante Dia Meroes Ante Dia Syenes &c. As for meridians there is ſaid ſufficient of them in the 7.Chapter,they are circles cutting thequinoctiall ſquare,and meeting altogither in the poles and ſerue to diſtinguiſh the longitudes of places by degrees of thequinoctiall.

Of the longitudes and latitudes of places.

Chapter 12.

THe longitude of a place is the arch of thequinoctiall, comprehended betweene the meridian paſsing by the yles of Canaria, and the meridian of the ſame place, and altereth by going full Eaſt or VVeſt from any place in euery 60.miles a degree : for Coſmographers haue appointed the firſt degree of longitude to beginne in thoſe yles called *Inſula fortunata* reconing the degrees from thenſe Eaſtwards,and ſo round thither againe. I ſay 60.miles to a degree, but that onely is vnder thequinoctiall, but in euery other parallel, the more he groweth towards the pole the fewer miles make a degree of longitude,as hereafter in my booke of Coſmography.

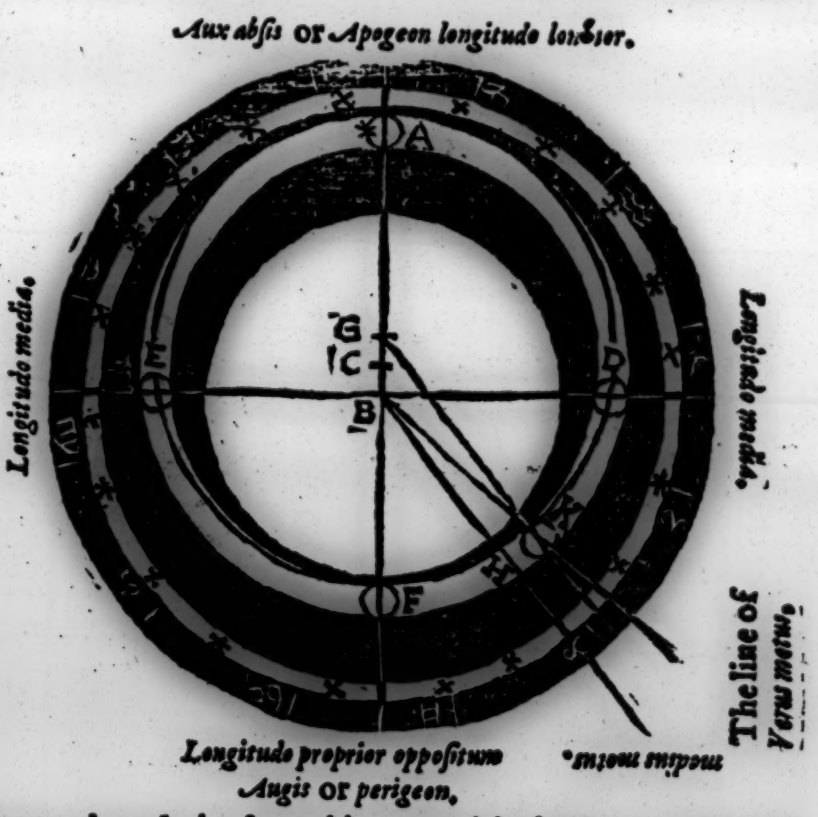
The latitude of a place is the parallel wherein the place ſtandeth reconed from thequinoctiall: but according to Coſmographers definition, the latitude of a place is the arch of the meridian of any place betweene the zenith and thequinoctiall, or rather the arch of the meridian of any place betweene thequinoctiall and the parallel which toucheth the zenith, and is alwayes equall to the eleuation of the pole aboue the horizon. The latitude altereth by going Northwards or ſouthwards in euery 60.miles a degree.Further you ſhall vnderſtand all places being ſcituate in,or vnder one parallel round about the earth haue one latitude,and for that cauſe they are commonly ſaid to haue one horizon(though indeede they haue diuerſe) and in truth one ſcituation of the horizon,my *Iewel* ſerueth thē all as hereafter ſhalbe ſhewed:the reaſon is becaute the ſun doth both riſe and ſet at like houres in one of thoſe places as in another, and likewiſe the planets and ſtarres. Yet as much as any countrie is diſtant from another in any one parallel by degrees of longitude,ſo much diſtance of time is there betweene the riſing and ſetting of the ſunne , moone or ſtarres, in euery ſuch perticuler place for euery 15. degree of longitude an houre, as for example : there is a famous citie in high Germany called Moguntia commonly Mentz , where printing was firſt deuiſed by one *Iohannes fauſtus* anno 1453.whoſe latitude is 50.degrees 10.minutes, not full equall with ours,and therefore lieth not altogither in the ſame parallel, it lacketh one degree and 30. minutes,yet neare ynough for this example except the difference were more: the longitude of that Citie is 25.degrees 4.minutes,and the longitude of our towne of Reading is about 13.degres. Now becauſe the longitude of Moguntia is greater then of Reading I know that it lieth more Eaſt by as much as 25 4/60 exceedeth 13.which is 12.degrees 4.min.& therefore the ſunne muſt needs riſe 12.degrees there before he riſeth at Reading, that is 4/5 of an houre, or more plainer three quarters of an houre and 3.minutes and little better, for euery houre is 15.degrees of thequinoctial & euery 4.minutes of an houre is one deg. yet for all this doth the ſunne riſe at the ſame houre there as he

as he doth here according to computation, becauſe the houres are al rekoned by noone, that is from the ſuns comming to the meridian, for who doubteth when it is noone heere aboue the earth, but that it is midnight vnder the earth and ſo contrariwiſe. Likewiſe when it is noone here, then to them that dwell in the VVeſt from vs, and iuſt in the ſame parallel that we do: it is ſunne riſing, and ſo when it is noone to them it is ſun ſet with vs. and therefore it cannot be noone there and heere altogither for it is neuer noone in any place vntill the ſunne come to the meridian of the ſame place that it ſtand full South vnto them.

Chapter 13.

Of the theorickes of the planets.

I Haue ſhewed before in the 2. Chap. that all the 7. planets haue their ſeuerall ſpeeres one vnder and within another: and in the end of the 9. Chapt. that their walke or proper motion lyeth continually vnder the Zodiacke in length and bredth. And now perhaps you would thinke that their walke, circuite or motion ſhould be performed about one centre vz. concentricke to the earth, as the zodiacke is, which is farre otherwiſe: For there is none of the planets which mooueth on the centre of the earth, but euery of them hath a ſeuerall centre farre and diuerſlie diſtant from it: which excentricitie is the cauſe of great toyle in ſeeking and making rules, tables, and ephemerides for the places of the planets: and alſo of faining and imagining of diuerſe circles, epicicles and hypotheſes, to bring thoſe irreguler motions vnto ſome certaintie: Inſomuch that of late yeares that ſinguler man Copernicus affirmeth that the ſunne is the fixed centre of the world, about whom the earth moueth (not the ſunne about the earth) and that all the reſt of the planets moue regularly about the centre of the ſunne ſauing the moone which like an epicicle moueth about the earth in the ſpeere of the earth 13. times in his yearely motion. But omitting the inuentions of Copernicus, and a number of the reſt, I will only heere ſhew a figure out of thoſe which haue alwayes bene before his time moſt in vſe to the end that you may the better vnderſtande diuerſe matters in my booke following. VVherefore Saturne being the highermoſt planet, I will proceede with his frame or theoricke repreſented by this figure following: wherein the vttermoſt ſpace or ſpeere is the zodiacke of the 9. ſpeere, the next is the 8. ſpeere or ſtarrie firmament, immediately vnder it is the frame or theoricke of Saturne conteining three orbes or ſpeeres, the two blacke ſpeeres are called *Deferentes Apogeon* or *Deferentes excentricum*, that is to ſay cariers of the Apogeon and excentricke: the white midlemoſt ſhut in betweene them is called thexcentricke, or Deferens epyciclum becauſe thepicicle carying the ſtarre of ♄ is carried therein. The Apogeon of ♄ is in theſe dayes about 13. ½ of ♐ thorough which D E a line drawne from the centre of the earth vz. B. and appointing the centre of the epycicle vz. A in the ſame line: diuide A B into 80. parts, 4. of thoſe and 20. minutes ſhalbe the ſpace C B. and vpon the centre C. is thexcentrike deſcribed, whereby you may well conceiue that when ♄ is in A, then is the centre of his epicicle farther of from B. the cẽtre of the world, then when he is in F. by the quantitie of the excentricitie vz. C B, & is then ſaid to be in his Appogeon aux or Abſis and longitudo Longior, that is fartheſt from the earth. In F. he is in *oppoſito augis*, or *perigeon* and *longitudo proprior*, that is neareſt the earth. In D & E, he is in *Longitudine media*, that is in the midde way betweene the Apogeon and Perigeon: that is to ſay nearer to B then A. and farther off then F. the prickt circle in thexcentricke is called the equants diſcribed on the centre G. as farre diſtant from C, as C is from B. whoſe office is to regulate the irreguler motion of thepicicles centre in his excentricke: and whereſoeuer thepicicle be a line drawen from the centre B. to the zodiack parallel to a line drawne from G. the equants cẽtre thorough thepicicles centre, the ſame line as B H. is called the line of *medius motus* and the degree in the zodiacke pointed thereby is called *medius motus* but by a line from B. the centre of the earth through thepicicles, centre is alwayes ſhewed *verus motus*. Further the bodie of the planet in reſpect of the epicicle is but a point & is carryed about on thepicicles circumference which cauſeth him to be direct, ſtationary, and retrograde: thepicicle alſo hath his Apogeon and Perigeon. A number of other matters there be about theſe theorickes which heere I neede not mention, as *Equaconus* arguments, proportionall minutes, and ſuch like: becauſe my purpoſe is heere but to giue a taſt, that you may not be ſtalled when any of theſe tearmes come in queſtion. VVherefore it ſhalbe ſufficient for you to know that all the reſt of the planets haue the like frame or theorickes in manner, though the ſunne his excentricke hath no bredth, neither hath he an epiciele: Mercurie he hath ſomwhat more to do by reaſon of his *Circulus paruus* and ſo of the reſt, ſome difference they haue ech of them: their excentricities and Apogeon alſo do all differ from other.



The ſeconde booke of the Mathe-

maticall *Jewel*: compiled by Iohn Blagraue of Reading, Gent.

ſhewing the compoſition and making of the Jewel: being a moſt ne-

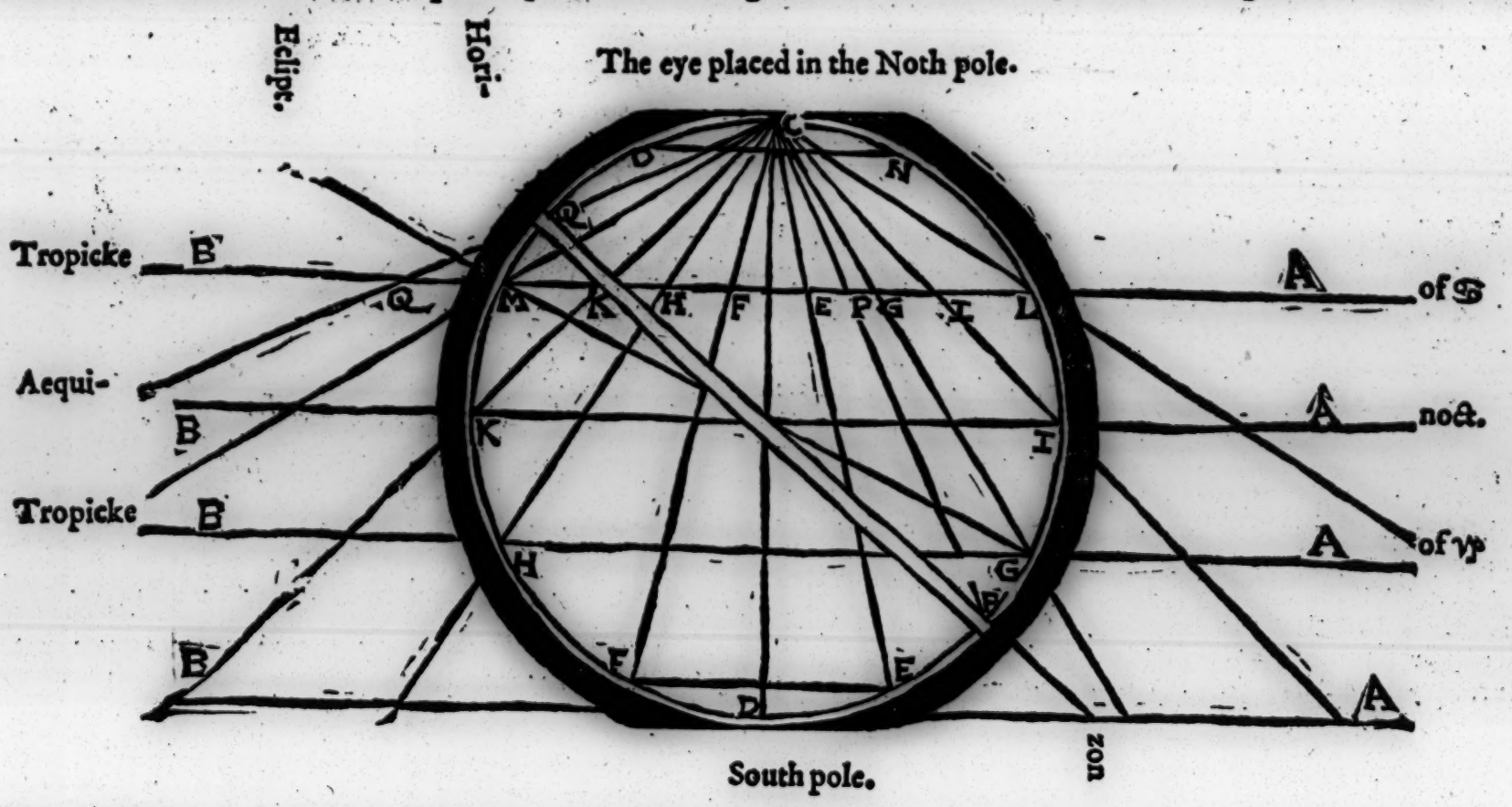
ceſſarie and profitable worke for all thoſe to be well ſeene in, which ſhall indeuour to deſcribe mappes or ſea cards, or to proiect artificially the globe or any part thereof into a plaine or platforme, as exact and perfite for any purpoſe as the globe it ſelfe.

Secondly it conteineth the inuention, definition and explanation of this Iewel, *and of* euery circle and thing therein contained with their vocables and names, moſt neceſſarie to be foreknowne.

Chapter I.

A neceſſarie diſcourſe which Gemma Friſius vſed as a preamble to his Booke, De Aſtrolabio Catholico by what reaſon the globe is brought into a flat or plaine forme.

BEcauſe ſaith he the making of a Sphere or globe is verie chargeable with his furniture and thereto requireth moſt exquiſite workemen, notwithſtanding cannot be made in all points without ſome errors, the vſe thereof to trauilers comberſome becauſe of caryage, hardlie kept and maintained from hurt, and being once blemiſhed or hurt not eaſily ſalued: for theſe & diuerſe other cauſes, ingenious perſons vſed long ſince great indeuour to reduce the globe with his circles into a plaine forme. But ſeing (as it is well knowne to the meaneſt geometricians) that a flat ſuperficies cannot be equally anſwerable to a globes Superficies in all points (nay ſcant in any point) for that there is no affinitie or proportion of a ſtreight line to a crooked, nor of a globe to a plaine: therefore there reſted no other meane to bring this purpoſe to effect but onely by the art Opticke, commonly called Proſpectiue: By which Arte artificiall painters vſe on plaine tables to ſet forth buildings or any thing elſe in the ſame ſhew of proportion, as it appeareth to the eye in all reſpects. Of which Art Albertus Durerus, a famous painter and Mathematician ſhewed two ſpecial and excellent reaſons; whereof the one and chiefeſt was thus: to place a plaine and cleare glaſſe ſet betweene the eye and the thing ſeene or obiect: And



ſo (the eye fixed in one very place) to draw vpon the glaſſe whatſoeuer he ſaw through the ſame. Now in trueth there is no way ſo apt, ſo proportionable and commodious, to bring the Globe or any thing that hath thickneſſe into a plaine flatte, but onely by proſpectiue lineaments, like to thoſe which the eye by imagination maketh on the glaſſe ſet betweene the eye and the thing ſeene. By this very Arte diuerſe men haue diuerſe wayes ſought to bring the globe in plaine: whereof the beſt & moſt commodious that euer was before this, was that of which Iohannes Stophlerus ſet foorth the making and vſe, although it be thought that he was not the firſt inuentor thereof. Touching the making whereof, he vſeth a long diſcourſe: whereas in ſhort it is but thus. Deſcribe a meridian circle, and place the eye at one of the poles vz. in C, and let any of the lines marked with A B. be your plaine glaſſe: marke where the lines iſſuing from C do cut your plaine A B. as in this firſt figure, for they doe limitte thereon the diametres of ſuch circles of the globe, as they comprehend betweene any two lines. This done draw any of the lines A B ſeuerally by himſelfe, whereon make all the notes there had. Theſe ſerue you for the dyametres of euery ſeuerall circle of the aſtrolabe, as in this ſecond figure appeareth. And thus haue you the manner of the drawing of the chief circle, of that noble inſtrument, a ſpeedier way then Stophler teacheth, which he either ouer ſlipt or ſaw not. But for the Almicantares and Azimuthes and other particularities to euery ſeuerall eleuation, it is no place heere to ſhew.

Behold the lineaments of Stophlers aſtrolabe taken out of the former figure, wherein the centre repreſenteth the North pole.

E F. The pole circle.

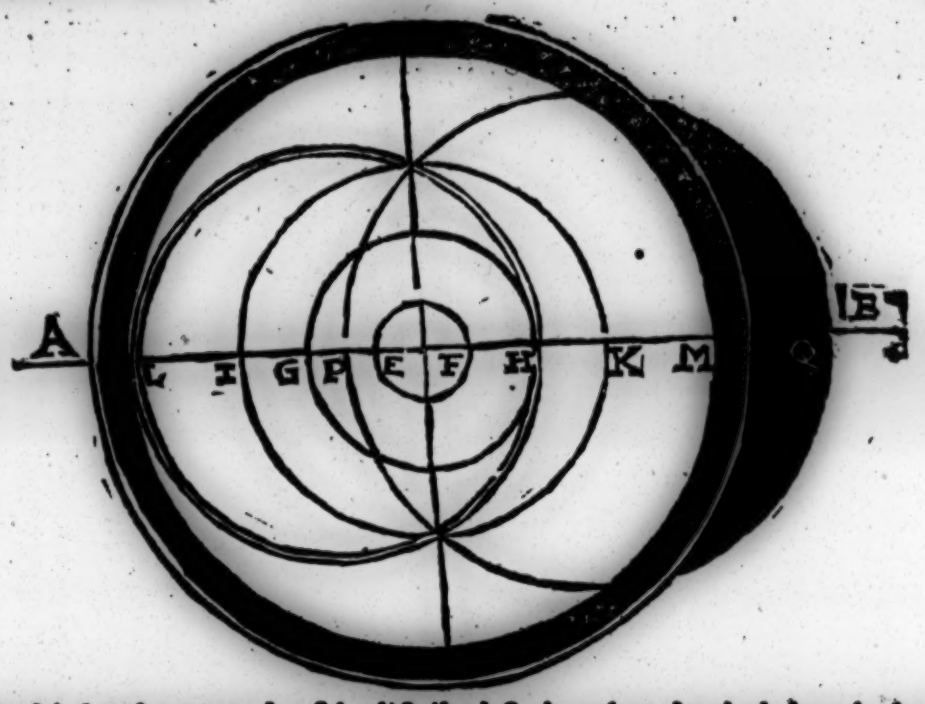
G H. The Tropicke of ♋

I K. The Equinoctiall.

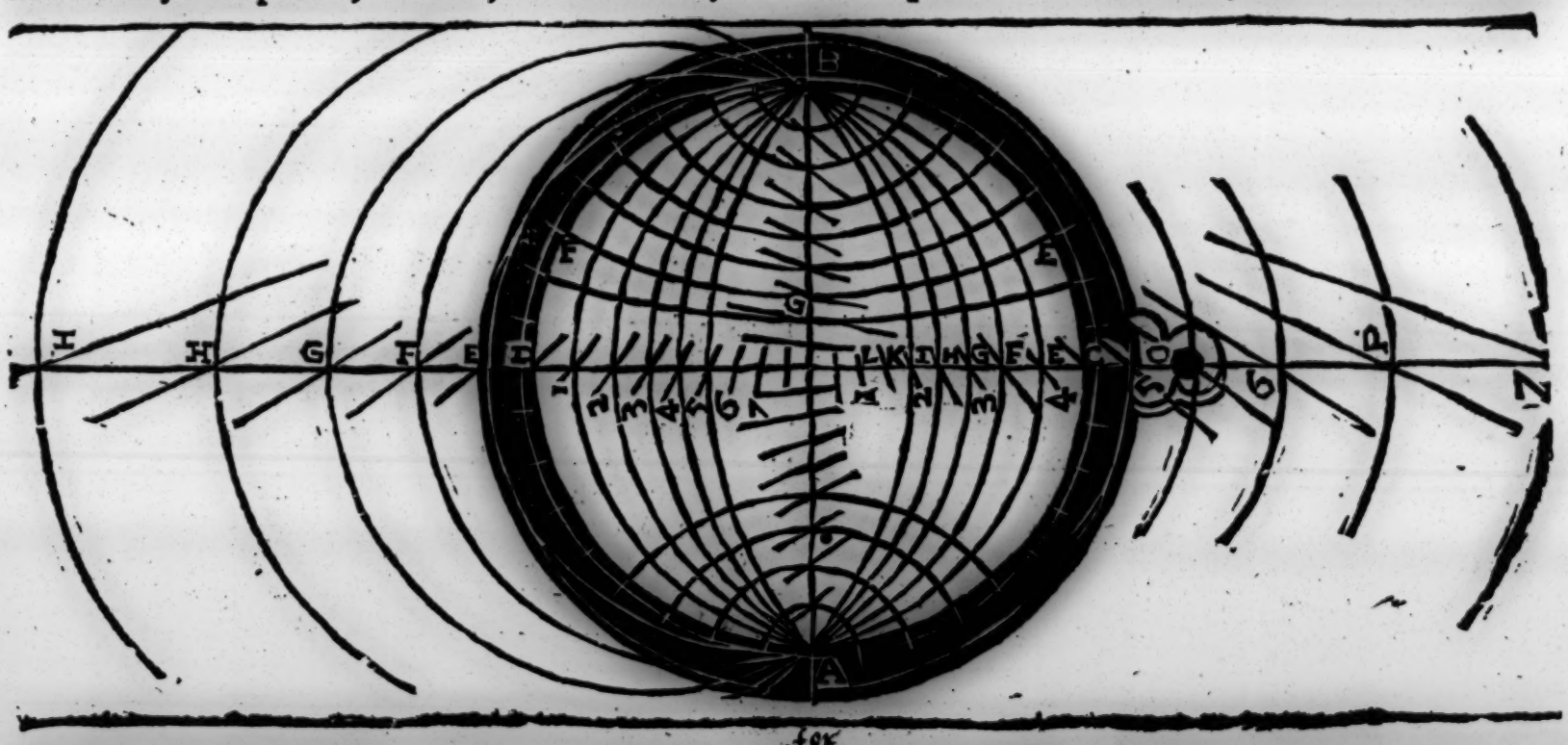
L M. The Tropicke of ♑ *which is alwayes in that aſtrolabe the innermoſt circle of the limbe.*

L H. Is the Zodiacke, and is made to moeue on the centre.

P Q. Is the horizon, whoſe part without the limbe is neuer vſed.



In verie like manner the plaine ſphere which this booke treateth of, is diſcribed, ſauing that the circle here is the Colurus Aequinoctiorum, which there is the meridian. His dyametres diuided, the one vz. A B, being the equinoctiall, the other vz. C D the axtree line. The eye is heere twiſe placed firſt in one of the poles as before: and againe in one of the interſections of the equinoctiall with the colure. From the eye in the pole ~~vz. M~~ A, is the equinoctiall line C D. diuided into 180. degrees, or parts by directing lines from the eye M A. to euery degree of the ſemicircle C B D againſt it. From the other eye in the ſaid interſection vz. in C, is the axtree line A B diuided likewiſe into 180. parts, by proiecting lines thence to euery degree of the ſemicircle A D B. lying againſt it. This done to make the meridians, you muſt ſeeke a centre for euery three prickes, of which, two muſt alwayes be the two poles vz. A B. the thirde, ſome one of the diuiſions



or parts of the equinoctiall line, thereby drawing the arch ~~from~~ for your purpoſe] as in this figure you ſee ſome drawne. To make the parallels, ſeeke the centre of euery three pricks, whereof two muſt alwayes be taken in the colure A C B D equal diſtant on ech ſide of one of the poles as here you ſee E F from the pole B. the thirde amongſt the parts of the axtree line of like diſtance from the ſame pole as you ſee G. and thus draw ſo many meridians and parallels as are parts in A B. and C D as by the lineaments within the blacke limbe of this figure, & by other figures following in this ſecond booke you ſhall more plainely ſee.

Theſe inſtructions did Gemma Friſius ſet downe for making his Catholicon Aſtrolabe & bids you to put to a rule, a Curſor and Brachiolum, and on the other ſide of the ſame plate (for he appointed to haue one or two ſeuerall plates loſe in the *Mater* like Stopler) willeth to place the theoricke of the ſunne, the kalender with the geometricall quadrat: and on the backſide of the inſtrument to haue generall horizons after Stophlers inuention, with Stophlers *Reete* and rule and ſo endeth. But I meane to haue but one plate in all, and none of all the other tackling but the kalender: and hope by the grace of God to make a perfecter inſtrument then yet euer was any: ſuch a one as you ſhall thinke worthie to be called a *Iewel* ſeruing generally through the whole world from pole to pole, which Stophlers could neuer doe, nor Gemma Friſius in all points.

Chapter 2.

What materiall this Iewel *were beſt to be made of.*

THis *Iewel* conſiſteth but of two plates, the one lydging and called the *Mater*, the other moouing thereon called the *Reete*. VVherefore for your *mater* if you will haue your *Iewel* in mettall and take no great paines in working, you may prepare a plate of pure tynne, and tynne glaſſe, of ech like quantitie melted together, and it will be ſtiffe, and looke faire like ſiluer almoſt: otherwiſe get faire new lattyn braſſe ſomewhat thicke for it is a great fault if your *Iewel* be too light in taking altitudes, becauſe euery ſmall winde will ſtirre him and other cauſes alſo. Or if you will go to the coſt ſiluer is beſt, and farre eaſier to worke then braſſe, I haue tried them al and made aſtrolabes of euery of thoſe mettals

tals with my owne hands. And for a ſhift a faire paſtborde paſted on a maſsie borde, will ſerue a young practitioner.

Then for the *Reete* it is beſt to be of the ſame mettal the *mater* is of, but as thinne as may be. A very plaine and cleere lantornes horne (were it not that it will caſt with heate and moiſture of the aire) had no fellow, which I once vſed as a great treaſure vntill I at the laſt diuiſed artificially to cut out the *Reete* as hereafter I will ſhew. And to be ſhort but for a mans fancy and feare of hurting when it ſhall come in children or fooles handling, there can be no way more delicate then to make the *Iewel* of paſtbord, becauſe the circles may be ſo liuely diſtinguiſhed with coloured ynkes, and the *Reete* ſo eaſily and exactly cut out. And no doubt being glewed and faſtned to the bottome of a ſhallow box made for the purpoſe of bone, or ſome maſsie woode, as Braſil, Lignum vitæ or Boxe, will there be kept ſafe many yeares from bruſing, ſoyling, wet and ſuch like. And he that will be ruled by me let him firſt make one of theſe ere he proceede to worke in mettall, and let him make his paſtboord of pure good paper himſelfe.

Chapter 3.

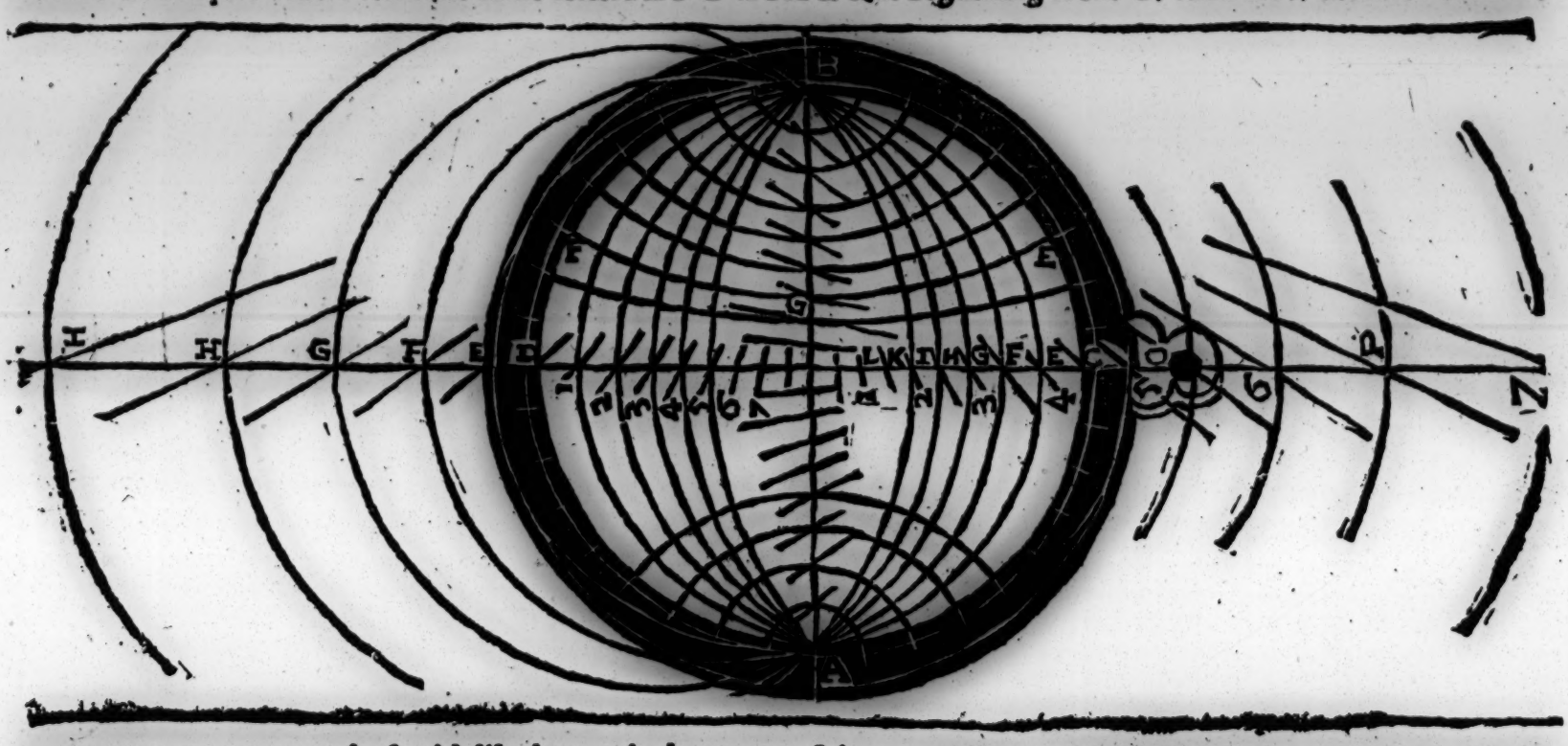
The meridians and parallels drawne how to finiſh the Mater.

THe *Mater* of my *Iewel* is nothing different from that of G. Friſ. as by the great figure you may ſee, but be your *Iewel* neuer ſo little, let the meridians and parallels not be aboue two degrees betweene, as in my great figure they are, and being aboue 8. ynches diametre but one degree, for euer the more circles the better and more effectuall to euery action. VVell then your meridians and parallels ſo drawne as Gemma Friſius hath taught, then would I haue euery third diſtinguiſhed with a greater ſtroke, or euery fift if you make them but one a peece. Thē let euery 15. meridian be ſeuered with prickes al a long their arches: likewiſe let the $23\frac{1}{2}$ parallel on ech ſide of the equinoctiall, which are the tropickes: alſo the $23\frac{1}{2}$ parallel next either pole, which are the pole circles, togither with the two diamet. be prickt, which with the clipticke line drawne byas betweene the two oppoſite ends of the tropickes, as you ſee, doe liuely repreſent vnto you all the chiefe circles of the Sphere, euen as though you had the Sphere or globe in your hand. To the ſaid prickt houre lines, you muſt ſet figures the axtree line being 6. of clocke at morning and euining the outermoſt meridian at C next the handle 12. at noone, and the ſame meridian at D. 12. at night, and ſo the reſt but in very ſmall inſtruments: theſe figures might be placed on the rule which applied to the equinoctiall line at all times ſupplieth the turne. So likewiſe vnto the cliptike line let the carecters of the ſignes be aſcribed vz. ♈ and ♎ to beginne at the centre ♋ at the end next the handle, which end muſt lye $23\frac{1}{2}$ degrees from the equinoctial towards the North pole as you ſee, and ♑ to beginne at the other ende, the reſt in order. Theſe carecters in ſmall inſtruments may be alſo ſupplied in the rule: So much for the *Mater*. As for the numbers of the limbe, let them be written as in the great figure of this work you ſee done.

Chapter 4.

How to finde the centres of the meridians three ſeuerall wayes.

AS for the limbe of my *Iewel* it is eaſily to be diuided by my 1. Booke 20. concluſion into 360. degrees, but foraſmuch as Gemma Friſius ſaith, that it is an eaſie matter to finde the centre of three prickes and ſo paſſeth away: he ſpake belike to Geometricians, for it is not ſo eaſie to him that neuer read the 5. propoſition of the 4. Booke of Euclide or the like: well, let that be one way: my 15. Concluſion will inſtruct you therein. A ſecond way is thus; you ſhall infinitly extend or draw forth the diametre C D, on either ſide: then to diſcribe all the meridians betweene C. and the centre of the *Iewel*, you ſhall firſt diuide the extenſion of that dyametrall line beyond D. by laying a ruler on the point A. directing the other ende from degree to degree of the quadrant A D. or from 2. degrees to 2. deg. euen in the verie ſame manner as you diuided the diametre C D. by the ſemicircle C B D. which diuiſions in the ſaide line I haue marked with E F G H I K L &c. beginning from D. then for your better helpe I note all the diuiſions of the line C D. betweene C. and the centre with the ſame letters E F G H I K L &c. beginning from C. And now the iuſt middle be-



tweene any two notes marked with like letters, is the centre of that meridian: which middeſt you may either meaſure out with your compaſſe, or finde by the 3. Concluſ. As for example: in this figure pitch your compaſſe in the iuſt midſt betweene H and H. and there doubtleſſe is the centre of the meridian A H B. So likewiſe iuſt betweene K and K is the centre for the meridian A K B. and ſo of the reſt. In the very like manner to make the other halfe of the meridians, you may diuide the other part of the diametrall line extended beyond C from A, by the degrees of the quadrant A C,

A C, and worke in all points as before, but in this figure I haue reſerued it to ſhew my third way becauſe I would ſaue a ſtampe. Note that theſe circles E F G H I K &c. ſtanding whole round as here they do, are the very ſelfeſame circles which are called generall horizons after Stophlers inuention, and are neuer aboue 90.

The thirde way to finde the centres of the meridians is the beſt and eaſieſt and therefore I alwayes vſed it, but except euery pricke and lineament before drawne be paſsing preciſely done, it is the worſt: for the bredth of a centre pricke loſt in ſome one place will bring you out 2. or 3. ynches in another, yet by this meanes you may know the better to examine your worke. VVell, then to deſcribe the other halfe of the meridians next D, by this third way, you ſhal find of my poore credite, that euery ſeconde note of the line C D, betweene C and the centre, is a centre to one or other of the notes betweene D and the centre. VVherefore for breuities ſake I ſet figures to all the notes of the line C D. betweene D and the centre vz. 1 2 3 4 5 6 7 8. beginning from D. Likewiſe I ſet the ſame figures to euery ſecond note in the other halfe of the ſame diametre, beginning from the centre of which the note 1. is the true centre to the arch A 1 B. and the note 2. to the arch A 2. B. the note 3. to the arch A 3. B. and ſo of the reſt: but now all the matter is how to come by the centres without the diametre C D. that ſhall you do very eaſily if you do but giue a note or nicke backwards on the diametrall line beyond C. from euery of the former centres after you haue drawne your arch: as for example, when on the centre 1. you haue drawne the arch A 1 B. then giue a nicke backwards beyond C. your compaſſe vnſtirred, there ſet 5. likewiſe after that on the centre 2. you haue made the arch A 2 B, giue a nicke backwards with your compaſſe at 6. and from the centre 3. your arch A 3 B being made, giue a nicke, backwards at 6. and ſo of the reſt. And now when euery ſeconde pricke of the ſemidiametre haue plaide their parts, then doeth euery ſecond of the nickes that you made backwards ſerue in very like ſort: as for example, the nicke 5. is the centre to the meridian A 5. B. and heere muſt you alſo giue a nicke backwards and there ſet 7. and thus continually making nickes backwards euery ſecond ſerueth your turne, and one centre ſtill finding out another.

Further it is a ſinguler thing to be noted and moſt neceſſarie for examination of your worke. Thoſe nickes or notes backwards do diuide the diametrall line beyond C, into the verie ſame parts as the other part of the ſame diametrall line beyond D. was before diuided from the point A by the degrees of the quadrant A D if your worke be exactly done: yet you ſhall hardly make them agree becauſe the miſſe of the pricke of a compaſſe driueth the nicke backwards marueilouſly out of his place, in ſuch meridians whoſe centres be farre off; wherefore as I haue often ſaid, you cannot vſe too great care in performing theſe things. Hereby you may contrariwiſe perceiue that if from A you had diuided the dyametrall line beyond C. by the degrees of the quadrant A C. then needed you not to haue made or giuen any nickes backwards but taken for your centres euery ſecond diuiſion.

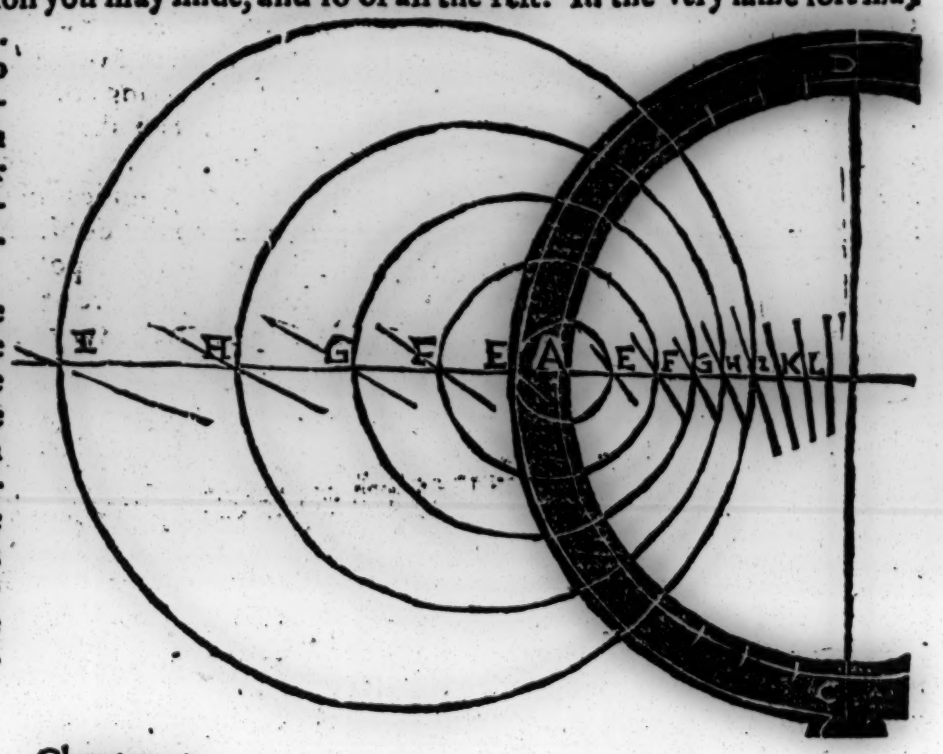
Chapter 5.

How to finde the centres of the parallels artificially.

YOu muſt here alſo firſt draw forth the diametre A B infinitely beyond A & B, as you ſee in this figure: which done you ſhall diuide his extenſion beyonde A, by helpe of a ruler laide on C. and euerie of the degrees of the quadrant A C. euen as in the laſt Chapter you did, then ſet at the diuiſions or notes theſe letters E F G H I K L &c. beginning from A outwards, and likewiſe at euery of the diuiſions of the ſemidiametre of the *Iewel*, place the ſame letters E F G H I K L M &c. beginning likewiſe from a towards the centre: now againe is the iuſt middle betweene any two notes marked with like letters, the centre of the parallel noted with the ſame letter as in this figure you may ſee in the iuſt middle betweene the notes G and G is the centre of the parallel S G T. and betweene H and H is the centre of the parallel Q H R. which by my 3. Concluſion you may finde, and ſo of all the reſt. In the very ſame ſort may you deſcribe the other halfe of the parallels next B.

VVell by this time you ſee there is more wayes to the woode then one, more wayes to finde the centres then by the rule of three prickes, which in truth is the verie worſt. In that Gemma Friſius neuer mentioned any thing of theſe wayes might be becauſe he was no mechanicall workeman and practiſer herein.

Theſe be pleaſant and eaſie wayes to finde the centres by, for ſmall worke: but for great worke painefull ynough: for which the deuiſe of three rules with ſinguler help ſupplieth your deſire. Theſe two Chapters yeelde more notable fruite then can be conceiued at the firſt vewe, as (if any man endeuour to proiect the globe into a plaine any way) he ſhall quickly finde: and yet ſhall my 7. booke much more. Note that theſe whole circles of the parallels being drawne rounde as they be in this figure do repreſent the verie almicantares of Stophlers Aſtrolabe, though not for his latitude.



Chapter 4.

How to make the Reete *with all his lineaments.*

NOw at the laſt the *Mater* of this *Iewel* finiſhed, it followeth that I ſhew the making of the *Reete* after my inuention ſo largely promiſed, which doth ſo throughly furniſh it, that where before this ſelfeſame *Mater* with ſuch tedious tackling as Gemma Friſ. had appointed him, was verie troubleſome & vntoward almoſt to euery action, lame as it were & like a nowne adiectiue could not ſtand of it ſelfe without Stophlers *Reete*, & his general horizons on the backſide: now it may be pronounced the only mathematical inſtrument of the world exactly, ſpeedily, & plainly performing innumerable ſinguler concluſions to wonderfull purpoſes without helpe of any other, and all onely by meane of this new *reete*. VVherefore to returne to my purpoſe, you muſt get a verie thinne plate the thinner the better, if it hold the

working

working whereon make the circle A B C D very iuſt in quantitie with the innermoſt circle of the *Maters Limbe* and marked with the ſame letters in like ſort, and within it make another circle to conteine a very narrow ſpace, for the 360. degrees compaſsing in the *Reete* which I will call the *Reetes Limbe*, being ſo diuided that euery 5. degree be ſome way noted from the reſt, and muſt in ſmall worke needs be ſo narrow that it wil beare no figures, for it is made more for neceſsitie to ſtrengthen the *Reete* then to any vſe except it be for one quarter to write in the numbers for the almicantares.

VVel, to proceede farther with our *Reete*, diuide the two diametres A B, and C D: ech into 180. parts, if it may be poſsible, euen as you diuided the ſame diametres A B and C D in the *mater*, being both of very iuſt length one to another. Then in the diametre A B from the point A. recon the ſunnes greateſt declination vz. $23\frac{1}{2}$ degrees, among the diuiſions, and there ſet E. and likewiſe from the point B. there ſet F. Likewiſe from the centre towards A, number $23\frac{1}{2}$ degrees there ſet G. and thence towards B as much, there ſet H. as in the great figure you ſee: which done, you ſhall by helpe of the 4. Chapter or 15 Concluſion drawe the arches C F D, and C E D. and within either of thoſe 2. other arches of ſufficient width to make a *Limbe* for our zodiacke, therein to write the numbers and carecters, or names of the ſignes, yet as ſcant as may be, eſpecially the *Limbe* of C E D, which cannot be too narrow: Then ſhall you diuide or graduate the one halfe of our zodiacke vz. C F D into 180 parts or degrees by keeping the rule on the point E, & thence laying him from deg. to degree of the ſemicircle of the *Limbe* C B D. euen as you diuided the diametre C D, of the *mater* from the point A in the 1. Chapter. Likewiſe diuide the other halfe of our zodiacke vz. C E D into 180. parts by laying the rule from the point H on euery of the degrees of the ſemicircle C A D. Reade more hereof in my 4. Booke 4. Chapt. Let euery 30. diuiſion or degrees croſſe the whole *Limbe* of the zodiacke, and euery 10. degree or 5. if you can come halfe way, the ſecond ſpace, which done, then beginning at C number towards E, and ſo round about both the *Limbes* by thirties in manner of the ſignes. At C is the beginning of Aries as in the great figure of the *reete* you ſee written: at D of ♎, at F. of Capricorne the reſt in order: therefore in the bigger halfe zodiacke C F D. you may write their names at large, and in the leſſer C E D, onely the carecters, if your inſtrument be ſmall, and ſo is the zodiacke of your *reete* made to finguler purpoſe. But looke in any wiſe that with great diligence and care you get the 4. points E G H F, moſt exactly and heedfully draw and diuide the arches C F D. and C E D, & that their common diametre or chorde C D be full as long and no whit longer then the diametre C D. or A B of the *mater*: or elſe all the reſt of your labour will be loſt. The premiſes performed, you ſhall nowe endeuour to make the azimuthes and almicantares: which to be ſhort is no more but to diſcribe halfe the meridians and parallels of the *mater* cut off in the diametre C D by the 1. Chap. yet on this condition that the ſpaces in ſmal worke may be 2. and 4. that is to ſay, let the firſt ſpace of the almicantares from C D be foure degrees wide, the ſecond 2. deg. wide, the thirde 4. againe the fourth 2. the 5. 4. and ſo to the toppe or zenith. But the azimuthes let them bee by two a piece, like as the meridians are. The cauſe hereof is for that the bigger ſpace muſt be cut foorth and made open as in the 11. Chapter ſhall be ſhewed: In great worke theſe ſpaces would be 1. and 2. or 1 and $\frac{1}{2}$ for euer the more circles the better.

Chapter 7.

How the zodiacke of this reete may be diuided diuerſe other wayes.

I Had thought heere to haue ſpent a long diſcourſe in ſhewing ſundrie wayes to diuide the *reete* by: which giueth the more credite to the proofe of any experiment when the ſame is to be done diuerſe wiſe, and ſomwhat it would haue made any man the redier in proiection of the ſphere for diuerſe purpoſes. But becauſe time is very ſcant with me and that I deſire an end I will ſleightly paſſe them, the rather for that amongſt them all the reddieſt and eaſieſt way is alreadie performed in the laſt Chapter. One way had beene with helpe of a table of right aſcentions, thereby laying the rule in the *Limbe* to the degrees of right aſcention anſwering euery 5. deg. of the zodiacke, ſo you may diuide him into 72. fiues, and euery of thoſe parted into fiue equall parts maketh 360. which by the 3. Booke and 11. Chapter you may well gather. Another way may the zodiacke be diuided, if you place him on the *mater*, ſo that the line C D of the *reete* lye euen in the line A B of the *mater*, So doe the parallels diuide him into 360. degrees: or if you liſt to go to the coſt you might make blind parallels croſsing C D. of purpoſe to diuide him by: this may you conceiue by the 4. Booke 4. Chapt. Another way alſo is by circles of longitude iſſuing from the poles of the zodiacke G H. like to thoſe which Stophler maketh his azimuthes, or thoſe which are drawne in the figure of my Booke. See what helpe here is where no neede is. But ſince it is manifeſted in my 4. Booke 4. Chapt. that the way of the laſt Chapter is moſt exact, ſlet vs keepe to that becauſe it is eaſieſt.

Chapter 8.

How to place the fixed ſtarres in the Reete.

IT is moſt neceſſary, commodious and pleaſant, to haue as many of the chiefeſt fixed ſtarres in your *reete*, as may conueniently be put in, without ouer much hurt to the other worke, and hinderance of ſeing the lineaments of the *mater* ynderneath. To performe this requireth 4. ſpeciall matters to be foreknowne, either by exact tables, or diligent obſeruations as ſhall be taught hereafter: that is to ſay, the mediation or culmination of the ſtarres, their declination from the Aequinoctiall: whither, North or South declining, and of what light or magnitude the ſtars are. The culmination of a ſtar differeth little from his right aſcention, the one is but the degree of the zodiack, the other the deg. of the equinoctial which commeth to the meridian or any right horizon with the ſtar. Thoſe 4. firſt knowne either by obſeruation as I will ſhewe in the 3. Booke and 39. Chapter: or by the table following which I haue taken out of Stadius a diligent writer, worke thus: Lay the rule from the centre of the *reete* to the degree of Culmination of the ſtar found in the table reconed on the zodiack, and draw there an obſcure line, then ſeeke in the table his declination, & number the ſame in the diuiſions of one of the lines A B, or C D. from the *Limbe* inwards, and then ſet one foote of your compaſſe in the centre, & extend the other to the ſame diuiſion, then turning the moueable foote about therewith croſſe the obſcure line, at this croſsing is the place of the ſtar deſired, or elſe becauſe the plate perhaps may haue too large a hole for the true centre you may take on one of the lines A B, or C D. the ſaide declination and ſetting one foote of the compaſſe where the obſcure line cutteth the *reetes limbe* or circle A B C D, with the other croſſe the obſcure line inwards towards the centre, it ſhall light in the former croſsing. VVherefore there ſhall you forme or faſhion the ſhape of a ſtarre with a point or apex ſomwhat farther out then the reſt, touching onely the preciſe pricke of this croſsing. And if this ſtarre be a North declining

clining ſtarre, then let his apex point outwards from the centre directly towards the *Limbe*, if South declining then inwards towards the centre. For example take the great ſtarre one of the firſt light called Hircus, the Gote, whoſe culmination by the table following I finde to be in the 13. deg. 0. minute of ♊, or his right aſcention 71. deg. 33. mynutes of the equinoctial, both which one line in my *Reete* laide from the centre, doth ſhew at once, if the point D of the *Reete* be laide to the point A of the *Mater*. But here his culmination ſhall ſerue, though from D, I might count his aſcention: therefore laying a rule on the centre and the 13. deg. 0. minutes of ♊ in my zodiacke I draw an obſcure line: then hauing likewiſe found in the ſaid table the declination of Hircus 45. deg. 5. myn. I ſet my compaſſe one foote in the centre, and open the other vnto 45. degrees 5. minutes counted in the line A B, or C D from the *Limbe*, and therewith turning about the compaſſe doe croſſe the obſcure line with a ſmall arch, then do I make by this croſsing the ſhape of a ſtarre with his apex touching the ſame point, and directly pointing outwards, becauſe in the table following I finde his declination North, thereby writing this name Hircus, as in the figure of the *reete* you may ſee. And yet can I helpe you to a neere way by helpe of your *Label* made as in the 12. Chapt. and faſtned in the centre of the *reete*. Lay the *Label* on the ſtarres degree of Culmination, and by the degrees of the *Label* number, his declination inwards from the *Limbe*, and there ſhall be the ſtarres place in the *reete*. Example, I will take another great ſtarre of the firſt light called Foma hand, or *Fuſio aquarii*, I finde by the table following his Culmination the 6. deg. 46. minutes of ♓: vpon which degree of my zodiacke, I lay the *Label* faſtned in the centre as I ſaide, then on the *Label* from the *Limbe* inwards, I number his declination, which by the table I finde 33. deg. 33. minutes, and at the end thereof I make a pricke, which is the place or point of the center of the ſtarre called *Foma hand* deſired, whereto I ſet the ſhape of a ſtarre prouiding that the apex point inwards towards the centre, becauſe I find in the table that his declination is ſouth from the equinoctial, as in the ſaid figure of the *reete*, you may alſo ſee. But heere I would haue you to note how wonderfully this *reete* excelleth Stophlers *reete*, which Gemma Friſius allowed of: for that here you may put in all the ſtars from pole to pole where in Stophlers *reete* you cannot poſsiblie doe it: neither can you put in any one ſtarre there that declineth Southwards aboue $23\frac{1}{2}$ degrees from the equinoctiall, except you will make the inſtrument vnprofitablie huge, for that purpoſe onely. As for example, the laſt ſtarre called Foma hand declineth 33. deg. 33. minutes ſouthward, which cannot be put in Stophlers *reete*, except the inſtrument be made larger by one quarter of the diametre of purpoſe, which will ſerue almoſt to no other vſe, and yet then can you not put in all by a number: but now againe to my purpoſe, when you haue in this manner as is before taught, put in as many ſtarres as you may, which to the number of 30. or. 40. in a ſmall inſtrument may well be, then muſt you linke them togither with ſmall branches handſomly, wherein their names muſt be written, but all in as ſmall roome as may be. And as for thoſe ſtarres which light among the almicantares, if you cannot there write their names, then muſt you ſupplie that want by writing them on the *Label*. Alſo for their bigneſſe of light or magnitude, you muſt either file them of three or foure faſhions which you ſhal hardly do in ſmal worke, or with the grauer, make them ſeueral ſhapes, or elſe you muſt deuiſe to ſet ſo many pricks by euery one as may ſignifie the bigneſſe of their light, either of the 1. 2. 3. or 4. magnitude, which you neede not exceede though they are reckoned to the ſixt. Note that in my great figure I haue ſet to euery ſtar a number, according to his being in the table following, whereby you may there eaſily finde his name.

Chapter 9.

How to finiſh and cut out the Reete *of this Iewel.*

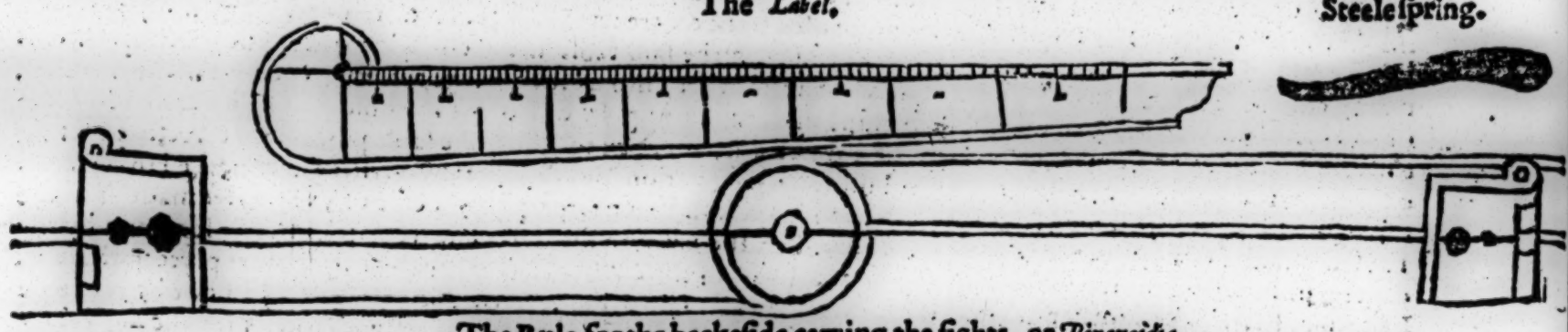
HAuing drawne all the lineaments exactly, and placed the points of ſo many ſtarres as you thinke good, as before is ſhewed: then muſt you graue or write in all the figures names, and carecters, of the circles, ſtarres, and ſignes: alſo the branches that ſhall ſupport your ſtarres with their ſeuerall ſhapes and that as exactly, as though they ſhould not bee filed or cut out: but in any caſe you muſt not ſtampe any figure, letter, or thing in a thinne plate: one blow may bring the whole out of frame. In any wiſe deſire not to make your *reete* of any thickneſſe, for in vſe it will be very vnpleaſant in many reſpects, the thicknes of a grote will ſerue very wel. VVhen you haue performed al this, then muſt you with fine files (or being in paſtbord, with a fine penknife) cut out al the ſuperfluous roomes of the *reete*, and euery greater ſpace of the almicantares, leauing nothing but the very zodiacke, the ſtarres and their braunches whereon their names are written, and euery leſſer ſpace of the almicantares, as in my great figure you may well perceiue. You ſhall finde it a very harde piece of worke to file out theſe things in mettall: eſpecially the almicantares, and to leaue the ſtarres there placed: and if you be not verie carefull, you may ſoone ſpill all: if you can get no file thinne ynough, you may grinde a file on one or two ſides till he come to your purpoſe. But to be the more ſure to keepe the almicantares from diſplacing, which is all my care, you ſhall leaue heere and there ſuperfluous ſtrings very ſlender in filing euery almicantare, which you ſhall not file away till you haue all done: and if you liſt at euery 10. 15. or 20. azimuthes, ſmall ſtrings quite through, as in the 4. Booke 1. Chapt. is ſaid, they will not onely keepe more ſure the almicantares at their true width, but alſo be a great eaſe in vſe of the *Iewel* for ſpeedie reckning of the azimuthes. And if there chance any ſtarres in your way, be chary to faſhion them as well as the roome will giue leaue: and alwayes be ſure to keepe your drilles, grauers, and all your tooles verie keene and ſharpe by often whetting, elſe ſhall you both worke tediouſly and hardlie keepe your worke from forcing in thicke plates, ſuch as the *Mater* muſt be made of: you may be ſomwhat bolder to ſtampe ſmall figures, letters and ſmall holes, and to peirch, but chiſill you muſt not, nor ſtrike with great pounches: I had forgotten it in the 8. Chapter, you muſt leaue a rule bredth vnder the line C D of the *reete* wherein to graue the numbers from 10. to 360. beginning at the centre and thence to C, there ſet 90. from C. to the centre backe againe, ſet 180. from the centre to D. 270. from D backe to the centre end in 360.

Chapter 10.

How to make the Label *for the* Mater *and the rule to the backeſide with ſightes.*

THere is neither of theſe two harde to make, for they are common to euery inſtrument, and may be better perceiued by theſe figures then by many words. The *Label* is but a rule in length equall to the ſemidiametre

tre of the *Iewels Limbe*, containing halfe the diuiſions of the line C D. whoſe loope about the centre hole muſt be ſlender as may be, and therefore I would haue you get a little ſpring of ſteele in length halfe the *Label*, to riuet to the centre next aboue it, which ſhall not onely keepe downe the *Label*, to ride about continually with the *Reete* which is ſomewhat needefull, but alſo it ſhall much ſtrengthen the loope at the riuet: let the *Label* be numbred



The Rule for the backeſide carying the ſights, or *Pinancida*.

from the centre to 90. at the *Limbe* and backe againe to 90. if you will. Then for the rule of the backeſide, let him be equall to the whole diametre of the backe of the *Iewel*, bearing two folding ſightes, ech hauing a great hole for ſtarres and a little hole for the ſunne beames in manner and faſhion of common Aſtrolabes: all which by theſe figures you may ſufficiently ſee. If you would be preciſe this rule ſhoulde be ſomwhat thinne: leaſt the verie weight of him ſhoulde cauſe your *Iewel* to hang awrie in taking altitudes, as we commonly ſee in the beame of a ballance, if the one end once get a little higher then the other, it then waigheth the other quite downe: which alſo *Cardanus* confirmeth in his Booke *De ſubtilitate rerum*, but then if you make your rule thinne you had neede ſet on a narrow *Limbe* on the backſide for the rule to ride vnder, leaſt it ſhould elſe take harme, which will alſo helpe well to giue the *Iewel* weight.

Chapter 11.

How to furniſh the backe of the Iewel *with the Theoricke of the ſunne and other neceſſaries.*

THis inſtrument, nay rather this *Iewel* doth alredie ſo fully ſupplie all the multitude of tables, which Stophler appointed like a box of trenchers iumbled togither within the *maters Limbe*, & all other knicke knacks, which G. Friſius would needes haue to be annexed and pertinent to his *Catholicon*, and generally all other inuentions for Aſtrolabes whatſoeuer, with much more then euer any of them coulde performe, yea or almoſt the globe or ſphere it ſelfe: that in manner it leaueth nothing to place on the backſide but onely the Theoricke of the ſunnes motion, with the kalender, and that for faſhions ſake more then any neceſsitie, if a man had alwayes an Almanach or *Ephemeris* about him. But becauſe thoſe are not alwayes at hande: and that ſuch hath beene euer the vſe in all Aſtrolabes: it is not a miſſe, not to digreſſe from the olde faſhion. And therefore thus ſhall you do. Make two *Limbes* ioyning togither the one for the zodiacke or circle of ſignes: the other for the kalender or circle of monethes. Let the zodiackes *limbe* containe 3. ſpaces, though in my figure I haue made a ſhift with 2. the 2. outermoſt ſpaces of like bigneſſe, one to receiue the names or carecters of the ſignes, the other their numbers from 30. to 30. which in my figure are both in one, the third ſpace to be narrow for the deg. being in the whole 360. This *limbe* being thus diſtinguiſhed and diuided by helpe of the 20. Concluſion croſſe, the whole with two diametres ſquare into foure quarters, but ſo that the one diuide the middle of the handle, where you ſhall beginne ♈ at the other end ♎, at the other diametre leftwards ♑ to beginne, and rightwards ♋. All which performed, draw a line from the centre to the 2. degree of ♎, deuide the ſame into 25. parts, and in the neareſt of them to the centre, make a ſmall centre hole placing it on the centre of ſome great circle redie diuided into 365. & $\frac{1}{4}$ equall parts by the ſaide 1. Booke 20. Concuſion, ſo that the 21. deg. of ♈ in your circle of ſignes lie, on the beginning of the ſame $265\frac{1}{4}$ diuiſions in your bigger circle, and ſo by helpe of your diuiding rule placed thereon, diuide the *limbe* of monethes in your *Iewel* into $365\frac{1}{4}$ parts or dayes from this excentricke centre, which *limbe* muſt be firſt drawne and ſeuered into 4. ſpaces, though in my figure I haue made but three. The firſt and outermoſt ſhall ſerue for the names of the monethes, the ſecond to ſet in the figures or numbers of the dayes to euerie moneth 10. 20. 30. or 31. as the moneth requireth, the third for the $365\frac{1}{4}$ equall diuiſions of the dayes: the fourth very large for to ſet in the feaſts or ſaints names.

Ianuarie muſt beginne at the ſaid 21. deg. of ♈, the reſt in order, all which I haue contriued in the circuler ſpaces without the circle A B C D of the great figure of the *reete* being loth to make a ſtampe of purpoſe, and therefore marke it well. If your *Iewel* were large ynough, then were it good to write the weeke letters A B C D E F G, in or to euery of the $365\frac{1}{4}$ diuiſions as they are in the Almanacke, and as in the ſame figure I haue begunne at Ianuary. And when you haue thus performed the theoricke with the kalender, yet will there be a great roome left: wherein they are wont to make the Geometricall quadrat, and the planetarie houres: but to how ſmall vſe both theſe are in little inſtruments, let them iudge that haue had of them. And as for that kinde of planetarie houres, the making of them being ſomwhat comberſome, the vſe of them troubleſome, and their beſt vncertaine by reaſon of the ſtricknes of the houre circles about 12. and the tropicke of ♈, I am therefore out of loue with them, and ſpecially for ſmall inſtruments, and rather becauſe ſo troubleſome a thing ſerueth but to one purpoſe, and to one latitude, and the ſame purpoſe is verie wel to be performed on the *Iewel*, as in the third booke is ſhewed. VVherefore the premiſes conſidered, if you will be ruled by my direction, I would haue you furniſh vp the backe of your *Iewel* one of theſe three wayes. The one is if you will take a little paines to make 6. circles one within another, conteining ſixe equall ſpaces: And in the three outermoſt of them to appoint two ſeuerall tables, one for to knowe the Dominicall letter for euerie yeare, the other for to finde the Prime and Epact. In the three innermoſt

ſpaces

spaces to appoint a table whereby to know on what day of the moneth Easter day shall fal for euer, in such sort, as in this table is expressed. A second way, is necessarie for him that studieth Astrologie, to place in this voide roome the three rounde tables which Claudius Dariotus in his book intituled: *Introductio ad astrorum iudicia*, doth shew if you can deuise to packe them one within another, as I haue done the other three tables, or at least one or two of the most necessarie. That booke is now in englisth: wherefore you may haue those tables printed for little. A third way is the best of all, and most necessarie for the traueiler, and pleasant to all sortes, and in especiall for any Gentleman, to place in this vacant roome the better halfe of the worlde in plaine, that it to say, from the north pole to the south tropicke, which in diuerse authors you shall see, as in Apians Cosmographie, who calleth it there, his Cosmographical glasse: some wil think the roome too little, indeede you had need in smal instruments make the circle of monethes narrow, by the space wherein the saints names are written, leauing them out quite, and then no doubt it may serue well, for I haue seene contriued the Cosmographicall description of the one halfe of the world in a round tablet, vnder 3. ynches diametre, and most curiouslie grauen in many notable cities and places in euery countrie, with their names ascribed very sensiblie.

Chapter 12.

How to set togither all this worke of the Iewel.

HAuing now performed all the seuerall parts of this *Iewel*, except you will place starres in the *mater*, by the 3. Booke 17. Chapt. which I thinke needeless, the handle onely excepted, which is an ordinary thing to make, consisting of two or three pliant ioynts, or Iymmoes, so ordered and appointed, that one of the diametres on the backside may hange very perpendiculer from the zenith alwayes in taking altitudes, as by certaine figures before you may partly see: I thinke there is now no man of any sense, but can set them togither: yet I thought good to note thus much by the way. First for the centre holes of the *Mater*, *reete*, rule and *Label*, let them be all of one euen bignesse, and as small as is possible to beare a riuet. For it is a foule fault to haue a great many degrees at the centre, taken vp with the riuette and the loopes. Secondly it would be so contriued if the workman be cunning, that part of the *reetes Limbe* might ride vnder the *limbe* of the *mater*, which for that cause would be set vpon the *mater*, in manner of the old Astrolabes: or if one did but hammer out 4. or 5. tets out of the *reetes Limb*, in euery quart. one, it shal suffice ynough to ride vnderneth which woulde be a great preseruatiue to your *reete*, least euery little strain of any thing that shuld get vnder it do lift him vp, shuld straight breake him off at the loope, or centre hole, being so thin as indeede he must be. I haue had experience to my cost: and thus my gentle *Iewel* I wish thee well to fare.

Here followeth the inuention, definition, and explanation of the *Iewel*, and of euery circle, part and thing therein conteined.

Chapter 13

VVhat moued the author to his inuention of this Iewel, *and whence he obtained it.*

AFter I had no small time traueiled as far forth as my leisure wold license me, in the most part of the Mathematicks, namely, in Geometry, Arithmetick, prospectiue Cosmography, Topographie, mensurations & such like, I think by the space of 8. yeares or more, and then at the last being drawne on by the studie of Cosmograpy, to the desire of Astronomy (which 2. cānot wel be in sunder) I thought it my best course about fixe yeares past, to craue the conference of one Thomas VVhite, Vicar and curate of Berkham being fiue miles from Reading, of whom I heard verie well of for his studie that way. I went to him, I founde him no lesse ioyfull of my comming, then I was carefull of his acquaintance, but his studie that way was altogither Astrologie, Phisicke, &c. where my desire was Astronomie, so that my hope of his helpe was almost on the sodaine frustrate: but there I founde such choise of Bookes for my purpose, that I repented not my comming, where he on the other side hauing heard greater speach then was cause perhaps of me, and my practise, in painting, drawing, grauing, and such like, would needes craue my helpe to make him a Catholicon Astrolabe, and to that ende he deliuered me Sconer, Stophler, Roiias, and Gemma Frisius Bookes, of their seuerall Astrolabes: but aboue all other he commended vnto me, as there was cause, Gemma Frisius.

Yeare of our Lord.	1582	83	84	85	86	87	88	89	1590	91	92	93	94	95	96	97	98	99	1600	1601	2	3	4	5	6	7	8	9
Dominicall letter for leape yeare.	E						G				B				D				F				A				C	
Dominicall letter.	g	f	d	c	b	a	f	e	d	c	a	g	f	e	c	b	a	g	e	d	c	b	g	f	e	d	b	a
Yeare of our Lord.	1577	78	79	1680	81	82	83	84	85	86	87	88	89	1590	91	92	93	94	95									
Prime.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19									
Epact.	11	22	3	14	25	6	17	28	9	20	7	23	4	15	26	7	18	29										
Prime.	16	0	0	13	2	0	0	18	7	0	0	0	15	4	0	12	1	0	9	0	17	6	0	14	3	0	11	0	19	8	0	0
Dom.let.	d	e	f	g	a	b	c	d	e	f	o	o	g	a	b	c	d	e	f	o	a	b	c	d	e	f	g	a	b	c	d	e
March	22	23	24	25	26	27	28	29	30	31	April.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21

Notwithſtanding becauſe I would be acquainted with all their inuentions, I peruſed them all, and founde that Stoners deuiſe was very ſtale,& the making comberſome & vncertaine by reaſon of the diffuſe traduction thereof. Stophler his was artificall, and in truth that which hath giuen light to all the reſt, yet requiring ſo many tables as there be diuerſities of the poles eleuations in the world,or at leaſt ſo many plates as you require countries: For the aſtrolabe it ſelfe ſerueth but to one latitude which to a traueller into diuerſe countries were both chargeable and comberſome:& to a ſtudent not altogether pleaſant conſidering he muſt take out & put in ſtil thoſe tables that are required. Then I come to Roijas and Gemma Friſius,which in effect haue al one ſcope,though Roijas be farre inferior to the other, becauſe his parallels be ſtreight lines, and his meridians not arches but rather certaine geometricall crooked lines called Ellipſes which come *ex ſectione piramidũ*,and thoſe parallels towards ech pole & the meridians towards the *Limb* grow ſo neare togither that one drownes,another which is a very great fault, whereas G. Friſius his parallels and meridians haue conuenient ſpaces, and are all of them true arches of circles, and in effect ſuch and no other then the circles in Stophler be, which he calleth almicantares and generall horizons,and therefore muſt needs be more proportionable to the circles of the ſphere *nam rectilineæ ad curuam nulla eſt affinitas*. Notwithſtanding al this, though Roiias and Gemma Friſius were ſinguler inuentions being generall to all the worlde, yet both of them were ſo comberſome in my fancie that I could not well brooke them: For beſides that, that they required one or two plates within the *mater* to be taken in and out,and alſo Stophlers moouer called the *reete*, to helpe him with his generall horizons, the cunning not little to make their *regula curſor & Brachiolum* exactly, with the ſcrew pinnes ioyntes and ſuch like,yet when all was done,the working of concluſions by them ſo tedious with often aſſaies and ſo vncertaine for ſlipping of any ioynt, or iogge, that without many profers and twiſe performance one could not be aſſured of any propoſition. All theſe when I had read and well conſidered I courſed many a ſundrie deuiſe for amendment thereof: at laſt though it were long firſt, it pleaſed God to ſuffer me to attaine vnto the very perfection of the thing I ſhot at, euen to perfect one plate and one moouer to ſerue to all purpoſes redily without profers,which I haue at the laſt done: but not without participating ſome of euery of the aſtrolabes before recited,ſo that my worke might iuſtly not vnlike to the firſt columpne or piller in architecture added by the Romans, be called *Aſtrolabium compoſitum* if it deſerued not a more ample name as the ſequele ſhall manifeſt.

Chapter 14.

Of the definition of the Iewel *and his principall parts.*

TVllie about the beginning of his firſt booke of offices, ſaith that euerie thing that any man will entreat of, ought firſt to be begunne from the definition: this inſtrument or *Iewel* therefore(if any man will aske what it is) is a certaine round ſuperficies or flat forme repreſenting the ſolide globe and ſphere, and euery circle in them contained or imagined,being ſo proportionablie, congruently,and artificially by proſpectiue lineaments protracted,that it performeth the whole vſe of the globe and ſphere in euery reſpect whatſoeuer,and rather more then leſſe and in effect by the ſelfeſame methode. This inſtrument or any ſuch like: ſome writers haue called *Planiſpherium* a plaine ſphere, but the moſt part *Aſtrolabum* or *Aſtrolabium* an aſtrolabe: the firſt ſpringeth of the traduction or deriuation of the thing it ſelfe,ſounding as it were *Sphera in plano* a ſphere or globe in plaine forme. The ſecond commeth of the qualitie or vſe called, *Aſtrolabium quia monſtrat lapſus aſtrorum*, bicauſe it ſheweth the gliding or courſe of the ſtarres. And in aſmuch as the vſe of the inſtrument is more often called in queſtion and practiſed then the traduction and deriuation, therefore the name aſtrolabe is more common then plaine ſphere. But I finding the vſe of this inſtrument ſo infinite and vnſpeakeable, haue not without cauſe named it the Mathematicall *Iewel*. To proceede further, whereas all other Aſtrolabes haue many,this onely hath but ſeuen principall parts. The firſt is the ring and ioynt whereby it hangeth called in latin *Suſpenſorium* the hangle becauſe it hangeth thereby, or *Anſa* the handle becauſe men commonly take and holde it thereby. The ſecond is the round brim that encloſeth the worke of the *mater* called in Latin *Limbus*, the *Limbe*, which is as much to ſay as the brimme, garde, or creſt of a thing. The third is the worke within the *Limbe* which I will call by the latin word *mater*(the mother) as well becauſe it containeth the whole worke of the ſphere within it, as alſo for that it is encloſed in with the *Limbe*. The fourth is the moouer on the *mater* called in latin diuerſe wayes *Voluellum* the mooucr becauſe it performeth his office by motion,alſo *Aranea*, the web,becauſe it is cut out ſomwhat like the ſpiders cobweb: laſtly, *rete* the nette,becauſe it is like a nette,but I will call it according to the latin *rete*, or rather the *reete* more engliſh like. The fift is the *Label*, called in latin *Index* or *oſtenſorium*, the ſhewer or pointer of ſome *Almuri*. The fixt is the backe of the Aſtrolabe, called *Dorſum aſtrolabii* for the moſt part,which I ſhall call the theoricke, becauſe the chiefeſt matter there,in common vſe,is the theoricke of the Sunne. The ſeuenth is the rule of the backſide, carying the ſightes called of ſome *Alhidada*.

Chapter 15.

Of the diuiſion or two ſeuerall diſtinctions of this Iewel.

THis *Iewel*,becauſe it is a thing gathered and compounded of all the writers of Aſtrolabes that are commonly extant,as I haue before ſaid may be likened vnto the worke of Bees in that, that out of the ſweete ſubſtance gathered of the freſh flowers,is made both the Hony and Honie combe, the one to ſupport the other, and very like in deede to a *Iewel* that is made of precious ſtones and golde one to hold in another. Euen ſo in this my *Iewel* there are two ſundrie inſtruments ſo vnited and compact that the one defaceth not the other, but of neceſsitie the one helpeth the other and in trueth without the one the other cannot be general. And euen as the Honie is moiſt and the combe ſtiffe, the golde will be forged, but the precious ſtones will not melt: ſo the one parte of this inſtrument performeth all his concluſions fixed, the other by continuall motion about the centre. I would not haue you take my meaning that the *mater* ſhoulde be one inſtrument and the *reete* another ſeuerallie, but they are ioyntlie two inſtruments for part of the *reete*, and the *Limbe* and all the circuler lines in the *mater* ſerue to the one

one, and but part of the lineaments in the *Mater* and *Limbe* and part in the *reete* ſerue to the other, as heereafter followeth.

Chapter 16,

Of the diſpoſitions of the circles both in the Mater, Reete, *and* Limbe, *for the firſt and chiefeſt diſtinction of this inſtrument.*

COncerning the two diſtinctions of this *Iewel* mentioned in the laſt Chapt. the firſt and chiefeſt is in effect agreeable to Roijas and Gemma Friſius inuention, and ſeruing to ſo many the vſes of their Aſtrolabes as are performed without Stophlers *reete*, and to innumerable other, as I will manifeſt, requiring part of my *reete*, part of the *Limbe* & all the *Mater* to ſerue his turne. Firſt therefore for this vſe the *Limbe*, firſt two ſpaces within the firſt great blacke circle is deuided into 4. times 90. degrees numbred from the quinctiall line to ech pole. All the lineaments of the *mater* are required to this diſtinction, which euery man may eaſily of himſelfe diſcipher, that hath any knowledge at all, or hath redde my firſt booke. For the circles meeting togither all at two points which are called poles, are meridians or houre circles, the other ſort croſsing them ſquare as it were growing leſſe and leſſe from the diametre or equinoctiall line C D, till they come to ech pole, are called parallels. The circles or lineaments in the *reete* ſeruing this, are called Almicantares and azimuthes: the almicantares called in latin *Circuli altitudinum* circles of height, are thoſe which are drawne in ech reſpect like the parallels in the *mater*, and are cut out like the ſpiders webbe. The Azimuthes called in latin, *Circuli verticales* verticall circles, are thoſe which are drawne cutting all the almicantares meeting altogither at one point, where the almicantares growe to nothing: which point I call the zenith point of the *reete*, and is marked with A. theſe Azimuthes are drawne no otherwiſe then the meridians in the *mater*, but that they are cut off in halfe: The firſt Almicantare is the ſtreight line croſsing the centre of the *reete* vz. C D. and from thence are the reſt numbred to 90. ending at A the zenith point: this ſtreight line or firſt Almicantare C D, is in truth the horizon or *Finitor*, & is to be ſet fixed to any latitude, but for diſtinction ſake I will henceforth call this horizon line the *Finitor*, or *Finitor* line, becauſe I muſt call other circles in the next Chapter horizons, yet in plaine engliſh they both are one. The rule bredth in the *reete* wherein in trueth are writen the deg. of the horizon, and therefore might altogither with the *Finitor* line be called the horizon or *Finitor*, yet in reſpect it muſt ſerue the ſteede of a rule verie commonly, I will call it the rule at many times.

Chapter 17.

The circles ſeruing the firſt and chiefeſt diſtinction knowne by the laſt Chapt. how to place them to the latitude of your countrie, where it ſhall reſt fixed to all concluſions, a few excepted euen in manner of the globe or ſpheere with a farther expoſition of the lineaments of the Iewel *ſo placed.*

YOu ſhall vnderſtande that in thoſe parts of the world, where the two poles be leuell with the horizon and neither of them eleuated, they are ſaid to haue horizon *rectus*, a right horizon, which is where the quinoctiall is zenith: wherefore place your *Finitor* line ſo, that he be euen with the axtree line, and you ſhall ſee the zenith to be in the quinoctiall line: ſo is your *Iewel* now ſet to the right horizon, and there being, performeth all or moſt of the concluſions of any right horizon whatſoeuer. But now to my purpoſe where one of the poles and axtree lines are eleuated any whit, there is it called horizon *Obliquus*, and looke how many degrees the pole is eleuated, ſo many degrees muſt you recken from the pole of your *Iewel* leftwards one of the *Limbe*, and thereto ſet the *Finitor* line or firſt Almicantare, and ſo is your *Iewel* ſet to the latitude, euen as the globe it ſelfe is ſet, and by the ſame reaſon. For example, I would ſet my *Iewel* to the latitude heere at Reading, either I get the height of the pole inſtrumentally as hereafter ſhalbe ſhewed, or out of ſome Coſmographers tables I finde it to be $52.\frac{2}{3}$ degrees. The pole on the left hand of my *Iewel* marked with A, is the north pole, from him I reckon downewards $52.\frac{2}{3}$ degrees on the left hande alſo, and thereto I place the *Finitor* line, ſo that the zenith be vpwards towards the handle: and let the *Iewel* remaine ſo fixed as it were, for there is he placed to our latitude and redie to reſolue any queſtion you will aske: there preſently may you ſee the ſunne riſing, the length of the day, the heightes of the ſunne, the houre, with a number of other matters, euen this booke full, *Vno momento*, as hereafter ſhall be ſhewed: therefore when I ſay henſe forwards place the *Finitor* to the latitude, remember this, that I meane place the *Finitor* line ſo many degrees vnder the pole, as the pole is eleuated in the region or place propoſed, although I might better bidde you place the zenith point of the *reete* ſo many degrees aboue the right ende of the quinoctiall, all commeth to one, but this is eaſier to vnderſtand. And now to proceede farther, the *Finitor* being thus placed vnto the latitude, and ſo remaining you ſhall vnderſtand that your *Iewel* ſymetrially repreſenteth the verie ſcituation of the globe or ſphere to the ſame latitude with all their circles and furniture. For to beginne with all, the *Finitor* line repreſents the horizon with the numbers thereof: the innermoſt circle of the *Limbe* is the meridian, repreſenting alſo the *Colurus ſolſtitiorum* and alwayes ſerueth the vſe of the meridian or noone line to euery latitude. The points in the ſame meridian, where all the meridians doe meete vz. A and B. are the poles that on the left hande of the handle vz. A. the pole artike or North B. the antartickе, the diametrall line lying betweene theſe poles videlicet A B, is the axtree line, and repreſenteth the *Colurus equinoctiorum* and the 6. of the clocke houreline to euery latitude, the line which cutteth the centre comming from the handle vz. C D, which croſſeth ſquare the axtree line A B, is the quinoctiall, the rule applyed thereto ſheweth his numbers and diuiſions; the circuler lines meeting at the poles A B are meridians and in vſe ſerue for houre lines, and circles of right aſcention, of which euery 15 is ſeparated with prickes to ſerue our common houres, the axtree line being 6. The circles of like width in the *mater* beginning from the quinoctiall C D waxing leſſe vnto ech pole, are called parallels or *Circuli parallelli*, and in vſe doe limit the declination of the ſunne, moone, planets, ſtarres, or any place or point of the heauens from the quinoctiall the $23.\frac{1}{2}$ parallel on ech

ſide

ſide thequinoctiall ſeuered, which prickes are the tropickes, and the 23. ½ parallel reconed from ech pole, are the pole circles: further the line drawne from tropicke to tropicke, cutting the centre and onely there touching the *Limbe*, is thecliptícke line: at the centre is the beginning of ♈ & ♎, the reſt in order accordingly, the rule applied to this eclipticke ſheweth the degrees, and euery 30. degrees make a ſigne. The axtree line being the 6. of the clocke line at morning and euening, as I ſaid, euery 15. meridian repreſents the other houre circles, the *Limbe* it ſelfe being 12. of clocke at noone and night, the reſt in like order, as they be figured. Then to come to the *reete* ſtanding ſtill in the ſame ſcituation, the almicantares are the circles of altitude, reconed from the *Finitor* as in the laſt Chapter: the azimuthes arè the verticall circles, the Eaſt azimuth paſsing from the centre directly to the zenith, I call the zenith line, which is alſo theaſt line, and firſt azimuth, and from him they are numbred towards the ſouth rounde to 360. Some haue called the Eaſt azimuth ſimplie the verticall circle. Now laſtly to make ſhort of all, the *Finitor* being ſtill at the ſaide latitude, this *Iewel* repreſenteth in all points the verie lineaments, that you ſhould ſee on a cleare plaine glaſſe placed perpendiculer, quite through the middeſt of the Globe in the plaine or flatte of the meridian circle, and the eye placed in the point where the Eaſt azimuth, and thequinoctiall do meete the horizon vz. in the centre. Or thus more plainely, if you had a Globe of chriſtall with all the meridians, parallels, azimuthes, almicantares, and the horizon drawne thereon, in the middeſt of which chriſtall globe, a plaine chriſtall glaſſe were placed, diuiding the globe in the plaine of the meridian into two hemiſpheres, and this done you did place your eye in the verie Eaſt, or VVeſt point of the horizon, and that you could drawe all the lineaments of the hemiſphere againſt you on the plaine glaſſe as your eye there ſeeth them, they ſhoulde be the verie ſame as the lineaments of this *Iewel* are.

Chapter 18.

Of the diſpoſitions of the circles both in the Mater, Reete, *and* Limbe, *for the ſecond diſtinction of the* Iewel.

THe ſecond diſtinction of this *Iewel* mentioned in the 15. Chapter, is in effect agreeable to the inuentions of Sconer, and Stophler, and ſupplieth the ſteede of Stophlers *reete*, and generall horizons, which Gemma Friſius would needs haue anexed to his Aſtrolabe, as an inſtrument by it ſelfe, but a great deale more effectual then it in many points, as hereafter wil appeare, though perhaps it ſeeme ſomewhat intricat at the firſt blemiſh, yet once in vſe, as eaſie as the other. For this ſecond diſtinction ſerueth the innermoſt ſpace of the *Limbe* and hath two diuiſions, the one beginning from the point A, and ſo leftwards rounde vnto 360. the other diuiſion is into twiſe 12. ſeruing to the ordinarie houses: the point A which in the firſt diſtinction was the North pole, is now 6. in the morning, anſt B there the South pole is nowe fixe at euening, the points C and D in the firſt diſtinction bounding the equinoctiall, are now 12. at noone and night, the reſt accordingly. Then to come to the lineaments of the *Mater* ſeruing this part, there is a great alteration, for the innermoſt circle of the *Limbe* which in the other part was the meridian, is heere thequinoctiall, the axtree line A B, is here the Eaſt line and repreſenteth the *Colurus equinoctiorum*, and thequinoctiall line C D is heere the meridian or noone line, and repreſenteth the *Colurus Solſtitiorum*, the centre repreſenteth the two poles of the world, and the meridians are here become generall horizons: the ſubſtance of the *Reete* ſeruing this part is but the zodiack and the fixed ſtars. The zodiack is diuided into ſignes and degrees, and the ſtarres haue their names aſcribed: and where, in the former part the *Reete* was to be fixed to any latitude: heere the *Reete* is euer mouing about the poles, being the centre in all his actions. And in few words if the plaine chriſtall glaſſe mentioned in the laſt Chapter did diuide the globe furniſhed with all neceſſarie lineaments in thequinoctials flatte, and your eye were placed in one of the poles, you ſhould ſee on the plaine glaſſe the verie lineaments of this part of the *Iewel*, rebounding from the vnder hemiſphere, only this except, that there would be two ecliptickes drawne on the chriſtall globe one croſsing another ſquare, if you woulde ſee both the halues thereof, as on my *Iewel* they are made.

Chapter 19.

A farther declaration concerning the ſecond part of this Iewel *for the zodiacke, generall horizons, and fixed ſtarres.*

I Declared before in the 17. Chapt. that the oblique horizons alter with euery degree of the poles eleuation more or leſſe, but now am to inſtruct you in two things for your better vnderſtãding, the one is, that there is accounted but 90. ſeuerall differences of horizons through the whole worlde (in reſpect of the latitude, becauſe when thequinoctiall is leuell with the horizon, then the pole being zenith can be but 90. degrees eleuated: and being there at the heigheſt muſt needs wax leſſe againe: you will ſay there be 90. to the North pole, and 90. to the ſouth, ſo there are yet in this part of the *Iewel* one 90. ſupplieth both. The ſecond thing I am to minde you of is, that as all great circles of the ſphere croſſe ech other iuſt in halfe, as in my 5. Booke ſhalbe ſhewed: ſo euerie horizon howſoeuer he lie in the world, yet deuideth he thequinoctial becauſe they are alwayes both great circles of the ſphere, into two equall parts, and the one halfe lieth on the North part, the other on the South part of thequinoctiall, and ſo likewiſe of the zodiacke being alſo a great circle. Further you ſee that in this *Iewel* there be but 180. meridians, or rather halfe meridians, euerie one of them being in proſpectiue a ſemicircle, euen the axtreeline A B it ſelfe. ſo that in trueth they are 180. ſemicircles which are 90. whole circles, and muſt be numbred from the axtree or rather here to be called the Eaſt line A B, on ech ſide to the *Limbe* ending there at 90. Then ſatiſfie your ſelfe thus that you haue by this meanes 90. whole circles called horizons euerich of them taken with his match equally diſtant on ech ſide from the axtree line A B, which line A B is a whole circle it ſelfe. Then muſt you take this of almes from proſpectiue, and looke for no demonſtration thereof at this time of me, except my leiſure were more then it is: for in trueth it is ſomwhat holpen by this maxim: *Oppoſitorum ſemper eadem eſt ratio*, otherwiſe it would proue but *rudiori minerua*. I ſay that thoſe halfe horizons which lie on the North ſide of the axtree line A B. that is to ſay, next the midnight point of the noone line, are the North parts of the horizons, that is to ſay, by reaſon of their interſection, lying

lying on the North ſide of thequinoctiall : which here as I ſaide is the innermoſt circle of the *limbe*: and the halfe horizons lying on the South part of A B next the handle of the *Iewel*, are the South parts of the ſame horizons: the centre of the *Iewel* being centre to them all : and you muſt imagine, that (they all croſsing thequinoctiall) the one ſort is on this ſide, the other beyond, and yet are foreſhortned by meanes of their erection and depreſsion from the eye proſpectiuewiſe : and ſo likewiſe for the zodiacke in the *reete*, the North halfe muſt be imagined to lie northwards, and the other halfe croſsing thequinoctiall to lie ſouthwards, and to be foreſhortned or thruſt togither as painters terme it by placing of the eye. Now by this time I hope you wel vnderſtand me how theſe 180. halfe meridians are become 90. whole horizons: I think you be not ignorant that theſe 6. ſignes ♈ ♉ ♊ ♋ ♌ ♍ are north ſignes that is, alwayes lying on the North ſide of thequinoctiall, and the other ſixe ♎ ♏ ♐ ♑ ♒ ♓ ſouth ſignes, and lying on the ſouthſide. then will I conclude the ſcope of this Chapt. which is, that you muſt know and alwayes remember (therefore note it well) that the North ſignes in the zodiacke muſt alwayes riſe and ſet on the North part of the horizons, & the ſouth ſignes on the ſouth part, & ſo likewiſe the North ſtars to riſe & ſet on the north part of the horizons, & the ſouth ſtars on the ſouth part: note that in my *Iewel* for diſtinction ſake, the north ſtarres do point outwards from the centre towards the *Limbe*, and the ſouth ſtarres inwards towards the centre. Further though I call them henceforwards north horizons, and ſouth horizons for breuitie ſake, yet I meane the north part and ſouth part of the ſame horizon, or horizons. For the ſtarres magnitudes looke in the 7. Chapter. Laſtly the verie point or tippe of any ſtarre in the *reete*, I will call his apex.

Chapter 20.

How to chooſe out which of the 90. horizons ſerueth to euery countrie.

BY the ~~10.~~ 8. Chapter you may gather thus much that the pole articke or point A in this ſecond operation is the Eaſt point, the point B V Veſt: at the handle South, or noone: and at the lower part, north or midnight: & are ſo to be written in thoſe parts of the *Iewel* vz. Eaſt, V Veſt, North, and South, and nowe to chooſe the horiſon to any place, get the latitude thereof either by tables, or elſe by obſeruation, as ſhall be ſhewed: and then remembring that the centre is now become the pole or poles, reckon ſo many horiſons from the centre on ech ſide as the degrees of latitude come to, and the horiſon there ending is your deſire. For example, I would know which of the 90. horizons doth ſerue heere for our towne of Reading : the poles eleuation or latitude (all is one) is $51\frac{2}{3}$ as I ſaid in the 17. Chapt. therefore for my horizon I reckon from the centre being here the pole, and that northwards on the noone line $51\frac{2}{3}$ degrees, and as much on the noone line ſouthwards. the horizon there, is the ſouth part of the ſame: ſo I conclude that thoſe two halfe horizons are the whole horizon, ſeruing Reading, & becauſe I moſt remaine in that place, I ſhall haue little neede of any of the other, and therefore I diſtinguiſh him out with prickes or ſome other deuiſe, whereby I may readilie ſee him at all times from the reſt, and ſo am I well for my horizon as long as I remaine at Reading, but if I trauaile ſouthwards or northwards to Northampton, Barwicke &c. then muſt I chooſe another horizon according to the latitude of the place, as before, to ſerue my turne as long as I am there.

Chapter 21.

How the Zodiacke and fixed ſtarres riſe and ſet on the horizon now found.

THe *Iewel* in this ſecond diſtinction is diuided into foure quarters by the Eaſt line, and noone line as you ſee. The firſt quarter, betweene north and eaſt, is the north eaſt quarter, the ſecond towards ſouth, the ſoutheaſt quarter, the third from ſouth to weſt, is the ſouth weſt, and the fourth from weſt to north, is the north weſt. Now thequinoctiall alwayes riſeth and ſetteth full eaſt and weſt, that is in the points A and B, and therefore as manie ſtarres, planets, or other points of the heauens as happe in thequinoctiall, as namely of the zodiacke the degree beginning ♈ and ♎, & therefore the ſunne there being doth alwayes riſe and ſet full eaſt and weſt, euen in the points A B, but of all other ſtarres, points, ſignes, and their parts or degrees that lie ſome north, ſome ſouth from thequinoctiall, ſome part muſt needs lie north, and ſome ſouth from it, and are therefore called north and ſouth ſtarres, points and degrees, according as they lie: then in reaſon you may well conceiue that if thequinoctiall do alwayes riſe eaſt, thoſe ſtarres or ſignes that lie north from it, muſt needes riſe on the north eaſt quarter of the horizon and ſet on the north weſt quarter. And likewiſe the ſouth ſignes and ſtarres muſt needes riſe on the ſoutheaſt quarter, and ſet on the ſouthweaſt of the horizon : and that which more is, if in any latitude any ſtarres lie more north or ſouth then thequinoctials height is, in the ſame countrie ſuch north ſtarres be continually aboue the horizon and neuer ſette, and ſuch ſouth ſtarres be continually vnder the horizon, and neuer ſeene in that country. An example would not be amiſſe here to helpe a beginner in the firſt entrance: admit I would ſee where about the great ſtarre called *Arcturus* or Bootes riſeth in our horizon, now becauſe I know him to be a north ſtarre by the 19. Chapt. I turne about my *reete* till his verie apex come to our horizon, founde by the laſt Chapter, in the north eaſt quarter of the *Iewel*, for there doth that ſtarre alwayes riſe: and then laying the rule to the degree of the ſunne in the zodiacke, it ſhewth in the *limbe*, the houre of his riſing: but that ſhalbe ſhewed hereafter. Nowe to know where *Arcturus* ſets, turne the *reete* till his apex doe touch the ſaid horizon in the north weſt quarter, and there is your deſire. Then admit I would ſee where the great ſtarre called *Fomahand* being a ſouth ſtarre, doth riſe and ſet, I turne about the *reete* till his apex do touch the horizon in the ſoutheaſt quarter, there he riſeth alwayes, then turne about the *reete* till his apex do touch the horizon in the ſouthweſt quarter and there the ſtarre ſetteth alwayes, the rule then applied to the degree of the ſunne ſheweth the houre. And in like manner is it with the north and ſouth ſignes, and their degrees : admit I would know where aboutes the 10. degrees of ♊ riſeth becauſe ♊ is north ſigne, I turne the *reete* till the 10. deg. thereof do cutte the horizon in the northeaſt quarter, and for his ſetting in northweſt. In like manner the ſouth degrees riſe in the ſoutheaſt quarter, and ſet in the ſouthweſt: take ſome paines and conceiue this Chapter well, and you ſhall haue a pleaſant vſe of your *Iewel*; you ſhall vnderſtand it the better if you conferre it with other Chapters of the third booke that hang hereon, as hereafter will follow.

Chapter

Chapter 22.

Of the backe of the Iewel, *and his lineaments.*

IN the backe of the *Iewel* I haue appointed the theoricke of the ſunne, and ſome one deuiſe or other within it, for euery man to make choiſe of by the 23. Chapter. The rule on the backſide ſerueth chiefly to take altitudes: the line hanging plumme from the handle, I call the perpendiculer or plumme line, and the line croſsing the ſame square, which in taking altitudes lieth (or at leaſt muſt lie, if you do well) parallel to the horizon, I call the line of leuell. Looke more in the fourth Booke and laſt Chapter.

FINIS.

The third booke of the Mathematicall *Iewel*: gathered and ſet foorth by Iohn Blagraue *of Reading Gentleman: conteineth al the propoſitions, and concluſions* of Iohannes Stophlerus, Iohannes Sconerus, Iohannes Roijas, G. Friſius, or any other, written on their ſeuerall Aſtrolabes, ſet downe heere in the ſelfeſame order that Gemma Friſius hath done them on his Catholicon, but all of them applied to this Iewel, and altered to a more plaine and briefe Method, both in writing, working, and vnderſtanding, by reaſon of the Authors new deuiſed *roote*, which performeth them fixed as redie, as by any tables, without any of thoſe tedious profers, or doubtfull aſſaies, which Gemma Friſius is driuen to vſe, by his *Curſor*, or *Brachiolum*: who alſo hath annexed diuerſe new additions to moſt of the Chapters.

Chapter 1.

How by this Iewel *to take the altitude of the Sunne, Moone, or Starres.*

THough this matter be too too wel knowne alreadie, euen to the common ſort: yet ſince other authors haue made it their firſt Chapter, I refuſe not to follow the ſame courſe. A degree in latin *Gradus*, is the 360. part of a circle: *Altitudo*, in Engliſh altitude, or height: the ſame in all Aſtronomical obſeruations, is taken by the degrees of a circle, or a relation vnto them. VVherefore to take the altitude of the Sunne, let hang the *Iewel* in your hand at libertie, turning the edge of the *Limbe* towards the ſunne, the rule on the backſide lying euen in the line of leuell, which line repreſenteth here your horiſon: then lift vp that ende of the rule next the ſunne (or put downe the other, all is one) vntill you may ſee the ſunne beames to peirce both the ſight holes, and looke what degree of the *Limbe* the rule cutteth, counted from the line leuell, the ſame is the Sunnes altitude for that inſtant. For ſtarres you muſt with one eye ſhut, looke through the greater ſight holes, made of purpoſe for ſtars, till you ſee the ſtarre thorow them both, and then doth the rule ſhew your altitude. Alſo a man may, if his ſight holes be too little, or the ſtarre dimme, take the ſtarre euen with the vpper edges of the ſights, ſo they be parallel to the holes, which of ſome is thought the beſt way. Further if your inſtrument be heauie, or your hand vncertaine, it is beſt to hange it by a naile on a tree, ſtaffe, or poſt, this taking of altitude will be much in vſe hereafter.

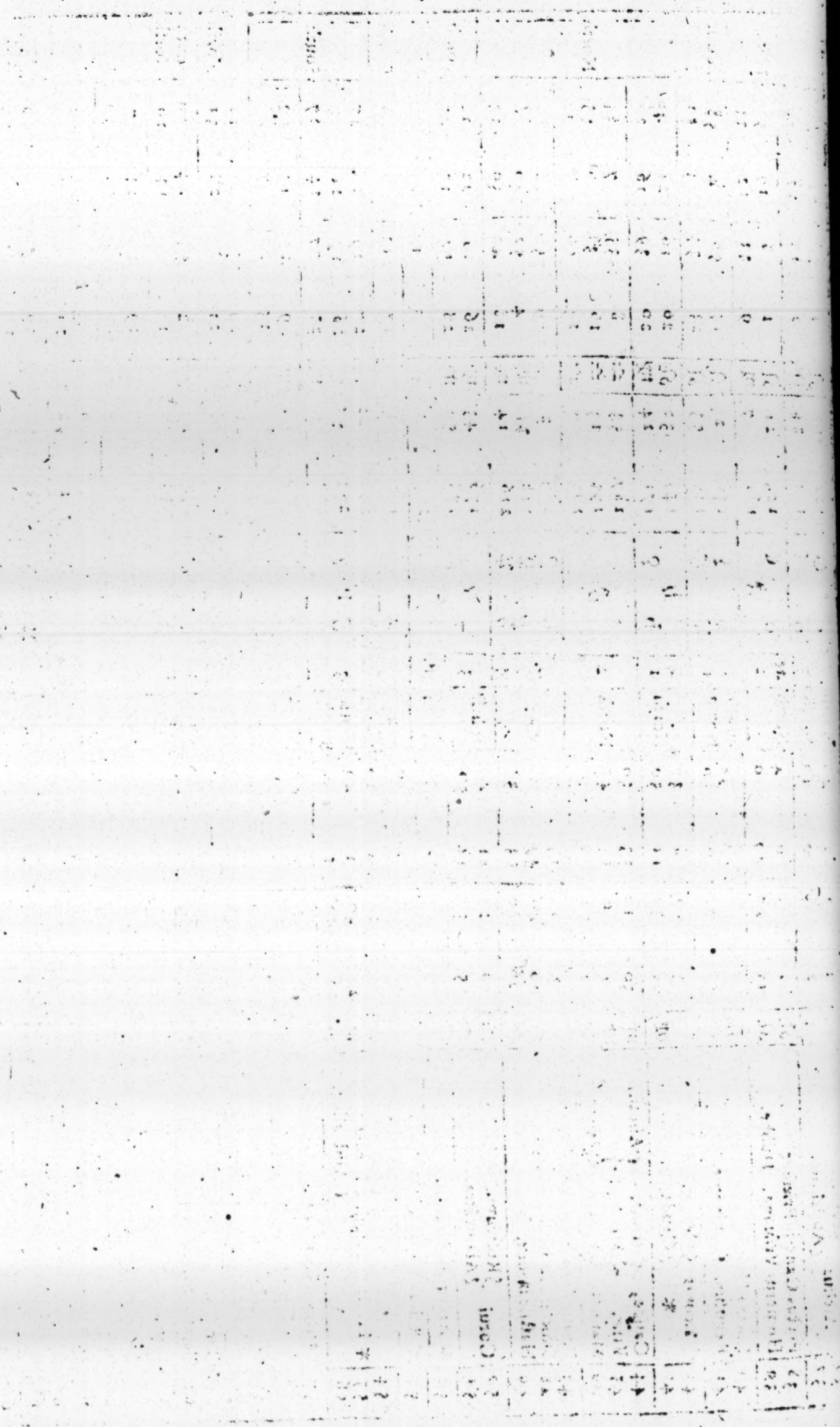
Note that when you take the altitude of the Sunne, Moone, or any ſtarre being in the Meridian circle, then is the ſame called the meridian altitude.

Chapter 2.

How to get or rather examine the Sunnes greateſt declination, and diſtance of the Tropickes, and thereby alſo to learne the latitude of any countrie.

Gemma Friſius. 4

THe Eclipticke lieth alwayes byas, or inclining to the equinoctiall, according to the ſunnes greateſt declination, which in theſe dayes is 23. degrees 30. minutes, or rather after late writers 23. degrees, 28. minutes, which to proue, take the meridian altitude of the ſunne in the higheſt of ſommer, being in the very Solſtice, and



34½	Caput ♊	Precedens	♋	14	20	9	40	North	2	☿	☉	119	47	♋	18	32	28	29	North
35	*	Sequens.	♋	17	40	6	15				☿	119	4	♋					
36	Asellus	Boreus.	♌	1	20	2	40	North	4	♂	☉	124	13	♌	1	58	22	28	
37		Australinus.	♌	2	19	0	10	South		♂	☾	124	34	♌	2	18	19	29	
38	Pectus Cancri. Præsepe.		♌	1	10	0	4	North	ne	♂	☾	123	21	♌	1	8	19	56	North
39	Humerus Vrsæ ma. Dubhe.		♌	11	40	49	0		2		♂	161	7	♍	9	33	62	33	
40	Lucida Hidri.		♌	21	0	20	30	South	2	♄	☿	138	24	♌	15	56	4	58	South
41	Regulus. Cor leonis.		♌	23	30	0	10	North	1	♃	♂	145	53	♌	23	34	13	31	North
42	Leonis	Ceruix.	♌	23	10	8	30	North	2		♄	148	30	♌	26	16	21	49	
43	*	Cauda.	♍	15	30	11	50		1	♄	☿	171	24	♍	20	39	16	36	
44	Caude,	Prima, Alioth.	♍	30	10	53	30	North	2		♂	187	53	♎	8	35	57	35	North
45	Vrsæ,	Media,	♍	9	0	55	40					195	13	♎	16	31	56	55	
46	Maioris.	Vltima. Benanac.	♍	20	50	54	0					202	9	♎	23	56	51	12	
47	Preuuendemiatrix Protrig.		♎	3	10	15	10	North	3	♄	☿	189	2	♎	9	50	12	38	
48	Sinister Humerus Bootis.		♎	10	40	49	0		3	♄	☿	212	35	♏	4	52	40	6	
49	Hastile Bootis.		♎	26	40	53	35		4	♄	♃	226	14	♏	18	42	39	13	
50	Corui	Rostrum.	♎	6	20	21	40	South	3		♄	176	43	♍	26	26	22	18	South
51		Ala dextra.	♎	5	30	14	50			♄	♂	178	59	♍	28	54	15	45	
52	* Arcturus Bootes.		♎	18	0	31	30	North	1	♂	♃	209	1	♏	1	10	22	0	North
53	* Spica Virginis.		♎	17	40	2	0	South	1	♂	☿	195	51	♎	17	12	8	48	South
54	*	Borealior	♏	27	20	1	10	North	3	♂ P ♄		235	22	♏	27	39	18	17	South
55	* Frontis Scorpij.	Media	♏	26	40	1	40	South				233	57	♏	26	17	21	3	
56		Australior.	♏	26	40	5	0	South				233	6	♏	25	27	24	17	
57	Lances	Australis.	♏	9	0	0	40	North	2	♄ P ♂		216	49	♏	9	13	13	52	
58	Chele	Borealis.	♏	13	10	8	30			♃	☿	223	17	♏	15	46	7	41	
59	Palma Ophiuchi.		♏	27	0	12	30	North	3	♄	☿	237	21	♏	29	34	8	18	North
60	Lucida Coronæ Gnosiæ.		♏	5	40	44	30		2	☿	♃	228	45	♏	21	45	28	31	
61	Cor Scorpij Antares.		♐	3	40	4	0	South	2	♄	♂	240	49	♐	2	53	24	50	South
62	Caput Engounasi.		♐	8	40	37	30	North	3		☿	252	35	♐	13	57	15	19	North
63	Caput Ophiuchi.		♐	15	50	36	0	North	3	♂	♃	258	17	♐	19	14	13	7	
64	Caput Iunonij Draconis.		♐	20	40	75	30			♄	♂	266	2	♐	26	22	52	10	
65	Fidicula Lucida Lyræ.		♑	8	20	62	0	North	1	☿	♃	275	0	♑	4	35	38	41	
66	Aquila Vultur volans.		♑	24	50	29	10		2	♄	♂	291	42	♑	20	4	7	34	
67	Caude ♑	Precedens.	♒	15	50	2	10	South	3	♃	♄	318	59	♒	16	32	18	10	South
68		Sequens.	♒	17	20	2	0					320	26	♒	18	0	17	34	
69	Romboides	Australis.	♒	9	20	32	0	North	3			303	28	♒	1	14	13	1	North
70	Delphini.	Borealis.	♒	11	9	33	50					304	6	♒	1	31	13	25	
71	Cauda Delphini.		♒	8	29	29	10					303	30	♒	1	16	10	5	
72	* Foma hand.		♒	27	59	23	0	South	1	♂	☿	338	30	♓	6	46	33	33	South
73	Cuspis Sagitte.	4	♒	1	10	39	20	North	4			294	58	♑	23	8	18	33	North
74	Cauda Cygni.		♓	0	9	60	0	North	2	☿	♃	307	7	♒	4	46	44	3	North
75	Crus Pegasi.		♓	23	10	31	0			♃	♂	340	39	♓	9	3	25	36½	
76	Cauda Ceti.		♓	26	40	20	20	South	3		♄	5	23	♈	5	52	19	53	South

A Table of the most notable fixed starres taken out of *Stadius*, rectified by obseruation to the yeare of our Lord 1558.

	The names of the fixed Starres.	Signes.	Longitude. deg.	Longitude. min.	Latitude. deg.	Latitude. min.	Part of Latitude	Magnit.	Their nature	Right ascention. Equinoct. degees.	Right ascention. min.	Signes.	Culmination. deg.	Culmination. min.	Declination. Equin. deg.	Declination. min.	Part of declination.
1	Cornu arietis Præcedens.	♈	27	40	7	20	North	2	♂ ♄	22	55	♈	24	55	17	29	North
2	Dexter humerus Cephei.	♈	7	40	69	0	North	3	♄ ♃	317	15	♒	14	47	61	6	North
3	Extrema Eridani Acarn.	♈	21	10	53	30	South	1	♄ ☿	43	0	♌	15	29	40	41	South
4	Andro medæ {Scapulum Cingulus} Scheder Mirach.	♈	16	20	24	30	North	3	☿	4	33	♈	4	58	28	50	North
5		♈	24	50	26	30	North	3	☿	11	38	♈	12	40	33	58	
6	Ceti {Iuba.	♈	28	40	4	10	South	4	♄	28	7	♌	0	13	7	6	North
7	Ceti {Venter.	♈	16	0	20	0	South	3	♄	24	14	♈	2	9	16	7	South
8	Dextrum latus Perſei.	♌	25	50	30	0	North	2	♄ ♃	43	17	♌	15	38	48	4	North
9	Caput Meduſſe. Algol.	♌	20	40	23	0	North	2	♄ ♃	40	28	♌	12	37	39	56	
10	Vergiliarum {Trapezum.	♌	24	40	5	20				50	52	♌	23	16	24	7	
11	Seu {Auſtralior.	♌	23	29	4	30	North	5	♂ ☾	49	42	♌	22	8	23	1	North
12	Pleiadum {Borealior.	♌	23	10	4	40				49	29	♌	21	55	23	5	
13	*Seuen Starres.* {Minima.	♌	23	40	5	5				49	53	♌	22	18	23	37	
14	Hiadum ſiue Suculārum. {Prima in naribus tauri	♌	29	40	5	45				58	43	♊	0	53	14	28	
15	Hiadum ſiue Suculārum. {Secunda inter hanc & oculum Boreum.	♊	1	21	4	35				60	10	♊	2	16	15	57	
16	Hiadum ſiue Suculārum. {Tertia inter eadem.	♊	1	50	5	50	South	3	♂	62	56	♊	3	0	14	49	North
17	Hiadum ſiue Suculārum. {Quarta & Paliliciū 5.	♊	2	50	2	0				61	10	♊	3	13	18	47	
18	* Oculus {in oculo Boreo & 6.	♊	2	40	5	0				61	37	♊	3	39	15	48	
19	Tauri. {Septima Palilicium.	♊	3	40	5	10		1		62	30	♊	4	29	15	49	
20	Humerus. {Dexter.	♊	23	0	17	0	South	1	♂ ☿	83	16	♊	23	50	6	20	North
21	* Orionis. {Sinister.	♊	11	20	17	30	South	2	♂ ☿	72	10	♊	13	34	4	49	
22	Aurige Humerus. {Dexter.	♊	23	50	20	0	North	2	♂ ♀	82	1	♊	22	41	47	17	North
23	* Hircus. {Sinister.	♊	16	0	22	30	North	1	♂ ♀	71	33	♊	13	0	45	5	
24	Hædorum. {Præcedens.	♊	13	10	18	0	North	4	♂ ♀	68	51	♊	10	28	40	13	
25	*Eriphon.* {Sequens.	♊	13	0	18	0	North	4	♂ ♀	68	39	♊	10	17	40	12	
26	* Sinister pes Orionis.	♊	10	10	31	30	South	1	♃ ♄	72	56	♊	11	56	9	12	South
27	Stella Polaris Cynoſura.	♊	21	10	66	0	North	3	♄ ☿	5	2	♈	5	30	88	24	North
28	Balthei vel Cinguli Orionis. {Præcedens.	♊	16	20	24	10				77	32	♊	18	33	1	18	
29	Balthei vel Cinguli Orionis. {Media.	♊	18	20	24	50	South	2	♄ ♃	79	25	♊	20	17	1	48	South
30	Balthei vel Cinguli Orionis. {Sequens.	♊	20	20	25	50				81	18	♊	22	1	2	39	
31	* Canobus in Argo naui.	♋	8	10	75	0	South	1	♄ ♃	93	24	♋	3	7	51	38	South
32	* {Maior *Seirius.*	♋	8	40	39	10	South	1	♃ ♂ ♂	96	58	♋	6	26	15	55	South
33	* Canis {Minor Procion.	♋	20	10	16	0	South	1	♃ ♂	109	28	♋	17	58	6	7	North
34	Caput ♊ {Precedens {Apollo	♋	14	20	9	40	North	2	☿ ♂	106	47	♋	15	28	32	18	North
35	* {Sequens. {Hercules.	♋	17	40	6	15				119	4	♋	18	32	28	29	

and againe in the deepeſt of winter, being in the winter Solſtice Deduct the leſſer meridian height, out of the greater, the remainder is the diſtance of the tropicks: halfe that, is the Sunnes greateſt declination. For example.

In the yeare 1579. about S. Barnabies day, becauſe the countrie folke account that the longeſt day in the yeare: for two or three dayes before, and after, I watched the ſunne comming to the ſouth, and there I tooke his higheſt alitude eth day, and I found that the 13. day of Iune the ſunne had the higheſt Meridian altitude, which was 62. degrees that I noted in my book. Then about ſaint Lucies day in winter, counted the ſhorteſt day in the yeare, I watcht the ſunnes loweſt meridian altitude, and found it the 13. of December to be 15. degrees. then of theſe two I ſubduct the leſſer vz. 15. out of 62. the greater: there reſteth 47. and that is the diſtance betweene the tropickes, the halfe of that vz. $23\frac{1}{2}$ is the greateſt declination of the ſunne, and ſo much doth the zodiacke decline from the equinoctiall, and as much doe the poles of the worlde, which are alſo the poles of the equinoctial, differ from the poles of the zodiacke.

Heere muſt you note that this greateſt declination of the ſunne hath not alwayes beene found one, of ſeuerall authors at ſeueral times: for Ptolomeus & Ariſtarchus, Samius and Timochares, that were 400. yeares before him almoſt, did finde by obſeruation that this declination was 23. degrees, 51. minutes, and 20. ſec. and from their time hitherwards, it is founde to decreaſe by little and little. For *mahometes Aratenſis* after Ptolomeus 749. yeares, found it 23 degrees 35. minutes: after him 420. yeares, Prophatius Iudeus 23. deg. 32. minutes. Thoſe of our times, as VVernerus, and Copernicus, haue found it 23. deg. and 28. minutes, and ſo is it thought to be at this time, of thoſe that haue made obſeruations with large inſtruments.

But now by this declination is had the latitude of any place, verie commodiouſlie. For example: becauſe before I found the ſunnes loweſt meridian altitude 15. deg. adde thereto the ſunnes greateſt declination nowe knowne vz. 23. degrees, 30. minutes, it maketh 38. deg. 30. minutes, the equinoctials height aboue the horizon, which you ſhall deduct from the quadrant of a circle vz. 38. 30. out of 90. there reſteth 51. deg. 30. minutes, the height of the pole, or latitude at Reading, where I made the foreſaid obſeruation. But being made by ſmal inſtruments, I haue ſomwhat erred, for it is heere $51.\frac{2}{3}$. The contents of this Chapter ſerueth moſt truly as before, to all thoſe that dwell in the temperate zones, that is whoſe zenithes are betweene the tropicke and pole circle. But to thoſe that haue either tropicke in the zenith, the loweſt meridian altitude of the ſunne, being in the winter ſolſtice muſt be taken out of 90. & ſo reſteth the diſtance of the tropicks. And to thoſe that haue their zenith betweene the two tropickes, the two meridian altitudes taken in the ſolſtices, muſt be ioyned togither, and the ſunne amounting muſt be taken out of the ſemicircle vz. 180. and ſo reſteth againe the diſtance of the tropickes, whoſe halfe vz. the ſunnes greateſt declination being added to the ſunnes loweſt meridian altitude, maketh the eleuation of the equinoctial aboue any ſuch horizon, which being take (as before) out of 90. degrees, leaueth the poles eleuation. Likewiſe whoſe zenithes are in the colde zones, who haue the ſun continually aboue the horizon certaine dayes, there muſt you take the ſuns greateſt meridian altitude, being in the Solſtice, and likewiſe his left meridian altitude the ſame day (for the ſun commeth twiſe viſiblie in 24. houres, to the meridian euen as our north ſtars do) thoſe two meridian altitudes ioyned togither, make the whole diſtance of the tropickes, thoſe that haue either of the poles their zenith, there the higheſt meridian altitude, is the ſunnes greateſt declination, and thoſe that haue the pole circles zenith, the greateſt meridian altitude, in the ſommer Solſtice is the diſtance of the tropickes.

Blagraue. All theſe and many more you may learne by the very beholding of the *Iewel*, and might haue done if nothing had beene written thereof, for the *Iewel* being placed from latitude to latitude, as in the 2. Booke and 19. Chapter is ſhewed: you may reckon euen vpon the *limbe* all theſe altitudes aboue the *Finiter*, ſeuerally at pleaſure, for the Almicantares are had *Ipſo aſpectu*, and truely all the circulations of the globe, more readie then if you had the ſphere in your hand, and therefore reade well the laſt part of the ſecond booke, to vnderſtand euery part of your *Iewel*.

Chapter 3.

How by the theoricke on the backe of the Iewel, to get the ſunnes place or degree in the zodiacke any day.

Gemma Friſius 5.

SEeke the day propoſed in the circle of monethes, on the backſide of your *Iewel*, and thereto lay the rule which in the circle of ſignes adioyning, ſheweth the degree ſought for.

Example.

This preſent day being the 7. of Iune 1582. I would know where the ſunne is, vz. in what degree of the zodiacke, I reſort to the backe of my *Iewel*, and ſeeke in the *Limbe* of monethes fore Iune: and there the 7. day, as they be figured, where on I lay the rule which I finde to cutte in the *Limbe* of the ſignes, adioyning the 25. degree, and almoſt 40. minutes of Gemini, ſeruing to the beginning of the ſaid 7. day of Iune, which is the degree of the ſunne ſought for.

But here by the way you muſt note, that the circle of the yeare conteineth but 365. dayes, where as in truth it is 6. houres more verie neare, which is a quarter of a day, therefore when you make your theoricke, you muſt know to what yeare it is made, either the firſt, ſecond, or third after the leape yeare: that knowne, your theorick ſerueth exactly ynough for that yere to which it is made, but to the next yere you muſt adde a quarter of the next day following to your day propoſed, and then take the degree of the zodiacke ſeruing that, to the ſecond yeare, take halfe the day following, to the third three quarters, and then the fourth is the yeare of the theoricke againe, and needeth nothing to be added: deuide the yeare of our Lord by 4. and if there remaine nothing it is leape yeare, if one two or three, as neuer it is aboue, it ſheweth the yeares paſt from the leape yeare.

Example.

This yere 1582. deuide 82. by 4. (for always the M. & C. years are leapyears) there reſteth two, wherby I

know that it is the second from the leape yeare, so that if my theoricke were made for the first yeare, then this being the second yeare, I adde to euery day, a quarter of the day following through the whole yeare, and so of the rest. These sixe houres foure times gathered make a day, and therefore euerie fourth yeare Februarie hath 29. dayes, and then is called the leape yeare, in latin *Bissextus* or *annus intercalaris*, because a day is then put in betweene, other minutes wanting are not here to be regarded.

Chapter 4.

How to finde the sunnes declination for euery day in the yeare, and also the declination of euery part of the ecliptick.

Gemma Frisius.

WHat the sunnes greatest declination is, it is said in the 2. Chapter. In like manner, euerie degree of the zodiacke hath his seuerall declination denominated by the parallel, cutting the same deg. wherefore the ecliptick line being drawne on the *Mater*, and deuided as in the 4. Booke, and 6. Chapter is shewed: Learne by the last Chapt. the sunnes degree in the zodiacke, for the time proposed, seeke the same degree on the ecliptick line, and then doth the parallel crossing the same, shew in the *Limbe* (or otherwise reconed from the equinoctial) the declination sought for. *Example.*

The first of may this yeare 1582. I would knowe the sunnes declination: by the last Chapter I finde the sunne to be in the 20. degree, and almost an halfe of ♉ I seeke that degree on the ecliptick of the *Mater*, and there do I see to crosse the $18.\frac{1}{2}$. parallel, reckoned from the equinoctiall, and because the same parallel lieth towards the pole artick, or north pole from the equinoctiall, I conclude the sunne declineth $18.\frac{1}{2}$ degrees northwards, and so may you see the declination of any degree of the ecliptick, whither the sunne be there or not: if the ecliptick be not on your *mater*, or be vndeuided, the rule or *Finitor* may supplie it.

How to do the same a new way by the zodiacke of my Reete.

Blagrave. Lay the *Label* on any degree in the zodiacke, and his degrees reckoned inwards from the *Limbe*, shew your desire. *Example.*

The last of March 1582. the sunne being in the 20. deg. 26. minute of ♈, I seeke the same degree in the zodiacke of the *reete*, and thereto lay the *Label*: then reckoning the degrees of the *Label*, betweene the *Limbe* and that degree of the zodiacke, I finde 8. degrees and better, the declination thereof: and because ♈ is a North signe it declineth North. Note that if your *Label* be not deuided, you neede but turne about the *Reete* till the zodiacks degree proposed, do cut the noone line, or axtree line, and either of them shall supplie his want.

Chapter 5.

How to finde the latitude or poles eleuation, any day proposed in any place, or countrie whatsoeuer.

Gemma Frisius.

THe latitude or poles eleuation, are in effect all one, and what they are is shewed in the first Booke 11. Chapt. first by the 3. Chapter seeke the degree of the sunne, for the day proposed, and by the 4. Chapt. the sunnes declination, then take the meridian altitude by the first, and reckon the same among the almicantares, from the *Finitor* vpwards, turning about the *Reete* till the same almicantare do touch in the *limbe*, and that rightwards from the pole artick, the sunnes parallel of declination: and so doth the *Finitor* stande fixed to the latitude of that place: the degrees in the *Limbe* betweene the *Finitor* and the pole artick, sheweth the eleuation thereof aboue the horizon of that countrie, or place.

Example. The 10. of Aprill 1582. the sunne being in ♉, by the 3. and his declination $11\frac{2}{3}$. by the 4. I tooke his meridian altitude 50. deg. which done, I told out the 50. almicantare, and turned about the *reete* til the said 50. almicantare did meete the $11\frac{2}{3}$ parallel (reckoned towards the north pole) on the *limbe*, and there the *reete* fixed, I reckon on the *Limbe* the degrees betweene the *Finitor* and the pole, or betweene the zenith and the equinoctiall, which are alwayes equall, I finde $51\frac{2}{3}$ degrees, which is the height of the pole, or latitude at Reading. Note that in all workings for the most part, the zenith point of the *reete* must be aboue the equinoctiall towards the North pole.

You may do the same another way without the *Iewel*, thus: if the declination of the sunne be north, subduct the same from the meridian altitude, as here it was $11\frac{2}{3}$, which taken out of 50. the remaine videl. $38\frac{1}{3}$ is the equinoctials height aboue the horizon, which alwayes subducted from 90. leaueth the latitude vz $51\frac{2}{3}$. But if the declination be south, adde the same to the meridian altitude, and so haue you the equinoctials, height which taken from 90. leaueth likewise the latitude.

Chapter 6.

How to know what it is a clocke at any time the sunne shining, and in any region in the world.

Gemma Frisius. 30.

I Cannot choose but set downe this Chapt. next. because it is one of the chiefest that any yong beginner can delight in, and by my *Iewel* is one of the easiest where, in Gemma F. instrument, it was one of the hardest, and

and therefore he appointed it the 30. Chapter, because a man should be first well acquainted with his instrument. The latitude of your place or abiding, being first had by the 5. or otherwise, as shalbe shewed in other Chapters following, you must get the sunnes declination for the day proposed, by the 4. and then at the instant that you would know the houre, take the sunnes altitude by the first: which done, the *Finitor* being set to the latitude reckon the sunnes altitude so taken amongst the almicantares, and marke where his parallel of declination cutteth the same almicantare, for there the houreline passing by, doth shew the houre and deg. euerie deg. being 4. minutes, thus in briefe always the altitude being taken the houreline passing through the crossing of the Almicantare, and the sunnes parallel is your desire.

Blagraue. As oft as I looke what it is a clocke by my *Iewel*, and see with what facilitie, certaintie, and rediness for euerie countrie I doe it, ouer that I was wont by Gemma Frisius instrument, with his Cursor and Brachulum, I can not but meruaile that hee and his sonne Cornelius G. coulde bee content with so troblesome a practise, which trulie I lothed euen vpon the first sight, which not onely in working this Chapt. but also in most of their whole booke is verie combersome, although in some Chapter more then in other, yet is Gemma Frisius to be excused, because he was the inuentor. *Facilius enim est inuenta addere quam de nouo componere.*

Example. The sixt day of October 1580. in the morning I went foorth of my studie at our mansion house at Bulmersh by Reading, where I was writing the first copie of this proposition at that present instant. The latitude being there $51\frac{2}{3}$ degrees, and at the same time the sunne being in the 23. deg. 8. minutes of ♎, his declination by the 4. was 9. degrees southwards, and 6. minutes, all these things to him that vseth his *Iewel* daily will alwayes be in memory, I say, I went forth then, and turning the edge of my *Iewel* to the sunne, I founde his altitude 12. degrees by the first, which is as much to say, as the sunne was that instant in the 12. almicantare, wherefore by the second booke 19. Chapter, setting the *Finitor* to our latitude vz. $51\frac{2}{3}$ deg. I marked where the 12. almicantare reconed from the *finitor*, did cut the $9\frac{6}{60}$ parallel, reconed from the equinoctiall line towards the south pole, for that the sunnes declination was south, and there did the houreline that crossed this intersection shew me that it was 3. deg. past 8 of the clocke, that is 8. of the clocke, and 12. minutes past. For euerie degree is 4. minutes in reconing the houre at all times, though euerie degree in euerie other respect hath 60. minutes, and euerie minute 60. seconds, and euery second 6. thirds &c. vnto tenthes. It was 12. minutes past 8. and therefore time to go to breakfast, for him that lay not last a bedde: I made hast into the hall where I found my companions at it, and so sharpe set, that had not my *Iewel* beene readier then Gemma Frisius Astrolabe, I had lost my breakefast with looking what it was a clocke, and therefore I had great cause to commend it at that time.

But what did I see more, euen the azimuth that the sunne was in at that instant, also crossing the intersection of the sunnes parallel, and almicantare, which being reckoned from the zenith line, did shewe me that the sunne was in the $32\frac{1}{2}$ vertical circle reckoned from the East, the azimuthes indeed do represent, as it were the points of the compasse, and before the compasse found, men sailed thereby, and therfore if you did distinguish them into the points of the compasse, by some notable difference, then might you see at all times on what point the sunne, Moone, or any starre were without helpe of the lodestone, there be 32. points, by which deuide 360. and the quotient sheweth $11\frac{8}{60}$ degrees, seruing euerie point seuerally, but this serueth to no great vse but onely on the sea.

Chapter 7.

The latitude of any place knowne, how to get the declination of the sunne, or any starre, planet, or comet, by obseruation.

THe latitude knowne, subduct that out of 90. there resteth alwayes the height of the equinoctiall in that horizon, for the latitude and the equinoctials height, are one the complement to the other, they both making 90. degrees. This knowne, take the meridian altitude of any starre or planet by the first Chapter, which if it be found greater then the equinoctials height, then to vs that dwell on this side of the equinoctial, the starre shalbe north declining, if lesse, it shalbe south: therefore take the lesser height out of the greater, the remaine, it is your desire.

Example. Heere at Reading the latitude being $51\frac{2}{3}$. the complement of 90. is $38\frac{1}{3}$. degrees, the equinoctials height. I tooke the meridian altitude of the great starre called Arcturus, or bootes, and found it $60\frac{1}{3}$ degrees, out of $60\frac{2}{3}$. there resteth $22\frac{1}{3}$ degrees, so much is the declination of Arcturus from the equinoctiall, and that, northwards, because his height was greater then the equinoctials, and so may you do for the declination of any starre, plannet, comet, or the sunne.

But there are a great many starres that to vs neuer come at the south, because they neuer come beyond the zenith, which are those whose distance from the pole, is either lesse, or equall to the distance betweene the zenith and the pole. The distance betweene the pole and the zenith, is alwayes equall to the equinoctials height, wherefore the meridian height of such starres may be northwards, twise taken, once aboue the pole and againe vnder the pole. For the declination of these, take the latitude out of the greatest meridian altitude, and the remainer out of 90. so resteth your desire. But if you will worke by his lesser meridian altitude, subduct it out of the latitude, and the remainder out of 90. there resteth then the declination sought for. Againe if you list, subduct the lesser meridian altitude out of the greater, and part the remainder in halfe, and subduct that halfe from 90. the same declination appeareth againe.

There be other starres which come past the zenith, and yet neuer set: their declination may be had both wayes. The *Iewel* it selfe yeldeth so plaine demonstration thereof, that it needeth no example, for the *finitor* being at the latitude, the parallels shew what stars may set, or set not: rise or not rise. I take the lesse, vz. $38\frac{1}{3}$ in that countrie.

To do the ſame, the ſtarre being in any place of the ſkie by obſeruation and helpe of the Iewel.

Blagraue. Place your *Iewel* to lie leuell or flatte with the horizon on a ſtoole, poſte, or otherwiſe, ſo that the line of leuell lie in the meridian vz. iuſt north and ſouth, which you may doe by helpe of a diallneedle, rectified or otherwiſe, as ſhalbe ſhewed in the 33. Chapter: this done turne about the rule till you may ſee the ſtarre deſired, euen with your ſights, and then marke what degree the rule cutteth in the *Limbe*, and reckon him from the nooneline, for that ſheweth the azimuth the ſtarre is in: this done, take vp your *Iewel* preſently and take the altitude of the ſtarre. Now the altitude and the azimuth being had both at one inſtant, ſet the *finitor* to the latitude, and ſeeke where the like azimuth, & almicantare do croſſe ech other, the parallel of the *Mater* cutting the ſame croſsing, ſheweth the declination of the ſtarre, being numbred from the equinoctial.

Example. I went one night to make this experience of the great ſtarre vnder Orion, called *Canis maior* the great dogge, when my *Iewel* was laide flatte with the horizon on a ſtoole, and the nooneline euen with the meridian, which I had firſt drawne in the day time vppon the ſtoole, then turning about the rule towards the ſtarre, I founde that my ſightes were not apt and heigh ynough for my purpoſe. wherefore I hanged a plumline, or corde ſomewhat VVeſt from the Starre, ſo that I might perceiue the ſame line or corde thorow the ſights, then I ſtayed vntill the ſtarre going forwards was come, ſo that I might ſee thorow the ſight holes of my rule euen with the corde, then I marked what degree in the *Limbe* the rule there cut, I found it $\frac{49}{49}$ degrees from the noone line, this number kept I in minde for the azimuth, and ſpeedily I tooke vp my *Iewel*, and tooke the Starres altitude, which that inſtant I founde to be 14. degrees. This done, I ſet the *Finitor* to our latitude vz. $51\frac{2}{3}$ and ſought where the $\frac{49}{49}$ azimuth, and the 14. almicantare did croſſe, and there I found the 16. parallel to cut them: whereby I concluded that the declination of *Canis maior* was 16. degrees, and that ſouthwards, becauſe I found the ſaid parallel on the ſouth ſide of thequinoctial, & what more might I there ſee, the houreline cutting there with the parallel, did ſhew me his houre diſtance from the ſouth, whereby I might haue knowne what it had beene a clocke, and his right aſcention alſo, as ſhalbe ſhewed in the 17. and 26. Chapters. Such is the redineſſe of this *Iewel*, that he would clappe thre or 4. of Gemma Friſius Chapt. into one, if I would ſuffer him, and ſometimes doth whether I will or no, except I ſhould make two Chapters of one thing, which I neede not for lacke of matter, as ſome haue done, that coyne a great many Chapters without ſubſtance.

Chapter 8.

How the latitude or poles eleuation may be had in any region by the ſtarres that neuer ſet.

TO thoſe onely that dwell vnder thequinoctiall, all the ſtarres of the firmament do both riſe and ſet euerie 24. houres: but to thoſe that haue either pole eleuated, by howe much the more it is eleuated, by ſo much the more number of ſtarres are continually aboue the horizon, and as many continually vnder, in ſomuch that where the pole is zenith, the one halfe is euer aboue, the other vnder. By thoſe ſtarres that are alwayes aboue any horizon may the latitude be thus found.

Take both the meridian altitudes of the ſtarre vz. the higheſt and loweſt altitudes in the meridian, as in the 7. there will be 12. houres betweene theſe altitudes, and yet oftentimes it may be done in a winters night, otherwiſe you muſt haue certaine monethes ſpace betweene. Adde the two altitudes, the halfe thereof is the poles eleuation.

Take here Gemma Friſius example. He ſaith at Louane the 13. of December 1547. about ſixe of the clock at night, he did diligently obſerue the greateſt altitude of the ſtarre in the taile of *Vrſa minor*, called *Cynoſura* which we call the pole ſtarre, to be 53. degrees, 58. minutes, the next day about ſixe of the clocke in the morning he obſerued his meridian altitude, and found it 47. deg. 43. min. theſe two added make 101. degrees 41. min. the halfe hereof vz. 50. degrees 50. minutes is the height of the pole at *Louanium*.

Blagraue. But the cauſe why I tooke Gemma Friſius example, is to ſhew you that it appeareth by his obſeruation, and the middle part of the Chapter laſt before, that the ſtarre which the common ſort and vnlearned do take for the pole of the worlde is diſtant 3. degrees 8. minutes from the pole, the pole it ſelfe being no ſtarre, but onely a point imagined 90. degrees diſtant from ech part of thequinoctiall. Farther you muſt note that this ſerueth but for ſtarres equall or leſſe diſtant from the pole, then the zenith: but where as the one meridian altitude ſhalbe north from the zenith, and the other ſouth, adde them as before, and halfe the product is the declination of the ſtarre, ſubduct that out of the greateſt altitude, there reſteth thequinoctials height, which ſubducted from 90. leaueth the latitude.

Chapter 9.

The latitude knowne, how to finde what day of the moneth it is, though it were loſt, as they ſay, by the ſunne ſhining at noone.

IN the 3. Chapter is ſhewed by the theoricke to get the ſunnes place by the day giuen. But here we ſuppoſe the day vnknowne: or elſe admit we would examine the account made by Alphoſus tables. Firſt gette the latitude as before, then take the meridian altitude of the ſunne, and thereby get his declination by the 7. and whether it be northwards, or ſouthwards: if therefore the declination be north, reckon it on the *Iewel* among the parallels towards the north pole: but if ſouthwards then among the ſouth parallels, and the ſame parallel ſo reckoned from thequinoctiall, ſhall cut the ecliptick in the degree that the ſunne is in at that inſtant.

Example. In Autumne 1580, on a day vnknowne for want of an Alminacke, I tooke the meridian altitude

tude of the ſun 32. deg. ſo that by the 7. I found his declination ſouth 9. deg. Therefore among the parallels ſouthwards I reckon 9. I finde that the 9. parallel doth cutte the $22\frac{1}{2}$ of ♎, or ♓, but becauſe it is in Autumne, therefore it muſt needs be of ♎, if you doubted whether it were Autumne or ſpring, obſerue the altitude againe a day or two after, and if it be leſſe it is Autumne, if greater, the ſpring. But now hauing the degree of the ſunne in the zodiack, to finde the day of the moneth it is but *Vice verſa* to the 3. Chapt. and 4. for euen as in the 3. hauing the day, you ſeeke the ſunnes deg. ſo heere hauing the deg. you may in the circle of monethes of the theoricke, ſee the day anſwerable accordinglie, thus: ſeeke in the theoricke $22\frac{1}{2}$ deg. of ♎, and there ſhall you ſee to anſwere the 5. day of October your deſire. *Hoc rudiori minerua.*

But this obſeruation although it be excellent, by which both the quantitie of the yeare, and ſunnes motion hath beene by old practiſers obtained, yet doth it require inſtruments verie large, and for all that about the Solſtices it is ſcarſe exquiſite, becauſe then the dayly aſcenſe of the ſunne cannot eaſily be diſcerned by inſtruments, by reaſon of the tranſuerſe ſight of the zodiacke.

Blagraue. I will ſhewe you another as reddie way, or reddier by my *Reete*, and the rather becauſe in ſmall inſtruments the ecliptick line may be either left out, or not exactly diuided in the *Mater*, and is not much different from my addition in the 4. Chapt. For hauing taken the altitude, and founde the declination as before, then reckon that declination on the *Label* from the *Limbe* inwards, vz. 9. becauſe it is Autumne: let the *Label* runne ouer the autumnall ſignes, which are ♎, ♏ and ♐, vntill the ſame 9. degree doe touch or croſſe the zodiack, & you ſhal finde that it wil light on the $22\frac{1}{2}$ deg. of ♎, euen as before, and if the *Label* be not deuided, turne the autumnall ſignes on the nooneline, and you ſhall finde the $22\frac{1}{2}$ of ♎ to touch the 9. deg. reckoned on the noone line from the *Limbe*: if it were for the ſpring time, then you ſhould take ♈ ♉ and ♊, if in winter ♑ ♒ ♓, in ſommer ♋ ♌ ♍, &c.

Further you ſhall vnderſtand that though Gemma Friſius appointeth this to be done, the ſunne being in the meridian, yet by my addition in the 7. Chapter, you may do it any time of the day, and as well or better towards night then at noone. For by the Azimuth and altitude of the ſunne, you may get his declination, as I there ſhewed you to get the declination of the ſtarre, and ſomwhat more eaſie is it in the ſunne: for you neede but hange a plum line in your hand betweene the ſunne and the *Iewel*, if the ſights be not long ynough, and then turne the rule about till the ſhade of the line fall equall with the fiduciall line of the rule.

Chapter 10.

To finde the right aſcenſion for any portion of the ecliptick, and what degree of the Equinoctiall doth coaſcend, or riſe togither with euery degree of thecliptick in a right ſphere.

Gemma Friſius. 12.

THecliptickeby reaſon of his oblique ſite to thequinoctiall, doth continuallie change his ſcituation in euery horizon: ſo that ſomtimes he is ſcituate ſomwhat right, and then goeth he ſlower, ſomtime more oblique, and then goeth he ſwifter. But thequinoctiall becauſe in euerie ſeuerall horizon, he keepeth one inclination, and forme of his angles in his motion, there doth alwayes riſe in equall time equall portions thereof: wherefore Authors haue meaſured the motion of thecliptick by thequinoctial partes. Therefore the portion of the equinoctiall, which riſeth with any part of thecliptike is called *Aſſenſio* in latin, in Engliſh a riſing, or aſcention. In a right ſphere, this riſing is called *Aſcenſio recta*, right aſcention: in an oblique ſphere, *Aſcenſio obliqua*, oblique or byas aſcention, and beginneth his number where 0. in ♈ cutteth the Equinoctiall.

This matter is eaſie by the *Iewel*. For hanging it by the handle or equinoctiall line of the *Mater*, the axtree-line repreſenteth the right horizon. And the meridians cutting euerie degree of thecliptick, if you follow them to thequinoctiall line (whereto the rule with his numbers muſt be ſet) do expreſſe what degrees of the one do riſe with the degrees of the other. Note alſo that euery meridian is a right horizon to ſome place or other.

Example. Take the *Iewel* in your hande, and ſet the rule to thequinoctiall line, becauſe of the diuiſions, and then if for the right aſcention of the 5. degree of ♉, you follow the meridian, cutting the ſame degree of the ecliptick line drawne on the *Mater*, till you come to the Equinoctiall: there the rule ſheweth you $32\frac{2}{3}$ deg. and a little better, which is the right aſcention of 5. deg. in ♉. And for the 5. degree of ♍. The ſame meridian ſheweth in the rule $212\frac{2}{3}$ degrees and better, for the vppermoſt numbers of the rule, ſerue the vppermoſt carectars of thecliptick line vz. from ♑ to ♋, and the lowermoſt the other vz. from ♋ to ♑. And now it is eaſie to know how many degrees of thequinoctial riſe with any whole portion of thecliptick: as for example, with the whole ſigne of ♉, the two meridians comprehending it, do ſhew in the rule 29. degrees 54. minutes of thequinoctiall.

Chapter 11.

The ſame more compendiouſly by the Reete.

Gemma Friſius. 13.

PLace the *Reete* ſo, that 0. in ♈ of the zodiacke touch the north pole in the *Mater*, and 0. in ♎ the ſouth: and now the *Limbe* heere repreſenting thequinoctiall, as I haue ſhewed in the 2. Booke 10. Chapt. is deuided accordingly, the numbers beginning from the ſaid interſection at 0. ♈ in the centre being now the pole or poles of the worlde. It muſt needs be therefore that euerie line imagined from the centre to the,

Limbe

Limbe repreſenteth a meridian, which cutting the zodiack of the *Reete* performeth the ſcope of the laſt Chap. For lay the *Label* on any degree of the zodiacke, and it preſently ſheweth in the *Limbe* his right aſcention: this needeth no example.

Note that oppolite ſignes and alſo ſignes equally diſtant from thequinoctiall points do aſcende in a right ſpheare in equall time as ♈ ♓ ♎ and ♍ likewiſe ♉ ♌ ♏ and ♒: alſo ♊, ♋, ♑, ♐.

Note alſo being that euerie Meridian is a right horiſon to ſome place, as in the laſt cap. I ſayd, therefore the like degrees of thecliptcke and thequinoctial muſt needes cut togither the meridian of any place as do cut in any right horiſon, but then are not called right aſcention, but cælimediation or *Culmination*, whereby you ſee that the right aſcention and *mediation* or *Culmination* are all one in ſubſtance: but yet to ſay the trueth, the right aſcenſion is properly the degree of thequinoctiall, the Culmination is the degree of thecliptike coaſcending the right horizon with any ſtarre.

Gemma Friſius 14. Contrariwiſe to any right aſcention propoſed any man may gather the degrees of thecliptick coaſcendaing performed, *viſa verſa*.

Chapter 12.

What point of thecliptcke it is, as well in the firſt quadrant as in the reſt alſo, that hath the greateſt difference betweene the arch of thecliptcke, and the arch of thecquinoctiall coaſcending with him.

Gemma Friſius 15.

THis working requireth heede. Lay the rule vpon the pole circle of the *mater*, moving it to and fro thereon, in ſuch wiſe that the pole circle do cut like degrees in the rule numbred from the centre, as the rule doth cut in the *Limbe* numbred from thecquinoctiall, and then reckon the degrees in the *Limbe*, from the pole to the rule, the ſame is the declination of the part ſought for, then by the declination ſo known, the point of thecliptickeis had by the 9. Chapter.

Example. Let the pole circle (to auoide profers for breuities ſake) cut $73\frac{1}{4}$ deg. in the rule, the ſame number you ſhal finde the rule to cutte in the *Limbe*. Then betweene the pole & the rule reckoned on the *Limb*, are $16\frac{3}{4}$ degrees, the declination of the point ſought for. Therefore by the 9. to that declination, there anſwereth $16\frac{1}{4}$ degrees of thecliptick e numbred from o in ♈ or ♎, ſo that in the firſt quadrant it lighteth in the $16\frac{1}{4}$ degrees of ♉ in the ſecond, the $13\frac{3}{4}$ of ♌ in the third, $16\frac{1}{4}$ of ♏, in the fourth, $13\frac{3}{4}$ of ♒. now becauſe the right aſcenſion of the $16\frac{1}{4}$ deg. of ♉ is $43\frac{3}{4}$ deg. of thecquinoctiall, and that that part or degree of thecliptick is diſtant from o in ♈ $46\frac{1}{4}$ deg. ſubduct the leſſe from the greater, there reſteth $2\frac{1}{2}$ deg. almoſt which is the greateſt difference betweene the arches of thecliptcke and thequinoctial in theaſſenſions of the right ſphere which hapeneth in the ſaid 4. places.

Note that if the greateſt declination of the ſunne vz. $23\frac{28}{60}$ deg. ſhoulde in time alter to be more or leſſe as it hath in time beene found leſſe and leſſe by *Ptolomeus*, *Vernerus*, *Copernicus* and others, then would the number afore vz. $73\frac{1}{4}$ deg. alter by reaſon the pole circle then altereth alſo, or elſe not.

Chapter 13.

Of the difference of aſcenſions.

Gemma Friſius 16.

IT appeareth by the laſt Chap. that the greateſt inequalitie of aſcenſion in a right ſphere, cannot be aboue $2\frac{1}{2}$ degrees betweene the riſing of the degrees of the quinoctiall and thecliptcke: but in an oblique horizon there hapeneth great varietie and difference, becauſe thecliptcke and the horizon make for greater mutations of angles, and inclinations, then in a right ſphere. VVhereby the ſigne ♈ hath a farre deale leſſe aſcentiſion in regions, declining to the North, then in a right ſphere: and contrariwiſe ♎ far greater: So hath ♉ leſſe then ♏, and ♊ then ♐, which in a right ſphere haue equal aſcentions. The cauſe is for that thoſe riſe aboue the horizon in a more oblique ſcituation, and theſe in a more right, inſomuch that the ſixe ſignes from the beginning of ♋ vnto thend of ♐ are called right aſcending, the reſt oblique aſcending. And the more the ſphere is bending towards the poles, the greater is this diuerſitie: this difference of aſcentions, either in exceſſe or decreaſe, from thoſe of the right ſphere, is called *Aſcenſionum differentia* (that is) the difference of aſcenſions, yet looke how much wanteth to ſignes oblique, aſcending of their right aſcenſions, ſo much againe the oppoſite ſignes right aſcending do encreaſe: ſo that the right aſcentions of two oppoſite figures are equall to the oblique aſcenſions of the ſame ſignes taken togither. But now to the matter.

How to finde the difference of aſcenſions, place the *Finitor* to the latitude, then ſeeke in thecliptick line that degree whoſe aſcenſionall difference you require, and followe the parallel, cutting that degree vnto the *Finitor*, marke what meridian cutteth the *Finitor* there with the ſame parallel, looke howe many degrees diſtant that meridian is from the axtree line, ſo many is the difference of aſcention of that degree of the eclipticke.

Example. I would know the aſcenſionall difference of the beginning of ♉, I place the *Finitor* to the latitude vz. $51\frac{2}{3}$ here at Reading: then following the parallel, cutting o. in ♉ vnto the *Finitor*, I finde betweene the axtree line & the ſame point of the *Finitor* $14\frac{5}{60}$ meridians, therefore I conclude that the difference of aſcenſion of o in ♉, is 14. degrees, 5. minutes. And becauſe ♉ is a ſigne oblique aſcending (as is ſaid) I ſay that he riſeth ſo much ſooner in our horizon then in a right horizon, as 14. deg. 5. min. commeth to. The like diuerſitie of aſcenſion hath the beginning of ♏, which is oppoſite to ♉, but that you muſt take away the diffe-

rence

serue from the right ascension in the first, where in this you must adde it, and so is it in all opposite signes and therefore to saue labour, the one being had the opposite is knowne.

Chapter 14.

The same very plainely by the Reete.

Gemma Frisius 17.

YOur horizon knowne by the 2. Booke 20. Chapt. If you would know the ascentionall difference of any degree of the North signes, lay the same degree to cut on the northeast quarter of your horizon, and the rule thereto, and then numbring on the *Limbe* the distance betweene the rule so placed, and theast, or 6. of the clocke line, the same is your desire. If you seeke this difference for south signes, you must lay the degree on the south east part of your horizon, and then doe as before, but you may as is saide, to saue labour, know by the one halfe the oppolites, yea by one quarter the whole for this matter, if you take them alike from o. in ♈ and ♎ for ♓ ♈ ♍ and ♎ haue like difference, ♒ ♉ ♏ and ♌, so ♑ ♊ ♎ and ♍, but not in like degrees as the oppolites haue.

Chapter 15.

To know the oblique ascensions.

Gemma Frisius 18.

BY the 11. and 12. you learned to finde the right ascension of any part of the zodiacke, and by the 13. the difference of ascentions, these knowne, then to the right ascentions of the south signes, add the difference, and from the right ascentions of north signes, take the difference of ascention, and so haue you your desire.

Example. The right ascen. of the 4. deg. of ♊ by the 11. & 12. is 62. degrees, the difference by the 13. is 28. deg. 18. minutes: now because ♊ is a north signe, I subduct the difference vz. $28\frac{18}{60}$ out of 62. the right ascension, there resteth 33. deg. 42. minutes the oblique ascension. Likewise the right ascension of his opposite deg. vz. the 4. of ♐ is 242. to it I adde the difference vz. 28. 18. there commeth 270. deg. 18. minutes, the oblique ascension, and so may you do for euerie eleuation or latitude.

These preceps though they shew barren, yet haue they great vses in dimensions of times, and inquisions of the heauenly motions.

Chapter 16.

Of descentions both right and oblique, that is, what degree of the equinoctiall doth descend with the ecliptycke.

Gemma Frisius 19.

TRuely the *Iewel* it selfe sheweth it, and therefore I will not stand vppon it, you must worke for them in the west parts of the horizon, as before in the 14. Chapter you did on theast, the ascensions and descensions in a right sphere are all one, and the difference in the oblique are all one, onely in this it altereth, that if you added the difference for the oblique ascension as in the 15. you must substract it for the descention, and so contrariwise.

A briefe rule. Adde 180. vnto the oblique ascension of the opposite degree to the degree proposed, and so haue your desire.

Example. For the oblique descension of the 4. of ♊ adde 180. to the oblique ascension of the 4. of ♐, which is 270. degrees 18. minutes, as before, it maketh 450. 18. minutes, the oblique descension of the 4. in ♊ if it had not exceeded 360. But now you must out of this vz. 450. 18. minutes, cast 360. the remaine vz. 90. deg. 18. minutes is the descension sought for.

Chapter 17.

The longitude and latitude of any starres proposed, to know their right or obliqne ascention, and how much they decline from the equinoctiall, and how to place starres in the Mater of the Iewel.

Gemma Frisius 20.

HEere Gemma Frisius bursteth out in these words. This instrument, saith he, hath such aboundant and wonderful vse, that I scarse know to set it downe orderly, and therefore am forced oftentimes to reduce many things into one Chapter. How then (gentle reader) shall my poore capacitie being euerie way too much inferior to Gemma Frisius, set downe and performe so great a charge orderly, as my *Iewel* casteth vpon me farre surmounting the other instrument, and well the worse, being, that this wrangling world yeldeth me, time but by snatches.

But to the matter: set the *Finitor* euen with the ecliptycke line of the *Mater*, and so do the azimuthes represent the longitudes, and the almicantares the latitudes of the ecliptycke, therefore reckon the longitudes and latitudes of your starre, which in diuerse tables you may haue amongst them: looke where they crosse, there is the starres place in the *Mater*, then marke what meridian and parallel of the *mater*, doe crosse the same place of the starre so founde, the one sheweth his right ascention, as in the 10. and the other his declination as in the fourth.

Example

Example. The faire red ſtar of the firſt light of the nature of ♂ called *Oculus tauri*, the buls eye, riſing alwaye ſo ſight about 2. perches vnder the ſeuen ſtarres: This ſtar I finde by Stadius tables, who is one of the neweſt writers, his longitude to be 63. degrees, 40. min. or in ♊ 3. deg. 40, min. all is one, and his latitude 5. deg. & 10. min. Southwards, this knowen, I ſet the *Finitor* to the Ecliptike lyne, ſo that the Zenith be towardes the South pole, becauſe the ſtarres latitude is South. Then on the *Finitor* or Eclipticke, which heere are all one frō ♈ towardes ♋, I reckon the ſtarres longitude, vz. 63. degrees 40. min. or the 3. deg. 40. min. of ♊, and in the azimuth iſſuing from that pont of the Eclipticke, I number his latitude, vz. 5. degrees 10. min. in the ſame point, that is to ſay, where the $63\frac{40}{60}$ azimuth, and the $5\frac{10}{60}$ almicantare do croſſe ech other on the *mater*, is the ſtarres place, and there may you graue him in and write his name: but becauſe in ſmall inſtruments it would hurt the worke. and alſo for that the ſtarres do remoue in 100. yeares almoſt a degree, therefore make but a pricke with yncke for the time being, and then turne away the *Finitor* vnto the latitude vz. $51\frac{2}{3}$ deg. being our eleuation here at Readinge. Now the rule beeing ſette to the equinoctiall may you ſee the $62\frac{1}{2}$ meridian, cutting the ſaid prick or ſtar ſo placed, which ſheweth his right aſc. to be 62. deg. 30. min and the $15\frac{5}{6}$ parallel ſhewing his declination to be 15. deg. 50. min. & that towards the north from the equinoctial though he were ſouth from the eclipticke, laſtly for his difference of aſcenſion, you may eaſily know as in the 13. by following the ſtarres parrallel to the *Finitor*, and there take the number of the hourelines to the axtree line which here are 11. and better, by the which and the 15. Capt. you may gather his oblique aſcenſion. For becauſe the ſtarre is on the north part of the equinoctiall, therefore ſubtract the difference vz. 11. from the right aſcention before found vz. 62. deg. 30. minutes and there reſteth 41. degrees, 30. minutes, the oblique aſcenſion: but if you adde the difference vz. 11. vnto the right aſcention vz. 61. 30 thereof commeth 92. degrees 30. minutes, the oblique aſcenſion. But if it happen ſo that the ſtarres parallel touch not the *Finitor* then being a north ſtarre, he neuer ſetteth in that region, or being a ſouth ſtarre, he neuer riſeth there.

Chapter 18.

The declination of fixed ſtarres propoſed with their latitude, which neuer altereth, how to finde their longitude in the zodiacke.

Gemma Friſius. 21.

THe like Hipotheſis did Ptolomeus vſe, & before him Hipparchus Samius, whereby they found that the ſphere of the fixed ſtarres, had alſo a motion contrary to the firſt mouer, and that it is perfourmed according to the motion of the Zodiacke, and about his poles. For inſomuch as the latitude of all the fixed ſtarres from the Eclipticke, hath alwayes beene found one, and neuer altering, both of old writers and alſo of thoſe which now do obſerue the motions out of the heauen it ſelfe, and that it is moſt certaine that the ſtarres do continually goe forwards (from the Equinoctiall points, deſcribing circles parallell to the Eclipticke) they cannot therefore performe thoſe motions vpon any other then his poles. And hereby it commeth to paſſe, that although they be always of like diſtance from the Eclipticke, yet they come neere or depart further off from the Equinoctiall.

It is ſhewed in the 7. Chapter how to get their declination from the equinoctiall: their latitude may be taken out of Ptolomeus tables, or any other, becauſe they neuer chaunge. Let vs therefore take an example out of Ptolomeus, who ſaith that Timochares which liued 432. yeares before him, did obſerue the ſtarre called Spica, in his time to be declining north from the equinoctial 1. deg. $\frac{2}{3}$ that is 24. minutes, but in the time of Ptolomeus, he was ſouth from the equinoctiall halfe a degree, my deſire is to knowe the true places of Spica, that is his longitude & latitude for both thoſe times propoſed. The latitude of Spica is 2. degrees ſouth from the eclipticke, therefore I place the *Finitor* to the eclipticke, the zenith towards the ſouth pole of the *mater*, for that the latitude is ſouth: and there do I note where the latitude of Spica being here the ſecond almicantare, and his declination for Timocharis time being $1\frac{2}{3}$ parallel northwards, do croſſe the azimuth paſſing on the ſame croſsing to the eclipticke, doth ſhew the 22. deg. of ♍, the longitude of Spica for Timocharis time. In like manner working, you may finde his longitude for Ptolomeus time, to be in the $26\frac{1}{3}$ degrees of ♍.

VVhereby alſo Ptolomeus did gather that the fixed ſtarres did go forwards one deg. in an hundred yeares according to the ſequele of the ſignes, becauſe betweene him and Timocharis paſſed 432. yeares. But nowe ſinſe Ptolomeus obſeruations, the fixed ſtars haue gone forwards 21. deg. almoſt, for now Spica is in the 17. of ♎ 36. minutes. Therefore to one degree there cannot alwayes be giuen an 100. yeare: ſince there are but 1406. yeares paſſed, but making eſtimation according to this rate, the ſtarres ſhould bee mooued one deg. in 67. yeares, yet we muſt account of ancient writers, as we would haue ours accounted of, and confeſſe that there is a certaine *Anomalion* or inequallitie of motion in the fixed ſtarres, which no man hath hither vnto better coequated then Copernicus.

Chapter 19.

The difference of aſcenſion of any part of the zodiacke or fixed ſtarre propoſed, or the oblique aſcenſion to know what the latitude of the countrie is.

Gemma Friſius 22.

NUmber the difference of aſcenſion in the parallel of the place, point, or ſtarre, propoſed from the axtree line leftwards, for north parts or ſtarres: and rightwards for ſouth, thereto ſet the *Finitor* which ſhall ſhew in the *Limbe* the poles eleuation, but if the oblique aſcention be propoſed, get the difference by the 15. vz. ſubducting the oblique aſcenſion out of the right aſcenſion, if the part or ſtarre be north: but if

it be ſouth, then ſubduct the right aſcenſion out of the oblique, ſo haue you the difference, and then worke as before.

Example. I would know in what region the beginning of ♋ doth riſe with 60. degrees of the equinoctiall In a right ſphere it riſeth with 90. degrees. Now becauſe it is a north ſigne, I ſubtract the oblique aſcenſion out of the right aſcenſion vz. 60. out of 90. there reſteth 30. degrees, the difference of aſcenſion: then in the parallel drawne by 0. deg. of ♋ which is the verie tropicke, I number 30. deg. reckoned by the houreline from the axtree line towards ♈, and thereto lay the *Finitor*, it ſheweth in the *Limbe* the 49. latitude almoſt: By this Chapter, the diſtinction of the Geographicall parallels, and climates are performed very eaſily.

This Chapter is alſo performed eaſily by the *Reete*, numbring in the *Limbe* the difference of aſcenſions from 6. of the clocke downewards for north ſignes and ſtarres: and vpwards for ſouth, and thereto ſet the rule, then vnder the fiduciall line of the rule, ſet the deg. or ſtarre propoſed, and marke which horizon it toucheth, ſo haue you your deſire.

Chapter 20.

To finde the amplitude of the riſing and ſetting of the ſunne or ſtarres.

Gemma Friſius 23.

ALl men know that euery horizon hath foure principall points, videl. eaſt, weſt, north, and ſouth. There is neither the ſunne, nor any ſtarre that riſeth full eaſt, except he be in the equinoctiall where the ſunne is neuer, but being in the beginning of ♈ & ♎, at al other times he is either north or ſouth & riſeth north-eaſt or northweſt accordingly as in the 2. Booke 23. Chapt. is ſhewed. Theſe points in the ſunne riſing differ euerie day, and are called *Amplitudo ortus*, and thus are had.

Place the *Finitor* to the latitude, and follow the ſtarre or ſunnes parallel vnto the *Finitor*, the deg. betweene that point and the centre is the amplitude.

Example. The ſun being 0. deg in ♋, follow the tropicke parallel vnto the *Finitor*, placed to the $51\frac{2}{3}$ latitude it cutteth on the *Finitor* 40. degrees reckoned from the centre which is the ſunnes amplitude of riſing.

To do the ſame another way on the Horizons.

Blagrave. Place 0. deg. of ♋ on the northeaſt part of the $51\frac{2}{3}$ Horizon, and marke what parallel of the *mater* reckoned from the pole, doth cutte the ſame horizon in the ſame point, and you ſhall finde the 40. parallel agreeable to the former operation. This coulde neuer bee performed in this manner by any other Aſtrolabe: the reaſon heereof is for that the north pole of the *mater* repreſenting the aſt point, by the ſeconde booke 23. Chapter, all the horizons are by the parallels artificially deuided into 360. degrees. You may alſo get the amplitude of the ſunne by obſeruation, thus: place the *Iewel* flatte with the Horizon, ſo that the nooneline may lie iuſt in the meridian fixed, then when the ſunne is riſing turne about the ruler, till you may ſee him throw the ſightes, and reckon the degrees which the rule cutteth on the *Limbe*, from the noone line, which if they happen to be 90. then is there no amplitude, for the ſunne riſeth full eaſt, but if it be more or leſſe then 90. ſubduct the leſſe from the greater, the remaine is your deſire.

Example. The ſunne being 0. in ♋ at the riſing, the rule ſhall cutte 130. deg. reckoned from the noone-line, therefore taking 90. thereout it leaueth 40. the amplitude in our latitude.

Chapter 21.

The amplitude of the riſing of any knowne ſtarre giuen, or of the ſunne, with his place in the Zodiacke to know the latitude of the countrie agreeable.

Gemma Friſius. 24.

ADmit the amplitude of the ſunnes riſing being 0. degrees in ♋ were giuen to be 50. degres, to know in what latitude that happeneth, reckon 50. deg. on the *Finitor* from the center, and applie the ſame to and fro till it cutte the ſunnes parallel, which then is the tropicke, and you ſhall ſee in the *Limbe* the pole eleuated $58\frac{2}{3}$ degrees almoſt, the latitude ſought for, and ſo may you do with the amplitude of any ſtarre and his parallel.

To finde the ſame by the Horizons.

Blagrave. Reckon the amplitude of the ſunne or ſtarre from the pole among the parallels, and to that parallel lay the deg. of the ſunne or the ſtarres Apex, and the horizon paſſing there ſheweth the latitude. This likewiſe could not be done by any other Aſtrolabe before this.

Example. Admit I had taken the amplitude of the great ſtarre *Oculus tauri*, as before in the addition of the laſt Chapt. or elſe that it were otherwiſe propoſed to be 45. degrees, I reckon from the North pole the 45. parallel, and turne about the *Reete* till the ſtarres Apex do touch the ſame, and there preſently doe I ſee the $67\frac{1}{6}$ Horizon, wherby I conclude that where the amplitude of *Oculus tauri* is 45. the latitude of that countrie is $67\frac{1}{6}$ deg.

Chapter 22.

To know the riſing and ſetting of the ſunne, and the ſemidiurnall arch of the ſunne and ſtarres.

Gemma Friſius 25.

PLace the *Finitor* to the latitude, and then marke where the ſunnes parallel doth cut him, the houreline lighting there ſheweth your deſire.

Example. The ſunne being in the 10. degree of ♉, I woulde knowe his houre and time of riſing and ſetting, his parallel of declination by the 4. is 15. northwards, therefore I follow the 15. parallel to the *Finitor*, and the houreline cutting him there, doth ſhew me 5. degrees before fiue of the clocke, the ſunne riſing, that is 40 minutes, or $\frac{2}{3}$ of an houre after 4. and by the afternoone figures the ſame houreline ſheweth 5. deg. after ſeuen of the clocke the ſunne ſetting, that is 20. minutes, or $\frac{1}{3}$ of an houre after 7. for euerie deg. of thequinoctiall is 4. minutes of an houre.

Now for the ſemidiurnal arch the portion of the ſonnes parallel betweene the *Finitor* and the *Limbe*, or meridian ſheweth it videl. an 100. deg. or 7. houres and 5. deg. then reckon backe againe from the *Limbe* to the *Finitor*, or double it, & ſo haue you the whole dyurnal arch vz. 200 deg. or 14. houres 10. degrees, which taken out of the whole circle, which is 360. deg. or 24. houres, it leaueth 160. deg. or 9. houres, and 5. deg. the length of the night.

The ſame may be donē eaſilie by the *Reete*, place the deg. of the ſun on theaſt part of the horizon of your countrie, as in the ſecond Booke 23. Chapter, and thereon alſo lay the rule, and it ſheweth in the *Limbe* the houre of the ſunne riſing, from which houre, number the degrees vnto the nooneline, ſo haue you the ſemidiurnall arch.

But for ſtarres it is beſt to be done the firſt way bicauſe the ſtarres parallel in all reſpectes performeth it as the ſonnes parallel did.

Example. The *Finitor* being ſet to our latitude vz. 51 $\frac{2}{3}$, I follow the parallel of *Oculus tauri*, which either by the 7. or 17. I find to be the 15 $\frac{10}{60}$ parallel vnto the *Finitor*, and there do I ſee to be betweene the *Limbe* and it 111. degrees, or 7. houres, 6. degrees, his ſemidiurnall arch, which doubled, maketh 222. deg. or 14. houres and 12. deg. his diurnal arch, which is as much to ſay, as ſo many houres and deg. the ſame ſtarre doth remaine aboue the horizon in our countrie, and the ſaid 222. degree, or 14. houres, 12. deg. ſubducted out of the circle, leaueth his nocturnall arch, 138. deg. or 9. houres 3. degrees.

Heere note that the difference of aſcenſion of the ſunne, or any ſtarre learned by the 13. added to 90. maketh the ſemidiurnall arch, for North ſignes and ſtarres, but for ſouth it muſt be taken out of 90.

Alſo the houres of ſunne riſing, and ſetting added, alwayes make 12. alſo if you double the houre of ſunne ſet, thereof commeth the length of the day, that taken from 24. leaueth the length of the night, as if the ſunne ſet at 7. of clocke, double that, it maketh 14. houres the length of the day, the reſt of 24. is the night.

Chapter 23.

The quantitie of the longeſt day of any countrie propoſed, how to get the elenation of the pole there, and of the diſtinction of the climates and parallels.

Gemma Friſius 26.

IT is ſaide that at Conſtantinople the longeſt day is 15. houres, 12. minutes, the queſtion is, what the latitude is, there deuide the day in halfe, there commeth 7. houres 36. min. or 9. deg. for euery 4. min. are one degree, the houre of Sun ſet. Now therfore apply the *Finitor* to the place in the Sommer tropicke, where ie cutteth the 7. a clocke houre line, and 9. degrees paſt, and ſo ſhal it ſhew in the *Limbe* 43. deg. the height of the pole at Conſtantinople.

To doe the ſame by the Reete *and Horizons.*

Blagraue. Lay the rule vnto 7. a clocke and 9. deg. paſt at night, in the *Limbe*, and thereto lay the beginning of ♋ which ſhal cut among the horizons the 43. as before, which is the latitude ſought for.

Alſo if an ignoraunt man were in a ſtrange countrey, and did bring home what houre the Sun did ſette any day in the yeere, you might therby know the latitude thus. Admit he were at Conſtantinople the 20. day of May, and that then eyther by dyalles or clockes, he perceiued the Sun to ſet at 7. a clock and halfe an houre paſt, and that thereby I would know the latitude there: I lay the ruler on 7. a clocke and 7. deg. paſt on the VVeſt part in the *Limbe*, thereto doe I lay the degree of the Sun for that day, which I finde by the theoricke, as in the 3. chap. or otherwiſe by tables to be 9. in ♊, and there preſently doth the ſame 9. deg. of ♊ chooſe out the Horizon for Conſtantinople, as before: vz. 43. which is the latitude ſought, but when the Sunne is neere the Equinoctiall points, vz. 0. in ♈, or ♎, it can hardly be done, by reaſon the horizons be there neere together. You may performe the ſame the former way, by reckoning the 7. houres, 7. deg. on the Sunnes parallel being the 22. & thereto lay the *Finitor*, it ſheweth the latitude. Note that the farther off the ☉ is from 0. in ♈ or ♎, the more exact.

Gemma Friſius.

The climates are parallel circles on the earth correſpondent to the parallel of the ſphere, in whiche the longeſt day of the yeere is increaſed or ſhorned halfe an houre. The Geographical parallels are the like circles, encreaſing the longeſt day a quarter of an houre. The Climates begin from the fourth of theſe parallels, of which the Equinoctial is the firſt, ſo that the middeſt of the firſt climate hath his longeſt day, 13. houres which middeſt the 5. of theſe parallels doth determine. Looke hereofin my 1. Booke, chap. 10.

Therefore by the quantitye of the longeſt daye, you maye knowe vnder what parallell or clymate any countrey is ſcituate in this manner. Out of the number of the houres of the longeſt day, take 12. the remainder, reduce to quarters of houres, to which adding one the number and order of the parallel deſired, is knowen.

As in example, here at Reading the longeſt day is 16. houres, and a quarter of an houre long, & ſomwhat better,

better, I woulde knowe in what paralel we dwell, I take 12. houres out of the same $16\frac{1}{4}$ houres, there resteth 4. houres $\frac{1}{4}$, which maketh 17. quarters of houres, to them I adde one, it maketh them 18. I say therefore that we dwel in, or vnder the 18. parallel. And now hauing found the number of the parallel, take out of it 3. & halfe the remainder sheweth the order of the climate, as in example before, take 3. from 18. there resteth 15. the halfe, thereof being 7. and a halfe, sheweth the middle of the seuenth climate and a halfe past that is to say, we dwell in or vnder the very beginning of the 8. climate.

But if any man would proceede beyond the pole circle, to know the latitude where the longest day is two monethes, or 60. dayes, take halfe that number videl. 30. and in the ecliptick line, reckon so many vz. 30. from ♋ towards the centre, it endeth at 0. in ♌: I reckon then the distance of the parallel passing thereby from the pole, and finde it 69. deg. $\frac{4}{5}$ which I say is the latitude where the day is 60. dayes long.

By this Chapter any man may make tables of the parallels, and climes, and if he list, note them in the *Limb* of the *Iewel*, and may see the diuersitie of them for Ptolomeus time, when as the sunnes greatest declination was almost 24. deg. which now is scant $23\frac{1}{2}$ degrees. Note also that some haue begun the clymats from the equinoctiall, and some otherwise, &c.

Chapter 24.

To know what houre of the day or night the Moone, or any planet or starre riseth or setteth on the horizon.

Gemma Frisius 27.

TO performe this proposition you must haue the day giuen with the deg. of the sunne, the latitude: and whither he be a North or South starre, then proceede thus: lay the apex of the starre vpon the northeast quarter of the horizon proposed, if it be a North starre, or on the Southeast quarter, if it be a South starre, then the rule laide to the degree of the sunne, sheweth in the *Limbe* the houre required.

Example. The first day of Ianuarie 1580. the sunne being by the 3. Chapter in the 21. of ♑, I assaide to know what time the great starre called *Canis maior*, the great dogge did rise on our horizon at Reading. Now because he is a south declining starre, I laid his apex on the Southeast quarter of the $51\frac{2}{3}$ horizon, and then turning the rule to the 21. of ♑, it shewed me in the *Limbe* 6. of the clocke and 5. degrees past and better: whereby I concluded that in our horizon or latitude at Reading, *Canis maior* did rise the said first of Ianuarie 5. degrees, or 20. minutes and better after sixe of the clocke at night. Likewise for his setting the same day (or rather night I might say) I laide the starres apex on the Southwest part of the same horizon, and the *Label* to the saide degree of the sun, it sheweth in the *Limbe* 3. of the clocke and 8. degrees past, that is 32. minutes after 3. of the clocke in the morning, at which time *Canis maior* did set againe vnder our horizon that day.

But if you would know the rising and setting of any starre that is not made or placed in the *Reete*, then by the 21. get his semidiurnall arch, and reckon the same on the *Limbe* from the noonepoint Eastwards, and thereto lay the rule, then turning about the *Reete*, place euen with the rule the deg. of mediation, or culmination of the star, that is, the deg. of the ecliptick, rising with the same starre in a right horizon by the 11. Chapter: that done, turne the rule without mouing the *Reete* vnto the degree of the sunne, & it shall shew as before, the houre sought for.

Admit for example that *Canis maior* were not in my *Reete*, therefore I gette his semidiurnall arch, which by the 21. Chapt. I finde to be in 68. deg. $\frac{3}{4}$ or 4. houres, and $8\frac{3}{4}$ degrees all is one, which being reckoned on the *Limbe* from noone Eastwards, I lay thereto the rule, then by the 11. finding that the degree of culmination of *Canis maior* is the $6\frac{1}{2}$ deg. of ♋, I lay the same euen with the rule, and there the *Reete* resting, I turne the rule to the degree of the sunne, as before vz. 21. of ♑, there I finde in the *Limbe* 5. degrees after 6. of the clocke, as before, the rising of *Canis maior*: and for his setting I reckon $68\frac{3}{4}$ deg. from noone westwards, and thereto lay his culmination vz. $6\frac{1}{2}$ of ♋, and thence turning the rule to the 21. of ♑ it sheweth there in the *Limbe* 8. deg. past 3. in the morning, his setting, as before. Remember alwayes in your working of starres that *Locus solis ostendit horam*.

Likewise for the Moone or any planet hauing no latitude from the ecliptick, which seldome happeneth but they haue, laide the deg. of the zodiacke on the horizon, as you did the starres apex, and then *Locus solis ostendit horam*: Looke more in the 33. Chapt. of the 2. Booke.

Blagraue. You shal vnderstand, if the moone or any planet be much in latitude, as they cannot be aboue 6. deg. and the moone not aboue 5. at the most, that then there may happen a great diuersitie of their rising and setting, if you should but onely lay their deg. of the zodiacke to the horizon as before, which the expert practiser for all the planets will easily remedie: but for the moone you haue more to do, for if you take her deg. out of the ephemeris, or the common Almanacks, the same is but her degree at noone for that day, for at 6. of the clocke at night, she wilbe 3. degrees farther in the zodiacke: the cause is, she walketh the whole zodiacke in a moneth, and euery 2. houres after a sort one deg. and somwhat more: wherefore you must adde for euery two houres past noone, one deg. & before noone subduct, and yet commonly in tables the day endeth at noone, & this day beginneth yesterday at noone.

Chapter 25.

To know what starres neuer rise or set to euery region, and which are verticall.

Gemma Frisius. 28.

FOr these take these general preceps, that star whose declination is equal to the latit. so that the declination & latit. be both north, or both south, doth still once in 24. houres come to the zen. as appeareth by the zen. point of the *Iewel*, which toucheth in our latit. the $51\frac{2}{3}$ parallel: also those stars whose declination southwards is equal to, or greater thē thequinoct. height, are neuer seene in that country, as for example set the *finitor* to our

latitude 51 $\frac{2}{3}$ you ſhall ſee on the *Iewel* that the 38 $\frac{1}{3}$ parallel ſouthwards doth but touch the *Finitor*, the reſt beyond him neuer are ſeene: contrariwiſe thoſe North ſtarres whoſe declination exceedeth the equinoctials height, do neuer ſet, as by the *Iewel* may apparantly be ſeene. Farther if you would know to what countrie any ſtarre or ſigne is verticall, ſeeke his declination, and where his parallel lighteth on the *Limbe*, thereto ſet the zenith point, and ſo the *Finitor* ſheweth the latitude.

Chapter 26.

Of the Coſmicall Eliacall and Acronicall riſing and ſetting of the ſtarres.

Gemma Friſius 29.

THe Coſmicall riſing is when as the ſtarre riſeth on the horizon with the ſunne neare one inſtant, and is had if you place the ſtarres apex on the Eaſt part of the horizon, the deg. of the zodiacke cutting then the horizon, is your deſire, then by the theoricke you may finde the day of the yeare anſwering thereto. But if the ſtarre deſired be not in the *Reete*, then reckon his ſemidiurnall arch, from noone Eaſtwards, and thereto lay his degree of culmination, had by the 11. and marke what deg. of the zodiacke, doth cut the horizon on the Eaſt part, the ſame is alſo your deſire.

The Coſmical ſetting, is when any ſtarre doth ſet at one inſtant when the ſunne riſeth, or neare it, which is as eaſily had as the other by laying the ſtarres apex on the VVeſt part of the horizon, and then the deg. of the zodiacke riſing on the eaſt is your deſire: if the ſtarre be not in the *Reete* reckon his ſemidiurnal arch Weſtwards, and lay thereto his culmination, and ſo againe the aſcendent is your deſire, euen as before.

The Acronicall riſing is when a ſtarre riſeth the ſunne ſetting, place the ſtarres apex on the eaſt part of the horizon, and then marke the deg. of the zodiacke, deſcending on the VVeſt, for when the ſunne comme th thither, then doth that ſtarre riſe *Acronice*, and this may you do by his ſemidiurnall arch alſo as before.

The Acronicall ſetting, is when a ſtar ſetteth with the ſunne, and is eaſie to be vnderſtood by the premiſes.

The *Heliacall* riſing of a ſtarre, is when he getteth out of the ſunne beames, and the *Heliacall* ſetting, is when he getteth into the ſunne beames: but heere you muſt note that the greater ſtarres may be ſeene when they are more neare the ſunne then the leſſer. Starres of the firſt light are not ſeene except they be 12. degrees diſtant from the ſunne, of the ſecond light 13. of the third 14. of the fourth 15. of the fifte 16. of the ſixt 17. Then for the planets, Saturne requireth diſtance 11. degree ♄ 10. ♂ 11 $\frac{1}{3}$ ♀ 5. ☿ 10. but theſe be not alwayes certaine, for the planets turning in their epicicles, doe now ſhew greater, and now leſſe, alſo the inclination of the zodiacke to the horizon may ſomwhat alter them, and the ſtate of the aire alſo.

Chapter 27.

How to know what it is a clocke in the night time by the ſtarres.

Gemma Friſius 31.

THe ſunne onely ſheweth the houre at all times, yet by the ſtarres or planets in his abſence we doe finde him out, and ſo get the houre.

Take the height of the ſtarre, and marke whither he be Eaſt or VVeſt from the meridian (for always the farther from the meridian the ſtarre is, the ſurer you worke) then ſet the *Finitor* to the latitude, and reckon the ſtarres height ſo taken among the almicantares, & where that almicant. and the ſtarres parallel do croſſe the houreline cutting there, ſheweth the houre diſtance of the ſtarre from the ſouth, which is called *Hora ſtellæ*, the ſtarres houre: but now by this to get the true houre, reckon this houre diſtance of the ſtarre on the *Limbe* Eaſtwards, if the ſtarre were on the eaſt ſide, or weſtwards, if on the VVeſt: and thereto lay the ſtarres apex by helpe of the rule, or his degree of Culmination, if the ſtarre be not in the *Reete*, and the *Reete* there ſtanding turne the rule to the deg. of the ſunne, it ſheweth the houre ſought for on the *Limbe*. This asketh much buſineſſe by Gemma Friſius Curſor and Brachiolum.

Example. The 6. of October 1580. I went forth and tooke the altitude of a faire great ſtarre called Hircus the Goate 20. deg. being then northeaſt, whoſe declination is 45. degrees North, then placing the *Finitor* to the latitude I marked where the 20. almicantare cut the 45. parallel, the houreline croſsing them there in their point of meetings ſhewed foure houres & 2. degrees, the houre diſtance of Hircus from the north part of the meridian Eaſtwards: and that which more is then Gemma Friſius Aſtrolabe could ſhewe at one inſtant, the azimuth croſsing there alſo with them, ſheweth in what verticall circle the ſtarre was, but to the matter: Becauſe the ſtarre Hircus was on the eaſt ſide of the North part of the meridian; therefore in the Northeaſt quarter, I reckoned on the *Limbe* from North *Media nox* Eaſtwards, the ſtarres houre vz. 4. houres 2. deg. and thereto by helpe of the rule I laide the ſtarres Apex, the *Reete* there ſtanding, I turned the rule to the degree of the ſunne which then was in the 23 $\frac{1}{3}$ deg. of ♎, and found it pointing on the *Limbe* 7. of the clocke, and halfe an houre paſt within little the houre ſought for.

Gemma Friſius 32. Heere Gemma Friſius maketh a Chapter to reckon the houres from noone vnto 24. ending the next day at noone, as the Aſtronomers doe, and likewiſe from ſunne riſing or ſunne ſet to 24. as diuerſe nations do, but ſeeing it ſo eaſie and friuolous for our countrie I omit it: I ſhall adde much and therefore had neede be ſhort in ſome.

Chapter 28.

Of the vnequall or planetarie houres.

Gemma Friſius 33.

EVerie artificiall day is accounted betweene ſunne riſing and ſunne ſet: the night accordingly from ſunne ſet to the riſing, the 12. part of euerie ſuch day is called a planetarie houre, ſo that as the dayes are longer and ſhorter, ſo are the houres, and therefore called vnequall. They are called planetary becauſe auncient Aſtrono-

Aſtronomers haue held opinion that the ſeuen planets in order haue dominion, theſe houres giuing that planette to the firſt houre of the day, after which hee is called as on ſunday ☉ hath the firſt houre the next ♀ then ☿ ☽ ♄ ♃ ♂, which endeth in the 7. houre, and then beginne agayne the 8. houre with ☉, and then forwards ♀ ☿, &c. as before with the 12. night houres alſo, and you ſhal finde that ☽ will hitte righte to begin the 1. houre of the day following, whereof the day hath his name, which is the cauſe that the week dayes are not named according to the order of the planets.

Blagraue. Gemma Frilius appoynteth a particuler quadrant on the backe of the Aſtrolabe for the planetary houres, eyther the ſame, or very like it which Stophler appointed, but I like not to haue any thing particular to one latitude, in ſo general a *Iewell*. I would rather doe thus: deuide the ſemidiurnall arch by 6. and by the quotient, deuyde the arche of the houre ſought from Sun riſing: as for example, the Sun being in the beginning of ♋ the ſemidiurnal arch is 124. deuide that by 6. the quotiẽt is 20. $\frac{4}{6}$ deg. admit at 10. a clocke before noone, I would know the planetary houre, the arch thereof to the *Finitor* is 94. deuide that by 20. & let the $\frac{4}{6}$ alone, thence commeth 4 $\frac{14}{20}$ that is to ſay, the 4. planetary houre, and $\frac{14}{20}$ paſt, that is as good as $\frac{2}{3}$ partes.

But for thoſe that canne no Arithmeticke, an eaſier waye is thus, and for that cauſe chiefly did I diſtinguiſh all my meridians by fixes, with great ſtrokes: beginne from the *Finitor* with the next fixt meridian that cutteth the paralel of the ſun, and thence tell euery 6. on the ſame parallel with a ſticke as they ſay, vnto the *Limbe*, and count how many they be, and looke how many fixes you haue on that parallel, ſo manye degrees reckon to euery planetary houre on the ſame parallel, as before in the laſt example: you ſhall finde 20. fixes, and foure degrees ouer: reſpect not thoſe 4. but go on with 20. for the firſt planetary houre from the *Finitor*, and 20. for the next: and ſo on till you come to the 10. houre deſired, and if you will work exquiſitely for euery planetary houre reckoned, account 4. and then ſee how oft you can take out 6. & ſo many degrees, adde to the end of your reckoning, and ſo haue you the planetary houre exactly, as in example before, at 10. of the clocke, it was found the fourth planetary houre, almoſt the 5. therfore I heape 4. together 5. times, it maketh 20. thence, I can take 6. 3. times: which 3. I adde to the planetary houre before found: ſo that now it is the 4. $\frac{17}{20}$ houre, that is almoſt the fift.

Alſo the *Finitor* being ſet to the latitude, he that would deuide the 3. arches of the Equinoctiall, and the two tropicks, into ſixe equall partes betweene the *Finitor* and the *Limbe*, and ſo draw arches from tropicke to tropicke, by helpe of eche of theſe 3 pricks, might ſo haue the planetary houres for that latitude, but this being particuler, is not conuenient in the *Iewell*. But if any man did deuide euery 3. parallel betweene the tropicks in 6. parts, and did but only ſet great perceiueable pricks at euery part, it woulde ſerue and not muche hurt. This matter is not weighty, and therefore let euery man doe his pleaſure.

Chapter 29.

How by the Iewell *to know the Meridian or greateſt altitude of the ſunne and ſtarres.*

Gemma Friſius. 34.

THe innermoſt circle of the *Limbe* in reſpect of the *Mater*, repreſenteth alwayes the Meridian in euery latitude: wherefore the *Finitor* being ſet to the latitude, marke where the parallel of the Sunne or any ſtarre doth touch the *Limbe*, the almicantare touching there with him, ſheweth ſtreight the Meridian altitude.

The ſame another way.

Blagraue. Lay the degrees of the Sunne or ſtarres apex to the noone line, and if he bee a South ſigne or ſtarre, you neede but reckon on the noone line, the diſtaunce from your horizon vnto him, the ſame is your deſire. As for example, the 7. of ♎ layde to the noone line, ſeeke out the 51 $\frac{2}{3}$ horizon, chap. 29. and you ſhal finde the diſtaunce betweene it and the 7. of ♎, reckoning on the noone line 39 $\frac{1}{3}$ degrees. But if it bee a North ſigne or ſtarre then laid to the noone lyne, you muſt reckon from the horizon to the *Limbe*, and thence backe agayne towards the horizons, till you come to the degree or ſtarre, as if you would know the meridian height of the 7. of ♈ or ♍, lay either of them to the noone line, then from the horizon to the *Limbe* in our latitude is alwayes 38 $\frac{1}{3}$ degrees, and thence reckoning backe, you ſhall finde 3. degrees, which added makes 41 $\frac{1}{3}$ degrees, the altitude ſought, and ſo for ſtarres. 35 $\frac{1}{3}$

Chapter 30.

How to finde the degree of Medium Cœli, *or Culmination called middeheauen at any time propoſed.*

Gemma Friſius. 35.

THe degree of the Zodiacke being in the Meridian at any inſtant, is called in Latine the *Gradus medii Cœli*, of ſome *Culmen Cœli*, and *Cor Cœli*, that is the top and heart of the heauen.

By helpe of the *Reete*, it is eaſilye had, laye the degree of the Sunne in the Zodiacke vnto the houre and time propoſed, by helpe of the rule, and then looke what degree cutteth the noone lyne, the ſame is your deſire.

Example. the Sun being 6. degrees in ♌, I would know the degrees of the mydheauen or Culmination, at 8. a clocke in the forenoone, I ſet the 6. of ♌ vnto 8. a clocke by the rule, and then in the noone line towards the handle, I ſee there 9 $\frac{1}{2}$ deg. of ♊ the deg. ſought for, and in the ſame noone line vnderneath I ſee the 9 $\frac{1}{2}$ of ♐, which is called *Gradus Imi Cœli*, the deg. of deepe heauen or the Angle of the earth.

To

To doe the ſame by the *Mater* without the *reete*, firſt to get the right aſcention of the degree of the ſunne, and then his diſtance from the South in degrees and minutes, take this diſtaunce for forenoone howres out of the right aſcenſion of the Sunne, if it may not be, adde 360. to the aſcenſion, & then ſubduct it, but for afternoone howres, adde the diſtance to the aſcenſion, & if the whole ſumme exceede 360. caſt away 360. ſo haue you the right aſcention of the culmination, then the degree of the Zodiacke anſwering, is had by the 10.

Example. the Sun being in the 6. of ♌ as before his right aſcention is $128\frac{1}{3}$. & becauſe the howre before is of the forenoone diſtante from the meridian 4. houres or 60. degrees. I ſubduct 60. out of $128\frac{1}{3}$. there reſteth $68\frac{1}{3}$. degrees, the right aſcenſion of the degrees of culmination: then by the 10. Chap. I finde the degree of the Zodiacke, anſwering thereto, to be $9\frac{1}{2}$. degree in ♊ as before.

Chapter 31.

To know how much any degree or point of the heauen is diſtant from the Meridian at any time by degrees of the Equinoctiall.

Gemma Friſius. 36.

LAY the degree of the Sunne vnto the houre propoſed on the *limbe* by helpe of the rule, and then turning the rule about to any degree, ſtarre or other point in the *reete*, it ſheweth in the *limbe* the diſtance thereof from the meridian, either in degrees or houres. This propoſion is neceſſary for directions, and needeth no example. Note that for ſtarres not in the *reete*, their deg. of culmination ſerueth the rule applyed thereto.

Chapter 32.

How by the Iewell to know the height of the Sun aboue the horizon euery houre and time of the yeere ſitting within dores: and thereby to make tables to any latitude, for making of particuler Dials, as Cylinders, ringes, quadrants, & ſuch like.

Gemma. 37.

G*Emma Friſius* heere commendeth the profite of this Chapter, ſpecially for making tables for particuler dials, ſaying that it hath a wonderfull facilitie of working by his Aſtrolabe, but truly by my Iewel the working is an hũdred fold more eaſie: For my *finitor* being once placed to ♄ latitude, he that vnderſtãdeth the 6. chap. the 21. & 26. may ſee his table as readie made vpon his *Iewel* as *G. Friſius*, or any man els can make him the other way with 3. or 4. houres paines: and that which more is, you neede but the *finitor* vnto any other latitude, and ſo is your *Iewel* a table readie made alſo to the ſame, ſo that you may heereby peruſe tables for many latitudes in leſſe time then you could peruſe for one latitude after *G. Friſius* by his curſor and Brachiolum.

Example. the 13. of September 1588. the Sun then in the 0. degree of ♎ I woulde know of what height the Sun is euery houre that day, I place the *finitor* to the latitude vid. $51.\frac{2}{3}$. here at Reading, then do I marke what Almicantares do cut the Sunnes paralell being here the equinoctial, in the croſſing of euery houre line: I ſee at the *limbe* being 12. of the clock the $38\frac{1}{3}$. degrees or Almicantare: at 11. whiche alſo ſerueth for 1. in the afternoone 37. almoſt, at 10. & 2. $32\frac{1}{2}$. at 9. & 3. 26. at 8. & 4. 18. & better, at 7. & 5. 9. & at 6. & 6. 0. becauſe the Sun then ſetteth, & is nought aboue the horiſon. This doth *G. Friſius* aduiſe to ſet in a table. doing the like with euery paralel, or eucry 3. paralel at the leaſt, of the Sunnes declination, & when you haue done, your table ſhalbe no readier then my *Iewel*: for the paralel of declination knowne, I can goe vpon hym from houre line to houre line, either for quarter, halfhoure, or other minute, & there ſee preſently what Almicantare doth limit the height, which your table cannot do, except you make him almoſt infinit to euerie minute, for which purpoſe only my *Iewel* were worthie to be accounted of.

Chapter 33.

How to know the height of any ſtarre aboue the horizon ſitting cloſe within dores, and thereby to learne to know the ſtarres in the skie.

Gemma Friſius 38.

THis chapter bringeth any man to the knowledge of euerie ſeuerall ſtar & their names, and for other vſes is commodious: if the ſtar be in the *reete* whoſe altitude yee ſeek, then lay the degree of the Sun to the houre, & part at which you deſire, the height by help of the rule, & then turne about the rule to the ſtarres Apex, which ſheweth in the *limbe*, the ſtarres houre, or diſtance from the South, by the 30. and if you reckon on the rule the degrees, from the ſtarres Apex to the *limbe*: you haue the ſtarres declination alſo, and whether his declination be North or South I ſhewed in the 2. booke 21. chap. This done, ſet the *finitor* to the latitude, & then on the ſtarres paralel reckon from the *limbe* his houre diſtance, before found, & mark what almicantare doth croſſe there, the ſame ſheweth the ſtarres height for that inſtante, & the Azimuth alſo cutting there, ſheweth the verticall circle, in which the ſtar is, which is more then was propoſed.

But if the ſtarre be not in the *reete*, then muſt you get his declination by tables, & likewiſe his culmination, and vſe his degree of culmination in ſteed of the ſtarres Apex, as in the 24. 25. 26. &c. and then proceede as before in all reſpectes.

Blagraue Here *G. Friſius* ſetteth no example, but truly I think it good, becauſe this chapter brought mee too knowe the ſtarres as well in the skie as I did on my *reete*. The fifteenth of October 1576. about eyght of the

of the clocke at night, whileſt I was reading of Copernicus reuolutions, and Stadius, conferring them together, I found there that the anncient Aſtronomers appointed the firſt deg. of the Equinoctiall to begin from a ſtarre of the ſecond light, neere the Eclipticke called *Cornu arietis*, whoſe longitude was in olde time in the very vernal interſection of the eclipticke with the Equinoctiall, which ſtarre by reaſon of the motion of the ſtarres *Apogea*, or more plainly of 8. and 6. ſpeeres called *Motus preceſſionis Equinoctiorum*, is gone forwards 27. deg. 40. min. ſo that now he is in the 27. deg. 40. min. of ♈. For this cauſe I was deſirous to know that ſtarre in the sky as well as on my *Reete*. The Sunne then being in the $27\frac{1}{6}$ deg. of ♎, I layde that deg. to 8. a clocke in the *Limbe*, thence turning the rule to the ſtarres apex, I found in the *Limbe*, the ſtarres houre diſtance from the South 3. houres and 13. degrees, almoſt towards the Eaſt, & on the rule his declination $17\frac{1}{2}$ deg. VVherfore I ſet the *Finitor* to the latitude, and on the $17\frac{1}{2}$ parallel Northwards, I reckoned 3. houres and 13. deg. inward by the houre lines, and I found there croſsing the $33\frac{1}{2}$ almicantare the heighte of the ſtarre: & there preſently I ſaw likewiſe the 15. azimuth reckoned from the Eaſt or zenith line, to cut the ſame place of the ſtarre. Then with ſpeede I ſet the rule with his ſights on the backſide, ready vnto ſo many deg. vz. $33\frac{1}{2}$ aboue the line of leuell, and going abroade I turned my face towards the 15. azimuth as neere as I could geſſe from the Eaſt Southwards, and there might I ſee the ſtarre ſought for through the ſight holes, and ſo by noting his place and the ſtarres about him. I knew him euer ſince.

Gemma Friſius 39. To fynde in what verticall circle or azimuth the Sunne, or any planet or ſtarre is, anye time.

Blagraue. Here Gemma Friſius maketh a ſpecial chap. for that which my *Iewell* yeeldeth vnasked, as by the 6. 31. 38. and diuers other chap. appeareth. Note that before the lodeſtone, found, men ſayled by the verticall circles of the Sun and the ſtarres.

Chapter 34.

How to finde the Meridian line, and the foure quarters of the worlde called cardines, *diuers wayes, and ſpecially by the Aſtrolabe.*

Gemma Friſius. 40.

BY the Sun you may doe it any houre, firſt getting the azimuth by the 6. and then lay the rule to the lyke degree Eaſtwards, or VVeſtwards on the *Limbe*, as the Sunne ſhal then be, and the rule there fixed, lay the *Iewell* flat on a ſtoole, or by ſome deuiſe leuell with the horizon, and turne him about till the Sunne ſhine through both the ſight holes, and then will the noone line of the *Iewell*, lye euen in the Meridian, by which you may draw your line, but here had you neede of a wyre ſet vp plumme in the furthermoſt ſight hole, and ſo lette his ſhade couer the other, becauſe the Sunne beyng aloft cannot pearce the ſightes, as in the ſeuenth is ſaid.

Another way. Erect a ſtile or wyre plum on any horizontall plaine in the forenoone, and hang the *Iewell* that the Sunne pearce the ſight holes, and marke both what deg. the rule cutteth in the *Limbe*, and where the ſhade of the top of the ſtyle endeth on the plaine, and there ſet a pricke, let the *Iewell* hang ſtill, till the Sun in the after noone pearce agayne the ſights, then ſet another pricke where the ſtiles ſhade endeth, then frō the ſtile draw a line iuſt betweene thoſe prickes, the ſame is the Meridian, and croſſe the ſame ſquare with another, and that ſheweth Eaſt and VVeſt.

Another way. Make a conuenient circle on a plain in the centre, ſtick a ſtile plum, marke both in the forenoone and after noone, where the ſtiles ſhade toucheth that circle, then draw a lyne from the centre to the middeſt of that arche betweene the touches, the ſame is the Meridian.

Another. The Aſtrolabe placed on an horizontall plaine, as before, and the Sunne ſhining, at Sunne ryſing, and likewiſe at Sunneſetting through the ſight holes, the degree betweene both, ſheweth the Meridian line.

Another. Get the amplitude of the Sun by the 19. chap. number it on the *Limbe* from the noone line, and therto lay the rule and the *Iewel* leuel with the horizon, marke at Sun riſing, that the beames pearce the ſights and then is the noone line your deſire.

Blagraue, Though Gemma Friſius hath ſet downe wayes inough to find the Meridian, yet all of them depend vpon the ſhining of the Sun, and in the day time. Therefore I wil ſhew two other wayes in the night, by the ſtarrs, nothing inferior to the other. For, in like maner as you did the firſt way by the Sun, may you do by any ſtarre hauing gotten his azimuth by his houre diſtance from the South by the 26. or 32. the deg. of the azimuth, reckoned on the *Limbe*, and the rule fixed therto, laying the *Iewel* flatte, as before in the Sun, turne him til you may ſee the ſtarre, euen with the ſightes which here had neede be wyres both.

The ſecond way is that which I vſed, before I knew any other almoſt to ſet dyalles by, and in my opinion is nothing inferior to anye of thoſe before. Thus woulde I do whē I had made my horizontal dial, & ſodered on the cocke or *Gnomon*, I wold chooſe a bright ſtarry night, & then bring forth my dial, & ſet him on the poſt where he ſhould ſtand, then had I a playne ſquare little boorde, or rather then fayle, a fayre ſquare trencher, this ſquare boord would I faſten or hold perpendiculer, cloſe to the cocke, and ſo turne about cocke, dyall, boorde and all together, till I might ſee the laſt ſtarre of the tayle of *Vrſa minor*, which they commonly cal the pole ſtarre, euen with the boord, and then was I ſure the cocke ſtood North and South, and ſo would I naile my dyal on.

But in this way you muſt not neglect two things, one is that the boord ſtand plumme vp with the cocke, at the time of view. The other which truely in thoſe dayes I tooke no regard vnto, becauſe I knew it not, for I could make dyalles before I ſtudied Aſtronomy: and that is this: you muſt by helpe of your *Iewel*, chooſe your time, ſo that the beginning of ♈, or ♎, be in, or neere the meridian either aboue or vnder the earth. For when

when ☌ & ♈ are in the meridian, the pole ſtarre declineth 3. degrees from the meridian once euery night that time may be had, halfe an houres miſſe will not alter much, or els watch at any time: when the firſt ſtar of the tayle of *vrſa maior*, called *alioth*, and commonly the thill horſe of charles waine, doth ſtand directly ouer or vnder the pole ſtarre, as it were both euen with a plumme line, then be ſure the pole ſtarre is directlie North to ſtrike your true meridian by.

Chapter 35.

The meridian founde, to finde the Azimuth of the Sunne or ſtarres by ſight without the houre.

Gemma Friſius, 41.

THE noone line of the *Iewel* placed in the meridian on any horizontall playne by the 33. you may turne about the rule to the Sunne or any ſtar by helpe of wier ſights, as in the 33., and it ſheweth in the *limbe* your deſire.

Chapter 36.

The verticall circle of the Sunne or any ſtarre, the degree of the Sunne, with their altitude abone the horizon, giuen howe to knowe the latitude and what it is a clocke.

Gemma Friſius. 42.

TAke the Azimuth by the 34. and the altitude by the 1. reckon the ſame among your almicantares and azimuthes on your *Reete*, & mark where thoſe two croſſe one another, thē turn about the *reete* til the ſame point of croſsing light on the paralel of the Sunne or Starres declination, & the houre line lighting there with them, ſheweth the houre of the Sunne or Starre. The *Finitor* likewiſe ſheweth the latitude in the 7. This chapter is ſet downe è *conuerſo* almoſt: if you vnderſtand the 6. the 26. Chapter and others, this is very eaſie.

Chapter 37.

The latitude, the Sunnes Azimuth and almicantare giuen to finde his place in the Zodiacke.

Gemma Friſius 43.

ABout the Tropickes this rule is vncertaine, as in the 9. chapter is ſhewed, where in the end of my addition this Chapter is perfourmed.

Chapter 38.

To know what houre the Sunne or any Starre commeth to any azimuth propoſed.

Gemma Friſius. 44.

PLace the *Finitor* to the latitude, and marke what houre line cutteth the azimuthe propoſed on the Sunne or Starres parallel, the ſame is your deſire, ſauing for Starres remember, *locus ſolis oſtendit horam*, as in the 26.

Blagraue. I haue made ſhort of theſe laſt chapters without examples, becauſe my *reete* with the azimuthes and almicantares doth readily expreſſe them: but in deede by *Gemma Friſius*, working in his Curſor and Brachiolum, examples had been needfull, and ſpecially in this chapter, which by *G. Friſius* requireth great paines.

G. Friſius 45. & 46. Theſe chapters by reaſon of my new *Reete* haue no matter.

Chapter 39.

A Comet, a Planet, or an vnknowen ſtarre being ſeen in the skie, how to get his longitude and latitude in the Zodiacke, and how by the right aſcenſion and declination of any ſtarre to get his longitude and latitude.

Gemma Friſius. 47.

WHen you ſee any Comet, Planet, or Starre vnknowne: Firſt take his altitude, and the ſame Inſtant get the houre and minute by the 26. or otherwiſe very certainely, and his azimuth by the 34. Then ſet the *finitor* to the latitude, and marke what parallel cutteth the croſsing of the Comets, almicantare and azimuth, ſo found, the ſame is his parallel of declination, and the houre line cutting the ſame croſsing, ſheweth his diſtance from the meridian by the 26. This done, place the degree of the Sun to the houre in the *Limbe* before found: and then in the noone line may you ſee the degree of culminatiō whoſe right aſcenſion ſeeke by the tenth. Nowe if the Comet bee on the Eaſt part adde to this right aſcenſion his houre diſtaunce, if on the Weſt, ſubſtract it, and ſo ſhall you gette the right aſcention of the Comet, but if in adding, the number exceede 360. caſt away 360. alſo if the houre diſtance may not be ſubducted, then adde 360, and ſo the product is the Comets right aſcenſion. And nowe hauing the right aſcenſion & declination of

of the Comet, marke where they crosse on another, and make a prick with ynke, for there is the Comets very place in the heauens then: to know his longitude and latitude, set the *finitor* to the ecliptick, and so the azimuth and almicantare cutting the pricke or Comets place before made, sheweth your desire, as in the 17. This is a notable practise and hath wonderous vse in Astronomie, and therfore worthie of an example at large.

Blagraue. VVherefore I will shew an example, whiche I my selfe for mine owne delight and experience did obserue. The nienth of October 1580 walking foorth at night, as I commonly dyd, to behold the constellations, I perceiued Southwardes a shimering or dimme light, in bignesse to sight, about an hand breadth, but because the skyes were cloudie, I coulde not well iudge wheather it were a Comet or some starre, with his light dissipated by the clowdes. The tenth of October, being a cleere starry night, I perceiued wel that it was a Comette, or rather, of some called a crinitall starre: it was then placed vnder the three starres of the sight, called to sight about two yardes, so that it made an equicrurall triangle, with the two vttermost of those three starres. The eleuenth of October, the nyghte was cloudie and no starres seene: the twelfth of October, a cleere starrie night againe, and then I sawe the Comet was gotten abone the saide starres about an ell to sight, and there made an equicrurall triangle, with the midlemost of the same three starres and the vtter starre VVestwardes, wherefore seeynge I was too wryte this Chapter: shortlye after seeynge the skyes so cleere, I thought good too take a little paynes for that cause.

But hauinge on that sodaine no meridian readie founde, neither my Astrolabe furnished wyth wyer sights, nor placed as in the 33. and 34. to get the Azumuth, I presently deuised by help of two long poles, set somewhat a slope, vnderteeled wyth two forkes, too hang from the toppes of eache of them a corde with a plummet pitching them too and fro, tyll at the last standing South from them a pretie way off, I might see the pole star euen with both the cordes, which deuise here serued me in steed of a meridian line, for then I went on the North side of the cordes, and watched tyll the Comet came euen with them: whiche happened presently after I was readie for him: then was I sure that he was in the meridian or South azimuth, which done, I presently tooke the height of the Comet $38\frac{1}{2}$. deg. also of the great star called Aldeburam, the bulles eye, by whiche according to the 26. I founde it eyght of the clocke the sunne then beeing in the 29. degree, 10. minutes of ♎.

Nowe because the Comets height, videlicet, $38\frac{1}{2}$. was equall in manner to the Equinoctials height, by the 7. I concluded his declination was 0. videlicet, in the Equinoctiall it selfe, and by the 29. his degree of culmination $24\frac{1}{2}$. degree of ♒, and by the 10. his right ascension 327. And then hauing his right ascension and declination, I seeke them out on the *Mater* by the 17. and marked where they crosse, and there make a pricke with ynke, for it was the Comets place at the time of my obseruation. Lastly, I turned the *finitor* to the Ecliptick, and there by the seuenteenth chapter, the azimuth cutting the pricke or place of the Comet, shewed his longitude 329. and better, whiche more plainelier is the 29. degree of ♒, and the Almicantare cutting the same pricke shewed me $12\frac{2}{3}$. degree, the latitude of the Comet Northwardes from the Ecliptick. By this chapter and the 17. a man might make a globe, with his constellations truer then some are that are solde.

Note that I sawe the same Comet or crinitall starre, about the East circle, and about 12 degrees high as neere as I coulde gesse vpon the sixe and twentie day of Nouember next following, for in truth, his proper motion was a mayne towardes the Sunne, though wandering, and then the Sunnes motion being towardes it, they had met and passed by eche other.

Chapter. 40.

Howe to knowe the quantitie of the angles which the Ecliptick maketh with the meridian at any moment.

Gemma Frisius 48.

IT hath been shewed before that the ecliptick doth alwayes diuersly or vnequally passe ouer as well a ryght horrizon or the meridian of any place, as the oblique horrizon, yet wyth a more diuersitie and vnlikenesse in the last, then in the first. The cause (as it hath been saide) is the deformitie of the angles or inclinations of the Ecliptick to the meridian. The determination of this chapter therefore is to teache how too knowe this inclination, which is very commodious in many respectes as shall bee shewed in their place.

First you must vnderstande that as oft as two great circles deuide, or crosse one another, they make either 4. right angles, or els 4. angles equall vnto foure right angles. Further the 2. opposite angles, at eyther point of the intersection are alwayes equal which in the Theoremes before my spericall triangle is manifested. VVherefore one of them beeynge knowen, all the reste are knowen lykewise, for doubling the angle known, & then taking that out of 360, degrees, which is the quantitie of foure right angles, there remaineth the other two angles, the halfe thereof is one of them: as too the purpose, when ☉ is in the meridian, all the foure angles are right angle when ♈ or ♎ is there, then the twoo lesser angles are alwayes equall to the complement of the Sunnes greatest declination: the other twoo, then are eafily knowen as before. Know also that the degrees of the Ecliptick alike distante from eyther of the Equinoctial points, do make like angles of inclination, but in sundry partes of the heauen. In this Chapter shalbe shewed to find the two lesser angles.

First therefore by the 29. learne the degree of culmination (that is the degree of the Ecliptick) in which you desire to learne the angles that he maketh with the meridian, then marke howe farre this degree is distant from the next Equinoctiall point: the like distaunce number on the artike circle from the poynt

where it toucheth the meridian or *limbe*: for at the ende of this reckoning muſt needes bee the pole of the Zodiacke when ♈ or ♎ is in the meridian. Then applying the rule to the ſame poynt in the pole circle, it ſheweth on the *limbe* the leſſer angle ſought for, if you reckon from the Equinoctiall to the rule.

For example, I woulde knowe the quantitie of the foure angles whiche the ecliptike maketh with the meridian in the beginning of the tenth degree of ♉. The diſtance of this poynt from the next equinoctium, videlicet, from ♈ is 39. degrees, then numbryng 39. in the pole circle from the *limbe* inwards, & therto laying the rule, I finde in the *Limbe* 71. degrees $\frac{1}{3}$. the quantitie of one of the leſſer angles, whiche doubled, maketh $142\frac{2}{3}$. this taken out of 360. leaueth $217\frac{1}{3}$. the quantitie of both the greater angles, each of them being $108\frac{2}{3}$. or thus $71\frac{1}{3}$. taken out of halfe 360. videlicet, 180. leaueth $108.\frac{2}{3}$. your deſire: Thus haue you all the foure angles.

Chapter 41.

To doe the ſame another way.

Gemma Friſius. 49.

LEarne the diſtaunce as before from the next Equinoctiall poynt, number the ſame in the rule from the center outwards, ſeek alſo the right aſcentiō of the diſtance, now learned as thogh it were diſtant frō ♈ and reckon the ſame among the parallels: then moue the rule vp and downe, vntyll ſuche time that the point of the rule before noted do light exactly on the ſaide parallell, reckoned from the Equinoctiall, and ſo doth the rule ſhewe in the *limbe*, the leſſer angle ſought for as before.

For example, I would knowe the angles which the $6\frac{1}{3}$. of ♍ maketh with the meridian, the diſtaunce thereof from ♈ is $82\frac{2}{3}$. the right aſcention of that diſtaunce from ♈, by the 10. chapter, is $81\frac{2}{3}$. Therefore numbryng on the rule from the center $82\frac{2}{3}$. and among the parallels $81\frac{2}{3}$. I turn about the rule tyll the $82\frac{2}{3}$. degrees thereof doe cut the $81\frac{2}{3}$. parallell, and then doth the rule in the *Limbe* ſhew 87. deg. the quantitie of the leſſer angle.

A thirde waye is thus, gette the declination of the poynt propoſed, by the fourth Chapter, number the ſame on the rule from the *Limbe* inwardes, and laye it on the pole circle, it ſheweth on the *limbe* your deſire.

As in the firſt example, the declinatiō of the tenth degree of ♉ is 14. degrees $\frac{1}{4}$. that numbred on the rule from the *limbe* inwards, and laide on the pole circle, ſheweth $71\frac{1}{3}$. as before, I coulde ſhew another way but it needeth not.

Chapter 42.

How to finde the horoſcope, called the Aſcendent at anytime appointed.

Gemma Friſius 50.

THE powers of the planets and ſtarres, as they differ in reſpect of their nature, ſo are they either increaſed or decreaſed according to their cituation in the heauens. For we ſee dayly that tempeſtes are chiefly eyther raiſed or ſtayed by the Sunnes riſing, ſetting or comming to the meridian, & that the moone by commyng to thoſe foure places, doth rule the ebbyng and flowing of the Seas, and ſometimes rayſe tempeſts. Beſides that, he that wyll marke it well ſhall finde, that the riſing and ſettyng, and comming to the meridian, of the great ſtarres, and ſpecially with the Sunne, as in the 25. woorketh great effects in altering the ayre.

And therefore olde writers called theſe 4. places, the Cardines, and other newer call them angles or points, of the 4. Cardines or angles. Ptolomeus appointeth the midheauen to be chiefe and ſignifieth glory and honour. The next too bee the aſcendent, and ſignifieth life. Then the deſcendent, and ſignifieth deathe. Laſtly, the *Imum cæli* called *angulus terræ*, the angle of the earth, and ſignifieth contrarye to the mydde heauen.

But nowe to the matter againe: the horoſcope or aſcendent is the degree of the eclipticke ryſing vppon the horrizon at any time propoſed, or in the beginning of any thing, and chiefly in the natiuitie of a man, and is very eaſily founde by helpe of the *Reete* thus.

Place the degree of the Sunne in the Zodiack, vnto the houre propoſed by helpe of the rule applied to the *limbe*, and ſtreight looke what degre of the Zodiacke, cutteth your horrizon on the Eaſt, the ſame is the aſcendent, and the degree cutting on the VVeſt, the deſcendent.

Example.

Admit a childe were borne the 12. of October 1580. at eight of the clocke at night in our latitude, and that you woulde knowe the aſcendent of this natiuitie. The Sun was then in the 29. degree 10. minutes of ♎, whiche degree, lay vnto eight of the clocke among the afternoone houres in the *limbe*, and then looke vnto the $51\frac{2}{3}$. horrizon what degree cutteth him on the Eaſt part, I finde the 1. degree of ♋, whiche is the aſcendent. In this example, the North halfe of the Zodiacke croſſeth both the North and the South parts of the horrizon, but becauſe it is not the North halfe of the Zodiacke that riſeth on the horrizon, therfore you muſt take the degree which lyeth on the Northeaſt part of the horrizon, which is as I ſaye, 1. degree of ♋. For on the South part of the horrizon, there lieth the 10. degree of ♉. whiche is not too bee accounted of. But to proceede further in the example you may alſo (the degree of the Sunne ſtanding at eight of the clocke as before) ſee the firſt degree of ♍, in the VVeſt parte of the horrizon deſcendente, where alſo the South halfe of the Zodiacke lyeth croſſe both halfes of the horriſon, ſo that the tenth degree of ♏, cutteth the North parte of the horriſon, whiche muſte bee refuſed, remembrynge alwayes that the North halfe

of

of the Zodiacke muſt cut the North halfe of the horrizons,and riſe and ſet on them, and no other,& ſo likewiſe, the South,on the South, as is ſhewed in the ſecond booke & 23. Chapter. Further hauing thus the aſcendent and deſcendent, you may alſo ſee the degree of midde heauen or culmination in the vpper part of the noone line $24\frac{2}{3}$.in ♒,and in the lower part,the degree of *Imum cæli*,or the angle of the earth, the $24\frac{1}{3}$. of ♌,and ſo haue you the foure angles or principall houſes of your figure for that natiuitie vz.the 1.4.7.& 10.houſes.

Chapter 43.

How to finde the horoſcope or aſcendent another way by the Mater.

Gemma Friſius 51.

FIrſt get the degree of culmination for the houre and time propoſed by the 29. and his meridian altitude by the 31. Next the quantitie of the angle,which the ecliptick in that degree maketh with the meridian by the 39.and 40. Theſe had,number in the *Limbe*,the alt. of the deg. of culmin.from the Pole towards the Equinoctiall,and thereto lay the rule.Then in the Equinoctiall line from the *Limbe* in wardes reckon the quantitie of the ſaide angle, founde by the 39.and 40.and from that place following the houre line there, vnto the rule,and from the rule vnto the pole:note the degrees contained on the ſame houre line, betweene the pole and the rule,for thoſe are the degrees of the Zodiacke contained betweene the degrees of culmination,and the neereſt part of the horrizon.For you muſt vnderſtande that notwithſtanding the horrizon and the eclipticke, beeing both great circles of the ſphere,doe alwayes deuide one another into equall partes,yet is not the degree of *medium cæli* or culmination,iuſt in the middeſt of the halfe Zodiacke aboue the horrizon,but onely when ♋ or ♍.are in the meridian by reaſon of the obliquitie of the Zodiacke:but is diuided by the meridian into vnequall portions,whereof the leſſe this chapter findeth out. But ſometime the leſſer portion is on the Eaſt part,and ſometime on the VVeſt,and is thus knowne. For when ♋ is on the Eaſt part the leſſer portion is on the VVeſt,and then muſt thoſe degrees bee ſubducted out of the degree of culmination,and when ♋ is on the VVeſt,the leſſer portion is on the Eaſt,& then thoſe degrees muſt be added andthereof commeth the horoſcope.

Example.

In our latitude of $51\frac{2}{3}$.admit the Sunne to bee in the 20.of ♋,and the degree of culmination at the houre propoſed to bee 20. in ♋,whoſe meridian height by the 31.is $60\frac{1}{2}$.degrees:the angle of inclination of the Eclipticke to the meridian in that degree is 82.by the 40.Theſe knowen number in the *limbe* from the pole downewardes,the meridian altitude vid.$60\frac{1}{2}$.and thereto lay the rule,then reckon on the equinoctiall from the *limbe* inwardes the quantitie of the angle of inclination, videlicet,82.which lighteth on the 82.houre line reckoned from the *limbe*,wherefore reckon the degrees vpon the ſame 82.houre line betweene the rule and the pole,and you ſhall finde $85\frac{1}{3}$.which is the diſtaunce betweene the *medium cæli* or culmination,& the parte of the horrizon next it.And now becauſe ♋ was on the VVeſt part,you muſt adde this $85\frac{1}{3}$.to the degree of culminatiõ,vz.the 20.of ♋,& ſo proceeding according to the ſequele of the ſignes.the numeratiõ falleth on the $15\frac{1}{3}$.♎,which is the aſcendent ſought for. Thus haue you two wayes to come by the aſcendent, the firſt eaſier, but this more artificiall by the reaſons of ſpericall triangles,and by my fift booke may be performed two or three wayes.

Chapter 44.

Of the twelue houſes,and what a circle of poſition is.

Gemma Friſius. 32.

ANtiquitie which ought to be imbraced,hath appointed 12.houſes or places, wherein the ſtarres & planets haue great powers,of which 12.the 4.angles or *cardines* are chiefeſt. For euen as the Suns recourſe doth not only alter the yeere beyng in the two equinoctiall points and ſolſtices, but alſo in euery of the 12.ſignes,which cauſeth euery moneth to differ from other: euen ſo reaſon and experience teacheth in diſtinguiſhyng the 12. houſes. But now all writers agree in ſetting out the 4. *cardines*, and greatly diſagree in the other 8. For ſome beginning from the aſcendent, deuide the Zodiacke into 12. equall partes by circles meetyng together at the poles of the Zodiacke,and to theſe doth Ptolomeus agree. Otherſome deuide the equinoctiall by circles from the poles of the world into 12.equall partes,as Alkabitius,& Iohannes de Saxonia. Otherſome(which in deed is the beſt)according to Regiomontanus, deuide the equinoctiall into 12. equall parts,by circles meeting together at the two interſections of the meridian with the horrizon, and this way ſayth he,ſtandeth with moſt reaſon,ſince they are diſtinguiſhed by circles,like vnto thoſe which appoint the 4.*cardines*,and meeting together with them at the ſame points. A fourth way did Campanus a famous mathematician appointe,deuiding the verticall circle of the Eaſt into twelue equall partes from the ſaid interſection of the meridian with the horrizon. VVel, howſoeuer the houſes bee deuided,thoſe circles which diſtinguiſh them,or rather ſemicircles (for twelue ſemicircles do perfourme it) are called circles of poſition and the like ſemicircles may bee drawen by anye ſtarre, degree, or poynt of the heauen, and are called alſo circles of poſition. Therefore in ſhort, a circle of poſition according too *Io. Regiomontanus*, whome wee meane to followe, for that all menne nowe doe ſo,is a ſeemicircle paſsing from the twoo interſections of

the horrizon with the meridian by any ſtarre, degree or point in the heauens. Notwithſtanding by the *Iewell* we ſhall teach to perfourme them to euery mans minde.

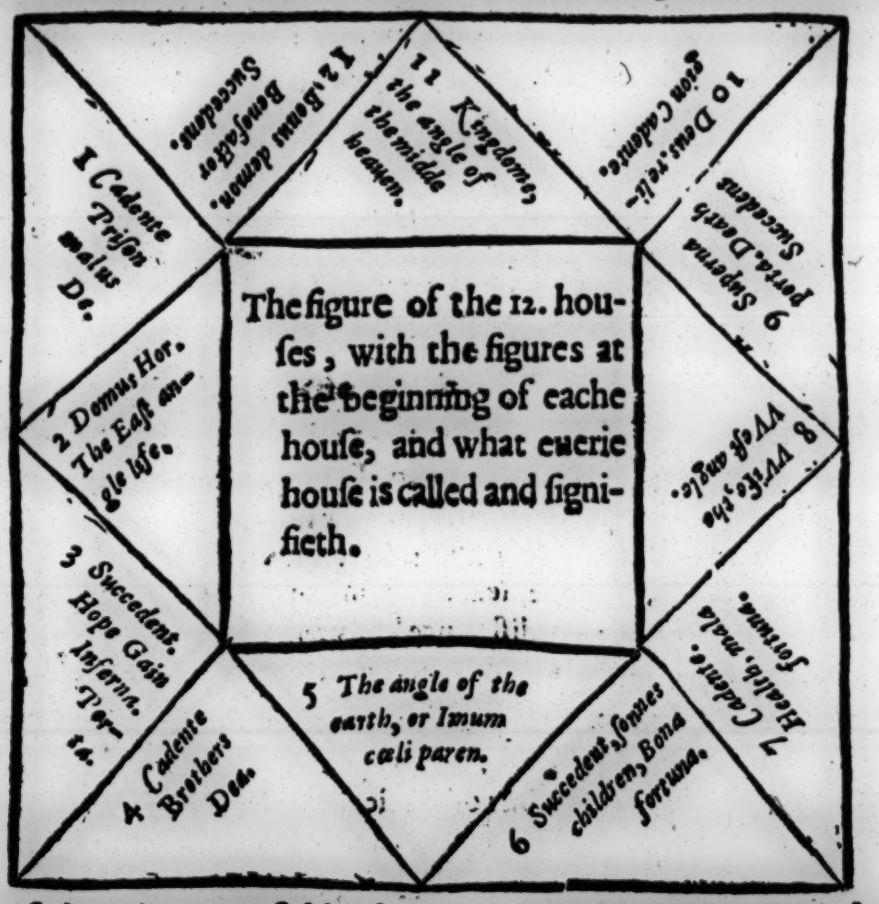
But nowe for the order of the 12. houſes, therein they all agree. The firſt houſe beginneth in the aſcendent, and ſo proceedeth vnder the earth, and the reſt in order by the *Imum cœli* vntoo the deſcendent, and thence by the *medium cœli* vnto the aſcendent againe, where the 12. houſes end. Euerye houſe hath 25. degrees forwardes, and fiue degrees of the houſe that went before. Alſo the 12. houſes are ſet downe in a Geometricall figure, as you ſee here to gether with their properties.

Chapter 45.

To finde the circle of poſition according to Io. Regiomontanus *and* Campanus, *of any point propoſed, and howe much the pole is eleuated aboue any ſuch circle.*

Gemma Friſius 53.

IT is ſhewed in the laſt chapter what a circle of poſition is. They are alſo as it were certaine Horrizons, vpon the which the point or ſtarre propoſed doth ariſe. And in truth euery of them doth repreſent ſome one horriſon in the worlde, whoſe latitude, or poles eleuation wee ſeeke, and ſo by ſphericall reaſon wee finde, what degree of the eclipticke is in euery of them. And therefore generally thus doe:



Get the declination of the point propoſed by the 4. or 7. and his diſtance from the meridian by the 26. and others. This done, ſet the Eaſt Azimuth or zenith line to the latitude in ſteed of the *Finitor*, and ſo doe the azimuthes repreſent all the circles of poſition. The zenith line beeing placed thus to the latitude, as I ſay, then in the parallel of declination of the point propoſed, reckon his houre diſtaunce, from the meridian before founde. and the azumuth there cutting, ſheweth the circle of poſition, if you count him from the Zenith line: which being had, then to knowe the poles eleuation aboue it, doe thus, reckon amongeſt the Almicantares from the Zenith point, the latitude of your countrie vpon the circle of the poſition found: and thertoo lay the rule, then reckon the ſpace between the ſame point of the circle of poſition, and the *Limbe* by the degrees of the rule, and ſo haue you the eleuation ſought for. By this chapter any man may eaſily make tables of poſitions for any countrie.

For example, in the 41. chapter I ſuppoſed a natiuitie the 12. of October 1580. at 8. of the clocke at night: I woulde now knowe in what circle of poſition the Sunne then was, I lay the Zenith line to our latitude, vz. $51\frac{2}{3}$. inſteede of the *finitor*, then for 8. of the clocke, I reckon 8. houres from the South in the Sunnes parallel being $11\frac{1}{2}$ by the 4. and there I finde to croſſe the $28\frac{1}{2}$. azimuth vnder the Zenith line, whereby I conclude that the Sunne was then in the $28\frac{1}{2}$. circle of poſition vnder the earth, in the Northweſt quarter. This done, reckoning our latit. vi. $51\frac{2}{3}$. on this $28\frac{1}{2}$. azimuth, & ♑ from the zenith point ~~found~~, therto lay the rule & you ſhall ſee betweene the *Limbe* and the ſame point vpon the rule 43. degrees. $\frac{1}{2}$. the poles eleuation, aboue the ſame $28\frac{1}{2}$. circle of poſition, ſome were wont to name the circles of poſition according to their latitude as that this ſhould be the 43. $\frac{1}{2}$. circle of poſition.

Blagraue, Heere you may ſee the wonderfull diuerſitie in vſe of all the circles in my *Reete*. Inſomuche that at any time when ☌ is in the South meridian, as he is on the *Iewell*, then placing the Zenith line, too the latitude, the azimuthes doe ſhew the circles of poſition of euery ſtarre placed in the *Iewell*, and point in the eclipttick or other part of the heauens without any more ado, wheras by G. Friſius Curſor and Brachiolum

you

you muſt ſtand ſetting for euery particular place with much trouble. Heere note that where the Azimuthes do not reach any point, you may but turne about the *Reete* ſo that the zenithes nadir be to the latitude, and then he ſupplieth the whole.

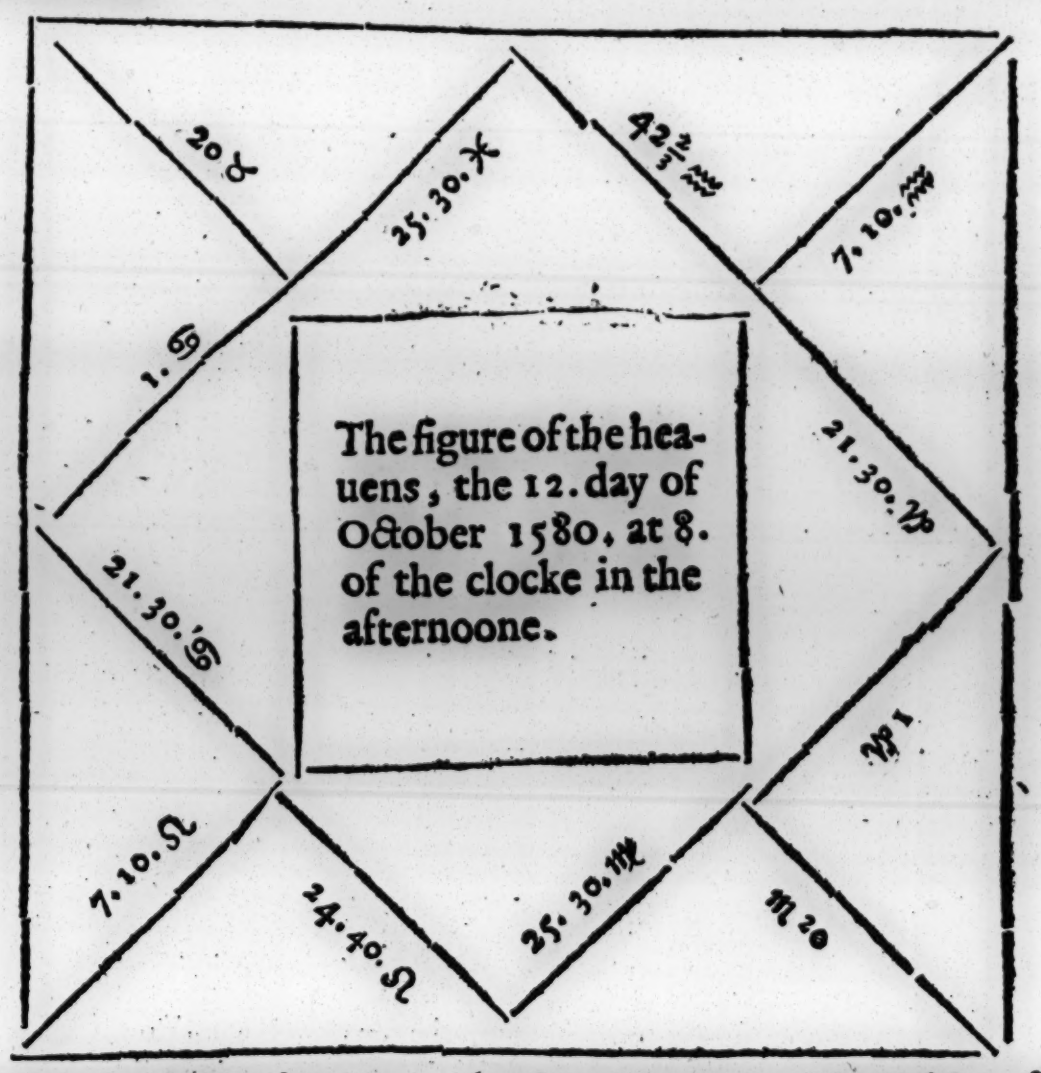
Chapter 46.

Of the other eight houſes according to Regiomontanus, and how to ſet a figure.

Gemma Friſius 54.

THe foure angles haue beene exactly taught before in the 41. and others, and the other 8. houſes alſo any man of capacitie might picke out by the laſt Chapter, but this working hath a pretie methode, which I will not hide: for you ſhall vnderſtand that all the 12. houſes are diuided with 6. whole circles of poſition, which diuide the equinoctial into 12. equal partes. Two of thoſe fixe are the horizon and the meridian which limit out the foure angles, the other 4. are two of ech ſide alike diſtant from the meridian, bending towards the horizon alike, and haue like eleuations ech to his match, and ech of them boundeth two houſes. So that euen as the meridian doth bound the midde heaueh, and angle of the earth, ſo doth the circle of poſition next the meridian on the eaſt part, appoint the 11. and 3. houſes, the ſecond next the horizon, the 12. & 2. likewiſe two circles on the weſt part, ſheweth the other foure houſes. Further knowe that oppoſite houſes are numbred by like degrees of oppoſite ſignes, ſo that hauing founde ſixe houſes, they are had all 12. Therefore 4. houſes knowne with the Horoſcope and midheauen, there needeth no more for this turne: & thoſe 4. are knowne if the two circles of poſition of the 12. houſes, which alſo is of the 2. & of the 11. which alſo is of the 3. be knowne by their depreſſions vnder the pole in this manner.

Place the zenith line to the latitude as in the laſt Chapt. and then marke what azimuth cutteth euery 30. deg. of the equinoctiall line, the ſame is the circle of poſition of euery ſuch houſe, to which get the eleuation of the pole by the laſt Chap. Thus reckon on the equinoctial from the *Limbe* inwards 30. deg. and there ſhal you finde to cut the 47. azimuth or circle of poſition, ſeruing to the 11. and 3. houſes, whoſe depreſſion vnder the pole by the laſt Chapter is $32\frac{1}{3}$ deg. Then reckon on the equinoctiall 30 deg. farther in, videl. 60. from the meridian, and there ſhall you finde to cut the $19\frac{3}{4}$ circle of poſition ſeruing to the 12. and 2. houſes, whoſe depreſſion vnder the pole by the laſt Chapt. is $47\frac{1}{3}$ deg. Theſe two eleuations vz. $32\frac{1}{3}$ and $47\frac{1}{3}$, keepe well in minde: for they ſerue you to get your 12. houſes for euer in our latitude: you ſhal do wel to chooſe them out among the horizons in the *Iewel*, and with prickes or otherwiſe diſſeuer them from the reſt, that you may redilie finde them, writing the number 12. and 2. to the $47\frac{1}{3}$ horizon on ech ſide, and 11. and 3. to the $32\frac{1}{3}$ horizon, for ſo I alwayes did.



Theſe things done, you ſhall thus haue your 12. houſes and the degrees of the ecliptick, anſwering them, lay the degree of culmination of the time propoſed on 6. of the clocke before noone in the *Limbe*, where he lyeth in a right line or right horizon, thenſe for the 11. houſe moue forwards the degree of culmination 30. deg. in the *Limbe*, that is vnto 8. of the clocke by helpe of the rule, then in the horizon of the 11. houſe videl. the $32\frac{1}{3}$ horizon, you ſhall finde the degree of the ecliptick, beginning the 11. houſe, as in the natiuitie mentioned in the 41. Chapt. lay the degree of culmination, being $24\frac{2}{3}$ of ♒, vnto 8. of the clocke before noone, you ſhal

ſe

ſee to cut the Horizon of the 11.houſe videl. the 32 1/3 the 25 1/2 of ♓, which is the beginning of the 11.houſe: Now for the 12.houſe I moue the deg. of Culmin.60.deg. that is to 10. of the clock,& there do I ſee in the horizon of the 12.houſe videl.the 47 1/3 the 20.deg.of ♉, which is the beginning of the 12. houſe, but then mouing the Culmin. to 12.of the clocke,there appeareth in our horizon videl.5 1 2/3 the Horoſcope or aſcend. videl.the 1.deg.of ♋.Now for the 2.houſe I moue the deg.of Culmin. 30. deg.forwards vz. vnto 2. of the clocke at the afternoone, then in the horizon of the 2.houſe, which is all one with the horizon of the 12.vz. the 47 1/3 horizon,I ſee the 21 1/2 of ♋.Then remouing the Culminat. 30.deg. farther videl.to 4.of the clock, there I finde in the horizon of 3.and 11. houſe videl. the 32 1/3 the 7. deg. 10. minutes of ♌, the beginning of the thirde houſe, and nowe haue I 6. of the 12. houſes. For the reſt, I neede but ſet downe in the oppoſite houſes, oppoſite ſignes & deg.and ſo is the figure ſet of that natiuitie,as you may here ſee.By this meanes you may make tables of the 12.houſes to euery latitude, perhaps truer then ſome that are printed.

Blagraue. Note that you muſt in remoouing the degree of Culmin.alwayes take the degree riſing in theaſt, on the ſaid horizons, and further that you be ſure to remember, when the ſouth ſignes riſe, to take the deg. cutting the Southeaſt part of that horizon, and when North ſignes riſe, the deg. cutting the Northeaſt part as in the 2.Booke 23.Chapt.and in the 41.is ſhewed.

Chapter 47.

Of the other eight houſes, according to Campanus and Gazulus.

Gemma Friſius 55.

HE that hath well vnderſtood the laſt 3.Chapt.neede no inſtructions more as I thinke,for Campanus, as is before ſaid,deuideth the circle of theaſt into 12.equall parts as Regiomontanus doth the quin oct. and that alſo from the ſame oppoſite points of the meridian, croſsing the horizon, wherefore laying the zenith line as before.to the latitude, then doth the *Finitor* repreſent the circle of the Eaſt, or the Eaſt azimuth being there redily diuided into all the circles of poſition, which the azimuthes do now repreſent, and euery 30.azimuth beginneth one of the 12.houſes in order from the aſcendent vnder the earth, and ſo forwards.Getting alſo their depreſsions vnder the poles in al reſpects as before,onely herein is the greateſt difference,that you muſt alſo here get the houre diſtance of euery houſe from the meridian,which the houreline on the *mater* ſhew vnalked, for the zenith line ſtanding at the latitude, you may ſee the 38 1/2 houreline reckoned from the *Limbe*,to cut the 30. circle of poſition reckoned on the *Finitor* from the *Limbe*, and likewiſe the 66 1/2 houreline to cut the 60. circle of poſition.The reſt are diſtant in like deg. from the meridian, though in ſeuerall quarters. And where in the 45. you laid the deg.of Culmination to 30.deg.or 8.of the clocke from theaſt, for the 11.houſe.you muſt inſteede thereof lay it on the firſt houre diſtance from the meridian found vz. 38 1/2.And in ſteede of remouing the Culmination to 60. deg. or 10.of the clocke, you muſt lay it to the ſecond diſtance videl.66 1/2 for the 12.houſe,then for the 2.houſe reckon 66 1/2 deg. from ſixe a clocke at night towards noone,and for the 3. 38 1/2 from the weſt vpwards alſo, for you muſt note that the axtree line here repreſenteth the meridian and the nooneline or horizon after a ſort,in working.

Blagraue. I leaue out here an example becauſe it differeth not from the vnderſtanding of the laſt Chap.but onely in reckening the houre diſtance, of the which I preſume, my *Iewel* being ſo furniſhed with all needeful circles,ſupplieth the meaning *prima facie* without tercuing about the buſh as Gemma Friſ.doth. And as for the other kind of houſes after Firmicus and Alkabitius and others, I vtterly leaue out becauſe my *Iewel* being full fraught with al neceſſarie circles,as I ſay,yeldeth greater matters with ſmall inſtruments,& alſo for that they are vtterly reiected,and none now in vſe,but the firſt way after Iohn Regiomontanus.

Chapter 48.

To know in what houſe any ſtarre is at any inſtant and to place the chiefe ſtarres in the 12.houſes.

Gemma Friſius. 56.

SOme haue had a delight to place diuerſe of the great ſtars in the figure being ſet,which muſt thus be done. Get the ſtars houre diſtance from the merid. at the inſtant propoſed by the 30. & number the ſame in his parallel of declin.then ſet the zenith line to the latitude,as in the 2.laſt Chap.and looke what azimuth or rather circle of poſition cutteth the ſame point in the ſaid parallel,as in the 44.then mark in the quinoct.line what deg.that circle of poſition lacketh of the meridian(for euery houſe is 30.deg.of the quin. after Io. Regi. whom we follow).The meridian is alwayes the beginning of the 10.houſe,wherfore ſo many times 30.as the ſtars circle of poſition is diſtant from the meridian on the quinoct.line,ſo many houſes it lacketh,or is beyond the 10.houſe:ſo that by knowing in which quarter of the skie the ſtar is,the houſe, & part is eaſily reckoned.

Blagraue. He that vnderſtandeth al before, needeth no example here,but thus much farther,I wil ſhew you as in the 45.you may plainly ſee that 4.circles of poſition with the *Finitor*, & the meridian,do ſhew the whole 12.houſes,which are as there is ſaid,the 47.and 19 3/4 azimuthes or circles of poſition, reckoned on ech ſide of the zenith line as he is there placed: wherefore if you did diſſeuer out with ſome diſtinction by prickes or otherwiſe thoſe 4.azimuthes, and ſet their figures to them, namely at the zenith line, 1. becauſe hee being the horizon beginneth the 1.houſe at the 19 3/4 azimuth vnder him:or leftwards two,for the 2.houſe:at the 47.azimuth 3.at the 90.which is the *Limb* 4.and ſo round backe againe by the ſame circles ending at 12.in the 19 3/4 azimuth aboue the zenith line. Theſe circles being thus diſtinct & numbered,are in truth your 12.houſes, the zenith line being at the latitude.So that now the ſtar or ſunnes diſtance from the meridian reckoned on their parallel

parallel as aboue, make there a pricke with incke, for there is the ſonne or ſtarres place for the time being, and now in what houſe he is, the ſaid azimuthes diſtinguiſhed and figured as before, ſhewe without asking. This could neuer be done by G. Friſius Aſtrolabe in this manner.

A ſhort example: the ſunne being in the tropicke of ♋, I would know in what houſe he is at 7. of the clock before noone. I ſet the zenith line in ſteede of the *Finitor* to the latitude, and then I finde that the $27\frac{1}{2}$ circle of poſition cutteth the tropicke with the 7. a clocke houreline, which according to the numbers euen now placed ſheweth me that the ſunne is farre within the 11. houſe about $\frac{3}{4}$. neere towards the 12. Or thus more plainly, follow the ſaid $27\frac{1}{2}$ circle of poſition vnto the equinoctiall, and there doth he ſhew that the ſunne is in the 20. deg. numbred on the equin. within the 11. houſe towards the 12. that is $\frac{2}{3}$. Likewiſe I ſhewed in the 26. chap. that the great ſtarre Hircus was 4. houres and 2. deg. paſt the north meridian eaſtwards in his parallel being 45. there make a prick, & mark in what houſe he then was, you ſhal finde him in the 11. houſe ſomwhat more then halfe in.

Chapter 49.

Of directions, what direction is, and by what meanes it is performed.

Gemma Friſius 58.

TO direct ſaith Io. Regiomontanus, is no other thing elſe then to turne about the ſphere vntil the ſecond place be brought to the place of the firſt, that is to ſay, vntill the ſecond place light on the ſemicircle of poſition of the firſt, ſome haue called this prorogation, ſome inambulation, ſome *Apheſin*. I thinke it rather ſaith G. Friſius *Dimiſſion*, or *Emiſſion*, becauſe one place of the heauens by turning of the ſphere is emiſed or ſent forth vnto the ſite of another place. Whereby Ptolomeus doth call thoſe places which hee termeth chiefe, that is to ſay, thoſe which may receiue, the chiefe ſignifiers of our life, and adde power to thoſe ſignifiers *Topous Apheticous*: and alſo calleth thoſe ſignifiers *Apheta*, as it were Dimiſſaries, or Emiſſaries. The Arabians call them *Hylech Plilippus*, prorogators. To be ſhort, *Apheta* called commonly both in latin & engliſh, *Significator*, or ſignifier, is either a ſtar, or ſome notable place in the heauens, hauing the chiefeſt dominion ouer the life of man, and Ptolomeus hath 4. almoſt, the Sun, Moone, the pars Fortune, & the Horoſcope. There be other places alſo accepted of authors, as the midheauen, and any planet hauing gotten ſome notable dominion: and are called *Apheta* and ſignifiers, vnto whom other places of the planets, or places filled with the beames of planets are dimiſed or ſent, that is to ſay, dyrected.

For declaration hereof, take G. F. own example of the natiuity of King Philip, ſomtime huſband to Queene Marie. He was born 1527. at a quart. paſt 4. of the cloke in the afternoone, the ſun being 9. deg. in ♊, & the lat. of the country being 40. The figure thereof being ſet there, is found $2\frac{1}{2}$ deg. of ♍ in the aſcendent, and in the midheauen $8\frac{1}{2}$ deg. of ♌. And now to the purpoſe, the Horoſcope is alwayes a notable place. For hence do they take their iudgement of the helth of the body, of life, and of iourneying, wherefore it is called *Apheta*, or a ſignifier. And Mars which then was in the 2. houſe, is called the promiſer, & the 2. place, which by the motion of the ſphere is brought vnto the firſt place, that is to ſay, vnto the horizon which is the circle of poſition of the horoſcope, & this is called direction or emiſion. The queſtion is chiefly how many deg. of the equinoct. ſhall paſſe ouer the circle of the horizon or the meridian, in the meane time, while the place of ♂ be brought vnto the place of the *Apheta*. For it is ſignified that there are ſo many yeres to come before the effect of the promiſer ſhalbe performed on the *Apheta*. This matter is eaſily done by the *Iewel* for any region or point of the heauens. Seeke the circle of poſition of the firſt place or ſignifier, and his horizon by the 44. and 45. or rather 47. and vpon the ſame horizon lay the deg. of the *Apheta* in the zodiacke, and marke then what deg. is in the midheauen. And then turne about the *Reete* vntil the 2. place of the promiſer come to the ſame horizon of the ſignifier, by and by moue the rule vnto the deg. which before was in the midheauen, it ſheweth on the *Limbe* the degrees paſſed of the equinoctiall, which ſignifie the yeares of the direction.

As in the ſaid example, place the Horoſcope being the ſignifier of life vnto his circle of poſition, that is, vnto the 40 horizon, you ſhal ſee on the line of midheauen ♌ $8\frac{1}{2}$ deg. then turning the *Reete* vntill the place of ♂ which then after Alphonſus, was in the 29. deg. 17. min. of ♍ be brought vnto the ſame 40 horizon, and then ſetting the rule vnto the 8. deg. 30. min of ♌, you may ſee that the *Reete* hath gon forwards on the *Limbe* 34. deg. $\frac{1}{6}$ almoſt, and ſo many are the deg. of emiſion or direction of ♂ vnto the horizon. But heere muſt you diligently note that for Aphetaes placed in the eaſt halfe of the heauens, that the horizons be taken on the eaſt part of the *Iewel*, & for thoſe of the weſt halfe, on the the weſt part, the reſt may be performed as before. Neither muſt you here neglect the latitudes and declinations of the planets, for which I taught you remedie in my addition to the 23. Chapt. And here another way to remedie the latitude of any planet, is thus: Get the deg. of right aſcen. and thereto lay the rule, whereon reckon his declination from the *Limbe* inwards, & there is the true place of the planet: this rule alſo ſerueth for all ſtars not being in the *Reete*, and alſo for ſtarres in the *Reete* in workings for time long paſſed or to come, becauſe fixed ſtarres go forwards in long. a deg. at the leaſt, in an. 100. yeares.

Chapter 50.

Of conuerſe or euerſe direction, or dimiſsion.

Gemma Friſius 59.

VVHen as the ſignifier is on the eaſt part Ptolomeus, always emiſeth or bringeth the 2. place vnto the firſt circle or place, or to the Aphetaes place, & reckoneth the deg. paſt in the meane time. And this he calleth proiection of beames, becauſe that either the planets places or their beames are brought vnto the Aphetaes place, and this is called direct, becauſe the ſecond place differeth from the firſt according to the ſequeale of the ſignes. But when the *Apheta* differeth from the midheauen towards the VVeſt, the ſeconde place ſhall be the VVeſt it ſelfe, and then ſhall the *Apheta* bee brought vnto the VVeſt part of the horizon

to

to know the direction of the *Apheta* vnto the Anareta, called Interſector, and is ſaide to be contrarie to the order of the ſignes, becauſe the ſecond place, that is to ſay, the deſcendent or point of the weſt, which then is the Anareta, is diſtant from the firſt, contrarie to the order of the ſignes. Of the firſt manner is ſpoken in the laſt Chap. Of the latter there needeth not many words, for it is euen like. For the deg. of midheauen being placed in the nooneline, turne the *Reete* till the *Apheta* be brought to the deſcendent, and then the rule laid to the deg. which was in midheauen, you may ſee howemany deg. the ſphere hath now gon forwards which are the degrees of the direction.

Chapter 51.

How farre, or into what part of the zodiacke, Dymiſsion or direction ſhall come any yeare.

Gemma Friſius 60.

IT hath beene plainely and exquiſitely ſhewed in what time the Dymiſsion or direction of any place is performed. But oftentimes the queſtion is before al the Dymiſsion be performed, into what deg. of the zodiack the direction ſhall come, and he that hath wel vnderſtood the premiſes, ſhal performe this Chapt. with little adoe: for the queſtion it ſelfe hath a certaine difficultie by the manner of pronuntiation, which I thinke beſt firſt to expound, for many either pronounce ill, or elſe vnderſtand not the matter wel: for whē as we ſay, how farre commeth the direction of the Horiſcope this yeare. It ſeemeth to be vnderſtoode that the Horoſcope (which now we cal the *Apheta*) ſhould be moued in the heauens: but the matter is otherwiſe when the *Apheta* is in theaſt part. For then we do not emit or ſend forth the Horoſcope, or any *Apheta*, but do moue the places following, or promiſers vnto him. Therefore when it is demaunded how farre the emiſsion of the *Apheta* placed on theaſt part, hath gone forwards: the queſtion is, what deg. of the zodiacke ſhal come vnto the ſemicircle of the *Apheta* in this or that yeare. But the *Apheta* ſtanding on the Weſt part: It is more rightly ſaid that his Dymiſsion doth occupie this or that deg. of theclipticke, becauſe the *Apheta* is knowne to be moued towards the Weſt. Now therefore togither with an example will we explane the matter.

Firſt in direct direction. Therefore in the natiuitie of king Philip, let vs appoint the *Apheta* to be the Horoſcope it ſelfe, let vs ſee then vnto what degree of the zodiacke the Dymiſsion, or direction ſhall come that yeare 1554. that is to ſay, what part of the zodiack ſhal then come vnto the Horizon circle by Dymiſsion vid. vnto the *Aphetaes* place. Firſt conſider how many yeares be paſſed from the natiuitie it ſelfe, which are 27. for he was born, anno 1527. Secondly, place the *Apheta* vnto his place, which is the Horizon, & note the deg. of culmination, then remoue the ſame deg. of culmination 27. degrees forwards in the *Limbe*, and there ſhall appeare in the 40. horizon ♏ 23. deg. 40. minutes almoſt, therefore this deg. of the zodiacke then commeth vnto the Horoſcope.

But now let the *Apheta* be in the weſt part of the heauens, as the part of Fortune is in the ſaid natiuitie, let vs ſeeke his Dimiſsion for the ſame yeare 1554. place the deg. of Culmination in the midheauen videl. ♌, 8. deg. 30. minutes: and numbering thence 27. deg. in the *Limbe*, and thereto laying the rule, place there the degree of Culmin. videl. the $8\frac{1}{2}$ of ♌. Now then marke diligently the diſtance of the part of Fortune from the ſouth, numbering the degree of the *Limb* betweene the nooneline and the rule placed on the part of Fortune. In our example they are 45. deg. 45. minutes almoſt, with this diſtance therefore, and the declination of the *Apheta* found before videl. 22. deg. 25. minutes north, ſeeke the circle of poſition of the ſame *Apheta* for that time, and theleuation to it, and how many degrees of thequinoctiall the ſame ſemicircle is diſtant from the meridian. Therefore by the 44. or 47. Chapt. the circle of poſition is found diſtant from the meridian in theaſt circle 41. deg. 48. minut. Theleuation of the pole is 25. 40. The ſame ſemicircle doth cutte off from the weſt quadrant of thequinoctiall 34. deg. 30. minutes as neere as may be gathered by a ſmall inſtrument. Now therefore we haue the circle of poſition of the part of Fortune for 27. yeares paſſed, the queſtion now is, what deg. of theclipticke was in this circle of poſition in the beginning of this natiuitie. For it may rightly be ſaid that the Dymiſsion or direction of the part of Fortune is come vnto it. But ſeeing this is a generall rule, it is better to ſhew it in a Chap. following.

Chapter 52.

To finde what degree of theclipticke occupieth any circle of poſition at any time.

Gemma Friſius 61.

THis Chapter is very commodious, yet comberſome by tables, & therefore the more pleaſant by the *Iewel*, it is but in effect the 45. and 46. euerſed. Get the circle of poſitions eleuation by the 45 and his diſtance from the meridian, as in the 47. Theſe had if the circle of poſition, ſhall be oriental and nocturnall (that is) vnder the earth, reckon on the *Limbe* his diſtance from the meridian found, from theaſt point towards midnight, and thereto lay the deg. of *Imum cæli*, or the angle of the arch, if orientall and diurnall then towards noone, and there lay the Culmination, if occidentall and noct. then from the VVeſt towards midnight and there lay the *Imum cæli*, if there diurnal, towards noone, and there lay the degree of Culmination as in the 46. Chapt. And ſo the deg. of theclipticke cutting the circle of poſitions horizon is your deſire.

Example. Admit the ſunne be in $8\frac{1}{2}$ deg. of ♏ the *Medium cæli*, or Culmination, then $6\frac{1}{2}$ of ♈. the *Imum cæli* or angle of the earth $6\frac{1}{2}$ of ♎, theleuation of the 48. poſition vnder the arth eaſtwards, being by the 44. $31\frac{2}{3}$ his diſtance from the meridian $29\frac{1}{2}$, which I reckon from theaſt towards midnight on the *Limbe*, and thereto lay the deg. of *Imum cæli* videl. $6\frac{1}{2}$ of ♎ then looking on theaſt part of the $31\frac{2}{3}$ horizon, I finde there the $10\frac{1}{2}$ of ♐, the deg. of thecliprick, occupying the 48. circle of poſition nocturnal. Likewiſe for the 48. poſition diurnall Eaſtwards the deg. of culmin. videl. $6\frac{1}{2}$ of ♈, laid $29\frac{1}{2}$ deg. towards noone, you ſhall finde in the ſame $31\frac{1}{2}$ horizon $18\frac{2}{3}$ of ♉, remembring alwayes to looke on the North Horizon for north ſignes, & ſo contrariwiſe, as in the 2. Booke and 23. Chapt. Looke farther the 4 Booke.

Chap

Chapter 53.

To know the quantity of the angle of the inclination of the Ecliptick to the horizon at any degree of the Ecliptick.

Gemma Frisius. 62.

THis chap. is very profitable about the occultations and appearing of ſtarres, and alſo for the deformation of Eclipſes. Firſt therfore place the deg. propoſed vnto the horizon on the Eaſt, then reckon the degrees of the Ecliptick betweene the aſcendent and midheauen, if they exceede 90. then take the deg. betweene the midheauen, and the deſcendent, or take the firſt out of 180. all is one: next get the meridian altitude of the degrees of midheauen. Theſe had, reckon on the rule that diſtaunce betweene the horizon and mydheauen outwards from the centre, and moue the rule vp and downe till the ſame poynt do cut the parallel anſwerable to the height of midheauen, and ſo doth the rule ſhew in the *Limbe*, reckoning from the Equinoctiall the quantity of the angle ſought for: which is alſo the leſſer angle, as in the 39. and therfore ſubducted out of 180. leaueth the greater, and this leſſer angle is alwaies the height of the Eclipticke 90. deg. or *nonageſs. gradus* from the horizon.

Example. In the yere of our Lord 1590. the 20. of Iuly the 19. houre 30. min. which is at halfe an houre paſt 7. the next morning (for Aſtronomers begin the day at noone, and reckon agayne til noone vnto 24. in their tables) there ſhall be a great Eclipſe of the Sun, the Sun and moone both being in coniunction in the $7\frac{1}{2}$. deg. of ♌: wherefore placing the deg. of the ſun vnto $7.\frac{1}{2}$. of the clocke 30. min. I ſee $10.\frac{2}{3}$. of ♍ in the aſcendent, and in the deſcendent $10\frac{2}{3}$. of ♓, and in the mydheauen 4. in ♊, I would now know the quantytie of the leſſer angle made betweene the Ecliptick and the horizon. From the aſcendent to the mydheauen, are $96\frac{2}{3}$. deg. take thoſe out of 180. there reſteth $83.\frac{1}{3}$. the meridian altitude of the degree of culmination, vz. of 4. of ♊, by the 28. is $59.\frac{1}{2}$. deg. This done, reckon the degrees betweene the midheauen and the deſcendent, vz. $83\frac{1}{3}$. on the rule from the centre, and lay the ſame degree on the $59\frac{1}{2}$. parallel, & ſo doeth the rule ſhew in the *Limbe*, reckoning from the Equinoctiall 60. deg. the quantity of the angle ſought for, which is the height of the *nonageſ. grad.* or 90. deg. of the Eclipticke aboue the horizon.

By this angle many things are brought about, as ſhalbe touched hereafter.

Blagraue. It may be that Gemma Friſius tooke the courſe before about this chap. for better vnderſtanding of the chap. following, or els I muſt ſay hee ouerſlipt another more eaſie way, as hee himſelfe ſaieth, learned men do many times, which I wil here ſhew you.

Place the deg. at which you require the angle, to be aſcendent: thence reckon vpwards 90. deg. on the Zodiacke, therto lay the rule, which ſheweth in the *Limbe* the diſtance of that 90. deg. from the South by the 30. and betweene the *Limbe* and this 90. deg. of the Zodiacke, reckoning on the rule, is the declination thereof by my addition to the 4. chapter. Theſe had, ſeeke on the *Mater* where this declination and diſtaunce do croſſe by the 26. and the Almicantare (the *Finitor* beyng at the latitude) cutting there, ſheweth the angle ſought for.

As in example before, the $10\frac{2}{3}$. of ♍, being aſcendent, I reckon on the Zodiacke vpwards 90. deg. it lighteth on the $10\frac{2}{3}$ of ♊, thereto I lay the rule, where I finde the diſtance of the ſame $10\frac{2}{3}$. of ♊, from the ſouth to be 7. deg. $\frac{1}{4}$. and his declination by reckoning the deg. on the rule betweene it and the *Limbe*, to be 22. and better, North. Theſe two being had, I ſet the *Finitor* to the latitude, and then marke where the $7\frac{1}{4}$. houre line reckoned from the *Limbe*, and the 22. parallel do croſſe, and there I ſee cutteth the 60. almicantare, as before, the angle ſought for, with leſſe trouble.

Chapter 54.

How to finde the altitude of the Sun or any point of the Ecliptick aboue the horizon at any time otherwiſe then hath yet bin ſhewed.

Gemma Friſius 63.

THe houre giuen, with the Sunnes place: the aſcendent is knowen by the 41. and by the 52. the altitude of the *nonag. grad.* or 90. degrees from the horizon, which I reckon of the *Limbe* on the *Reete*, and thereto lay the labell: this done, note how many degrees the Sun, or point giuen, is from the aſcendent or deſcendent, ſo many deg. reckon on the rule from the centre, where the Almicantare cutting, ſheweth the altitude ſought for.

As in the example of the laſt chap. the houre being 7. and 30. min. the aſcendent thereby $10\frac{2}{3}$. of ♍, and ſo the altitude of the 90. degree 60. VVherefore theſe being had, I would alſo know the altitude of the Sunne at that inſtant, being then as is ſaide in the $7\frac{1}{2}$. of ♑, to perform this, I reckon in the *Limbe* or vttermoſt azimuth of my *Reete*, the altitude of the 90. deg. of the Eclipticke, vz. 60. deg. and thereto lay the *Labell*, then I reckon on the *Label*, from the centre the ſpace betwen the aſcendent, videl. $10\frac{2}{3}$. ♍, & the ſuns place, vz. ♌ $7\frac{1}{2}$. whiche is 33 deg. $\frac{1}{4}$. And there do I ſee the $28\frac{1}{3}$. almicantare to cut the *Label*, which is the height of the ſunne for the time of that Eclipſe. In like maner may you come to the height for that inſtant, of any deg. of the eclipticke by their diſtance from the aſcendent or deſcendent, the *Labell* ſtanding ſtill there which in calculating the Eclipſes ſtandeth in great ſteede.

Chapter 55.

To find what angles the circle of altitude maketh with the Ecliptick at any point thereof aſſigned.

Gemma Frisius 64.

THis chap.serueth chiefly to descry the sunnes Eclipses,wherein is almost the greatest labour and diffyculty of Astronomie and Arithmeticke but shal be easily performed by helpe of this *Iewel*. First note that Gemma Frisius meaneth by the circle of altitude,the vertical circle or azimuth, wherein the sun or any other deg. proposed,is. Then to proceede,gette the distaunce of the point proposed from the zenith,by subducting his height learned by the last chap.out of 90.Likewise learne by the 52. the distance of the *nonagesi. grad.*or 90.deg.of the Eclipt. from the zenith: these 2. being known. reckon the greater of these on the rule from the centre,& then moue that note on the rule vnto the paral. of the lesser distance,and so doth the rule shew in the *Limbe* the quantity of the angle,reckoning from the Equinoct.As if in the last question of the Eclipse,I wold know of the angle which the circle of altitude passing by the Sun,doth make with the Eclipt. I seeke the distaunce of the Sun from the zenith,by subducting $28\frac{1}{3}$.from 90.$61\frac{2}{3}$. Likewise subducting the height of the 90.deg.vz.60.from 90.there resteth 30.his distaunce from the zenith also. Now doe I reckon on the rule $61\frac{2}{3}$.deg.and there make a note which I lay vnto the 30.parallell, and so is the angle sought for found to be $34\frac{1}{2}$.shewed on the *Limbe* by the rule.

This is alwaies one of the lesser angles,as in the 39.chapter,and is towardes the East, when the poynt of the Eclipticke assigned, is on the East part of his 90.degree,and on the VVest if otherwise. VVherefore this angle found,vz.$34.\frac{1}{2}$. subducted out of 180.there resteth $145\frac{1}{2}$.the quãtity of the greater angle as in the 39.

Furthermore know this for a certain truth,that the angle found in this chap.is that which they cal the angle of latitude,because the parallaxis or diuersity of aspect,which happeneth in latitude is subtended or lyeth vnder it, and this angle if we subduct him out of 90. there shall bee left the angle which they call the angle of longitude:but whereto these belong,and to what vse they are sought, shall bee shewed here following.

Blagraue. In this chap.the Equinoctiall representeth the Ecliptick,and the rule,the vertical circle of the sun, the poynt noted in the rule the zenith,or rather the like distaunce from the Sunne:the other way or downwards the centre of the *Iewell* representeth the point of intersection with the sunne:the line between the note in the rule(which as I saide is the zenith)and the Equinoctial,is the distance betweene the zenith,and the 90. deg.but rather between the like deg.on the azimuth beneath the sun. The reasons hereof I will shew somewhat more large in my 5.booke,and omit them here,both because too much tedious matter together might cloy a young student,and also for that I meane to shew,how to find al spherical triangles on the *Iewel*,which Gemma Frisius instrument could neuer doe.

Chapter 56

How to finde the parallaxis or diuersity of the sight of the moone in the circle of altitude,

Gemma Frisius, 65.

IT is by demonstration affirmed in Ptolomeus works & other famous authors,that the globe of the earth is but a point in respect of the heauen it selfe:that is to say an vnsensible bigneſſe in comparison to the heauens. And that whether a man be in the centre of the earth,or on the circumference,there commeth therby no diuersity of any apparents in the heauens: hereof it commeth that all dialles are made, as though we dwelt in the centre of the worlde, neyther hath any man found diuersity therein, were he neuer so ingenious or practick.But as these things are most true,and stand vpon good demonstration,yet in some of the planets it is not so.For although the fixed starres,and the highermost planets,because of their vnmeasurable distance from vs,make no diuersity of sight to vs,from that which is seene in the centre of the worlde : yet in the lowermost planets how much the neerer they be vnto vs,so much the more is perceiued the euariation of their place appearing from that which they haue vnto the centre of the world.For you must vnderstand that all calculations of the motions of all the starres and planets perfourmed by tables or Ephimerides,are done & made to the centre of the world.Now because the speere of the moone is neereſt vnto vs,there is perceyued a manifest diuersity betweene her place,which we see dwelling here on the circumference of the earth, & her place which a line drawn from the centre of the world through the moon,doth shew:because the semidiameter of the earth,according to which quantity we dwel from the centre,hath a sensible bigneſſe vnto the dystaunce of the moones speere from vs. This diuersitie is perceyued not only in the moone, but also in *Mercury* and *Venus*,and somewhat in the sunne, though verye little, and scarce perceiuable : this diuersity of sight called of *Ptolomeus*,*Parallaxis*, is greatest in the moone : in the rest it is almost neglected,neither is it alwaies alike in the moone:which is a sure argument that the moone is not carried in Homocentre speeres, as some woulde haue auouched.But how much the more the moone departed from the centre of the world, and becommeth *Apogea*,so much the lesse happeneth this parallaxis,and how much the neerer she commeth to the earth being *Hypogea*,so much the greater is this parall. so that the greatest parallaxis of all, is one degr.6.min. And the least which happeneth, the moon being in perig. is but 50.min.omitting a few seconds : in the sunn this parall.is 3.min.onely.This parallaxis,when it is simply spoken of,it is meant the moone or planet beyng in the *Finitor* or horizon, in the zenith they can haue no parallaxis:for there the beame proceeding from our sight,is all one with that which commeth from the centre of the world,but the more neere to the horizon,the greater. These things are plaine to those that vnderstand the theorickes,and are excellently demonstrated by Ptolomeus.

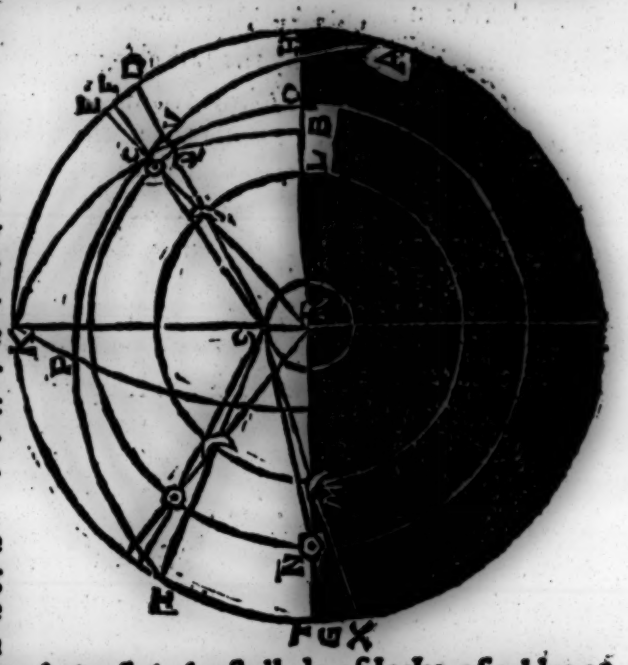
The scope therefore of this chap.is by the greatest horizontall parallaxis, giuen how to get the parallaxis of the moone for the altitude she is in.

Gette therefore out of some tables of Eclipses, or out of Copernicus, the moones greatest or horyzontall

gontall parallaxis for the ſight, ſhe is in her ſpeeres, and likewiſe the moones height aboue the horizon by the firſt or 32. though commonly in an Eclipſe, ſhe differeth not from the ſunnes height. Then number the horizontall parallaxis on the *Limbe* from the Equinoctiall vpward, and therto lay the rule: which done, reckon on the rule from the centre, the moones diſtãce frõ the zenith, or inwards from the *Limbe* the moones altitude all is one, the parallel cutting the rule there ſheweth your deſire.

This inſtitution bicauſe it is moſt pleaſant, by the which not without admiration of moſt men, Eclipſes are prognoſticated long time before they happen, deſerueth an example.

As in the Eclipſe like to be 1590. mentioned in the 50. chap. the ſun & the moone ſhall be in the $7\frac{1}{2}$. de. of ♌, this I ſay is their place calculated to the centre of the world: in the which luminaries ſhalbe coniun. Now therefore the queſtion is, what the ſun & moones viſible place ſhalbe, thereby to gather the euariation of the Eclipſe. For by reaſon of the euariation of the ſight or parallax. they ſhall not be ſeene here coniunct, as they are found to be in reſpect of the centre of the world: the altitude of the ☾ is found $28\frac{1}{3}$. as well her latitude being 20. min. North, as her longitude accounted of, the greateſt parallaxis I found 52. min. Therfore in the *Limbe* I number not 52. min. as I ſhould doe, but I reckon 5. deg. $\frac{1}{4}$. almoſt: ſo that ech deg. ſupplieth 10. min. the 5. deg. being here for the 50. min. and the other $\frac{1}{4}$. almoſt for 2. min, which is done becauſe els a huge inſtrument would be too little to perform this matter exactly. The rule being ſo ſet at $5\frac{1}{4}$. deg. almoſt, I reckon inwards thereon the height of the moone. vz. $28\frac{1}{3}$. and there I ſee to croſſe the rule the $4\frac{1}{2}$. parallel, now taking for ech of the 4. parallel 10. min. and for the half 5. min. there commeth 45. min. the moones parallaxis according to that height and place: that is to ſay, the moone ſhall be ſeene here 45. min. lower then in truth ſhe is, in reſpect of the centre of the world in that Eclipſe.



Likewiſe appointing 5. deg. for 3. min. the greateſt parallax. of the ſun, euery deg. is 40. ſecondes: there place the rule, & reckon the ſaid $28\frac{1}{3}$. deg. inwards it ſheweth the $4\frac{1}{4}$. parallel, whiche commeth to 8. ſcore and ten, or 170. ſeconds, deuide that by 60. there commeth 2. min. & 50. ſeconds, the parallaxis of the ſun. Here you muſt take heed that you ſeeke not to work greater angles in this manner: and in theſe leſſer, that you exceede not 10. deg. on the *Limbe*: for this eaſe ſerueth but in ſmall angles, where the queſtion is of minutes, which in greater wil not holde proportion, but if a third or 4. part of a minute be wanting. the errour is not much, but ſo much of a degree is not allowable.

Blagraue. Gemma Friſius ſaith before in this chap. that the moones greateſt horizontall parallaxis is 1. deg. 6. min. beeyng in her Perigeon, and that the leaſt horizontall parallaxis is the moone being in her Apogeon 50. min. For the reſt he referreth you to tables. But now becauſe I woulde not haue any deſirous hereof, to be ſtalledr of lacke of tables of ſight in the Theorickes: I will ſhew you after a blunt ſort how to gette the horizontall paral. of the moone according to her ſight between her apogeon and perigeon, by helpe of an Almanacke.

Firſt I would haue you vnderſtand that at euery change and full of the moone, ſhe is in, or very neere her apogeon, and at euery quarter, in, or neere her perigion, therefore when as if it be true as Gemma Friſius ſaieth, that the greateſt paral. in the perigeon is 1. degree 6. min. and the leaſt in the apogeon 50. min. ſubducting the leſſe from the greater, there reſteth 16. min. the difference. The ſpace of time betweene the quarter and full, or change, is 7. dayes, 9. houres or thereabouts, which maketh 177. houres, but account it here (for it hurteth not greatly) 180. houres. Now therefore this knowen, reckon the difference of theſe horizontall paral. vz. 16. min. on the *Limbe* from the Equinoctiall, or rather as you were wont 8. deg. ſo euery deg. repreſenteth 2. min. thereto lay the rule: and by helpe of an Almanacke learne howe many houres the moone is diſtaunt from the next ful or change, and reckon on the rule for euery two of theſe houres one deg. on the rule from the *Limbe* inwards, and there marke what parallel cutteth: for it ſheweth the minutes to be added to the moones leaſt horizontall paralaxis, videlicet 50. I would needes knowe her horizontall parallaxis, I looked in the almnacke, and there I ſawe ſhe was at the full the 22. of October, at 6. of the clocke at nighte. I caſte howe many houres were paſſed and found 50. houres, then laying the rule on 8. deg. in the *Limbe*, which here repreſent ech 2. minutes. I reckoned thereon from the *Limbe* inwardes, for thoſe 50. houres 25. deg. there I ſaw the $7\frac{1}{4}$. parallel to cut, which here repreſenteth but $14\frac{1}{2}$. min. to be added to 50. it maketh $64\frac{1}{2}$. min. or 1. deg. $4\frac{1}{2}$. min. the horizontal parallaxis for the time ſought for. VVhich being had, I might therby as before get the moones parallaxis for the height, ſhe was at *ſed hæc rudiori Minerua*, by reaſon of the moones Epicircle, which cauſeth ſome errour.

Chapter 57.

How to finde the parallaxis of the moone or any other planet, or ſtarre in longitude and latitude of the Eclipticke.

Gemma Friſius 66.

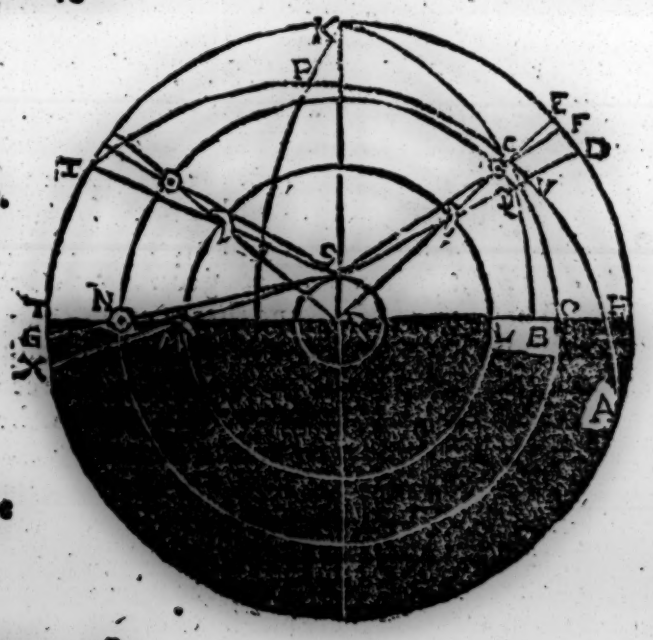
THE parallaxis beyng now found in the circle of altitude, in whiche circle his muſt needes happen: You

ſhall vnderſtand that ſometime it happeneth in longitude, whereby the ſtarres ſeem to haue another longitude then is found in tables calculated to the centre of the world, inſomuch that if the ſtarre ſhalbe from the aſcendent, leſſe then 90. deg. he ſhal ſeem to haue a greater longit. thē in truth he hath, but being ſituate VVeſt wardes from the 9 degree he appeareth to ſight in leſſe longitude: but in the middeſt or 90. deg. of the Eclipticke, he maketh o parallaxis in longitude, but all in latitude. For then the circle of altitude is all one wyth the cyrcle of Longitude of the ſtarre yſſuyng from the pole of the Zodiacke by the Zenyth. But when the parallaxis falleth wholly in Latitude then the Eclipticke muſte of neceſſitye cutte the Zenith and bee made one circle with the circle of altitude, this happeneth only to them that dwel either vnder one of the Tropickes, or betweene both. In other places the parallaxis doeth partly alter the longitude, and partly the latitude, ſo that the neerer the moone ſhal be to the 90. deg. of the Eclipticke, ſo much the greater is the parallaxis of latitude and of the longitude leſſe: and contrariwiſe the further ſhe is from the 90. degree, the greater is the parallax of longitude, and the leſſe of latitude. And how much either of them is, may thus be had.

Seeke by the 54. the angle which the circle of altitude maketh with the Ecliptick in the place of the moon or of the ſun: and by the laſt chapt. the parallaxis in the circle of altitude, reckon the quantity of the angle on the *Limbe* from the Equinoctial, and thereto lay the rule, then on the rule from the centre outwards, reckon the parallaxis found by the laſt chap. reckoning for euery 5. minutes therof one deg. on the rule, as you there did on the *Limbe*, the parallel cutting there, ſheweth the parallaxis of latitude, accounting accordingly euery parallel for 5. min. Likewiſe following the houre line cutting the point of the rule with the ſame parallel vnto the Equinoctial line it ſheweth the parallaxis of long. reckoning from the centre, taking euery houre line for 5. min. alſo. As in the 54. chap. the angle of the Eclipticke with the circle of altitude was found $34\frac{1}{2}$, the parallaxis in the ſame circle by the laſt chap. was then 45. min. I reckon on the *Limbe* $34\frac{1}{2}$ deg. and thereto lay the rule: then on the rule I reckon from the centre 45. min. or rather 9. degrees, ſo that euery deg may heere repreſent 5. min. then doe I finde to cut the rule in that 9. degrees, the 5. parallel which here repreſenteth but 5. times 5. min. videlicet 25. ſo many min. is the parallaxis in latitude. Then following the houre line, cutting the ſaid 9. deg. of the rule, with the ſaid 5. parallel he ſheweth on the Equinoctial line $7\frac{2}{3}$ degr. reckoned from the centre, which taking euery deg. for 5. min. are 39. min. almoſt, which is the parallaxis in longitude of the moone at the time of the ſayde Eclipſe. And becauſe ſhe is then Eaſtwards from the 90. deg. therfore the longit. of the moone ſeen, is greater then the true longytude. And therby it is euident that the viſible coniunction ſhal goe before the true, ſo long time as the moone by ouergoing the Sun may get 39. min. which is the euariation of ſight or parallaxis in longitude.

Beholde this figure.

R The Centre of the world,
S The place where we dwell from the Centre.
K The Zenith, H T the horizon.
K B The circle of altitude, A, I the Eclipticke.
R E The true place of the Moone in the Eclipſe.
S D The place ſeene.
Q C The parallaxis in the circle of Altitude.
C V The parallaxis in Longitude.
V Q The parallaxis in latitude.
N O The ſpeere of the Sun.
L M, The ſpeere of the Moone
T X The horizontall parallaxis of the ☽.
T H G Of the ☉, A C B, the angle made. by the circle of Altitude and the Eclipticke. P the 90. degree of the Eclipticke.



Chapter 58.

Howe to finde when the Eclipſe of the Sun ſhall be.

Gemma Friſius 67.

FIrſt ſearch diligently when the time of the Suns Eclipſe ſhall be, which then in truth is, when as the Sun and the Moon are ſeene in one longitude. It hath byn already ſaide ingenerall if the coniunction of the lights happen between the aſcendent and the 90. degr. of the Eclipticke, that the viſible coniunction goeth before the true, as much as the parallexis in longit. commeth to: but if the place of coniunction be VVeſt from the 90. deg. then the viſible coniunction (that is the midſt of the eclipſe) ſhal follow the true coniunction according to the proportiō of the parallaxis in longitude. Therfore the time of the Eclipſe is gathered by the parallaxis in longitude. As in the example before, the parallaxis is found 39. min. in longitude, ſo much the moone doth exceede the Sun almoſt at the houre of the true coniunction. The queſtion is now, howe muche time it wil come too, while the moone is ouergoing the Sun this little ſpace. Therefore we muſt gather how muche the Sunne and how much the Moone doe goe in an houres ſpace, and howe much the motion of the Moone is greater then the Sunnes in an houre, and that according to ſight, if a man wil handle this matter curiouſly. The houre motion of the moone is thus had. The time of the true coniunction of the ☉ and ☾ knowen by tables for that purpoſe, and the parallaxis in long. found by the laſt chap. if the coniunction be to come, adde one hour vnto the time of the true ☌ and again for that hour ſeek the altitudes & parallaxis of the

the ☉ and ☾ as before. Let the parallaxis of longitude be added to the true place, if the moone be Eastwards from the 90. deg. of the Eclipticke, but subduct it out of the true place, if the ☾ be VVastwards, and so shall you haue theyr *loca visa*, places seene. But if the true coniunction be past: which happeneth when the place of the true ☌, is in the oriental quarter: then subduct one houre, & seek the *loca vera* the true places of the ☉ & ☾, for that time, and their parallaxes in longitude and latitude. And againe, adde or substract the parallaxes of long. to or from the *loca vera* the true places of the lights, and so haue you them *loca visa* their places seen. The subduct the seene place of the time going before out of the seen place of the time following, so haue you the *motus visus* the motion seen: that is, how much the planet goeth forwards in the Eclipticke according to sight. But if you delight curious calculating, you must do this, as wel in the Sun as in the moon. And the you must take the Sunnes motion seene, out of the moones motion seene. So shall remaine the superation or exceeding of the moone aboue the Sunne in one houre according to sight. Now therefore by the proportion of the parallaxis in longitude vnto this houre superation you may gather the time betweene the true and seene coniunction or Eclipse: appointing the first number of the proportion, the houres outgoing or superation of the moone. The second one houre or 60. min. The third, the parallaxis in longitude. So shal come forth the min. of the time sought for, which being added too or subtracted from the time of the true coniunction, sheweth the time of the Eclipse. These min. are added when as the place of the coniunction happeneth in the VVest quadrant of the Eclipticke, and are subducted in the East quarter. But this profitable institution must needes haue an example.

VVherfore, in the Eclipse hitherto intreated of the parallaxis of the moone in longitude, is found 39. min. These I adde because they are in the East quarter, vnto the true place of the moone, which is $7\frac{1}{2}$. of ♌, and so the place of the moone according to sight, which I call the place seene, or *locus visus*, shall be 8. deg. 9. mi. & 30. seconds in ♌. The parallaxis in longit. of the ☉ I will here omit for breuities sake, whereby the place of the Sun shall still remayne 7. deg. 30. in ♌, being both the true place and here allowed for the seene place. Nowe because it is most certaine that the middle of the Eclipse which as before sayd, is the visible coniunction, doth goe before the true, we are againe to get the true places of the ☉ and ☾, with their parallaxes for one houre before the true coniunction, that is, for 6. of the clocke 30. min. And then is the Sunnes true place 7. deg. 28. min. of ♌, the true place of the moone 6. deg. 56. min. of ♌: because the houre motion of the moone is 34. min. The Horoscope or ascendent for this tyme is 0. deg. 20. min. in ♍ the deg. of culmination 19. degr 30. min. of ♉, whose meridian altitude is 56. Further, the distance of the deg. of culmination from the horoscope is 100. deg. 50. min. of the Eclipticke, whereby the angle made betweene the horizon and the Eclipticke, is found to be 57. degrees 10. min. by the 51. Likewise the Sunnes altitude againe to bee 18. deg. $\frac{1}{2}$. and the moons altitude to be 18. deg. 20. mi. The angle of the Ecliptick with the circle of altitude is $\frac{55}{53}$. almost by the 53. & this is the angle of latitude, therfore subducting that frō 90. there resteth 55. the angle of longit. of the moone. The parallaxis of the moone in her circle of altitude is found 52. min. almost by the 54. and in the sun 2. min. 30. seconds: therefore by the 55. the Sunnes parallaxis in longitude is almost 2. min. which beeing added to the true place of the Sunne, maketh for this houre the Sunnes place seene 7. deg. 30. min. of ♌. Lastly, the moones parallaxis in longitude is 42. min. and better, by the 55. Therefore her place seene, is by addyng those min. found to be 7. deg. 38. min. of ♌. Now therefore let vs take the place seene of the Sun going before, vz. 7. deg. 30. min. of ♌, out of the place following, vz. 7. 30. ♌, so there remaineth 0. This nowe is the Sunnes houre motion appearing or seene, which for want of paines commeth to nothing. In the like manner subduct the former motion of the moone appearing, vz. 6. deg. 56. of ♌ out of the later which were gathered to be 7. deg. 38. min. of ♌, and so is the houre motion of the moone founde to bee 42. min. For so muche shall the moone goe forwardes in one houre according to her motion appearing or seene. And because the Sunne maketh 0. his houre motion seene, and the moone 42. min. taking 0. out of 42. the moone shal ouergoe the Sunne 42. min. in an houre. The parallaxis of the moone in longitude at the time of the true coniunction was found 39. min. and of the sunne (if I had respected it) about 1. min. 20 seconds, but you must take this but as an example, and worke it exactly your selfe, therefore the moone had ouergone the Sunne then 38. min. 40. seconds according to sight. Last of all, because the moon doth ouergo the Sunne in an houre 42. min. she shall make 1. deg. 20. seconds (which is the difference of the Sunne and moones place seene at the time of the true coniunction) in 2. min. $\frac{6}{2}$. of an houre, with $\frac{6}{42}$. of a min. ouer. This time therefore because the coniunction seene, goeth before the true, we must take out of the time of the true coniunction, so shall remayne the time of the coniunction or Eclipse, appearing vnto vs. vz. 7. of the clocke and almost 28. min.

Chapter 59.

Another manner to finde out the parallaxis in longitude and latitude, as wel in the ☉ as the ☾.

Gemma Frisius. 68.

I Haue often times in this my booke spoken of Gemma Frisius Cursor and brachiolum, but perhappes euery man knoweth not what they are, the Cursor therefore is a rule, in length a semidiameter of the Instrumēt, and is so wrought with a rabbott or a doue tayle, as workemen call it, at one ende: that hee alwayes standeth square vpon the rule, and so slippeth vp and downe from one ende to another, his one side is deuided as halfe the rule, the other into an 100. or rather a 1000. equall parts, the brachiolum is a little arme at the end of the cursor, &c. This Chapter was to be performed by Sinicall working, and therefore impertinent to the *Iewel*.

Chapter 60.

To finde the latitude of the moone.

Gemma Frisius. 8.

THe Sun neuer digresseth from the Eclipticke, but the way of the moone lyeth by, as 5. degrees from the Eclipticke

cliptick, crossing him alwayes in 2. opposite points, of which the point Northwards is called *Caput draconis*, the other Southwards *Cauda draconis*. In these points only is the moone in the Ecliptick, and in or very neere them, the moone is at all Eclipses, they haue a continuall motion contrary to signes sequele, and perfourme it in 18. yeeres, and almost 5. monethes once round.

Therfore by tables or the Ephimeris, you must get the place of the moone for the time proposed, and likewise the place of *Caput draconis*, if now the moone be departed from the *Caput* towards *Cauda*, the latitude shall be north: but from the tayle towardes the head, South. This knowen, reckon 5. deg. on the *Limbe* from the equinoctiall line, thereto lay the rule, then by subduction get the distaunce of the moone from one of knottes vz. *Caput drac.* or *Cauda dracon.* to which soeuer she shall be neerest, reckon this distance on the rule from the centre, and so the parallel cutting there, sheweth the latitude sought: this needeth no example.

Chapter 61.

How much of the Sunne shall be darkened.

Gemma Frisius 69.

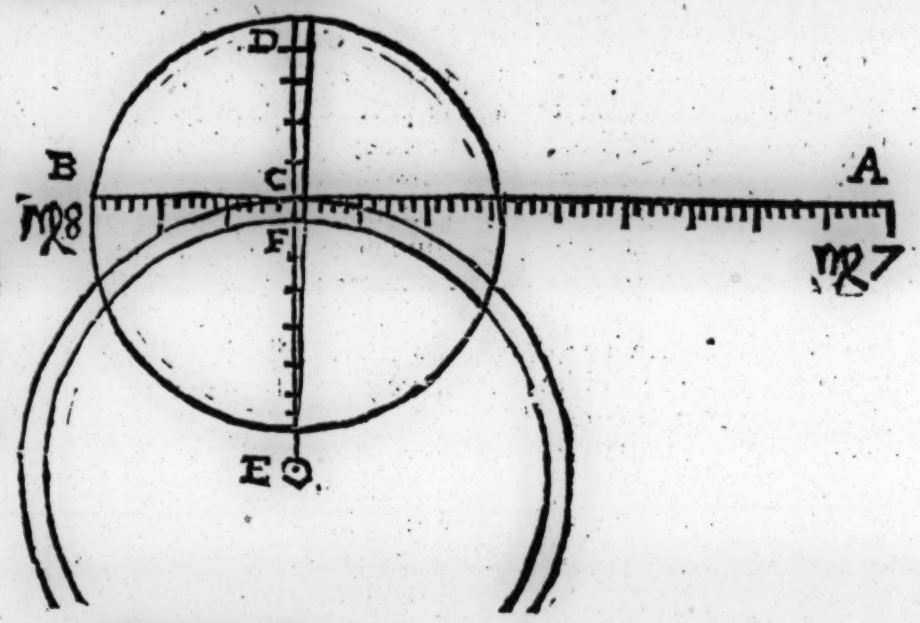
WHen as the Synode or coniunction of the lights happeneth neere one of the knots called *Caput* and *Cauda draconis*, then is there greate likelihood of an Eclipse of the Sun to be. But it is a more certayne token if the place of the middle coniunction called *Synodus media*, happen lesse then 20. deg. 40. min. before the *Cauda*, or after the *Cauda* lesse then 11. deg. 20. min. Likewise if it happen before the *Caput* within the space of a 11. deg. 22. min. or after it within 20. deg. 40. min: then is it possible that the Sun may be eclypsed, without these boundes there can be no Eclipse. But the most certaine token of all is, if the latitude seene of the moone doe exceede the quantity of both the semidiametres of the sunne and moone seene, at the time of the Eclipse seene, then shall the Sun suffer no Eclipse. Of these things the whole inquisition of the greateneſse of the Eclipse doth depend, vz. of the semidiametres of the lights, and the latitude seene at the time of the coniunction *visa*, as well of the ☉ as the ☾. But as much as belongeth to the inuention of the semidiametres appertayneth not properly to this instrumẽt, but is gotten by their proper tables. And here generally it is inough to shew, that the semidiameter *visus*, of the Sun neere his Apogeon, that is in ♋, is seene to be 15. min. & 49. seconds almost, for which you may take 16. min. in sleight calculating, the sunnes Semidiam. *visus*, in the perigeon is 17. min. Therefore when as the greatest differeth from the least but one min. there is no cause to seek ouer curiously for the other places, although the *mutatio excentrotetis*, maketh a litle diuersity in this behalf, but being almost insensible, is here by me neglected. Now for the moones *Semidiametri apparentes*, the least is 15. min. and the greatest that may happen in the coniunctions 17. min. 49. seconds, which differ little from 18. min. These therefore make ynough for our purpose.

For this take Gemma Frisius owne example, first mentioned in his 62. he recyteth of an Eclipse of the Sun to be anno 1560. the 21. day of August, at one of the clock 22. mynutes after noon, by Stophers tables, the Sun and Moone then being 7. deg. 45. min. in ♍, the latytude at Louan beeing 51. degrees almost, where Gemma Frisius remayned. Nowe because the ☉ in this Eclipse is more neere his apogeon beyng ♋, then his perigion beeyng ♑, take for his Semyademetre sixteene mynutes. The Moone is from hir Apogeon 5. *dodecatemoria*, that is 5. common signes and three degrees, and by that account beyng neere her perygeon, hath for her Semidiameter apparens 17. min. These Semidiametres therefore gathered into one summe, make 33. mynutes: and if now the *Latitudo visa* of the moone were greater then this Sunne, then as I sayde before, there coulde be no Eclypse. The *Latitudo visa* of the moone is thus gotten. You must by the 55. gette the parallaxis in latitude for the time of the *Coniunctio visa*: and likewise of the sunne, if there be any. Also the laxitude of the Moone for the same instaunt, calculated as in the last Chapter by her true place and distaunce from the neereſt knotte. If therefore this *Latitudo vera* be North, and the parallaxis of latitude lesse then it, deduct this from the true latitude, and so remayneth her *Latitudo visa* Northwardes: but if the parallaxis of Latitude, be greater then the North Latitude, subducting the North latitude out of the parallaxis of latitude, there shall be left the *Latitudo visa* Southwardes. But if the *Latitudo vera* shall be South, adde the parallaxis of Latitude to the *Latitudo vera*, and so shall you haue the *Lititudo visa* Southwardes. This therefore beyng gotten, you must take regarde also to the Sunnes paralaxis in Latitude, that is alwayes Southwards, becauſe the Sunne is alwayes in the Eclipticke, and all parallaxis happen downewardes. This therefore is all one wyth the parallaxis of Latitude. And it is alwayes added, when as the moones *Latitudo visa* is Northwardes, and is subtracted in her South *Latitudo apparens*, vnleſſe the same be lesse. For then it is taken from the *Latitudo visa* of the Sunne, and so remayneth the moones latitude from the Sunne appearing to our sight: which if you subduct from the number gathered of the Semidiametres of the sun & moone, there shalbe left the part of the sunnes diametre darkened: and multiplying that by 12. and deuyding the product by the sunnes diametre, you shal haue the poynts or dygites (as they cal them) of the Sunnes diametre darkened, as in this example the *Coniunctio apparens* or *visa*, by the 56. is at one of the clocke and 16. minutes, the horoscope 3. degrees in ♐, the angle of the horizon: and the Eclipticke 44. deg. and 45. mynutes by the 51. the angle of latitude 88. degr. almost by the 53. The altitude of the sunne 44. degrees 42. minutes almost by the 31. whereby the paralaxis in latitude of the moone is 43. mynutes almost by the 55. So also the paralaxis of the ☉ in latitude is two minutes, the *Latitudo vera* of the moone is 23. mynutes Northwardes. Nowe becauſe the parallaxis of latygude is 43. minutes, and the *Latitudo vera* 23. minutes North, subduct it out of 43. there remayneth 20. minut. the moones *Latitudo visa* Southwardes: but the Sunne also is seene distaunt from the Eclipticke 2. minutes *per aspectum*. Therefore the Moone is seene more Southwardes then the Sunne eighteene mynutes: thys is the *Latitudo apparens* of the Moone from the Sunne, which subducted from the number gathered of the semydiametres gotten before to be 33. minutes, there are left 15. minutes for the part of the sunnes diametre darkened.

Lastly,

Laſtly, theſe 15. being multiplied by 12. digites of the diametre of the ſun, make 180. which deuided by the whole diametre of the ſunne, or by 30. there are produced 6. points or digites of the ſuns diametre, which is one halfe: ſome call them points or digites of the Eclipticke.

A The end of the 7. deg. of ♍.
B The end of the 8. deg. of ♍.
A B One deg. of the Eclipticke deuided into 60. min.
C The *locus viſus* of the Sun.
C F The *Latitudo viſa* of the Sunne
C E The *Latitudo apparens* of the Moone.
F E, The *Latitudo apparens* of the ☾. from the ☉.
F G, The part of the Suns dyamet. darkned diuided into digites.
E F, The Semidiameter of the ☾.

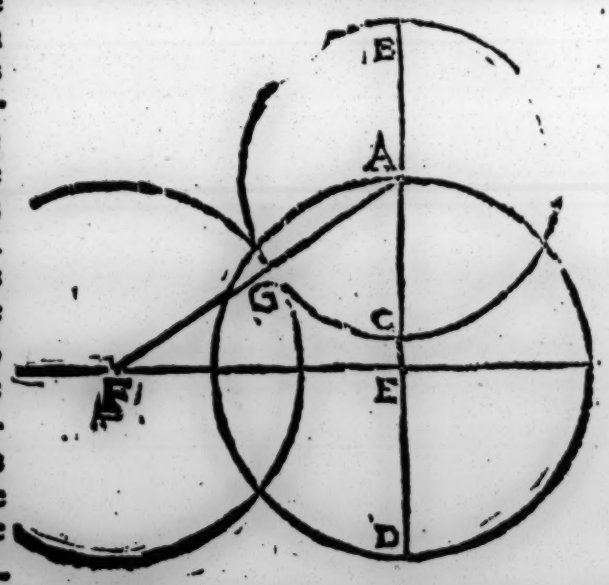


Chapter 62.

How long the Eclipſe of the ſunne ſhal endure.

Gemma Friſius 70.

MAke therefore a circle about the centre A, according to the bigneſſe of the Sunne, ſo that A B, bee 15. min. and the whole diameter B C, 30. min. and A E. The *Latitudo apparens* of the Moon from the ſun being in this laſt example 18. min. Therefore E is the *centrum apparens* of the moone. A D, the whole diametre of the moone, vpon the point E let there bee E F perpendicular: and then open the compaſſe according to the number gathered of the Sunne and moones diametres, which in the example before was 33. min. And ſo placing the one poynt of the compaſſe in the centre A, let the other touch the line E F in the poynt F. Now it is moſt euident that the Moone being in F, can couer no more of the ſunne, and that there is the end or beginning of the ſuns eclipſe. And when the moone ſhalbe in E, then is the middle of the eclipſe, which you may the eaſier beleeue, ſeeing G F is the Semidiameter of the ☾, the Sun and Moone both touching in G. Therefore E F is halfe the ſpace which the moone doth run in the whole Eclipſe of the Sunne: how much that is, you may get by our compaſſes onely on the ſcale or els multiplying A F into it ſelfe, and A E likewiſe, and then taking the ſquare of A E, out of the ſquare of A F, there ſhall be left by the laſt propoſitiõ ſaue one of *Euclides* 1. book, the ſquare of E F, whoſe roote is the quantity of the line E F. As in this example, A F is 33. min. the ſquare of this is, 1089. the line A E, being the *Latitudo apparens* of the Moone from the Sunne is 18. min. the ſquare hereof is 324. this taken out of the firſte ſquare, vz. 1089. There reſteth 765. the ſquare of the line E F, whoſe ſquare roote is about 27. min. $\frac{1}{2}$. the quantity of the lyne E F. This ſpace of the *motus apparens* of the moon is reduced vnto time by the *motus horarius apparens* of the moone founde, as in the 56. which in this example wilbe 28. min. Now ſay by the rule of proportion 28. min. of the moones *motus viſa*, make one houre or 60. min. How much then ſhall 27. min. $\frac{1}{2}$. and ſo following the rule, you ſhall finde 59. minutes, whiche are commonly called *minuta caſus* or *minuta incidentia*, that is to ſaye, howe muche it is from the beginning of the Eclipſe vnto the middeſt, and thence vnto the end whereby doubling theſe min. of incidence, you haue almoſt the time of the whole eclipſe, which here is two houres lacking 2. min. But yet I know well that theſe 2. times (that is from the beginning to the midſt, and thence to the end of the Eclipſe) are not alwaies equall vnleſſe the Eclipſe happen in the 90. deg. of the Eclipticke from the aſcendent, and that becauſe of the vnequality of the moones *motus apparens*, for the which in the orientall part the parallaxis of longit. is added to her true place, and is deducted in the occidentall.



He therefore that woulde performe this matter curiouſly ſhoulde as in the 56. get the *motus apparens* of the moone for one houre before the Eclipſe, and alſo for one houre after the middeſt of the Eclipſe, and thẽ meaſure the ſpace of the beginning of the eclipſe vnto the middle called *ſpacium incidentiæ*, by the *motus horarius*, going before, and the ſpace between the middeſt, and the end of the Eclipſe called *redditus luminis* by the houre motion of the houre following. And thus haue you all the reaſon hereof, although perhaps I haue not beene curious in the examples, ſince I haue ſet downe the inſtruction ſo plaine.

Blagraue. Note that I haue ſet downe theſe tearmes *motus viſa*, *Latit. viſa*, and ſuch like in Latine words, leaſt any man ſhould in Engliſh ſenſe ouerſlip them as words of courſe, being indeed words of art, and ſuch as this matter dependeth on,

Chap

Chapter 63.

Of the Eclipſe of the moone, the greatneſſe and duration of the ſame.

Gemma Friſius 71.

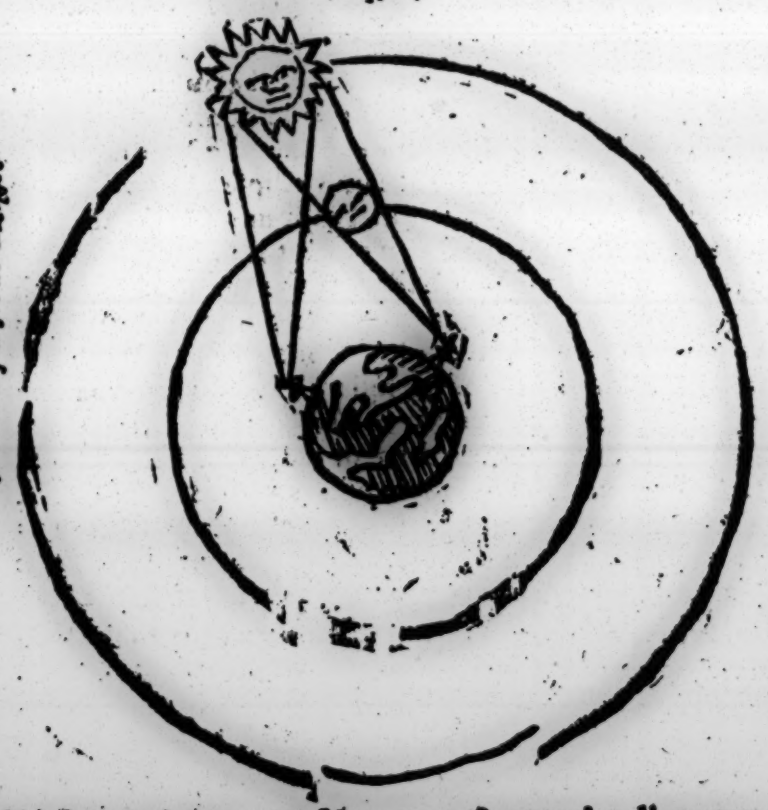
THe Eclipſe of the moone hath an eaſie calculation, the cauſe whereof is, that it dependeth not vppon the difference of ſeeing it in any reſpect (as before in the ſunne.) For as oft as there happeneth an Eclipſe of the moone, becauſe that ſhe beyng oppoſite to the Sunne, lighteth into the ſhadow of the earth, which extendeth it ſelfe farre beyond the ſpeere of the moone, and taketh away the light which ſhe had of the Sunne: ſhe by this meanes eclipſeth in like bigneſſe and manner to all places of the world: and continueth like time, and is ſeene euery where at one moment, although that it be accounted at diuers houres in dyuers places, according to the diſtaunce and difference of the meridians, as hereafter I will ſhew in the longitude of places.

Blagraue. It appeareth here that Gemma Friſius had a meaning to intreate of the longitude of places whiche Oh would God he had liued to haue performed, that it might haue beene my good happe to haue ſeen his vttermoſt, in that behalfe, by which meanes I ſhould haue eyther ſpared altogether the greate endeuour which I of neceſſitie muſt now for the credites ſake of my Iewell employ in that behalfe, or at the leaſt haue receyued ſuch light from his doynges, that I might with eaſe haue gone through, with it. But now Gemma Friſius is gone, lamenting will not helpe: he left his ſonne Cornel. Gemma behynde, who publiſhed his Fathers woorke of the Catholicon, and ſtryued to augment it by his owne induſtrye: but God knowes, againſt the ſtreame. For a man may eaſily knowe by the very matter and manner of it, his doyngs (of whiche I haue omitted the moſt part) from his fathers.

Gemma Friſius.

But now to come againe to the matter, as the Eclipſe of the moone is vniuerſall at one inſtaunt, it is farre otherwiſe in the Sunne: for one ſelfe ſame Eclipſe of the Sunne to ſome appeareth great, and laſting: to other ſome little and gone in a moment: to ſome the North part, to ſome the South part of the Sunne is ſeene to be darkened: and that in infinit diuerſities. The cauſe of ſo great variety, is the diuerſity of the places whence the ſun is beholde: becauſe that in the eclipſe of the Sunne he looſeth not his light, as the common ſort do imagine: but only that the moone commeth betweene vs and the ſun. VVhereby all countries ſee not the Eclipſe in one maner. Inſomuch that ſome doe ſee the ſunne in his ful light, when that others ſee him almoſt wholly eclipſed. For the ſeeing of it, according to the ſeituation in ſundrye places, is the greateſt cauſe of the diuerſity of the Eclipſe of the ſunne. But in the Eclipſe of the moone, the view euery where doth vary it nothing, neyther in lõgitude nor latitude, for the moone is darkened indeede, where there the ſun is not (as is already ſaid:) and therfore are we forced to ſeek ſo many variations of aſpect in the ſunne which in the moone needeth not. It ſhalbe ſufficient here to take the time of the true oppoſition of the ſun and moone for your meridian, either out of their proper tables, or out of Ephimerides, rightly calculated: & for that time to get the true place of the ſun in the ecliptick, whoſe oppoſite is the iuſt place of the moon: & then ſeek the latit. of the mooue very diligently, as is ſhewed in the 58. chapter. Laſtly you muſt gette the diameter or ſemidiameter of the moone: and likewiſe the ſemidiameter of the ſhadowe of the earth, of what quantity it is in the place of the moones paſſage. The ſame is ſomtime greater, ſomtime leſſe, for two cauſes: for when the ☾ is neere her perig. ſhe findeth the ſhade of the earth greater then in other places: the ſhadow of the earth runneth out taperwiſe vnto a point at the laſt, & becommeth leſſer by ſo much the further as it extendeth frõ the earth. Secondly, the ſhadow of the earth may be greater in the ſelfſame place of the moons paſſage, by reaſon of the diſtance of the ſun from the earth. For the neerer the ſun commeth toward the centre of the earth, ſo much the ſhadow of the earth growth the narrower & ſhorter, contrarywiſe, by the ſunnes bearing backe from the Earth, it is extended and enlarged in the ſame place of the ſpeere of the moone, in which it was narrower before, as by theſe figures you may ſee.

The Sunne eclipſed.



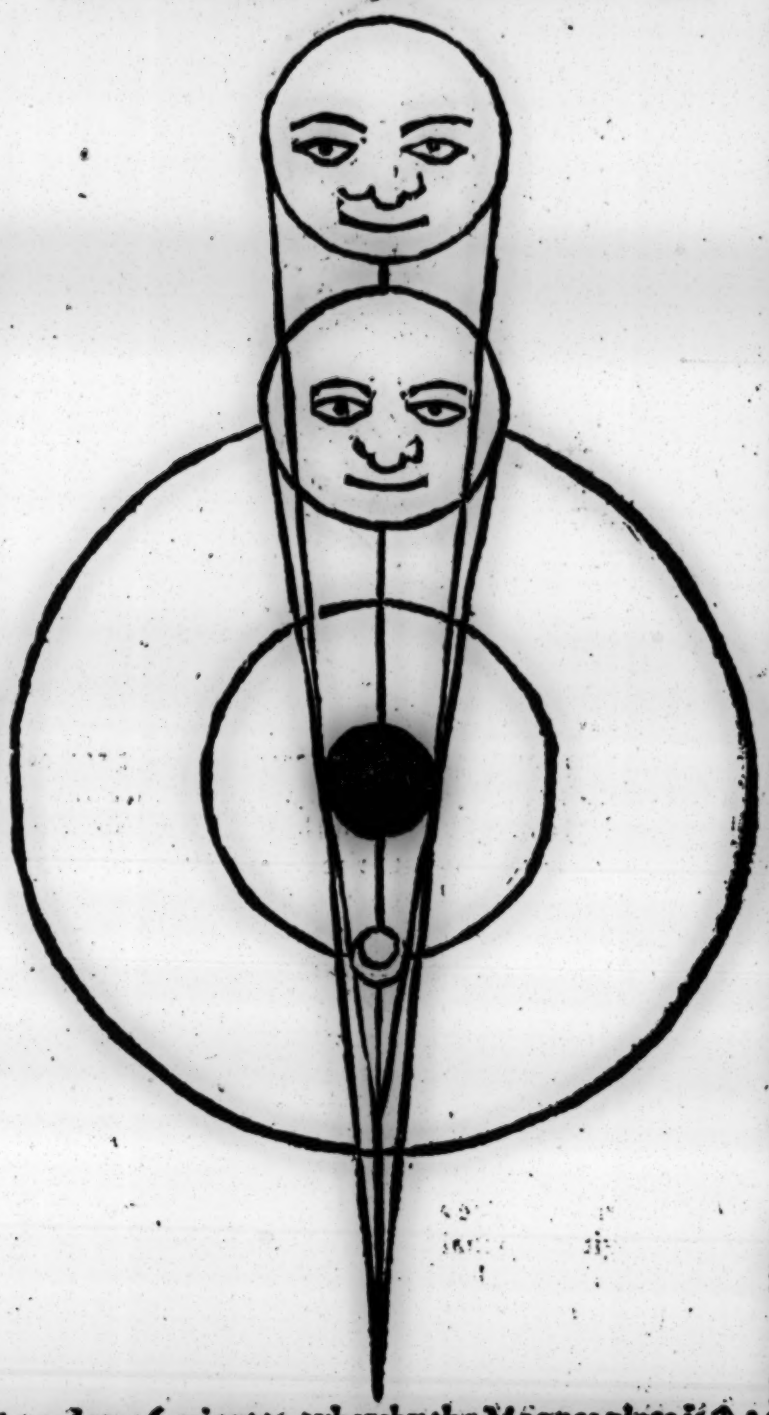
Therfore to our purpoſe you muſt know by tables the time of the true oppoſition of the ſun & the moone in the Eclipſe: and the place of the ſun & the moon for the ſame time, together with the latitude of the moon. And then the ſemidiameter of the moon, & of the ſhadow, through which the moons paſſage is. And here wil I ſhew in brief as much as ſhal ſuffice the learner to get the ſemidiamet. of the ſun, the moon, & of the ſhade by the *motus horarij*, or diurnall motions of the Sunne and the moone. It is found out by the induſtry of artificial men, that what proportion 20. beareth to 11. the ſame doth the ſuns diurnal motion vnto his diamet. appering. Therfore multiply the ſuns diurnal motiõ gottẽ out of any tables into 11. & deuide the product by 20. ſo haue you the diam. of the ſun. The cauſe is, for that both the greatnes of the diamet. *apparens* of the ☉, & ☽ ſwiftnes of the ſuns motiõ are increaſed & diminiſhed in like proportion, according to ☽ ſite of the ☉ in his excentricke.

The

The sunne in his Apogeon or farthest distance from the earth.

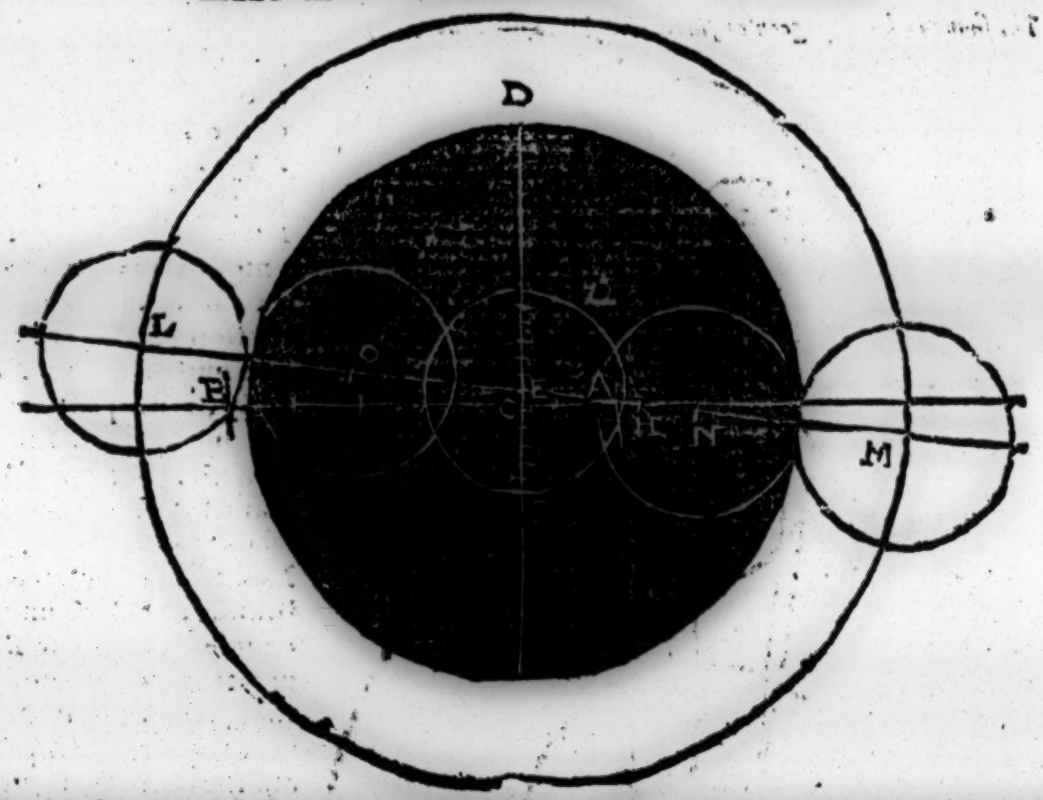
The sun in his perigion or neareſt to the earth.

The figure of the Eclipſe of the Moone.



The diametres alſo of the Moone do likewiſe keepe a proportion with her motion, ſo that the diametre *Apparens* of the Moone is almoſt equall to her *Motus Horarius*; Puruacchius ſaith, what proportion 48. beareth vnto 47. the ſame hath the *mot. horarius* of the moone vnto her *Diameter apparens*: therefore if a man take the *motus horarius*, or houre motion of ☽ for her diametre, he ſhal not miſſe a minute. But to haue it exactly, multiply the *motus horarius* of the Moone into 47. & deuide the product by 48. & therof ſhal come the moones diametre. Laſtly the diametre of the ſhadow of the earth you ſhal thus get. Multiplie the diametre of the moone founde as before into 13. the product deuide by 5. and ſo ſhall you haue the greateſt quantitie of the ſhadowe in that paſſage of the Moone, that is to ſay, the ſunne being in the Apogeon: but the ſunne being in other places, the ſhadow abateth in the ſame conſtitution of the ☽, as is ſaid, by ſo much the more as the ſun commeth neare to the earth: and how much the diametre of the ſhade is in another place, that is found by the *motus horarius* of the ſunne, for by how much greater it waxeth in other places, the ſhade of the earth is made leſſe, by ſo much ten times told: Theſe rules are both generall, exact, & moſt excellent, which by a familiar example I will manifeſt.

In the yeare 1555. the 4. of Iune the 14. houre, and 42. min. there is like to be a great eclipſe of the Moone, which is to be knowne in that the place of the ☽, at the time of the true oppoſition is not farre from *Cauda Draconis*. For the place of the ſunne is according to common tables ♈ 22. deg. 46. minutes, whereby the Moones place is ♎ 22. 46. alſo the *Caput Draconis* is in ♈ 23. deg. 8. minutes, therefore *Cauda* is in ♎ 23. 8. ſo that the true place of the Moone is but 21. minutes from *Cauda Draconis*, wherby there is not onely knowne that an Eclipſe ſhall be, but alſo a verie great one: for as oft as the Moone is aboue one deg. 8. min. in latitude any way, then ſhall the Moone auoide the ſhadow in her race. But heere the latitude of the Moone is ſcarce two min. which is a token that the Moone ſhalbe verie neere the centre of the ſhadow of the earth in this Eclipſe. Now to know the quantitie & continuance of the ſame, we muſt know the ſemidiametre of the Moone, & of the ſhade of the earth by the rules before ſhewed. The *Motus Diurnus* of the ☽ in that place is 13. deg. 5. therefore her *Motus horarius* ſhalbe 32. min. 40. ſec. and ſomwhat leſſe: then this ſhalbe the diametre of the moone: but if you wil know it exquiſitly, multiply 32. min. 40. ſec. by 47. & deuide the product by 48. & ſo ſhal you haue 32. min. 20. ſec. almoſt, & the ſame is the iuſt diametre of the Moone: halfe that vz. 16. min. 10. ſec. is her ſemidiameter: which multiplie by 13. and the product deuide by fiue, you haue the diametre of the ſhade of the earth in the place of the Moones paſſage in her ſphere, the ſun being in his apogeon, the multiplication of the diametre maketh 25220. ſec. the diuiſion produceth 5044. ſec. that is 84. min. 4. ſec. ſo that the ſemid. of the ſhade is 42. min. 2. ſec. And becauſe the ſun is neare his apogeon, the ſhadow of the earth cannot be diminiſhed any quantity, to be accounted of by reaſon of the ſuns comming neare vnto the earth, otherwiſe you ſhould heere ſeeke the *motus horarius* of the ☉ being in his apogeon and alſo for the place where he now is, and the difference of theſe motions decuplied, that is multiplied by 10. ſhould be taken from the greatneſſe of the ſhade here found, & ſo ſhall you haue the exquiſite bigneſſe of the ſhade of the earth, but here for methodes ſake we thinke not good to proſecute euery leaſt matter, but leaue ſuch curioſitie to thoſe that haue more leiſure, or who are wholly addicted to theſe ſtudies. By theſe things found, you may onely by drawing of a figure knowe the greatnes and duration of the eclipſe, draw therefore the ſtreight line A B. which ſhall be the 22. deg. of ♎ deuided into 60. min. Then in the 46. min. in which ſhall be the centre of the ſhadow of the earth, that is to ſay, oppoſite to the ☉ at the time of the true oppoſition, place the one foote of your compaſſe, and there according to the quantitie of the ſemidiametre of the earth now founde videlizet 42. minutes, make a circle for the ſhadow of the earth, the centre being C, whereon draw the line D E perpendiculer to A B, which is the circle of the latitude of the Moone, and becauſe the latitude of the ☽ is found 2. min. northwards, take in your compas 2. of the 60. parts of A B, & place the ſame from C. towards D in the point E. & that is the centre of the Moone: in this centre according to the ſemidiametre of the ☽ vz. 16. min. 10. ſec. make the circle of the moone, which being done you ſhal ſtreight ſee before



your eyes that the whole moone is drenched in the shadow of the whole earth without any calculation. And if you deuide the whole diametre of the moone into 12. digits, you shall apparantly see how many digits or points the eclipse of the moone shall be. For looke how many digits of the moones diametre D F. shall containe (with the distance from the vttermost shade of the earth vnto the edge of the moone fartheſt drenched within the darkneſſe) ſo many digits or points as they now call them shall the eclipſe be ſaid to be. For D is the point of the shadow of the earth, moſt bending from the ecliptick towards the which the moone departeth, but by numbers we shall thus proceed. Becauſe E F is 16. min. & 10. ſec. and the latit. of the Moone E C. 2. min. therfore C F shalbe 14. mm. 10. ſec. to which there being added C D, being 42. min. 2. ſec. the whole D F shal be 56. min. 12. ſec. Now by the rule of proportion, if 16. min. 10. ſec. be worth 6. digits, what shall 56. min. 12. ſec. be worth: & ſo following on the rule, you shal get 19. digits & 41. min. of a digit ſuch as the whole diametre of the moone maketh 12. To be short the whole moone containing 12. dig. shalbe wholly drenched in the shadow: & that ſo deepely that the shadow shal exceed the moone 8. dig. almoſt, which is as much to ſay, as that the eclipſe of the moone shalbe almoſt 20. digits. The moone her ſelfe conteineth not aboue 12. points, but the depth of the moone within the shadow of the earth shalbe almoſt 20. dig. wherby you shal vnderſtād that when as the greatnes of the eclipſe exceedeth 12. digits, then shal there be an eclipſe with tarriance, as they call it, that is, the moone shal tary a certaine ſpace in the darknes before she shal receiue her light againe of the ſunne & how many the more digits there shalbe found, by ſo much the longer shall the tariance of the moone be in the darkneſſe, & the whole eclipſe endure: both which exactly to deſcrie, thus muſt you do. Open your compaſſe to the quantitie of the ſemidiam. of the earth and the ☾ ioyned togither as in this example, vnto 58. min. 12. ſec. and on the centre of the shadow of the earth vz. C, deſcribe a hidden or blind circle, and note wel the touching of this circle with the way of the Moone. I cal the way of the moone a line drawne by the centre of the moone vz. E, parallel if you liſt to the line A B. or if you be diſpoſed to proſecute theſe matters more narrowly: let this line make with D F the blunt angle D E M. of 95. deg. as heere the line L E H M. is, for this is the true way of the Moone whereby H is the place of the next knot. This line is cut of the blind circle in the points L & M, in which points the Moone being, maketh the beginning and end of the eclipſe. Laſtly, from the 2. interſections of the shadow of the earth, and the way of the moone, ſet the ſemid. of the moone by helpe of your compaſſe in the way of the Moone vz. in N and O. For in the one of theſe places the Moone is altogether entred into the shadow of the earth and in the other, beginneth to get out thereof. Therefore you haue 5. moones if you liſt, the firſt on the centre L. where the ☾ begins to eclipſe: the next on the centre O, when she is wholly darkned: the third on the centre E, when she is in the middle of the eclipſe, the fourth on the centre N. when she beginneth to get light againe: the laſt on M. where the eclipſe endeth, ſo that L O is the ſpace, that the moone goeth from the beginning vntill she be all darkned, and O N of her whole darkning, which they call the tariance in Latin *Mora*: therefore O E is halfe her tariance called (*Dimidia mora.*) All which parts you may meaſure with your compaſſe by the parts of the ecliptick before made: and deuiding ech of them by the *Superatio horaria* of the moone you shall haue all the times ſought for, or the whole taken togither vz. L M. And by numbers may thus be performed. For that E C is 2. min. in latitude, and ſtandeth almoſt perpendiculer on L M. the way of the ☾, therfore the ſquare of L C shal be equal to the ſquares of L E, and E C. therfore taking the ſquare of E C out of the ſquare of L C, there shal remaine the ſquare of L E by the laſt propoſition ſauing one, of the firſt booke of Euclide: For L C is the ſum gathered of the ſemidiametres of the moone, and the shade vz. 58. minutes, 12. ſeconds, but letting paſſe the ſeconds, the ſquare of L C shall be equall to 3364. herehence taking the ſquare of C E, vz. 4. there is left 3360. the ſquare of L E, whoſe ſide is ſomwhat leſſe then 58. and that is the line L E, ſo much alſo is E M. Let no man carpe heere becauſe we ſaide that C E was perpendicular to the Moones way, when as before we drew him perpendiculer to the ecliptick, the beſt learned men will neglect ſo much, becauſe it bringeth no perceiueable error. Therefore the line L E being knowne to bee fiue 58. minutes almoſt, take out of that the ſemidiametre of the Moone vz. 16. minutes 12. ſeconds, the line L O shalbe 41. min. 48. almoſt, which they call *minuta incidentiæ*, and N M is almoſt equall vnto it, called *minuta repletionis*. Laſtly ſubduct L O being the minuts of the eclipſe waxing out of L E, there shal be left O E halfe the tarriance vz. 17. min. 12. ſeconds almoſt. Now becauſe the *Motus horarius* of the Moone

was 32.min. 40. ſeconds, and the ſunnes *Motus horarius* 2.min. 22. ſec. take the one out of the other there reſteth 30.min. 18.ſec. the ſuperation of the moones gate aboue the ſunne in one houre. By this deuide all the parts of the Moones way before founde and you ſhall haue the tymes correſpondent vnto them. As becauſe the Mynuts of the waxing of the Eclips were 41. mynuts 48.ſo deuide thoſe by 30.mynuts 18.ſeconds by reducing each number vnto ſecondes,ſo ſhal amount one howre 22.mynuts,the time of the waxing and likewiſe of the wearing of theclips. Likewiſe deuide the minuts of *Dimedia mora* vz. 17.my.12.by 30.min. 18. ſeconds there commeth,33.my.of an howre,wherby the whole darckning is 1.howre 6.my. the whole time of the Eclips 3.howres 50. mynutes.

Chapter 64.

How to get the parallaxis of the Moone out of the heauens at any houre propoſed, the latitude knowne with the longitude and latitude of the ☾.

Gemma Friſius 72.

THe Moones place giuen at any inſtant propoſed, calculate hir altitude on the *Iewel* by the 32. take the ſame inſtant the altitude of the Moone, by obſeruation, if theſe two agree there is no paralaxis, which can neuer bee except the Moone be neere the zenith, the difference of theſe two is the paralaxis, but this requireth exact calculation and obſeruation.

Chapter 65

To know the greateſt, or horizontall parallaxis of the Moone when ſhe is to be ſeene.

Gemma Friſius 73.

GEt the paralaxis of the ☽ for the altitude ſhe is in at the inſtant propoſed, then reckon the ſame altitude on the rule of the Iewell inwardes, and then moue to and fro till that poynt of the Rule doe cut the paralell anſwering the paralaxis of altitude,and ſo doth the *Finiter* ſhew in the *Limbe* the horizontall paralaxis ſought for. In this dooing you may accompt euery degree of the paralels for 10.mynuts.as in the 54. and 55.Cap. The greateſt horozontall paralaxis is the ☾ being moſt in her perigeon 63. mynutes almoſt. Hereof ſhall I treat at large in my worke of Longitudes.

Chapter 66.

The Longitude and Latitude of two Starres giuen, how to know the diſtance betweene them.

Gemma Friſius. 74.

BY this Chap.and others following is ſhewed to find the diſtance of ſtars, & alſo of townes, cities, & places as eaſily,and redily as on the globe it ſelfe, which no doubt muſt needs make the inſtrument very delectable. The longit.and latit. of any two ſtars had,ſubtract the greater long. out of the leſſe, ſo haue you the difference,which being had,number in the *Limbe* from thequinoctial the greateſt latitude of the two ſtars ſouthwards,if the lat it. be ſouth,or northwatds,if north: and thereto lay the Zenith point of the *Reete*,then in the meridian reckoned from the *Limbe* anſwerable to the difference of longit.number the lat.of the other ſtar and that from thequinoctiall accordingly,and the azimuth cutting there, ſheweth the diſtance ſought for, if you reckon thereon thence to the zenith point.

Example. I would know how many degrees *Canis maior* is from *Oculus tauri* I finde by Stophler (for any tables will ſerue the turne, as well old as new for that their difference of long. and their latit. neuer alter) that the longit. of *Oculus tauri* was then in ♊ $2\frac{18}{60}$ deg. that is from 0.in ♈ $62\frac{18}{60}$ deg. and his latitude,5.deg.$\frac{10}{60}$ ſouthwards. Likewiſe *Canis maior* in ♋. $7\frac{18}{60}$ which commeth to $97\frac{18}{60}$ deg.from ♈, and his latit. to be $36\frac{10}{60}$ ſouthwards alſo. Now ſubducting the leſſe longit.vz.$62\frac{18}{60}$ out of the greater vz.$97\frac{18}{60}$ there reſteth 35. the difference of longitude. Firſt therefore I reckon in the *Limbe* from thequinoct.line, the greateſt latit. of theſe vz.$39\frac{10}{60}$ towards the ſouth pole,becauſe the latitude is ſouth,and thereto I ſet the zenith of my *Reete* then in the 35. meridian counted from the *Limbe*, I reckon the leſſer latitude from thequin. line vz. $5\frac{10}{60}$ deg. ſouthwards,alſo becauſe that lat.was likewiſe ſouth: this done,I ſee the $52\frac{1}{4}$ azimuth to cut the place of the ſecond ſtarre whoſe deg. betweene the ſame place and the zenith by helpe of the almicantaries, I finde to bee $46\frac{1}{4}$ therfore I boldly pronounce that the diſtance betweene *Oculus tauri* and *Canus maior* is $46\frac{1}{4}$ degrees.

Chapter 67.

How the diſtance of any two ſtarres vnknowne may be had.

Gemma Friſius. 75.

THis may you do by the 38.Chap.getting their longit.and latit. out of the heauens,& then proceeding by the laſt Chap.But yet an eaſier way without al that circumſtance is thus: take the altitude and azimuthes of both ſtarres at one inſtant, or at the leaſt the diſtance of their azimuthes without reſpect of the meridian: theſe had the working is all one with the laſt Chapt.but onely chaunging the names, the difference of their azimuthes, for the difference of longitude: and their altitudes for their latitudes, and where as the equinoctiall did in the laſt Chapter repreſent the Eclipticke, hee muſt heere repreſent the horizon and the parallels, circles of altitude, &c. This needeth no example, for if you will ſuppoſe the altitudes

of two ſtarres equall with the latitudes in the laſt Chapt. and the diſtance of their azimuthes equall to the difference of longitude there, the matter is already performed.

Chapter 68.

To know how long the taile of a Comet is.

Gemma Friſius 76.

WOrke altogither as in the laſt Chapt. with the head and laſt point of the taile of the Comet, as though they were two ſeuerall ſtarres and the diſtance found is your deſire.

Chapter 69.

Another way to ſerch the diſtances of ſtarres one from another.

Gemma Friſius. 78.

GEt their declination by the 7. Chapt. and their difference of right aſcenſion by the 17. or 38. or by obſeruation, theſe had, worke altogither as in the 64. Chapt. onely chaunging the names, as is ſaid in the 65.

And thus haue you diuerſe and excellent wayes to examine the diſtance of ſtarres.

Blagraue. Truely theſe are very prettie concluſions & pleaſant, but yet not ſufficient to correct and amend the places of fixed ſtarres and planets, and the whole globe, as G.F. affirmeth, except they be performed by a very huge inſtrument at the leaſt of 10. foote diametre. But the onely way inſtrumentally to performe theſe things in my opinion (*Saluo meliori iudicio*) is by their croſſe ſtaffe to take their diſtances, and thereby to rectifie their places of longitude and latitude, vpon a great quadrant, as I will ſhewe you hereafter in my booke of logitudes if God giue me leaue and leiſure once to come to it, to the which no doubt theſe Chapters are good preambles and introductions, notwithſtanding for thoſe that take delight in varieties, I will ſomwhat helpe to fit their fancies. If you would know the diſtance of any two ſtarres placed in my *Reete*, turne it about till ſome one meridian touch the apex or point of either ſtar, and the degrees of the ſame meridian included betweene them, numbred by helpe of the parallels ſheweth your deſire, but then you muſt be ſure that theſe ſtarres be both North, or both South, declining from the equinoctiall. In my *Reete* I appointed all the North ſtarres to point outwards from the centre, and the South inwards.

Example. Oculus tauri, & *Canis minor* are north ſtars, and in my *Reete* their apex pointeth outward, turne the *Reete* about & at laſt you ſhal find the $17\frac{1}{3}$ meridian to touch both their apex & the diſtance betweene them on the ſame meridian $46\frac{1}{2}$ deg. But if one ſtar be North & the other South, yet wil I wiſh you wel: then turne about the *Reete* vntill the apex or points of either ſtarre do touch ech a ſeueral meridian, but equally taken on ech ſide of the axtreeline, and then muſt you reckon from the one of them to the pole, and from the pole backe againe to the other, and ſo haue you your deſire.

Example. Of *Oculus tauri* & *Canis maior*, the one in my *Reete* pointeth outwards, the other inwards, wherfore I turne the *Reete* till at the laſt I finde *Oculus tauri* to cut the 46. meridian on the left ſide of the axtreeline, and *Canis maior* the 46. meridian on the right ſide of the axtree, where I finde from *Oculus tauri* to the pole $23\frac{1}{4}$ deg. and from the pole to *Canis maior* $23\frac{1}{2}$ deg. theſe put togither make $46\frac{3}{4}$ deg. the diſtance of theſe two ſtarres.

Chapter 70.

The longitude and latitude of two ſtarres giuen, to get the angle of ſtation of one from another.

Gemma Friſius 80.

THe angle of ſtation is the diſtance of the azimuth, wherein any ſtarre is from the meridian of ſome called the angle of poſition. Place therefore the two ſtarres giuen according to their difference, longitude and latitude giuen in all reſpectes, as in the 64. Chapt. and the very ſame azimuth that there ſheweth their diſtance, ſheweth alſo the angle of ſtation, of one of them from another, being counted from the *Limbe*, and therefore I neede ſet no other example, but looke in my booke following of Coſmographie, and you ſhal perceiue it plaine, the like may you performe by their difference of aſcenſcion and declination, or their altitude and difference of azimuthes.

Chapter 71.

To know whither three ſtarres or cities be in one great circle.

Gemma Friſius. 81.

THis Chapter is verie pleaſant and no leſſe profitable in amending the places of the fixed ſtarres in the globe celeſtiall. Firſt therefore gette the angles of ſtation by the laſt Chapt. of one of the ſtarres from ech of the other, and if theſe angles be equall then are they in one great circle, elſe not.

Blagraue. This Chapt. of Gemma Friſius is true, ſo that you get the angles of poſition of the middlemoſt ſtarre, and of one of the other from the third, but if you get the angles of poſition of the other two from the middlemoſt, then if the angles of poſition being equall do make one ſtreight line, that is to ſay, if the one be Northeaſt, then if the other be Southweſt or ſo forth, you haue your deſire, euen like as in the compaſſe any two oppoſite points make one ſtreight line.

Chapter 72.

That the taile of a Comet extendeth it ſelfe directly from the Sunne, and howe you may try it.

Gemma Friſius 82.

THE Philoſophers affirme that a Comet is ingendered of a ſoft clammie ſlime, or viſcous ſubſtance, apt to nouriſhe fire drawen vp into the highermoſte region of the ayre, and there is ſet on fire, by the firy element, to whiche he is very neer: & that his taile cometh of the flowing or dragging of ꝑ matter, like as in ſtarres that ſhoote, where the matter of exhalation is, by his ſhutting drawen in length. But as concerning the tayle, that cannot bee ſo, both becauſe his motion is verye ſlowe, and alſo for that it hath alwayes been obſerued of learned men, as namely, Apian and others, that the tayle of euery Comet extendeth it ſelfe directly from the Sunne, ſo that the Comet his taile, and the Sun are in one great circle, as at all times by the laſt chapter, you may proue when any Comet happeneth.

Cornelius Gemma. It hath pleaſed God, as it ſhould ſeeme, to take away Gemma Friſius, when he hadde perfourmed thus farre, and before he had furniſhed this chapter, whiche his ſonne Cornelius G. took in hand to accõpliſh & proſecute the ſame very tediouſly, to little purpoſe, as hee doth all the propoſitions & chapters afterwards, which I mean to abreuiate in as much as ſhalbe neceſſary, & the rather becauſe I ſhall ſtuffe out this worke with ſuch ſtore of matter, that there will be no place for ſuperfluities. Hee taketh occaſion here vtterly to condemne all the Philoſophers, who ſay that there is a firie region or element next aboue the ayre, affirming that all fire and heate remayneth in the Sun. And further, that Comets are ſet on fire by the Sunne only: his greateſt reaſon is, for that if the comet were ſet on fire by force of the firy element, then muſt of neceſſitie his taile bee rounde about alike, but becauſe the tayle is alwayes found directly from the Sunne, therefore it is manifeſt, that the Sunne is the only cauſer of his flamme, appoint the Sunne, the Comet and his taile, inſteede of the three ſtarres in the laſt chapter, and the working is all one, if you liſt to make experience thereof.

Chapter 73.

To knowe what point of the heauen is in the Meridian of any place at any inſtaunt propoſed, and what it is a clocke in any other countrie.

Cornelius Gemma 85.

PLace the degree of the Sunne to the houre propoſed in the *limbe* by the help of the rule then number in the *Limbe*, from the noone line, the difference of longitude of that countrie from yours Eaſtwardes, if it lye Eaſtwardes, or VVeſtwardes, if otherwiſe: and thereto lay the rule, and it ſheweth the degree of the Zodiacke, or any ſtarre lying in the ſame meridian. Nowe if you turne about the *reete* til the degree of culmination of the inſtant propoſed lye vnder the rule, and then remoue againe the rule too the houre propoſed, marking what degree of the Zodiacke he there cutteth, laſtly, remouing the rule and the *reete* together, being ſtill on this laſt degree, vntill the degree of culmination come againe to the noone line, it ſheweth then in the *Limbe* the houre of the other countrie ſought for.

Example.

The Sunne being in ♉ 1. degree, I woulde knowe at ten of the clocke in the morning, what degree of the Zodiacke is the meridian at Rome: and what of the clocke it is there, whiche to doe, I number the difference of the longitude of Rome from vs, vz. 23. degrees, and that Eaſtwardes from the noone line, becauſe Rome lyeth Eaſtwardes, which I knowe, becauſe his longitude is more then ours: whereto I lay the rule, which ſheweth in the Zodiacke ♈ 23. degrees 45. the ſame degree is then in the meridian at Rome, and euery ſtarre or point els hauing that degree his mediation: which knowen, bryng the degree of culmination which then is ♓ 29 degrees vnto the ſame difference of longitude, and then turne the rule againe too the houre propoſed, vz. ten of the clocke, there ſhall you finde the 6. $\frac{2}{3}$. degrees of Aries. Nowe turne about the rule and the *reete* together, vntill the degree of culmination, vz. ♓ 29. come againe into the noone line, and ſo doth the rule ſhewe 11. of the clocke, and 9. degrees paſt, whereby I conclude, that it is 11. of the clock, & 9. degrees paſt at Rome, when it is but 10. at London.

Blagraue. The ſame may you doe another way, if the *limbe*, or vttermoſt Azimuth of the *Reete* were deuided into 12. equall partes or houres, the Zeneth point being twelue at noone, & his Nadir 12. at night, and then placing the Zeneth point on the difference of longitude, the rule laide on the *limbe* of the *Mater* to any houre ſheweth in the *limbe* of the *reete* among theſe newe houres, the houre of the other countrie.

Chapter 74.

How to direct a ſhip by the ſtarres in any voyage.

Cornelius Gemma 86.

VVHen you ſet fcorth from the ſhore, ſet the degree of the Sun vnto the ſame houre or inſtaunt, and thence turne the rule vnto the difference of longitude, as in the laſt chapter, of the place you meane to ſayle vnto: then numbering on the rule from the *limbe* inwardes, the latitude of the ſame place you goe to, there is the Zenith of the ſame. Nowe if there chaunce any ſtarre to be in this Zeneth, direct your ſhip full vpon him, & you ſhall goe as certainly, as if Mercurius were your guide. But if the ſtarre be in the ſame meridian but not in the Zenith, reckon howe many degrees on the rule hee is diſtaunt

The fourth booke of the Mathematicall *Iewel*: newly compyled and set foorth by Iohn Blagraue *of Reading Gentleman*: *conteineth diuers* Astronomicall Propositions, and vses of the Iewell, of his owne inuention, no lesse pleasant and commodious, then those whiche by other Authours haue beene founde heretofore on their sundrie Instruments.

Chapter 1.

An aduertisement of the Authour concerning necessarie Instrumentes.

HAuing in my thirde booke finished all the Propositions and conclusions, whiche haue been written by any authour heretofore vpon their seuerall Astrolabes lately: and lastly set downe, gathered togeather, and wonderfully encreased and augmented by that worthy and learned man Gemma Frisius, on his Cetholicon Astrolabe, and applyed all those too my *Iewel*, together with diuers of my owne additions, whiche performeth them an hundred folde more playnely, sensibly, and compendiously, then hys Catholicõ, as he that wyl trie both shall testifie: It were much for me now to come after so many learned men, and to bring in place any thing more worth the liking. Some wil think I go about to finish Apelles picture, that no mã yet durst vndertake: & perhaps say, who is bolder then blind bayard? or *Asinus ad Lyras*: knowing my rusticall education, and simple capacitie: VVell, bee as be may is no banning, such a ones words are not worthie the scanning, hee that feareth euery grasse must not pisse in euery medow. Babling tongues be hanging type, I wey not theyr glauering, I acknowledge my learning little, and truly my leasure lesse, notwithstanding as much as lyeth in my lot I will shewe my good intent in these volumes following. The rest I leaue to those that haue better wherewithall: to whom I yeelde my labours but as a preparatiue. The shoemaker can goe no further then the shoe, neither can the tayler make his coate bigger then the cloath wyll yeeld. Such as it is, yet I doubt not but it will bee liked of some at the least. Therefore too the matter, I woulde wish any man that woulde more effectually go to worke in these matters, euer and besides his little portable *Iewel* of mettall, which I taught to make in the first booke that must be his companion in all companies, and his page in all places, to haue neuertheiesse at home in his chamber or studie, a great *Iewell* of two foote diametre, if not of mettall, yet made of a round boorde, couered with a fine paste boord, whereon hee may distinguish the necessary lines with coloured ynkes very beautifully. The *reet* he may make of like paste boord, which hadde neede too bee made of good strong paper royal, foure papers in thicknesse, for neither is your bought paste boord broad enough, nor good enough: the cẽter of this paste boord *Reete* wold be lined with a thin plate of brasse for wearing: I made my selfe one of them that serued me to great purpose, and therefore I am sure I tell you true. Let your spaces of your meridianes and your paralelles in the *mater*, be degrees a peece at the most, and so likewise your azimuthes and almicantares in your *reete*, yet it is better that those almicantares, that you cut out be eche a degree, & the other that you leaue in like barres or the spiders web, to be ech but halfe a degree: for if you will haue your worke sufficiently transparent, two partes at the least must be cut out, and a thirde left in. Also bee sure that you deuise to leaue strings enough, in placing your staires, and cutting out the lower part of your *reete*, that the paste boord may beare out it selfe, and drawing them like braunches handsomely, the *reete* will shew the fayrer though they be not necessarie. I did also at euery 15. azimuth, leaue a string, a degree in breadth from the *finitor* to the Zenith, to keepe my almicantares a part at their true width, take heede you wishe not you had done so too when it is too late. In euerye of these stringes I did write the numbers of my Almicantares from 5. to 5. &c. tyl 90. And then if you had two other, the one made for a globe terrestriall, and the other celestiall, the meridianes and parallels, 5. degrees wide a peece, as hereafter in my Cosmographie shall be shewed, and beyng all three of equall bigneese, one *reete* may serue them all, they will be verie pleasaunt. These had, you are in heauen alreadie, except you chance to be in loue with a quadrant, and the crosse staffe, which in or before my booke of longitudes of regions, I wyll teach to make, without which you cannot be, if you will pearce vnto the exactneese of euery thing.

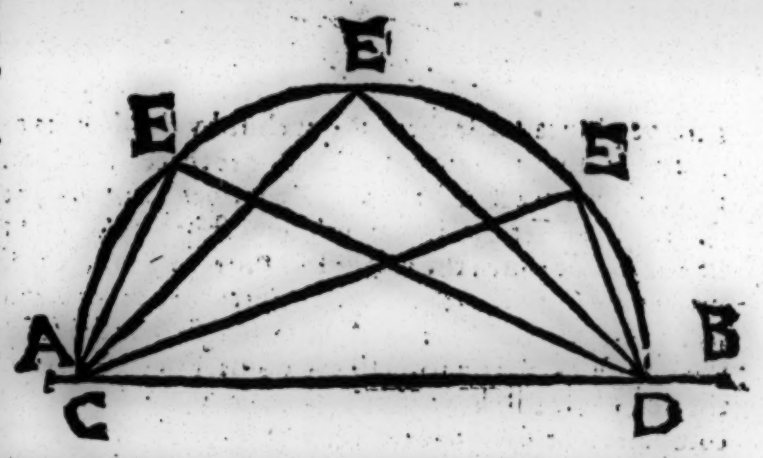
Chapter. 2.

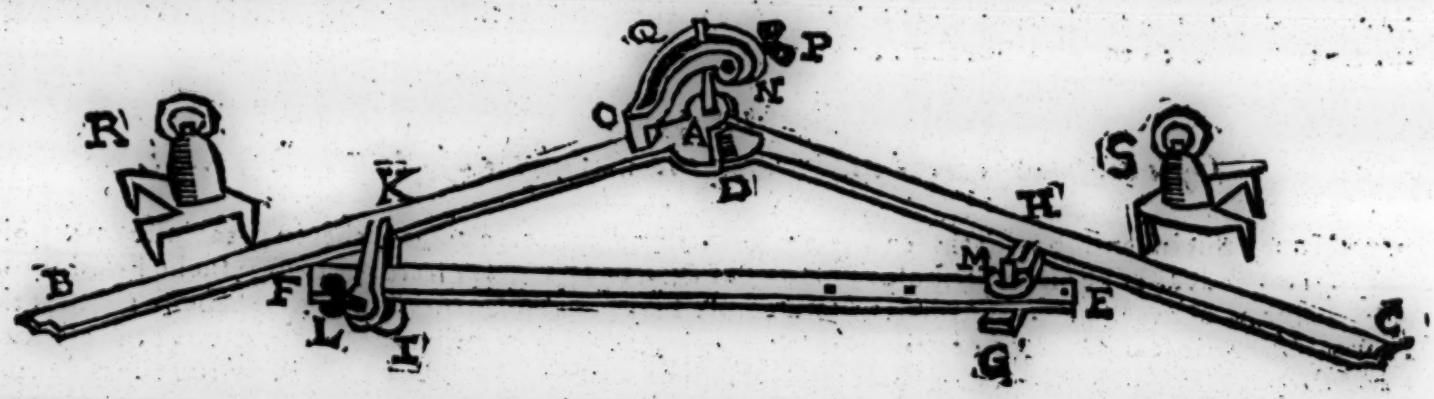
How to make an instrument of three streight rules better then any compasses to describe any arch with, be his center neuer so farre off.

IN making my *Iewell* on a boord of two foote diametre, I was not a little troubled to drawe the arches of the meridians and paralels, which come neereest too the center, and therefore I should doe you wrong if I should leaue you to seeke in that behalfe, truly I hadde deuised to make a beame compasse of a lauelin or

speare

ſpeare of 18. foote long, and yet neither would that length reach all my centers nor draw any arche without ſagging: inſomuch that I was driuen to finde to make them without any kinde of compaſſes by ſhorte ſtreight lines drawen from 10. degrees to 10. degrees, &c. the which I had thought wyth a long circumſtaunce heere to haue ſet foorth for theyr better helpe, that ſhould attempt the like. But in the meane time came a little booke to my handes of one *Guydo Vbalbus de Theorica Aſtrolabii*, wherein I founde a very ſleight inſtrument, made of three rules, to perfourme theſe great arches eaſily, which I ſought by great ambages. VVhē I ſawe it, I was almoſt out of conceite with my ſelfe for the time to thinke that my groſſe witte coulde not deuiſe it before: for I no ſoner ſawe a glimſe of it, but ſtreight I perceiued the reaſon without reading of anye woorde in the booke: and more then that I coulde ſtreight goe too the tree where *Vbalbus* gathered that frute though he himſelfe concealed it: namely, the 20. propoſition of the thirde booke of Euclides elements which ſayth thus, *If in one portion of a circle there be made diuers angles touching the circūference of the arch* (the corde of the ſame arche, beeing baſe vnto them all, for that is his meaning as appeareth in the Cōment) *all thoſe angles are equall one to another*. Alſo in the 30. propoſition of the ſame booke hee ſheweth that all angles in a ſemicircle touching, the arch be right angles, as in my ſecond concluſion appeareth, which proueth alſo ϕ former in reſpect. A ſemicircle is but an arche. For your better vnderſtanding beholde, theſe two figures in the Arche, Y X the three angles marked wyth A, hauing all of them the corde of the arche to their baſe, are euery one equall eche to other: likewiſe in the ſemicircle C D, the three angles marked with E, are all right angles, this is manifeſtly prooued in Euclide by demonſtration, & you your ſelf may experimentally examine the equalitie of theſe angles if you liſt, by the 10. cōcluſiō in my firſt book. But let this paſſe, I wil ſhew you the making of ϕ Inſtrument ſomewhat more commodiouſly then *Guydo Vbalbus* hath ſet foorth, and leſſe comberſome, which by this bare figure following will appeare better then by manifolde woordes, and therefore beholde it wel.



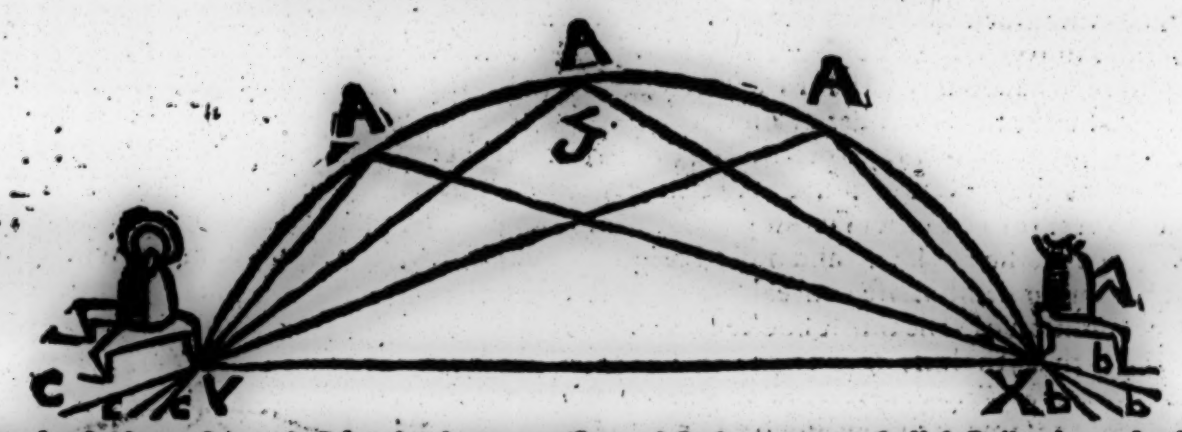


A B, and A C are twoo ſtreight rules ioynted together like a payre of compaſſes, at A, but ſo that the ioint A, muſt bee as ſmall as may be poſsible, the riuet being hollowe, and as bigge as may containe a crowes quill: then betweene A D are broade cheekes to helpe ſtrengthen the ſmalneſſe of the riuet. H G ſhoulde bee very neere one thirde part in length of A B or A C, hauing neare the ende G A a rounde pin erected wherevpon the thirde rule E F is put, and then the ſcrewed boxe M keepeth hym on the rule. E F hath three or foure holes towardes the ende E, to keepe him backe from ſhutting ouer the rule A B. Alſo K I is equall in length to H G & is double, or rather ſlit, ſo that the lower iaw is equal in thicknes to H G, & the vppermoſt thinner: whiche the skrew pinne L doth force together, to holde faſt the rule E F, when neede is. N O is not much vnlike to K I: yet for faſhions ſake it is made ſomewhat like the cocke of a qualifier or gunne, and ſtandeth an inche aboue the rule A B with a foote, his office is to hold a penne or a ſteele pointer faſt, being put in the hollowe riuet A, that it ſtirre not in grauing, to which purpoſe one checke muſt bee thicke, and the other thinner, as K I was. Q is the ſteele pointer or grauer, whiche beeing put through the chappes of N O into the hollwe A, is there holden faſt by the skrewe pinne P. Thus muche for your inſtrument: But to helpe him to worke, he muſt haue the aſsiſtaunce of two dogges as I eall them, ſuch as you ſee R and S, whiche haue each a waight of leade faſtened on their backes, their height muſt not exceede 3. quarters of an inche, that N O may ryde ouer them, the formoſt foote, whiche I call the noſe of the dogge muſt bee made like the blade of a knife, and the edge a little blunted, and ſet forwardes, but verye ſtreight downe, and the backe befiled away vnto a point.

Chapter 3.

The vſe of the former inſtrument in drawing of any Arche.

THE vſe of this trianguler compaſſe, is by three prickes giuen too deſcribe an arche, cutting them all three: whiche too doe you muſt ſette the twoo dogges their noſes in the two vttermoſt prickes, whereto applie the edges of the two rules containing the angle A, ſo that the pointer or grauer that is in the hollowe riuet A, bee in the midlemoſt pricke, and the three rules there faſtened by helpe of the



skrewe pinnes, leade them about close by the dogges noses, and so the poynter shall describe the arche desired. Example, in this figure let X Y Z bee three prickes assigned: first I set the two dogges noses in the two vttermost prickes X Y, the skrewe pinnes L and M, being first at libertie, I place the pointer Q which standeth in the riuet A, in the midlemost pricke Z, and there holding hym downe, I open the rules vntyll the side A C doe touch the dogge standing in Y: and the side A B touch X, which done, I wring hard the skrew pinnes L and M, and so is your instrument set fast in his angle. Nowe neede you but like a mower with hys Sythe leade along these three rules, so that the side A B doe still touche the dogge standing in X, and A C the dogge M Y, and so shall the pointer in A, describe an arche as true as any compasses whatsoeuer. For the dogs shal still keep you so within your bounds that you shall not choose, but mowe your swathe true compasse. In this figure the lines A B, and A C, do represent the sides of the rules, which because they include one angle whose base is X Y, therefore in respect that X Y doth alwayes keepe the Angle A, at one base the angle A must needes finde out the circumference of the arche as he goeth, as appeareth by the figures of the last chapter, by this you may gather also, that the two rules A B and A C, must bee at the least as long as X Y, if they be longer, it is superfluous for this arche, but perhaps may serue another the better, and thereby for my purpose you may cõclude that your two rules need be no longer then the diametre of your *Iewell* because the two prickes fartheste distaunte are the extreames of the diametre, so that one of these made of 3. rules, eache fixe inches long, wil make a *Iewel* of sixe ynches diametre: the best way is to haue two, one of 9. inches, another of twentie or foure and twentie.

Chapter 4,

Of a certaine secrete in diuiding the meridians of great Instruments.

THough this inuention with three rules bee so excellent to drawe great arches in great instruments that all other deuises be almost friuolous, yet wyll I shewe you a great secret or two, whiche I vsed for my helpe before the same was knowen, not because I woulde haue you vse it in that behalfe: but for that I haue founde great pleasure, & necessitie of it in working other inuentions and proiections of the spheare, For Astronomicall dials, mappes, and other matters more then I will speake of till time serue: in the meane season make account of it. If you remember, in the seconde booke and eight chapter, I shewed you to deuide he two halues of the Zodiacke from their seuerall poles: assure your selfe in very lyke manner, prooue it when you will, may you deuide any meridian of the *Mater* into the same diuisions that any of them is deuided by the parallels, and that without drawing the parallels, if in the noone line you seeke the center or rather pole of any suche meridian as farre backwardes within the center of the *mater*, as the meridian that shoulde bee deuided lyeth within the *limbe*. For example, if you woulde deuide the 30. meridian, counted from the *Limbe* on the South halfe of the *mater*, seeke then where the 30. meridian, from the center on the North halfe of the *mater*, doth crosse the noone line: and there is your pole or center most assuredly, whereon keeping the one end of a rule, and laying the other ende on euery of the deg. of the South halfe of the *limbe*, eache after other: you may thereby deuide the saide 30. meridian, euen as eyther halfe of the Zodiacke was deuided from their seuerall poles in the said second book and eight chapter. And when this 30. meridian is thus deuided, you shall finde those diuisions equall to those, which are made by the crossing of the parallels, which you may sufficiently gather by the last way, which I taught in the seconde booke and nienth chapter, to deuide your Zodiack by. I say in this maner may you denide any of the meridians if you seek his pole in the noone line, as farre beyonde the center as the meridian is within the *limbe*, and thereon as from a center deuide him by the opposite halfe of the *limbe*. For in very truth, as I say, that point is the very pole of euery such meridian, as in my next booke of spherical triangles will appeare. And thus may you not onely deuide the meridians but generally euery arche whatsoeuer, representing a great circle or semicircle of the spheare, which is specially to bee accounted of, and wil, beyng ioyned to the 4. and 6. chapter of the seconde booke, yeeld exceeding fruite and pleasure in proiecting of the globe. Before the inuention of the said three rules I was determined in making of great instruments to deuide euery 5. or 10. meridian in this manner and then by short streight lines to drawe the parallels.

Chapter 5.

Another secrete founde by the Authour in deuiding the parallelles of great Instrumentes.

FOr asmuche as the time hath beene with mee, and may bee againe with you in making great quadrants of 7. or 10. foot side, or great astrolobes, that you wold be cõtent to know how to diuide som one parallel or other

other into the very ſame diuiſions, whiche hee is, or at leaſt ſhould be deuided into by the meridians, though you thinke perhaps you ſhall not greatly vſe it, and I thynke I ſhall neuer haue the neede therof, I haue had, yet will I not omit to ſhew it, leaſt at a time you woulde wiſh you knewe it. Admit therefore for example that you had drawen the fortie & fiue parallel, how coulde you now know where euery meridian beeing truly drawen ſhoulde croſſe, you cannot tell: then ſay I, thus doe. Let the point of the 45. degree of the *limbe*, that is to ſay, where the 45. parallell toucheth the *limbe*, be vnto you inſteede of a center, on which point laying the one ende of a rule, mooue the other ende from degree to degree of the further half of the noone line beyond the center, and thereby deuide the one half of the ſaide 45. parallel, which deuiſions (if your rule be ſtreight and you worke truly) ſhall without all doubt be equall to thoſe diuiſions which the meridians do make. And ſo may you deuide any of the parallels eſpecially thoſe betweene the 45. parallel, and the pole exactly, but thoſe that lie towardes the Equinoctiall, cannot ſo exactly bee deuided in this manner, by reaſon that the rule wyll cut them ſo byas, or oblique that the pointes of interſections can ſcant bee founde. I feare mee, I ſhall haue occaſion to vſe theſe in making my great quadrant, and therefore I leaue out further demonſtration till then. The time was when I had writte more of theſe matters then I meane nowe to publiſhe. For I haue been driuen by helpe of thoſe diuiſions too drawe the meridianes partly by ame.

Chapter. 6.

Of drawing and dyuiding of the eclipticke line on the Mater.

THE eclipticke is, or ought to bee drawen croſſing the center of the *mater* to touch the one ende of eyther Tropicke at the *Limbe*, videlicet, leing byas to the Equinoctiall according to the Sunnes greateſt declination $23\frac{28}{60}$. degrees, as in the ſecond booke, and fifth chapter, and ought to be deuided into the very like partes or degrees as the Equinoctiall is. VVherefore if you croſſe this eclipticke line ſquare on the center, with a blinde line it ſhall touch in the *Limbe* the one ende of eyther pole circle, where doubt you not, but the poles of the eclipticke are. Therefore from eyther of thoſe points you may deuide him, by laying the rule on euery degree of the halfe *Limbe* fartheſt off from the ſame point, as in the ſecond booke, and fourth chapter, the Equinoctiall line was deuided from one of his poles: then ſet too the Caracters of the Signes, the beginning of ♈ and ♎, to be at the center: & of ♋ at the *Limbe* or endof the Eclipticke on the right hand, ♑ at the left: the reſt in order as I hope you knowe by this time. You ſee here that the reaſon of this differeth not from the deuiding of the meridians mentioned in the fourth Chapter. For the eclipticke being the great circle of the ſpheare, and lying 90. degrees within the *limbe*, towardes the center, euen croſſing the center: therefore hys gole muſt needes be 90. degrees from the center, which is at the *limbe*.

Chapter 7.

By the verticall circle or azimuth of the Sunne or Starres, taken to knowe what it is a clocke, very certainely and exactly.

HAuing now inſtructed you for the making of a great *Iewell* of two foote diameter, it is conuenient that I ſhewe ſome vſes needing hys capacitie, notwithſtanding that all the thirde booke wyll bee gladde of his acquayntance. I would haue no man preſume too much of a ſmall inſtrument be he neuer ſo exquiſitely made: in matters requiring great certaintie, there can no man worke ſo truly with a ſmall as with a great. For although that ſmall inſtrumentes in mathematicke ſpeculation be ſufficiently certaine to lead a manne to knowledge of thinges, yet there is no doubt but *Maiora ſemper ſunt certiora in mathematica practica*, neither is it poſſible to vſe any ſo great as in ſome caſes woulde be required. It is ſhewed in the ſixth chapter and third booke, how to knowe what it is a clocke by the altitude of the Sunne, but becauſe the Sun, and likewiſe the ſtarres in an houre before and after noone, euery day riſe and fall very little: therefore a ſmall myſſe in the altitude taking, may make a quarter or halfe an houres errour: beſides that, the interſection of the Almicantare, with the parallel of the Sunne, or any ſtarre neere the South, lyeth very byas, and ſpecially in South ſignes and ſtarres, whiche may make more errour for want of the point of the true interſection then may well be allowed in ſome caſes. VVherefore the trueſt and moſt certaine way to come by the knowledge of the houre, and ſpecially *ad minutum*, which in the rectifiyng of the ſtarres, planets, &c. and getting the longitude of regions is very needefull, is by azimuth, becauſe he alwaies croſſeth the parallel very diſtinctly, and is (the meridian line firſt had) to be found very certainely. It is ſhewed in the third booke 36. chapter, by the azimuth and altitude taken, to knowe both the latitude and the houre: but here by the azimuth, and latitude giuen, is had both the houre and the altitude of the Sunne or ſtarre, prouided that the degree of the Sunne in the Zodiacke, with the ſtarres declination be alſo knowen. Take either by helpe of a Topographicall inſtrument or els by the thirde booke and 35. chapter, the azimuth of the Sunne or any knowen ſtarre, which had, ſet the *Finitor* to the latitude, and marke where the like azimuth of the *vertex* dooth cut the parallel of the Sunne or Starre, the meridian croſsing there, ſheweth the houre and the almicantare croſsing there, ſheweth the altitude, yet in the ſtarres this houre is but *Hora ſtellæ*, the ſtarres houre, by which you may get the true houre, as is ſhewed in the thirde booke and 26. Chapter, remembring alwayes that *Locus Solis oſtendit horam*, this needeth no example, it differeth not much from the ſaide 36. chapter. Further all the difference from the ſixth chapter of the thirde booke is, that here you woorke by the azimuth as there you did by the almicantare.

Chapter 8.

By the latitude of any planet and his distaunce from any knowen fixed starre had: how to get his exact longitude.

BEcause in the first chapter of this booke I counsailed you to adde vnto your great *Iewell* the crosse staffe, I thought good to shew you, partly by this chapter, that he will serue you to necessary purpose. VVherefore according to Gemma Frisius instruction in his booke *de radio Astronomico*, you shal by the crosse staffe take the distance of any planet you shal see in the heauens, whose longitude you desire, frō some one knowē fixed starre, that is to say, such a stare whose longitude and latitude is knowen: which had, you shall then resort to the *Ephemeris* or some other like tables, and get the latitude of the planet for that day, nowe place the Zenith poynt of your *reete* so many degrees of the *limbe* aboue the equinoctiall, as the latitude of the fixed star commeth to: the *reete* there fixed, marke where the Almicantare counted from the zenith point, equall to the distance taken by the crosse staffe, doth cut the parallel counted from the equinoctiall, equall to the planets latitude had in the *Ephemeris*, and the meridian crossing there with them, followed too the equinoctiall line shall shewe you theron the deg. and min. of the difference of longitude betweene the starre and the planet, which if the planet be Eastwardes of the starre, you shall adde to the starres longitude, if VVestwardes, subduct it, and so shall you haue the longitude of the planet desired: remembring this by the way, that if the latitudes both of the starre and planet be one way, that is to say both North or both South, then must you after you haue placed the zeneth point at the *Limbe* fixed as before, take the said parallel on the same side of the equinoctiall, as the zenith point is: but if the one bee North, the other South, then take the said parallell on the further side of the Equinoctial.

For example. The 14. day of October 1582. about 6. of the clocke in the morning, I tooke the distāce of the firie redde planet, ♂, from the great starre of the light called *humerus dexter orionis*, Orions right shoulder, by a crosse staffe of my fashion for want of G.F. and founde the same 31. degrees 37. minutes. and 30. secondes, for my staffe was large. The latitude of the said starre of Orion, is, and euer hath been, and therefore this working is the more certain, 17. degrees southwardes from the ecliptick, which by my tables of fixed starres, you may finde: the latitude of ♂, by *Leouitius Ephemeris* was then one degree verye neere Northwardes, these had, I reckon on the *Limbe* from the equinoctiall towardes the South pole of my *Iewel*, the said 17. degrees, being the latitude of Orion: thereto I place the zenith point of the *reete* fixed: then among the parallels towardes the North pole, I reckon one degree beeyng the latitude of Mars, found in the *Ephemeris*.

Nowe then where the $31.\frac{37}{60}$. almicantare counted from the Zenith doth cut the saide first parallel (for I leaue out the secondes in small iostruments) there doe I see the $26\frac{1}{4}$. meridian counted from the *Limbe* to crosse, the same was the difference of longitude betweene the said starre of Orion and Mars at that instant. The longitude of that starre after Stadius is 23. degrees in ♊, and at that time I sawe Mars Eastwardes of this star. Therefore I adde this difference here found, vz. $26\frac{1}{4}$. to this longitude of the saide starre being 23. in ♊, and counting it according to the sequele of the signes, it falleth out too bee the 19. degree & 15. min. of ♋. which I conclude to be the place of Mars, his longitude at that instant. But some will here say to what purpose is this if wee goe to the *Ephemeris* for the latitude of the planet, then were it as good to take the longitude there also. True it is in some respect, but yet shall you see commonly that *Ephimerides* will differ in the longitude of the planets one from another, besides, when *Ephemerides* are made by diuers men to diuers longitudes, then may they differ, and yet be both true. VVherefore by this chapter you may with greater speede then Gemma Frisius or any other hath before taught, for ought that I euer read, both examine the truth of your *Ephemeris*, and of diuers of them, which is the truest & most neer to the longitude of your countrie. And though in their *Ephemerides* they shoulde differ muche in the longitudes of the planets, yet they cannot erre greatly in the latitude, if they doe, a little cannot hinder your purpose in working after this chapter, so you take your distaunce truely, as for the purpose you shall finde there that ♂ goeth but 2 minutes in latitude euery day, so that where before you tooke one degree or 16. minutes: if you had taken 60. minutes, which is the latitude of ♂ two dayes before, and wrought as before, you shoulde haue committed no sensible errour in the longitude. Though this experience be most pleasant to examine the places of the planets, yet meddle not with the Moone: for her tickle swift course found in tables, and her vntowarde parallaxe in takyng her distaunce, will disappoint you. But if it please God to lend mee life, and that this wrangling worlde alow mee leasure, wherein these sixe or seuen yeeres I haue fared like a tennise ball tost vp and downe, I wyll in my boke of longitude of the places, so sufficiently prouide you for euery shuttle nycitie of the Mone, that he shall haue no starting hole to deceiue you. You shall be fully instructed to meete her at euery turne, yea euen to leade her in a string, as they say, and that at pleasure. Then shall her defuse motion in her busie speeres with her euerdaring parallaxe (which hath heretofore stalled the most, and incumbred the best learned, yea and deceyued some of them, as I shall there manifest) be so broade and playnlie opened by my simple industrie, that all sortes shall wonder at the facilitie thereof. And therefore let all Cosmographers, Nauigators, Shipmasters, Trauailers, and such like, clap their handes for ioy of this *Iewell*.

Chapter 9.

How at all times you may beholde on the Mater *of your* Iewel *the whole circumference of the ecliptike, according to his cituation, abone the horizon, which bringeth with it pleasant vses as followeth,*

FIrst place the deg. of the Sunne in the Zodiacke, to the houre proposed, reckoned on the *limb*, & there presently shall you see the asc. on the East part of your horrizō by the 3. book 42. cha. & his amplitude by the first

firſt part of my addition to the 20. Chapt. of the third booke: The degree of culmination or midheauen, you haue there alſo by the 3. Booke 30. Chapt. and his meridian altitude by my addition to the 3. Booke 29. chap. there preſent all theſe are had at one inſpection, by applying the degree of the ſunne to the houre propoſed. Then to proceed, nnmber the meridian altitude of the degree of culmination on the *Limbe*, from the pole artike towards the right hand, if the aſcendent be a north ſigne, but leftwards if he be a ſouth ſigne: to the end of this numeration place the *Finitor*, which done, reckon on the *Finitor* from the centre outwards, the aſcendents amplitude and that alwayes leftwards or towards *Media noctis*, and mark what meridian of the *Mater* doth cut on the *Finitor* the ſaid degree of amplitude, the portion of the ſame meridian from the *Finitor* to the pole, repreſenteth the arch of thecliptick from the aſcendent to midheauen, & the meridian of like diſtance frō the axtree line on the other ſide which alſo cutteth the like amplitude on the *Finitor* his portion betweene the pole and the *Finitor* repreſenteth the arch of thecliptike from midheauen vnto the deſcendent.

Example. Our moſt towardly and vertuous young Prince and king, Edward the ſixt was borne 1537. the .11. of October the 13. houre 51. min. the ſunne then being in ♎ 28. deg. wherefore I would ſee the conſtitution of thecliptick to the horizon on my *Iewel* that thereby I might ſet the figure of his natiuitie, as I wil ſhew in the 13. and 14. Chapt. following. Firſt therefore I ſet the deg. of the ſunne vz. ♎ 28. vnto the 13. houre 51. min. which is one of the clocke after midnight, and 12. $\frac{2}{3}$ degrees paſt, where I ſee in our horizon being 51. $\frac{2}{3}$ the aſcendent ♍ 5. deg. almoſt, whoſe amplitude is 16. deg. $\frac{1}{2}$, and likewiſe the degree of culmination ♉ 26. deg. whoſe meridian altitude is 57. $\frac{1}{3}$ deg. Theſe had, I reckon the ſaide meridian altitude vz. 57. $\frac{1}{3}$ deg. on the *Limbe* from the pole artick rightwards, that is towards noone, becauſe the aſcendent is a north ſigne, thereto I ſet the *Finitor*, whereon from the centre northwards or leftwards, I reckon the aſcendents amplitude vz. 16. $\frac{1}{2}$ deg. and there do I finde to cut the *Finitor*, the 14. meridian counted from the axtree line, whereby I conclude that the arch of this 14. meridian betweene the pole and the *Finitor* is now the arch of thecliptick betweene the midheauen and the aſcendent, ſo that the pole being now the 26. of ♉, rekon thenſe from the ſame 14. meridian degree after degree as they follow in order on thecliptick, which by helpe of the parallels you may eaſily do, and you ſhall finde that the laſt number at the *Finitor* will end in the 5. deg. of ♍ which is the aſcendent. Laſtly the arch of the 14. meridian on the other ſide of the axtree line betweene the pole & the *Finitor*, is here the arch of thecliptick betweene midheauen and the deſcendet, cutting on the *Finitor* like deg. of amplitude, and being accompted from the pole, that is from 26. of ♉ backwards, contrarie to the ſequele of the ſignes, it wil end in 5. of ♓, which is the deſcendent. This 14. meridian or any like, I muſt for diſtinctions ſake hereafter call thecliptick circle.

Chapter 10.

By the Ecliptick circle conſtituted as in the laſt Chapt. how to get the altitude and azimuth of the ſunne or any point of thecliptick, and thereby alſo of the other planets ſo they haue no latitude.

THecliptick circle being ſcituate as in the laſt Chap. to the *Finitor* for any inſtant propoſed, reckon thereon the deg. of longitude of the ſunne or any other planet, not hauing latitude either from the midheauen if he be on the Eaſt part, or from the deſcendent, if on the weſt, becauſe ſo may you reckon according to the ſequele of the ſignes, better then from the aſcendent vpwards, or from midheauen towards the deſcendent: where your reckoning lighteth out on this ecliptick circle, the azimuth and almicantare croſsing there, ſheweth your deſire.

Chapter 11.

By the ſame conſtitution of thecliptick circle how to finde his Nonageſimus gradus *with the altitude thereof, being the quantitie of the angle made betweene thecliptick and the horizon.*

IF from the aſcendent you reckon on thecliptick circle vnto 90. deg. there is your *Nonageſimus gradus*, & the almicantare cutting, or rather touching there, is the altitude.

Chapter 12.

How by the ſame conſtitution of thecliptick to ſee all the circles of poſition liuely on the Iewel, *and what degree of the zodiacke is in any of them.*

THis is performed onely by ſetting the zenith line to the conſtitution inſteede of the *Finitor*, as for example: If in the 9. Chapt you had ſet the zenith point of the *Reete* 57. $\frac{1}{3}$ deg. vnder the pole, there do the azimuthes repreſent all the circles of poſition croſsing thecliptick or 14. meridian whereon you may reckon what degree euery of them cutteth which is your deſire.

Chapter 64.

How by the ſame conſtitution of thecliptick circle to ſet a figure verie ſenſible and apparantly.

FOr this purpoſe you muſt haue your foure domifying poſition circles diſtinguiſhed on the *Reete*, as in my addition to the 3. Booke 49. Chapt. is ſhewed, and you ſhall finde thoſe 4. with the zenith line in this conſti-

conſtitution, and the *Limbe* to cut in the ſaid eclipticke circle, the degrees beginning euery of the 12. houſes whereby you may eaſily ſet the figure, remembring that by the one halfe, the whole 12. are had.

Chapter 14.

How to ſet a figure with more pleaſure and eaſe, the laſt Chapter well vnderſtood, ſo that you may beholde your whole 12. houſes in their being altogether on your Iewel.

HEere my appointment was that the azimouthes and meridians ſhould chaunge their offices, ſo that the 14. azimuth ſhould repreſent the eclipticke circle of the 9. Chapter: and the meridians ſhould repreſent the circles of poſition by placing the zenith point of the *Reete* 57½ deg. aboue the poſe, and (the domifying poſition circles, being firſt diſtinguiſhed among the meridians) there ſhall they cut the 14. azimuth in manner of the laſt Chapt. and thenſe following the almicantares cutting there with them, vnto the *Limbe*, make prickes or notes on the *Limbe* of the *Reete* for euerie of them, and ſo are your 12. houſes had without any numbring at thoſe prickes, of which the zenith point of the *Reete* is alwayes one, yea and the beginning of the 10. houſe: then if about the *Limbe* of the *Mater* the 12. ſignes bee ſet and deuided, and amongſt them you chooſe out the *Gradus mediacli* for the inſtant propoſed, and thereto ſet the zenith point of the *Reete* fixed, there do the reſt of the prickes euery one chooſe out the deg. beginning the houſe he ſtandeth for, which are eaſily counted from the 10. houſe, here you ſee there is no numbring at all.

Chapter 15.

How to get the altitude of the Nonageſimus gradus *more readily then in the* 11. *Chapter is expreſſed.*

THe houre and minute giuen, get the aſcendent and his amplitude, and alſo the degree of culmination with his altitude, and thereby place the conſtitution of the eclipticke circle, as in the 9. Chapter is expreſſed aboue the *Finitor*, and then marke what almicantare doth but onely touch the eclipticke circle, the ſame reckoned from the *Finitor* is the altitude deſired.

I haue giuen ouer theſe 6. Chapters laſt paſt being indeede ſome of them very tedious and long, as one verie loth to trouble my booke with ſuperfluous matter, for in truth they tended chiefly to the ſetting of a figure, which when all is done, will (though perhaps not ſo liuely, yet more ſpeedily) be performed by the third Booke 46. Chap. but I haue let the titles ſtand, to the end that ſuch as liſt may the eaſier boult out the matter of themſelues.

Chapter 16.

How to place the eclipticke aboue the Finitor *after another ſort then is ſhewed in the* 9. *Chapter, and on the ſame conſtitution to finde out euery needfull circle, whereby moſt excellent concluſions are to be performed.*

THis conſtitution did Gemma Friſius finde out by his Aſtrolabe, as in the 2. Booke 54. Chap. appeareth, yet ſeruing there onely to get the bare altitude of the ☉ or planets, not hauing lat. which by the 6. Cap. may alſo be done, but my *Iewel* by the explanation of this Chapt. and the helpe of his almicantares and azimuthes, performeth the ſame to great and wonderfull purpoſes as will more appeare in my Booke of the longitudes of places then in this 4. Booke. Wherefore either by the ſaide 54. Chapt. place the *Finitor* as there is ſhewed, or elſe by the houre giuen, get the *Nonageſimus gradus* either by my addition to the 3. Booke 53. Chap. of the 3. Booke, or by the 11. or 15. of this booke, and reckon his altitude on the *Limbe* from the equinoct. line vpwards, thereto laying the *Finitor*, but on this condition that if the matter you purpoſe be in latitude North of the ecliptick, then let the almicantares ſtand vpwards towards the north pole, if ſouth then downwards towards the equin. & there is your *Iewel* in a ſinguler conſtitution ſeruing many pleaſant & profitable concluſions as ſhall follow, wherfore I think it moſt neceſſarie to ſhew or explane the reaſon therof, the equin. line here repreſenteth the *Finitor*, and the parallels & meridians repreſent almicant. & azimuthes, the *Finitor* repreſenteth the ecliptick, his point at the *Limbe* the *Nonageſimus gradus*, and at the centre the aſcendent and deſcendent. The almicantares and azimuthes are circles of longitude, and latitude of the zodiacke. The *Limbe* is the circle of longitude of the *Nonageſimus gradus*, which iſſuing from the pole or poles of the ecliptick alwayes paſſeth by the zenith, the centre of the *Iewel* is the aſcendent and deſcendent, and not the Eaſt as it is in the 9. Chap. and all other vſes of the *Iewel* moſt commonly. But here the Eaſt and weſt points are diſtant from the centre as much as the amplitude of the aſcendent or deſcendent commeth to, and likewiſe the miridian as much diſtant from the *Limbe*, ſo that in briefe the *Iewel* here repreſenteth the globe wreſted from the meridian vnto that azimuth, which cutteth the *Nonageſimus gradus*, it is moſt manifeſt that all great circles in the ſphere are concentricke to the ſphere, and deuideth other in halfe, howſoeuer they be drawne or croſſe ech other as I haue often ſaid, and as in the definitions & theoremes before my ſpherical triangles following ſhal appeare, wherefore when as the *Nonageſimus gradus* is apointed in the *Limbe*, the aſcendent and deſcendent being alwayes 90. deg. diſtant equally on ech ſide from it, muſt needes be in the centre, which of neceſſitie muſt be the place of croſsing of the *Finitor* with the eclipticke being both of them great circles, and twiſe 90. being a ſemicircle. But although that the eclipticke do alwayes deuide the horizon in equall parts, yet he doth it continuallie in chaungeable places by reaſon that hee lieth byas to the equinoctiall, hauing for that cauſe no vniformitie

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in his motion, by reaſon whereof the altitude of the *Nonageſimus gradus*, is continually variable euerie moment, and ſo alſo is the altitude of the degree of culmination, or any other part of the zodiacke. Likewiſe the aſcendent and deſcendent cutteth euerie moment in ſundrie pointes of the horizon: whereas the Equinoctiall, and all the other parallels cutte the horizon continually in one point and their meridian altitudes are alwayes one, inſomuch that the *Iewel* being ſet at any time to that conſtitution, which this Chapter teacheth, the Eaſt and VVeſt point cannot be in the centre of the *Iewel*, but onely when the beginning of ♈ or ♎ is aſcendent for then the *Nonageſimus gradus* being ♋ or ♑ muſt needes be in the meridian: at all other times the aſcendent cutteth the horizon either North or ſouth of the Eaſt, which is called the amplitude all in the thirde booke 20. Chapter appeareth, and according to this amplitude ſhall the Eaſt point on the *Iewel* be remoued.

Chapter 17.

How by the conſtitution of the 16. Chapter to get both the altitude and azimuth of the ſunne or any point of the ecliptick, or of any other planet or ſtarre how much ſoeuer they ſhall be in latitude.

HOw to get the altitude of the ſunne, or any point of the ecliptick, by this conſtitution hath beene taught by G.F. as in my 3. book 54. Chap. appeareth, but for any planet, ſtar or point of the heauens hauing lat. you are there neuer the neare, nor yet for the azimuth although it be had in the 11. Chapt. after another conſtitution for any point of the ecliptick. Now therefore by the houre and minute propoſed, haning ſet your *Iewel* to the conſtitution of this laſt Chapt. you ſhall by the 3. booke 20. Chapt. learne the amplitude of the aſcendent for that inſtant, which if it be of a North ſigne, you ſhall reckon on the equinoctiall line, which heere repreſenteth the *Finitor* from the centre rightwards towards noone, and there is your Eaſt point, and the meridian or houreline cutting there, repreſenteth the Eaſt-azimuth, but if your aſcendent be a ſouth ſigne, then count the amplitude leftwards towards *Media nox*, and there ſhalbe your Eaſt point and your Eaſt azimuth, which point and azimuth being found, you may eaſily reckon from thenſe your azimuthes, and that alwayes rightwards towards the *Limbe*, and thenſe backwards againe, and ſo round as occaſion ſhall ſerue, remembring that the 90. azimuth from this Eaſt point ſo counted muſt needs be your meridian.

As for example, in the natiuity of our vertuous king Edward, the altitude of the *Nonageſimus gradus* was 58½ deg. by the 11. or 15. wherefore that reckoned on the *Limbe* from the equinoctiall and the *Finitor* thereto laid, then doth it repreſent the ecliptick circle placed to this ſaid conſtitution, the aſcendent and deſcendent being in the centre of the *Iewel* by the 16. but becauſe the aſcendent is of a north ſigne in this natiuitie vz. the 5. of ♍ and the amplitude thereof by the 3. booke 20. Chapt. 16½ deg. I reckon on the equinoct. which now repreſenteth the *Finitor*, the ſame 16½ deg. and that from the centre rightwards towards noone, and there I conclude to be the eaſt point and that the 16½ meridian is the eaſt azimuth, and then if from this 16½ meridian you reckon to the *Limbe*, and thenſe backe againe vntill your reckoning come to 90. the houreline there ending repreſenteth the meridian or ſouth azimuth, being here the 16½ counted from the *Limbe* backe againe, ſo that hereby you may ground this generall rule, looke howe much the amplitude or eaſt point goeth rightwards from the centre, ſo much the meridian commeth backe from the *Limb* leftwards, and how much the amplitude goeth leftwardes ſo much the meridian commeth ſhort of the *Limbe* in reckoning, wherefore when you haue ſet the ecliptick circle to the conſtitution of the 16. Chapter, and haue any planet or ſtarre propoſed, whoſe altitude and azimuth you deſire to knowe thereby, get his longitude and latitude in the zodiacke by an *Ephimeris* or other tables: reckon his longitude on this ecliptick circle being here the *Finitor*, and that from the centre or *Nonageſimus gradus* at the *Limbe*, as in the ſaid 54 Chapt. is ſhewed, and then on the azimuth there being, reckon his latit. by helpe of the almicantares, where you ſhal ſee the parallel reckoned from the equinoctiall ſhewing his altitude, and the houreline cutting there, reckoned from the point or houreline repreſenting the eaſt azimuth found by the 16. Chapt. ſheweth the azimuth deſired.

Example. I ſhewed in my addition to the 33. Chapter of my 3. booke, the glorious planet Venus being the 10. of October 1580. in longitude 5. deg. 27. min. of ♎, in latitude 5. degrees ſouthwards, the ſunne then alſo being in ♎ 27. degrees 8. minutes that ſhe roſe about 6. minutes after 5. of the clocke in the morning. Nowe woulde I knowe what height ſhee was at 10. of the clocke the ſame morning and that by the rule of the 16. Chap. to ſee how it agreeth with the 3. booke 32. Chapter, firſt by the houre giuen videlicet 10. of the clock, and my addition to the thirde Booke 53. Chapter or the 11. of this booke, I get the *Nonageſimus gradus* to be 29½ degrees of Leo and his altitude 46. degrees, therefore by the 16. Chapter I reckon 46. deg. on the *Limbe* from the equin. towards the north pole, thereto I place the *Finitor* with his almicant. downewards towards the ſouth pole, becauſe the latit. of ♀ was ſouth: the aſcendent then by the 3. booke 42. Chapter was ♍ 29½ degrees, and his amplitude by the 3. booke 20. Chapt. 33½ deg. which I reckon on the equinoctiall line from the centre leftwards, becauſe ♍ is a ſouth ſigne, and there is my eaſt point, and the 33½ houreline cutting there, is my Eaſt azimuth. Well then becauſe in ſucceſsion of the ſignes after ♌ followeth ♍ ♎ &c. therefore ♌ being in the 90. degree, I know that Venus was then alſo in the eaſt quarter, that is to ſay eaſtwards of the 90. deg. For which cauſe I reckō on the *Finitor* from the *Limb* inwards, that is to ſay from the 90. deg. being ♌ 29½ vntil I come vnto ♎, 5. deg. 27. min. & it endeth in the 37. deg. almoſt of the *Finitor* counted from the *Limb*, & on the azimuth there ſtanding, I reckon the lat. of ♀ vz. 5. where I ſee to cut the 31⅓ parallel, whereby I conclude that ♀ was then ſo high, wheras if you had not reſpected the lat. but performed it by the long. after the 2. booke 54. Chap. that is to ſay, to haue taken the parallel cutting the *Finitor* it ſelfe in the 37. deg. from the *Limbe*, you ſhould haue founde ♀ 36. degrees heigh, which is no ſmall error if any man would thinke to

know

know the houre not respecting the latitude. But nowe to know in what azimuth she was then in, from my East azimuth found as before, which is the $33\frac{1}{2}$ houreline leftwards of the centre, and there I number rightwards towards noone vntill I come to the houreline cutting the place or latitude of ♀ with the said $31\frac{1}{3}$ parallel and my reckoning commeth to $79\frac{1}{2}$. the azimuth desired, wherein Venus was that instant. And thus may you do for any fixed starre or other planet.

Chapter 18.

How by the constitution of the eclipticke as in the 16. Chapter to get the longitude and latitude of any starre or planet, that is to be seene in the skie, and thereby also his right ascension and declination.

GEmma Frisius as appeareth in my 3. Booke 39. Chapt. sheweth to get the longitude, and latitude of any starre or comet out of the heauens, as this Chapter doth purport, after another manner then heere shalbe shewed, but in my opinion not so exactly, yet with farre more trouble though both wayes do well to examine a truth one by another. Wherefore thus shal you do when you see any planet, starre, or comet, whose longitude and latitude you desire: gette his altitude and his azimuth by some instrument, and also the houre and minute by the 4. Chapter if it may be: these three must be had all at one instant, and that verie exactlie which done, by the houre had, set the *Finitor* to the constitution of the 16. Chapter, and then by the East point founde by the saide 16. or 17. Chapter, seeke out the houreline representing the azimuth as there also is shewed, whereon reckon so many degrees from thequinoctiall line as the planet or starres altitude taken come to, and the azimuth and almicantare cutting there, sheweth the longitude and latitude desired accounting them as in the 16. and 17. Chap. is shewed.

Example, Of ♀ in the last Chapt. Admitte I had taken her azimuth $79\frac{1}{2}$ and her altitude $31\frac{1}{3}$ at tenne of the clocke in the morning the seconde of Ianuarie 1579. I being nowe in Gloster shire in the Christmasse time with the good knight and my deare vncle Syr Iohn Hungerforde, who departed this worlde the 19. of March, Anno 1582. there chaunced a great snow a day or two before, and therewithall followed rimie dayes, by which meanes, although there were no cloudes to be seene in the whole skie, yet by reason of the rimie thinne vapours that did hang in the lower region of the aire that much brake the force of the sunne beames, I did perceiue the sunne, the moone, and Venus who then was betweene the ☉ and ☾, all three of them at two of the clocke in the afternoone the same day: or rather somwhat before, and so all the day after, which I shewed there to diuerse of my friends and kinsmen which were there as I was, to passe the time with that worshipfull knight, whose hospitalitie was most bountifull. Wherefore admitte the tenth of October 1580. were some such like day, if you will graunt no absurditie, and that then I had taken the altitude of ♀ $31\frac{1}{3}$ deg. and her azimuth $79\frac{1}{2}$ at 10. of the clocke shee being to sight Eastwards of the *Nonagesimus gradus* and woulde thereby knowe her longitude and latitude, which to doe, I lay the degree of ☉ videlicet ♎ 27. degrees 8. minutes vnto 10. of the clocke in the *Limbe*, where I finde the ascendent ♏ $29\frac{1}{2}$, his amplitude $33\frac{1}{2}$, the *Nonagesimus gradus* ♌ $29\frac{1}{2}$, his altitude 46. by which I set the *Finitor* to the constitution of the 16. Chapter in all respectes as is done in the last Chapter, then by reckoning the amplitude of the ascendent videlicet $33\frac{1}{2}$ on the Equinoctiall line leftwards from the centre, because the ascendent was a south signe: there I finde the East point, from which towards the south I reckon among the hourelines, the azimuth of ♀ videlicet $79\frac{1}{2}$ which lighteth out in the 46. meridian counted from the *Limbe*, in which 46. houreline, I reckon from the Equinoctiall line the altitude of ♀ vid. $39\frac{1}{3}$ which if your almicantares chance to be turned towards the North pole, you shall perceiue not to reach theclipticke or *Finitor*: and therefore turne about the *Reete* that the almicantares may stande downewards towards the south pole, and then doth the said $31\frac{1}{3}$ deg. of the 46 meridian cutte the 5. almicantare, which sheweth that ♑ was then 5. degrees in latitude, and also it cutteth the 37. azimuth which followed to the *Finitor*, shewing her longitude if you account from the *Limbe* or 90. degree thither, that is to say, because ♀ was in the west quarter: therefore the redi'est way is according to the sequele of the signes count it from the end of the *Finitor* at the *Limbe* being ♌ $29\frac{1}{2}$, thence numbring downe towardes the ascendent at the centre vnto the said 37. azimuth, it will fall out to be the $5\frac{1}{2}$ of ♎, the longitude of Venus. And thus may you doe with any fixed starre or other planet, the Moone excepted, whose parallexis maketh many alterations in this behalfe.

Chapter 19.

The latitude and azimuth of the sunne, or any planet or starre given, how by the constitution of the 16. Chapter, to get their exact longitude.

THe sunne neuer hath latitude, and therefore we shall soone dispatch with him, the latitude of any other planet or starres, you must get by tables, as in the eight Chapter you did, which had, get at any instant the azimuth of the planet or starre exactly by some large instrument, either by the third booke 35. Chap. or by this next Chapter following, and the houre and minute at the same instant, by which set the *Finitor* in the constitution of the 16. Chapter with his almicantares to the North pole, if the latitude of the same planet

or ſtarre be North or to the South pole, if South, then by the 16. or 17. finde out the houre line repreſenting the azimuth ſo taken, and mark where the ſame houre line cutteth the almicantare repreſenting the planet or ſtarres latitude, and there ſhal you ſee the azimuth crosſing, pointing on the *Finitor* the deg. of the longitude ſought for, if you reckon the ſame from the centre or deſcendent, the planet or ſtarre being on the VVeſt quarter, or from the *Limbe* or *nonageſ. grad.* if on the Eaſt as in the ſixteene and 17. chapter, is ſhewed: as in my former example of *Venus*, admit that the ſaid 10. of October 1580. I found by the Ephemeris her latitude 5. deg. Southwards, and that the ſame day at 10. a clocke I took her azimuth $79\frac{1}{2}$. deg. from the Eaſt by a large inſtrumēt, then by the 16. chap. I ſet my *Iewell* to the conſtitution of the Eclipticke with the almicantares towardes the South pole, becauſe the latitude was South: which done, according as is ſhewed in the ſame 17. chap. I found the 46. houre line counted from the *Limbe* to repreſent the $33\frac{1}{2}$. azimuth: and now whereas this 46. houre line cutteth the 5. almicantare, there doe I ſee the 73. azimuth reckoned from the *Limbe*, to croſſe, whoſe degree reckoned on the *Finitor* from the *Limbe* or 90. degree, bicauſe that *Venus* was on the Eaſt quarter, falleth out to be the 5. deg. $\frac{1}{2}$. of ♎, which I conclude to be the longitude of ♀ at that inſtaunt. Hereby maye any man rectify the places of the fyxed ſtarres very eaſily, becauſe their latitudes neuer alter, and their azymuthes are eaſily taken at all times.

Chapter 20.

How to get the true and exact azimuth of any planet, comet, or ſtarre vnknowne.

YOu ſhall at one inſtaunt take the altitude as well of the planet or comet, whoſe azimuth you ſeek, as alſo of ſome one knowne ſtarre, not farre diſtaunt from him, which noted, you ſhall by helpe of your *Iewell*, or els of the croſſe ſtaffe, take the diſtaunce between this knowne ſtarre and the ſame planet: which noted, alſo you ſhal then reckon on the *Limbe* of your *Iewell* from the Equinoctial line towardes the pole, the greateſt altitude before taken, whereto ſet the Zenith point of the *Reete* fyxed, and among the parallelles counted from the Equinoctiall towardes the ſame pole, count the leſſer altitude: then among the almicantares counted from the Zenith poynt, reckon the ſaide diſtaunce taken: now where this almicantare and the ſaide parallel do croſſe, the meridian of the *Mater* cutting there, counted from the *Limbe*, ſheweth the horizontall diſtance betweene the ſayde knowne ſtarres azimuth, and the azimuth wherein the ſaid comet or planet was at the taking of the altitudes, which azimuthal diſtance or difference, you may adde or ſubtract too or from the ſtarres ſaide azimuth according to the coaſt, which you ſhall ſee one of them to ſtande from other in the skye. I truſt you can not be to ſeeke of the ſtarres azimuth, except you forgette the third booke 27. and 33. chap.

This working differeth little from the 8. chap. and therefore needeth no example, for here by the two altitudes and diſtaunce you get the difference of the azimuthes as there by the two latitudes and diſtance you got the difference of longitudes. But here alſo in the moone her parallaxis will make you miſſe certaine min. which in my booke of longitudes, I wil teach you to remedy.

Chapter 21.

The moone ſhining on any ſun diall to know in the night, what it is a clocke, very ſpeedily by helpe of the Iewell.

MArke what houre and part the ſhade of the moone giueth in the diall, then goe to an ordinary Almanacke, & looke in what ſigne and deg. the moon then is, lay the ſame deg. of the Zodiack by helpe of the *Label* to the like houre, & part on the *Limbe* of the *Iewel*, which the ſhade of the diall gaue you: and thence remouing the *Label* to the deg. of the ſun, it ſheweth you the houre & time ſought for: and that very eaſily without taking the moones altitude, or other troubleſome working, as in the 3. booke 27. chap. is required. But by the way you muſt not forget firſt to rectify the moones place by adding or ſubducting one deg. for euery houre, that you imagine the time propoſed to be diſtant from noone, as I haue often warned you.

Chapter 22

A ſpeedy way to know what it is a clocke by the ſtarres without taking their altitude.

VVHen you go forth any night, firſt behold the North pole, & therby directly back oppoſite therto, you ſhal eaſily fynd the ſouth: then mark whether you ſee any notable ſtarre there placed at that inſtāt: if any ſuch ſo chaunce, and that the ſame ſtarre be on your *Reete*, laye his apex on the noone lyne of your *Iewell* towards Meridies, and placing then the *Label* on the deg. of the ſunne, it ſheweth in the *Limbe* the houre deſired.

The like may you doe with any ſtarre, you ſhall chaunce to ſee on the North part of the meridian by laying his apex on the noone line towards *Mediam noc.*

Alſo you may with a little exerciſe iudge by ſight very neere how many houres anye ſtarre that you ſee in the skie lacketh of, or is paſt the South, if the ſcantlet be not ouer great, whereby laying his apex by helpe of the *Label*, to ſo many houres of the *Limbe*, wanting or paſt the noone line, then *locus ſolis oſtendit horam.* Truely this Chap. beſides the eaſe and ſpeede that it hath, giueth a great deale more credit among the vnlearned, then the other way, for they murmure if one be long hunting after the houre, and thinke the deuiſe of ſmal repu

tion not respecting art. It needeth no example.

Chapter 23.

The vse of the three tables comprehended in the 2. figure of the second booke 13. Chap.

I Thinke it not amisse here to shew the vse of the saide 3. tables to fyt the fancy of some men which may delight therein. First therfore you must remember that the whole rundle containeth 6. spaces, one compassing in another; of which 6. spaces, in the three vttermost are contained 2. seueral tables: the first seruing to get the dominical letter, the 2. the prime and Epact for euer. And in the 3. innermost of the same 6. spaces is onely contained the third table seruing to get Easter day for euer.

By the first table to get the Dominical letter for euer.

SEeke the yeere proposed in his vttermost space, and directly vnder it in the third space, you haue there the Dominicall letter, but if it be leape yeere, then is the letter in the middle space your desire: and when this table is worne out, you may begin his date againe where you left, that is to say, where 1582. is, there set 1610. at 1583, set 1611. &c. so forth, to be continued vnto the worlds end.

By the 2. table to get the Prime and Epact for euer.

SEeke the yeere proposed in the second table, and directly vnder it in the second space is the prime, and in the 3. space the Epact answearable to that yeere, this table you may also beginne againe like the former & so continue him for euer.

By the third table to finde Easter day for euer.

FIrst learne the Dominicall letter of your yeere proposed by the first table: and by the second the prime of the same yeere, these had, seeke the said prime number, in the vttermost space of the 3. table, and in the middlemost space seeke the sayde dominicall letter next vnto the prime rightwards from it in numbring, and there in the 3. space you shall finde directly vnder that letter, what day Easter day shall fal on that yeere.

Example of all three tables.

THis yeere 1584. I would know the Dominicall letter, the Prime, Epact, and Easter day, I seek the sayd yeere, videl. 1584. in the first table vnder it I fynde E the Dominicall letter, because it is Leape yeere: then in the 2. table I seeke the yeere 1584. there I finde vnder it the Prime to be 8, the Epact 24. Lastly, in the vtter row of the 3. table I seeke the prime of this yeere 1584. videl. 8. vnder which in the middle rowe is the letter C, but reckoning on in order rightwards till I fynd my Dominicall letter E, vnder it in the innermost space I see 20. and because this worde Aprill went last before in the same space, I conclude the 20. of Aprill to be Easter day.

The conclusion of this fourth Booke.

MY leisure on the one side not seruing me to beate about ordinary propositions, and finding on the other side my *Iewel* to cal me away vnto such more effectual matters, which rather require seueral treatises, as in my 5. and 6. bookes, and also in other 6. bookes (which after these if God spare me life and leisure, I meane shortly to set forth) shal appeare. I here therefore conclude this 4. booke of mine, least by seeking to enlarge it, I should cõfusedly place many things as I once had done. For I haue taken out diuers chapters about the Moones parallax, and other matters: reseruing them as more fit for my booke of longitudes and other like places: and were it that I had conuenient leisure to translate and alter the whole worke, this 4. booke should not be at all, but be diffeuered into the rest of the 12. bookes, neither should some part of the rest goe forth as it doeth, but that of necessitie for want of leisure I am forced to let it passe, hoping that little bias wil be borne withal, meaning to amend in the next.

The

The fifth book of the Mathematicall *Iewel*, newly inuented, compiled and set forth *by* Iohn Blagraue *of Reading, Gentleman: teaching most* plainely to performe on the Iewell the whole art of ſpericall triangles, without any whit of that great toyle by ſynes ſupplementes, tables, proportions, Arithmeticall calculations, *and ſuch like: by which Ptolomeus, Regimontan. Copernicus, and* all other Authors before me, haue wrought them, here performed by ocular inſpection with ſurpaſsing facility, and maye bee called the key of the knowledge of the ſphere, in that it vnlocketh all ſpherical queſtions whatſoeuer: and is a ſinguler introduction vnto the Sinicall woorking.

Chapter 1.

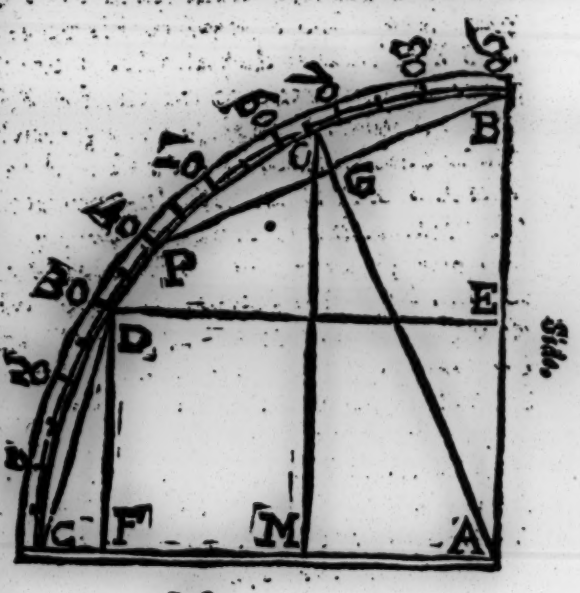
Of the Expoſition of a Sinicall quadrant, and all thinges thereto appertayning.

A Quadrant is in briefe the quarter of a circle, encluded with two ſemidiameters, as you ſee here this figure A B C. The arch thereof, vz. B C, I call the *Limbe* which is deuided as the order is, into 90. degr. The ſemidiameter erected, vz. B A is the ſide of the quadrant, the other ſemidiameter, vz. A C is the groundline or baſe of the quadrant, from which the *Limbe* beginneth to be numbred, vz. from C towards B.

An arch in Latine *Arcus*, is any porcion of the circumference or rynge of a circle, as here, B O, O P, P D, B C, &c.

A Chorde in Latine *Chorda*, is as it were the diameter to the arche, or the ſtring to the bow, as here the ſtreight line B G P, is the chorde of the arch B O P, and the line D C of the arche D C.

The Complement of an arch in Latine *Complementum arcus*, is that portion of a circle, which any arch lacketh of a quadrant or 90. degrees, as the complement of the arch B D is the arch D C, and the complemēt of D C, is B D. So that alwayes the arche and his complement make a quadrant.



A Syne in Latine *Sinus*, is a ſtreight line commenſurable eyther in length or power vnto the diameter of a circle, and to be meaſured by his parts.

The Syne totall in Latine *Sinus totus*, is the ſide or baſe of the quadrant, as A B, or A C, deuided into an 100. equall parts at the leaſt, but rather into a 1000. 10000. or 100000. parts for the more the better.

A right Syne in Latine *Sinus rectus*, is anye perpendicular falling from the arch or *Limbe* of the quadrant on his baſe, as here you ſee D F is the right Syne of the arch D C, and O M of O C, and is denominated of thoſe parts of the Syne total, whiche he contayneth or more briefly. a right Syne is halfe the cord of any arch doubled, as the right Syne of B O is the line B G.

A backe Syne in Latine *Sinus verſus*, is any perpendicular falling from the arch or *Limbe* of the quadrant on his ſide, as you ſee D E, which is called the backe ſine of the arch D C. By thi you may ſee and learne if you marke it, that the backe Syne of any arch, is the right Syne of the ſame arches complement, for D E is the backe Syne of the arch D C, but it is the right Syne of B D. Wherefore in briefe, a backe Syne is halfe the chorde of the arches complement doubled.

The Supplement of a Syne is that which any Syne lacketh of the quadrantes ſide or Syne totall, as B E is the ſupplement of the right Syne F D, or A E, which is equall vnto it: likewiſe F C is the ſupplement of the backe Syne E D, for E D and F C make the Syne totall.

Note that ſome Authors make but three diſtinctions of Synes, as I haue here done, vz. *totus rectus & verſus*, Other ſome as namely *Apian* and *Munſter* make foure, vz. *Sinus totus*, then *Sinus rectus primus*, which here I call the right Syne, and *Sinus rectus ſecundus*, which I call the backe Syne, and then *Sinus verſus* to be that which I call the Supplement of the backe Syne, videl. F C, being as the arrow to the bow: but truely I like not thereof, and therefore I haue thought good to call both F C and E B Supplementes as before, for if ſo be that F C be *Sinus verſus*, then may E B be as wel *Sinus verſus ſecundus*. The matter is not great for the names, ſo you vnderſtand the

meaning, for I shall neede none of them all in this my woorke of tryangles. Therefore I will not striue, yet I thought good here to set these downe, least happily their names come in question vnwares to the amazemēt of some young beginner, as sometime it happened to me for *Ignoratis terminis, ignoratur & ars.*

Note further, that in all the Figures of this woorke of tryangles all streight lynes cutting the Centre of any circle, and al arches howsoeuer they be drawne, are to be taken for great circles or portions of them, that is to say of circles of one selfesame quantity, ~~and of one selfesame quantity~~, and of one selfesame spheare.

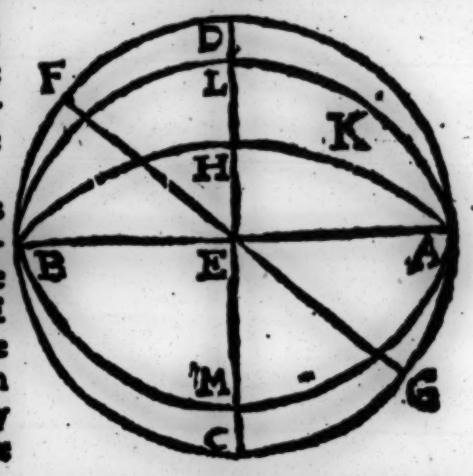
Chapter 2

Of certayne definitions needfull to the vnderstanding of this woorke.

WHat a spheare or globe is with hys axetree and poles, is shewed in the 1. booke 3. chap.

1 Great circles of the spheare or of any spherical body, are those whose diameters are equall to the spheares diameter concentricke to the spheare, and are eche deuyded by supposition into 360. equall and lyke partes or degrees, suche are the Equinoctiall, the Eclipticke, and all the Meridians or any other circle imagyned on the spheare, equall to any of them. As for example, in the spheare A B, C D, all the cyrcles A E B, D E C, F E G, and A L, B M, are equall, E beyng centre to them all, and to the spheare it selfe.

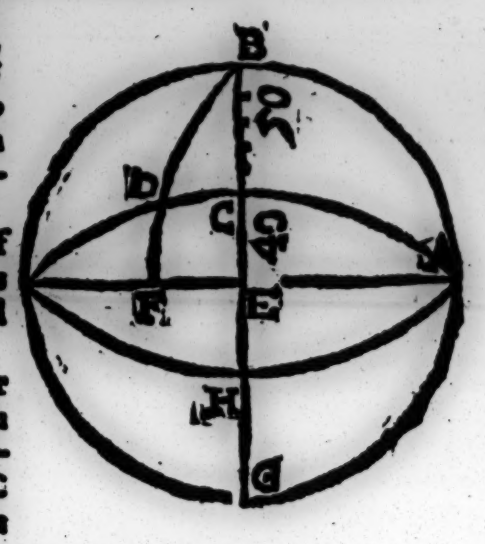
Though the spheare hath properly but one axetree and two poles in respect of his dayly motion: yet must it be here graunted that euerye great circle of the speare whatsoeuer, hath his peculiar axetree and poles, though no motion on them performed: the axetree of a great circle is a dimetient or diametre of the spheare, drawen perpendiculer to the flat of the same great circle through hys centre, as the line A E B, is the axetree of the circle D E C: and so of the rest. But this kinde of axetree line seldom commeth in question only his extreames, which are called the poles of the same circle, are in this treatise altogether vsed and are thus defyned.



2 *I. Regio. lib. 4. cap. 2.* The poles of euery great circle are 2. opposite points eche of them distaunt from euery part of his circumference, nienety degrees or a quarter of a circle, as the pointes A and B are the poles of the circle D E C, and are distant 90. deg. from euery of the poynts D L H E M C, of the same circle. Likewise the poyntes D and C are the poles of the circle A E B, and the poles of the circle A B C D, doe lye both couched in the point or centre E, which of necessitie is 90. degr. distaunt from euery part of the circle A B C D: briefly the pole of any circle is the poynt of the spheare, from whence as a centre, the same circle is described.

3 *Io. Regio. lib. 4 cap. 1.* A perpendicular arch to any great circle is a portion of a great circle passing by the poles of the great circle whereon it falleth, or at the least such a portion as will passe by, or cut his poles, if he be drawne forth, as in the spheare A B C D, the arch K H, is perpendicular to the circle D E C, because if it were drawn foorth, it wil cut his poles A and B. Likewise B D, B H, B E, & B C, are perpendiculer arches vnto D E C, because they all yssue from the pole of D E C, vz. B, and so are E D, E A, E B, E C, E F, and E G. perpendiculer arches to the circle A B, C D, by the same reason: and so of the rest.

4 A sphericall angle, is an angle included by the touching or crossing of two great circles or their arches and is denominated by the base of the angle at 90. deg. from the crosing as in the spheare A B G, the 2. arches A B, & A D, do include a spherical angle denominated by the arch B C, which arch B C is in euery 90. degr. from A. So that if the arch B C, containe 50. degr. then is the angle B A D so much: likewise the arches B D, and B C, doe include an angle, whose denomination is E F. But to define it more artificially, a sphericall angle is an angle contained between two great circles, and denominated by that portion of the greate circle, described on the point of their crosing, which is included betweene them.



5 A Sphericall triangle is a spherical flat, included by three arches of great circles, as A B D, is a spherical tryangle included by the three arches A B, B D, and D A: likewise A B D, and B C D, and D A F, are spherical triangles, and so of the rest.

6 A right angled spherical triangle is, that hath one right angle, or more, as A B C, is a right angled tryangle, whose angles B and C, are both right angles by the thirde definition, because A B and A C, stande perpendicular on B C. Also A B E, is a triangle of three right angles, and B C D, is a tryangle of one right angle vz. C. because the arch D C, standeth perpendicular on B C, by the 3. diff.

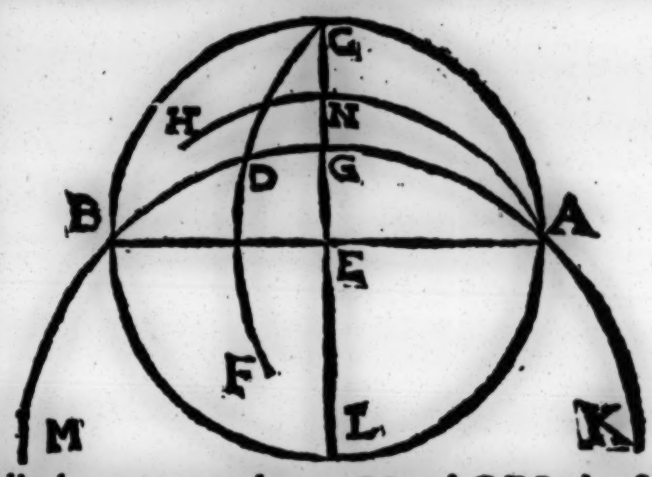
7 Equall spherical angles are those whose denominations are equall, being in equal spheres.

8 In euery spherical triangle, the side lying against any angle, is called the side, subtending the same angle, the other two sides are called the sides including the same angle.

Chap. 3.

Of cereaine Theoremes needefull to be premiſed for the better vnderſtanding of this my preſent worke of Sphericall trian-gles.

Io. Reg. lib. 3. cap. 19. ALl great circles of the ſphere, howſoeuer they be drawne, deuide eche other in halfe which is thus proued. Any two great circles being drawne on a ſphere howeſoeuer ſhall both bee concentricke to the ſphere by the firſt definition, and therefore of neceſsitye concentricke one to another: (that is to ſay) made both vpon one centre, euen the centre of the ſphere: wherefore they muſt needes croſſe one another on the extreames of a diametre of the ſphere. Now ſince by the 9. Geometrical definition of the fyrſt booke, a diameter cutteth alwaies any cyrcle in halfe, therefore of neceſsity anye two circles croſsing on another on the extreames of a lyne being diameter to them both, muſt needes deuyde eche other in halfe, ſince the ſame diameter deuideth them both in halfe. So is it with al the great circles of any ſpheare, for a line drawne betweene both croſſing of anye twoo circles, cutteth alwaies the centre of the Sphere, and is diameter to them both moſt manifeſtly as in this ſphere, A L B C. the great circles A C B, & *A G B*, deuide eche other in halfe on the diameter *A B*. Marke this Theoreme well, for hee often commeth in queſtion.



2 If two greate cyrcles or arches of great circles croſſe eche other, the two poyntes of theyr circumferences furtheſt diſtaunt, ſhall bee in eche circle at nientie degrees from the croſsing, this is manifeſt by the fourth diff. as in this ſphere *A L*, *B C*, the twoo arches A D, and A H, croſsing in A, I ſay that they ſhal be fartheſt inſunder in N and G, both which are 90. deg. diſtant from A by the ſaid 4. diff.

3 *Io. Regi. lib. 3. cap. 17. 18. 20.* If 2. great circles croſſing one another, doe cut ech others poles, they make at eche croſsing 4. right angles. This is moſt manifeſt by the 3. diff. becauſe they then muſt needs ſtand perpendiculer one to another: *A E B*, and C E L, doe ſo deuide themſelues.

4 Any two great circles of the ſphere, howſoeuer they croſſe, make at ech point of croſsing foure angles equall vnto foure right angles, the reaſon is this: looke what the one angle lacketh of a right angle, the other hath it euen as it is in ſtreight lynes croſsing eche other by the firſt booke 15. propo. of *Euclide*. as at the two pointes of croſſing *A B* of the two great circles K A, *B M*, and C A, *L B*: at the one, videl. A, the angles C A G, G A L, C A K, and K A L, are equall to foure right angles, for as C A G, and K A L, are eche leſſe then a right angle, ſo C A K, and G A L, are ech greater by ſo much, as one of the other want; likewiſe is it of the 4. angles *C B G*, *G B L*, *L B M*, and *M B C*, at the croſsing *B*.

5 Of euery two great circles croſsing ech other, the two oppoſite angles at ech croſsing, are equall, as the two great circles *A C B*, and *K A G B*, croſsing *M A* and *B*, I ſay that at the croſsing A, the angle *C A B* is equall to the oppoſite angle K A L: and C A K, equall to *B A L*. The proofe heereof differeth not from the ſayde firſt booke fyfteene propoſ. of *Euclide*, and therefore any one of the foure angles knowne, take him out of the ſemicircle, videl. 180. degrees, he leaueth the other knowne. As for example, if the angle *C A B*, be 50. deg. take that out of 180. it leaueth 130. deg. the quantity of the angle *B A L*, and ſo much is the angle C A K alſo by this Theoreme.

6 All great circles or arches of great circles drawne forth, being perpendiculer to anye one greate circle, muſt needs meet and croſſe at his poles: this is manifeſt inough by the 3. defynition, for otherwiſe they could not bee perpendiculer.

7 Two ſemicircles, or equal arches can include no triangle: for then were it repugnant to the defynition of a triangle, which muſt haue 3. ſides.

8 *I. Regiom. lib 3. cap. 37. 38.* In euery tryangle any two ſides muſt needes be longer then the third, for if two ſides ſhould be equall to the third, then ioyned together, they make an arch equall to the thirde ſide, and then can they not include a tryangle by the 7. theoreme: if ſhorter, then could they neuer reach the third ſide, thys is demonſtrated in the 1. booke 20. propoſ. of *Euclide*.

9 *Coper. 2. trian. ſph.* Euery of the ſides of a ſpherical triangle, muſt needs be leſſe thē a Semicircle: for if you admit one ſide of a triangle to be a Semicircle, as A L B, then cannot the other two be leſſe then ech of them a quadrant, becauſe they cannot els reach home, if ſo as A G, and B G, or A C, and B C, you ſee that at theyr meeting G or C, they become a ſemicircle, as A G B, or A C B, and then by the ſeuenth theoreme can include no triangle. If you would imagine the other two ſides greater then 2. quadrants, then would they meet without the ſpheare, and there can make no ſpherical triangle by the 5. definition.

10 *Ioh. Regiomont. lib. 3. cap. 39.* The three ſides of euery ſpherical tryangle muſt needes bee leſſe then twoo ſemicircles, this is ſufficiently proued by the nienth theoreme. For otherwiſe they muſt needes meete wythout the ſpheare in an irreguler forme, or within the ſpheare like the three ſides A L, B C, with C D, and A D: which 3. you ſee make no trianguler forme.

11 *Ioh. Regiomont. lib. 3. cap. 49.* It is demonſtrated in the 1. book 32. propoſ. of *Euclide*, that in euery right lined tryangle, the three angles are alwayes equall vnto twoo right angles. But in all ſpherical tryangles, the three angles are alwayes greater then twoo righte angles, as by the ſixte definition partely appeareth

How

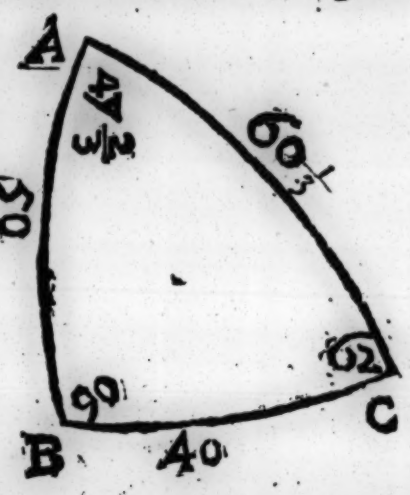
12 How many great circles ſoeuer croſſe in one point of the ſpheare, euery of their poles are to be found in that great circle, vnto which that poynt of croſsing is pole: for example of the circles, C A N A G A E A & L A. I ſay that euery of their poles lyeth in the circle C E L, vnto which the point A is pole by the ſeconde diff. This is moſt manifeſt, for by the 3. diff. the pole or poles of any circle muſt needes be found in another circle, ſtanding perpendiculer thereon: and ſeeing by the 3. and 6. theoreme, all thoſe circles are perpendiculer to C E L, and C E L perpendiculer to them, therefore by the 2. and 3. diff. all their poles muſt needes lye in C E L.

Theſe definitions and Theoremes premiſed, ſhalbe inough for my purpoſe.

Chapter 4.

If of a right angled ſpherical triangle, two ſides ſhalbe knowne how to get the quantity of the third ſide, and the other two angles.

A Right angle as well ſphericall as playne, contayneth 90. degrees: wherefore he is alwayes knowen. Lette there be propoſed the right angled triangle A B C, whoſe two ſides A B and C B contayning the right angle B, let them be giuen, vz. A B, 50. deg. and B C 40. deg. Now wil I ſhew you by the *Iewel* to get the quantity of the third ſide C A, and of the angles A and C beeyng yet vnknowne.



Number on the Equinoctial line of your *Iewell* from the centre outwards the quantity of the ſide A B geuen, videl. 50. deg. then on the meridiã meeting there, being the 50. meridian counted from the centre, reckon from the Equinoctiall towards the pole, the ſide B C giuen, videlicet 40. degrees, therto lay the rule, and note the degrees thereof cut, counted from the centre, by the ſaide 40. degrees of the ſaid 50. meridian, which you ſhall finde $60\frac{1}{3}$. deg. the ſame is the quantity of the third ſide ſought for, videl. A C, and that which more is, there doe you ſee your whole triangle on the *Iewell*, as plainly as on any ſpheare, included by thoſe three arches, videl. $60\frac{1}{3}$. of the rule, 50. deg. of the Equinoctiall, and 40. deg. of the ſaid 50. meridian, which being well marked and vnderſtood of you, then following the *Labell* to the *Limbe*, it cutteth there alſo $47\frac{2}{3}$. deg. which muſt needes bee the quantity of the angle A by the 4. difference, being that the ſide B C videlicet 40. degrees ſubtendeth it, and thus haue we one of the angles ſought for, the other you ſhall thus eaſily finde.

Reckon the ſide B C videlicet 40. degrees on the Equinoctial from the centre, then on the 40. meridian there, reckon the ſide A B, ſubtending the angle wanting, videl. 50. deg. thereto lay the *Labell*, and you ſhal ſee $60\frac{1}{3}$. deg. thereof cut as before: the *Label* cutting in the *Limbe* 62. degr. whiche is the quantity of C, the other angle ſought for. Hereby you may perceiue of your ſelfe, that it is no matter with which of the two ſides gyuen, you begin in performing this chap.

But you will ſay what if the ſide A C. videlicet $60\frac{1}{3}$. deg. ſubtending the right angle B with one of his including ſides as the ſide B C, be giuen, A B being vnknowen. Here muſt you begin on the *Labell*, and reckon theron from the centre, the ſubtending ſide A C, videl. $60\frac{1}{3}$. then mouing vp and down the rule, til the ſame $60\frac{1}{3}$. degree, doe cut among the parallels the degree of the ſide B C geuen, vz. the 40. parallell counted from the Equinoctiall, where you ſhall finde to meete you the 50. meridian as before is ſayde to make vp your tryangle.

In briefe.

If the two ſides including the right angle be giuen, reckon the one on the Equinoctiall outwards from the centre: and thence in the meridian there, reckon the other vpwards: thereto laying the rule, whoſe degr. cut, counted from the centre, is the third ſide: and it ſheweth in the *Limbe* one of the angles ſought: but if the ſubtending ſide with one of the including ſides be giuen, reckon the ſubtending ſide firſt on the rule from the centre outwards, and lay that degree on the parallel aunſwearable to the other ſide giuen: and there ſhall meet you the meridian that maketh vp the tryangle, the rule ſhewing as before, one of the angles at the *Limbe*.

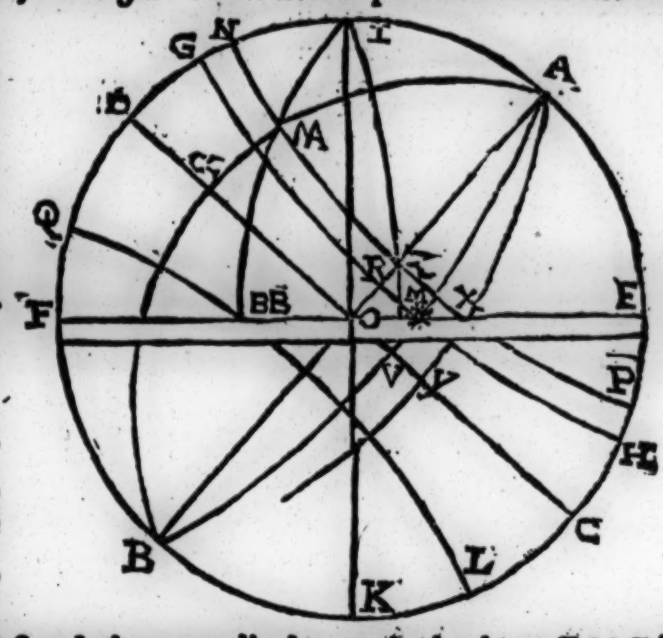
Chapter 5.

The declination of the Sunne, Moone, or any other planet or ſtarre from the Equinoctial, giuen together with the Latitude of the countrey, how by the 4. chap. to get the amplitude, the difference of aſcenſion, the oblique aſcenſion, and the diurnal arch of any of them.

Here ſhall you partly perceiue a taſt of the great fruite that commeth ſince of one chapt. of ſphericall tryangles, which extendeth it ſelfe to infinite queſtions, therefore it is no maruel, that ſo many greate volumes haue bin written of them.

I wil take now for example the great ſtarre of the firſt light, called *Oculus tauri*, the bulles eye: his declynation I finde after *Stadius* in the table of ſtarres in my 2. booke, to be $15\frac{2}{60}$. degr. Northwardes, which in thys preſent

preſent figure is the ſpace, G D or V M whiche is equal to it, the height of the North pole aboue the horizon being the ſpace E A, which is 51 $\frac{2}{3}$. deg. here at Reading: theſe two giuen, I wil, I ſay, perform all the premiſſes: and thus I goe to worke. The circle H G is the 15 $\frac{42}{60}$. parallel of declination Northwards, in which the ſtarre is fixed, I ſay fyxed, not exempting his diurnal motion, mentioned in the 1. booke and 9. chap. *ex raptu primi mobilis*, wherefore at his riſing hee muſt needes cut the point M of the horizon E F, the ſpace M O being his amplitude of riſing in this countrey, which to fynd, I ſubduct the declination of the ſtarre 15 $\frac{42}{60}$. deg. videl. M V out of 90. vid. A V, there reſteth his complement of declination 74 $\frac{18}{60}$. deg. videl. A M. And ſo haue you two ſides of the right angled ſpherical triangle A E M, giuen vz. A M 74 $\frac{18}{60}$. and A E 51 $\frac{2}{3}$. deg. V Wherefore by the 4. chap. you now gette the thirde ſide, E M, 63 $\frac{1}{2}$. deg. the complement whereof, vz. M O, being 26 $\frac{1}{2}$. deg. is the amplitude ſought for. That A E M is a right angled triangle is manifeſt, for that the arch A E ſtandeth perpendiculer on the horizon E M F, by the third def. in reſpect that being drawne foorth, it cutteth the pole of the horizon E M F, videl. the Zenith point 1.



[12] Then for the difference of aſcenſion and the oblique aſcenſion learne by the fourth chap. likewiſe the quãtity of the angle E A M being an angle of the ſame triangle, which by the 4. def. is denominated by the arch of the Equinoctial C V, videl. 69. deg. whoſe complement videl. V O is 21. degr. which by the 3. booke 13. chap. muſt needes be his difference of aſcenſion, and is the quantity of the angle M A O. This arch V O, videl. 21. ſubduct from his right aſcenſion, which by the 3. booke 17. chap. is 62 $\frac{1}{2}$. deg. there reſteth 41 $\frac{1}{2}$. degr. his oblique aſcenſion being a North ſtarre, but if it had bin a South ſtarre, you ſhould haue added this arch by the third booke 15. and 17. chap.

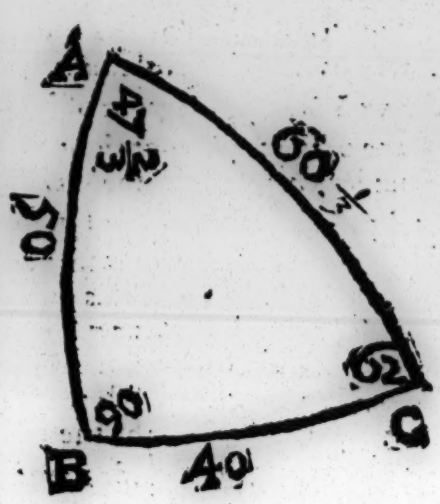
Laſtly, for his diurnall arche, adde the arch V O, videl. 21. vnto the quadrant O D, vz. 90. thereof commeth 111. the arch V D, or rather M G, his ſemidiurnall arch: that doubled, maketh 222. his diurnall arch, which 222. deg. deuided by 15. deg. leaueth in the quotient 14 $\frac{12}{15}$. houres: or more playne 14. houres 3. quarters and more: ſo long doth *Oculus tauri* bide aboue our Horizon betweene his riſing and ſetting. Note that in the ſun this Semidiurnall arche deuided by 15. ſheweth the houres of his riſing and ſetting counted from noone.

Chapter 6.

To performe the 4. chap. on the Iewell *by another manner of woorking.*

FOr this purpoſe let the right angled triangle in the 4. chap. be propoſed againe, whoſe ſide A C ſubtending the right angle B, let bee giuen: vz. 60 $\frac{1}{3}$. deg. and the ſide C B, vz. 40. deg. being one of the including ſides. Now to find the other ſide and angles after another manner, then in the 4. chap. is ſhewed, doe thus.

The Zenith line of your *Recte.* firſt placed euen in the Equinoctial line, and there fixed: ſeeke the length of the ſubtending ſide giuen, A B, videl. 63 $\frac{2}{3}$. deg. among the almicantares counted from the zenith poynt, and the length of the other ſide giuen, C B videl. 40. deg. amongſt the parallels of the *Mater.* counted from the Equinoctiall: in the croſsing of thys 60 $\frac{1}{3}$. almicantare, and 40. parallel, you ſhall finde the 50. meridian to cut and alſo the 42 $\frac{1}{3}$. azimuth: both counted from the *Limbe*, which two with the portion of the Equinoctiall or zenith line lying betweene them, doe ſhew the three ſides of the triangle ſought for in their right conſtitution. The ſaide 50. meridian cutteth in the zenith line which now lyeth euen in the Equinoctiall 50. deg. for the ſide A B, and in the ſaide 42 $\frac{1}{3}$. azim. 60 $\frac{1}{3}$. deg. for the ſide A C, both counted from the zenith poynt: and the ſame 42 $\frac{1}{3}$. azimuth, cutteth 40. deg. of the ſame 50. meridian counted frõ the Equinoctial for the ſide B C. Beſides all this, if you follow the 42 $\frac{1}{3}$. azimuth to the *Finitor* line, he there cutteth off 47 $\frac{2}{3}$. degrees, counted from the centre, the angle ſubtended by the ſide B C, videlicet, the angle A, by the fourth def. euen as it was founde by the 4. chapt. But now for the thirde angle videl. C, you muſt ſhift the ſides thus. The Zenith line lying ſtil in the Equinoctiall, reckon thereon from the Zenith point, the ſide B C, videl. 40. deg. there cutteth the 40. meridian, vpon which reckon the quantity of the ſide ſubtending the angle you ſeek, videl. A B, 50. degrees, and there ſhall you ſee to cutte the 38. azimuth counted from the *Limbe*, which followed to the *Finitor*, ſheweth 62. deg. counted from the centre, euen as in the 4. chap. the quantity of the angle C by the 4. def.



A. C vz. 60 $\frac{1}{3}$.

But

But here you will say: what if the two sides including the right angle be giuen, I aunsweare, there is no difference in the working to finde the subtending side from that which I did last to fynd the third angle: for the one including side reckoned on the Zenith, the other on the meridian, there standing, you shall see the azimuth cutting there that meridian to make vp the third side.

Chapter 7.

To performe the same by the Iewell *after a thirde manner.*

LEt the former triangle A B C remaine, whose side subtending the right angle, videl. A C, $69\frac{1}{3}$. degr. and the side A B videl. 50. deg. one of the including sides let be knowne, the thyrde B C vnknowne, wyth the angles A and C, which three after another manner then yet hath beene shewed, I will teache you to fynde.

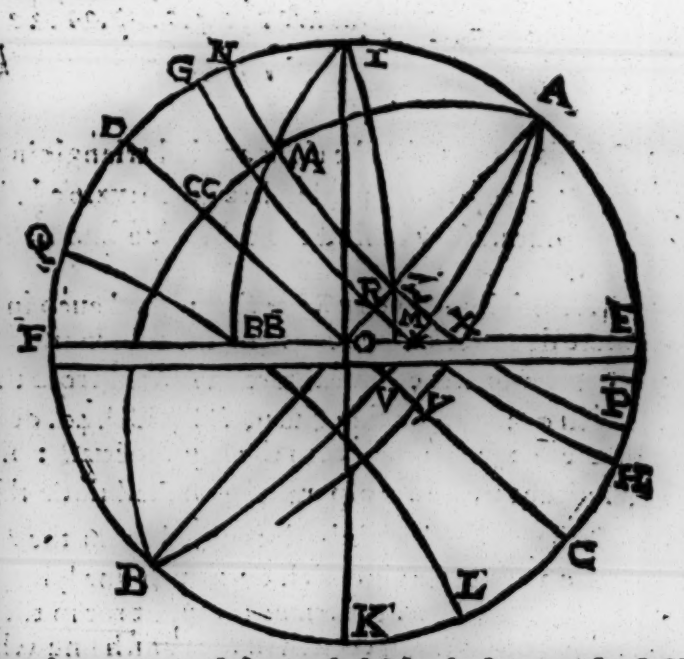
Reckon the including side giuen, videl. A B, being 50. deg on the *Limbe*, from the pole towardes the Equinoctiall, thereto set the rule fixed, then reckon among the parallels from the pole, the quantity of the subtending side giuen, videl. A C, $69\frac{1}{3}$. deg. and where this $69\frac{1}{3}$. parallel doth cutte the rule, there crosseth the $47\frac{2}{3}$. meridian, which cutteth off in the rule 40. degrees, the third side sought for, videl. B C: the same meridian followed to the equinoctiall by the fourth def. sheweth there $47\frac{2}{3}$. degrees, one of the angles sought, videl. the angle A, subtended by B C. And now for the angle C, you neede but shift the sides: that is to saye, reckoning the side subtending it, videlicet A B, 50. degrees on the rule inwardes, and the other including side B C, videl. 40. deg. on the *Limbe* from the pole: the rule thereto applied, you shall see the 62. meridian to cut the said 50. deg. of the rule, which followed to the Equinoctiall by the 4. def. sheweth there 62. degrees the angle sought for.

If the two sides including the right angle, had been giuen vz. 40. and 50. you see by this last woorking for the angle C, that the one reckoned on the *Limbe* from the pole, the other on the rule inwards, and applied together, the third side leaps into them being alwaies a portion of some one of the meridians.

I could yet shew you 2. wayes more to worke with a right angled triangle, but more combersome then commodious, yet in the 24. chap. I shalbe driuen to one of them, & for your farther liking of these past, I wil frame another proposition to apply them to vse as followeth.

Chapter 8.

The quantity of the longest day in any countrey giuen, to know the latitude, and in what climate the same is.



FOr example: heere at Reading our longest day is 16. houres 24. min. whereby the semidiurnall arche must needs be 123. deg. as in my 3. booke 22. chap. is shewed, admit the latitude vnknowne, which semidiurnal arch is in this figure. X N, or rather Y D: whiche if you take out of 180. videl. D Y C, there resteth 57. deg. the arch X P, or rather by the 4. def. Y C. VVherefore you haue here the right angled spherical triangle, A E X, the angle E being a right angle by the 3 def. whose subtending side A X is knowne. If you subduct the declination of the tropicke, vz. Y X, $23\frac{1}{2}$. deg. out of 90. vz. Y A to be $66\frac{1}{2}$. deg. and one of the angles not right: vz. E, A X being 57. deg. as is saide: wherefore by the 9. chap. the side A E shall be found $51\frac{2}{3}$. deg. the latitude sought for: but this proposition seruеth hitherto vnto this next chapt. following, being the speedier course in this question, but if you woulde haue done it by the 4. 6. or 7. chapt. which require 2. sides giuen, then should you out of the semidiurnall arch X N or Y D, take 90. vz. D O, there resteth O Y $33\frac{1}{3}$. deg. and then haue you a right angled spherical tryangle, O X Y, Y being a right angle by the 3. def. and 3. theoreme, whose two sides including the right angle, videl. Y O, 33. degree, and X Y $23\frac{1}{2}$. deg. are knowne: wherefore, by the first part of the 4. chap. or the last part of the 6. or 7. the side X O is had, which is the Sunnes amplitude in that country, he beyng in ♋, then take X O, now known out of 90. vz. O E, there resteth X E, wherfore now you haue the former triangle A E X of 2. known sides, videl. X E, one of the sides including the right angle E, and the complement of X Y vz. X A, subtending E. Therefore by the first part of the 6. or 7. or the last of the 4. you maye eafilye knowe the side A E beeyng the eleuation of the pole or latitude sought for, then for the climate you may learne by the thirde booke 22. chap.

I might haue shewed you an easier waye by the tryangle Y O X, and neuer driuen you to seeke further for the angle X O Y beeyng the arche, E C by the fourth def. is the depression of the Equynoctiall vnder the Horizon E F, whose complement E A, is the eleuation sought for, or more eafilye the

the angle D O F: being equall to the angle X O Y, by the 5. theoreme, is the equinoctials height aboue the horizon his complement being D I, by the 1. Chapt. is the latitude sought for by the first booke 12. Chap. Thus you see how aboundantly the sphericall triangles are like to serue your turne in euerie question.

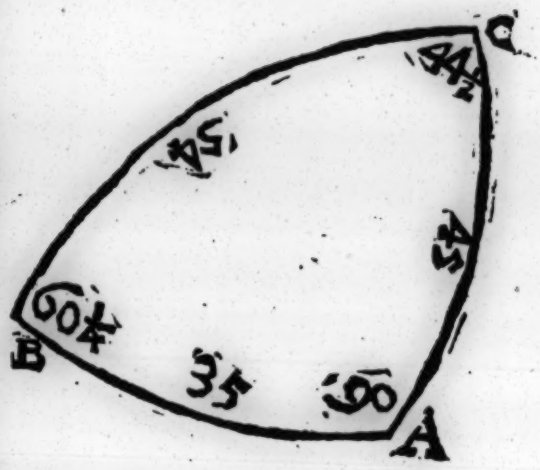
Ioh. Regimon. lib. 4. Cap. 27. Copernic. 4. triangle spher.

Chapter 9.

If one side of a right angled sphericall tryangle, with one of the two angles not right be giuen, how to finde the other two sides and angle.

LEt there be proposed the right angled sphericall triangle A B C, whose sides A B and A C doe include the right angle A of which A B is knowne to be 35. degrees, and the angle C, subtended by the side A B, knowne likewise to be $44\frac{1}{2}$ degrees: the third angle B with the sides A C, and C B knowne.

If you will worke after the 4. Chapter, reckon on the *Limbe* from the equinoctiall vpwards, the angle knowne vz. $44\frac{1}{2}$ deg. thereto fixe the rule, then reckon the subtending side giuen vz. A B 35. degrees among the parallels, and you shall finde the 35. parallel to cutte the rule in the $54\frac{1}{2}$ degrees counted from the centre, the same is the quantitie of the side B C, subtending the right angle, in which cutting you shal also see the 45. meridian to crosse which followed to the equinoctial, as in the 4. Chap. giueth on the equinoctiall line 45. degrees, for the third side A C, thus hauing al three sides, the third angle at B, you may easily haue by the 4. 6. or 7. Chapt.



You will say, what if the said angle C vz. $44\frac{1}{2}$ degrees, and the side A C 45. degrees, had beene giuen, I will shew you to get the other two sides and angle by the doctrine of the 7. Chapter, because I will vse you to them all. Reckon this side giuen vz. 45. on the *Limbe* from the pole, thereto fixe the rule; then on the equinoctiall from the *Limbe* inwards, reckon the angle giuen vz. $44\frac{1}{2}$ degrees, and follow the meridian there to the rule, and you shal finde on the same $44\frac{1}{2}$ meridian betweene his crosing of the rule, and the pole $54\frac{1}{2}$ deg. for the side B C, and the degrees of the rule betweene the same crosing and the *Limbe* 35. deg the side A B.

Yet me thinkes I heare you say what if the subtending side B C were knowne $54\frac{1}{4}$ deg. with the angle B $60\frac{1}{2}$ degrees, and the rest vnknowne, I will then shew you to performe this by the 6. Chapt. because we wil set them all on worke.

Place the zenith line euen with the equinoctiall line fixed, as there is shewed: and then on the *Finitor* line from the centre vpwards, reckon the angle giuen vz. $60\frac{1}{4}$ degrees, and vpon the $60\frac{1}{4}$ azimuth bounding there, reckon the side B C giuen, vz. $54\frac{1}{2}$ from the zenith point, and there shall you see to crosse the 35. meridian counted from the *Limbe*, shewing in the zenith line 35. deg. for the side A B, and from that crosing you shall finde on the same 35. meridian downe to the zenith line 45. degrees for the side A C, and thus haue I serued euery turne.

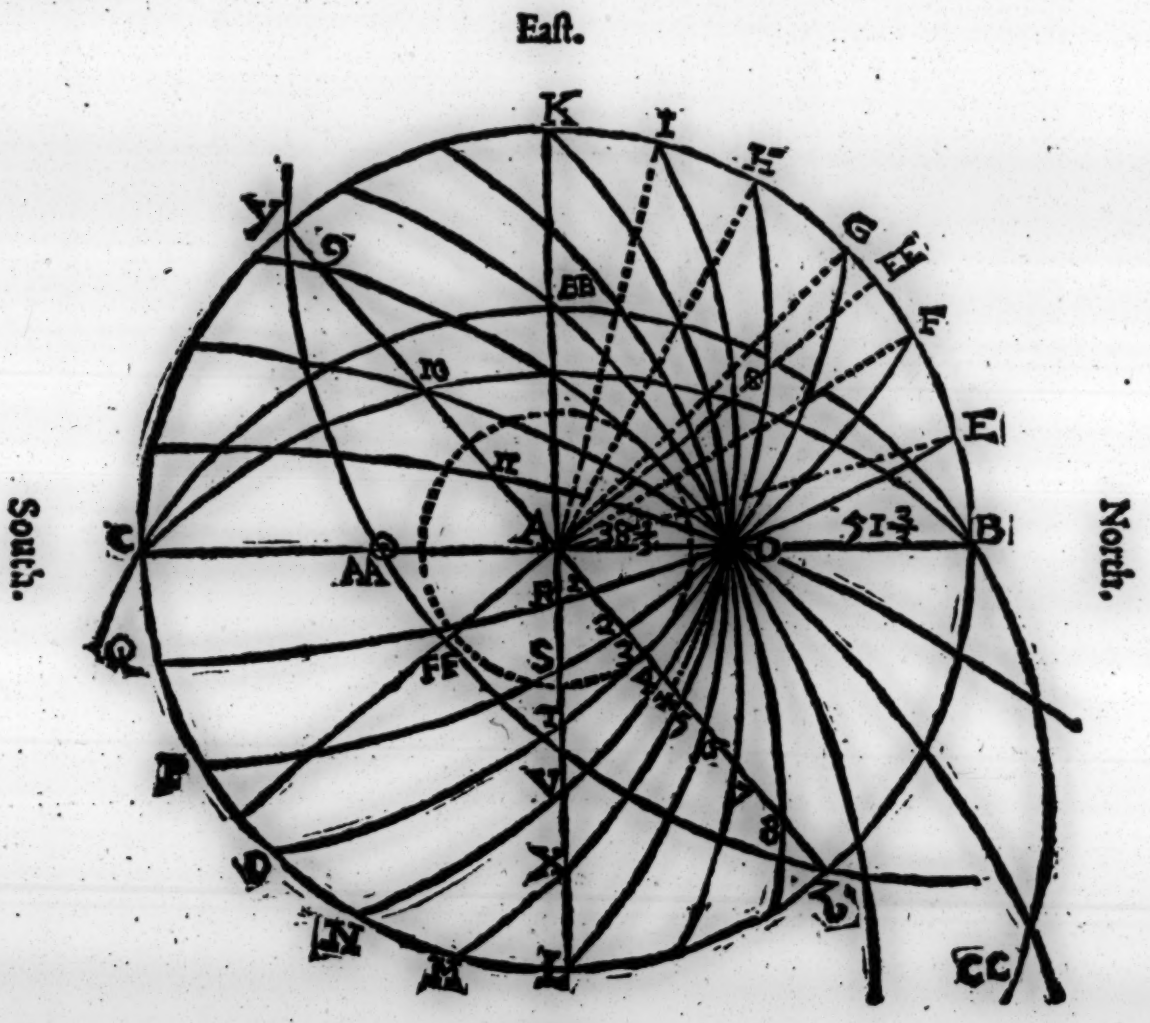
Chapter 10.

If you would make an horizontall or verticall Dyall to any countrie, how by this last Chapt. to finde what number of degrees euery houres space containeth.

YOu must not here be ignorant that those meridians of the sphere which deuide the equinoctiall into 24. equall parts being euerie 15. counted from the meridian of any countrie, are called specially the houre-lines, these cut any oblique horizon into 24. vnequall parts, as in this figure you may see D E, D F, D G, D H &c. rounde, where the circle B L C K representeth the horizon B C the meridian, and D the North pole eleuated $51\frac{2}{3}$ degrees aboue B, the North point of our horizon heere at Reading: to which intersections B E F G H I &c. if there be streight lines drawne or imagined from the centre of the horizon vz. A, your diall is made, and with a circle concentricke to the horizon, you may cut him off to any quantitie, as here you see the little circle, but hereof I must needs more aboundantly speake in the beginning of my 6. booke, and therefore I referre you thither.

In the meane time to delight you with the finguler profite of these sphericall triangles, I will shew you by them to finde the quantitie in degrees of the horizon betweene euery of these intersections, and because euery quarter of an horizon diall hath like spaces, so that by one quarter the whole may be made, I wil for your ease and mine, beginne in one of the north quarters, though we commonly leaue them halfe out: where, for the 1. houre you may see there is the right angled spherical triangle E B D (for that D B lighteth perpendiculer on the arch E B by the third diff.) whose side D B is knowne to be $51\frac{2}{3}$ deg. the poles eleuation, and the angle E D B must needes be 15. degrees by the 4. diff. because 15. degrees of the equinoctiall are taken to one houre, wherefore working by the seconde part of this last Chapter, hauing now one side and one angle giuen, you shall finde the side E B $11\frac{1}{2}$ deg. Then for the second houre you haue the right angled triangle F B D, whose

one ſide B D is knowne, & one angle vz. F D B, the quantitie of 2. houres which muſt needs be twiſe 15. that is 30. degrees, theſe had by this laſt Chapter, as before you ſhall finde the ſide B F $24\frac{1}{6}$ degrees; then for the third houre you haue the right angled triangle G B D, whoſe angle G D B is thrife 15. vz. 45. degrees, by which and the ſide B D, you ſhall get the ſide G B 38. degrees, and ſo for the reſt. Note that B D, vz. $51\frac{2}{3}$ deg. taken out of B A vz. 90. leaueth D A $38\frac{1}{3}$ degrees, the complement of B D, which added to A C vz. 90. alſo maketh $128\frac{1}{3}$ the arch D A C, by helpe whereof I might haue begunne with one of the ſouth quarters, as for the firſt houre you haue the right angled triangle D C Q. whoſe ſide D C is $128\frac{1}{3}$ deg. and his angle C D Q 15. deg. knowne, as in the former way, and therefore by the 9. Chapter you ſhall get the ſide C Q. $11\frac{1}{2}$ deg. euen equall to E B, for as I told you in euerie quarter, the match ſpaces are equall, that is to ſay, if they be of like diſtance from the nooneline, alſo commonly in the ſphere *Oppoſitorum eadem eſt ratio*, and ſo I leaue you for this diall.



Now for the verticall Diall which we commonly call the ſouth wall Diall. The circle L A K, being the verticall circle of the eaſt croſsing the meridian B A C, ſquare in the zenith A, is in trueth the horizon of this wall, aboue whoſe flatte ſouthwards the ſouth pole is eleuated with our antipodes, but Northwards, the north pole D is eleuated, and that viſiblie in our countrie according to the quantitie of the arch D A being the complement of B D, as I ſaide, which flat L A K the 24. hourelines doe vnequally deuide in L X V T S &c. but one halfe of the interſections are quite vnder our horizon, and therefore this Dial hath in our countrie viſilbie but 12. houres: but to come to the matter, you haue for the firſt houre of the quarter A L, the right angled ſphericall triangle D A R, whoſe ſide D A is $38\frac{1}{3}$ degrees, and his angle A D R 15. degrees, wherefore the ſide R A cannot be vnknowne by the ſaid ſecond part of this laſt Chapter, which ſhalbe the quantitie of the firſt houre. Likewiſe for the ſecond houre you haue the triangle D A S, his ſide D A knowne, and the angle A D S, being 30. deg. and ſo forth with all the reſt.

Chapter 11.

Ioh. Reg. lib. 4. ca. 34. & lib. 5. cap. 3. & 4. Cop. 3. triang.

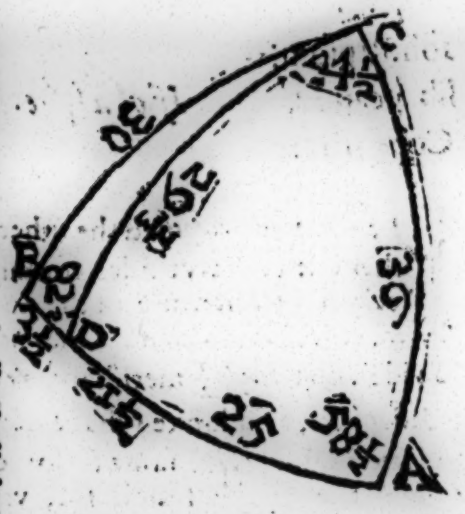
If the three ſides of any ſphericall triangle whatſoeuer ſhall be giuen to finde the quantitie of euerie of his three angles.

OF this matter Ioannes Regiomontanus maketh three long diſcourſes in three ſeuerall Chapters, with verie tedious buſineſſe by his Sines, Supplements &c, but my *Iewel* you ſhall ſee will make ſhort worke.

For

For if the three ſides giuen bee of a right angled triangle, then I perſwade my ſelfe that you are ſufficiently inſtructed by the 4. 6. and 7. Chapt. to finde all the angles, but yet whether the triangle giuen be right angled or not right angled, (for it is hard to know that by the bare ſides giuen) you ſhall and may worke in manner following to come by the three angles.

Let there be propoſed the ſpherical triangle A B C, whoſe three ſides are knowne vz. A B 25. degrees A C 36. and B C 30. but the angles A B and C, all vnknowne, which to know you ſhall reckon any one ſide (as for example) the ſide A B vz. 25. degrees on the *Limbe* from the pole any way, thereto lay the zenith point of the *Reete* fixed, then number one of the two ſides remaining as B C vz. 30. among the almicantares, counted from the zenith point of the *Reete*, and the third ſide A C vz. 36. deg. among the parallels, counted from the pole, now where this 30. almicantare, and 36. parallel do croſſe, there ſhall you find to croſſe the 58½ meridian, and the 82½ azimuth both counted from the *Limbe*, whoſe portions lying betweene the pole and the zenith, if you marke them, doe liuely repreſent your triangle ſought: for the portion of this 58½ meridian betweene this croſsing and the pole being as you may there ſee 36. degrees, numbred by helpe of the parallels, is the ſide A C: and the portion of the 82½ azimuth lying betweene this ſelfe ſame croſsing, and the zenith being there 30. degrees, is the ſide B C, and the portion of the *Limbe*, betweene the zenith and the pole being 25. degrees is the ſide A B, I ſay theſe three arches liuely repreſent vnto you the forme of the tryangle, whoſe ſides were giuen, and that as expreſly as on the Sphere it ſelfe. VVherefore if you aske counſell of my fourth diff. you will followe this 58½ meridian downe to the Equinoctiall, where it ſheweth you 58½ degrees counted from the *Limbe*, the angle ſubtended by the ſide B C, vz. the angle A. In like manner the 82½ azimuth followed to the *Finitor*, ſheweth 82½ degrees, the angle ſubtended by A C, vz. the angle B. And yet doe we lacke the angle C, which will be the eaſier had, now the reſt are knowne: wherefore place one of the ſides not ſubtending the vnknowne angle C, videlicet, either the ſide A C, or B C betweene the zenith and the pole, as before you did the ſide A B, and heere for example A C: then among the meridians, reckon from the *Limbe* the quantitie of the angle ſubtended by the ſecond ſide B C, vz. 58½ degrees vpon this 58½ meridian, reckon from the pole the third ſide ſubtending the angle you ſeeke, vz. A B, 25. degrees, and there ſhall you finde to cut the 44½ azimuth, which followed to the *Finitor* line ſheweth 44½ degrees, the quantitie of the angle C: and heere againe doe the three portions of the *Limbe*, the 58½ meridian, and the 44½ azimuth repreſent your triangle propoſed.

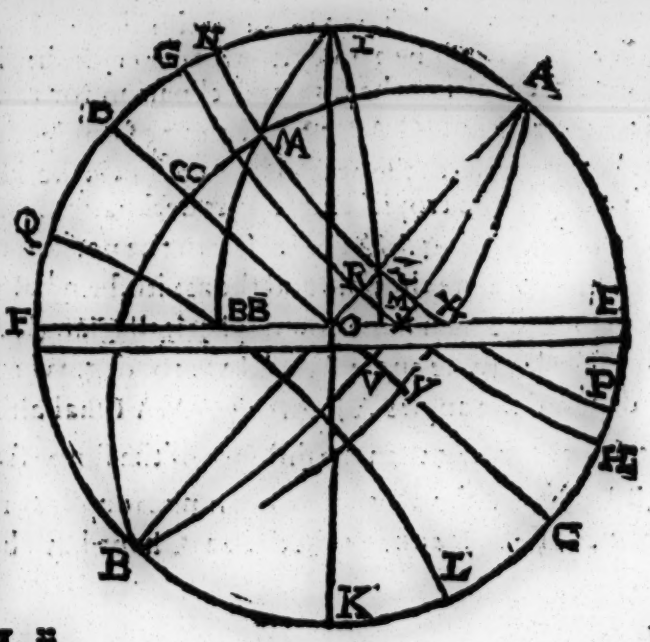


In briefe: Of the three ſides giuen, place the one betweene the pole and the zenith: of the other two reckon the one among the parallels counted from the pole, the other among the almicantares counted from the zenith: where theſe two croſſe, there ſhall meete an azimuth and meridian, whoſe portions from their meeting to the pole & the zenith, with the portion of the *Limbe* lying betweene the pole and the zenith, ſhall repreſent your triangle, and the two diſtances betweene the *Limbe*, and the ſaide azimuth and meridian are two of the angles. For the third angle you are to ſhift the ſides, that is to ſay, to place one of the other ſides betweene the pole and the zenith, and then worke as before.

Chapter 12

The declination of the ſunne giuen at any time togither with her altitude aboue the horizon, how by meane of the 11. *Chapt. to get what houre and minute it is, and in what azimuth the ſunne then is.*

FOr example admit that the 12. day of Iune when as the ſunne is in or verie neare the tropicke of Cancer being in this figure P N, and therefore his declination 23½ deg. vz. N D, I had then taken his altitude 53½ deg. take the complement of his declination, which is 66½ and alſo of his altitude which is 36½ degrees: the firſt is the diſtance of the ſunne from the pole, the ſeconde from the zenith at the time of the altitude taken videlicet A AA, and I AA, which with the arch A I, maketh vp the triangle A I, A A of the three knowne ſides vz. A AA, 66½ I AA, 36½ and A I 38½, being the complement of the latitude, wherefore by the eleuenth Chapter the angles ſhall be all knowne, of which the angle A I, AA is the diſtance of the ſunne from the ſouth ſo founde 30. degrees, that is two houres: wherefore if it were morning, then was it tenne



the quantitie of the angle A.

The quantitie of the angle 1.

of the clocke, if afternoone, two of the clocke. Also A I, A A, muſt needes bee the diſtance of the azimuth wherein the ſunne then was from the North, found by the 11. 130. deg. if out of that you take 90. there reſteth 40. deg. vz. 0. B B, the diſtance of the ſame azimuth, from the Eaſt or weſt, and that towards the ſouth. I am the briefer becauſe I meane onely to giue you a taſt of the vſe of theſe triangles, knowing this alſo to be verie eaſie to conceiue, to them that vnderſtand the 4. bookes paſt euen by the bare inſpection of this figure.

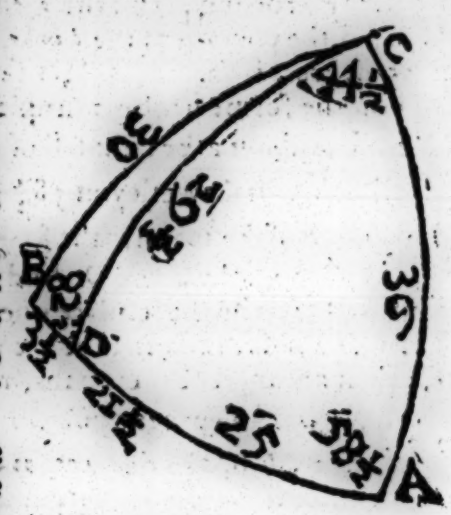
Ioh. Regiom. lib. 4. cap. 28. Copern. 11.

Chapter 13.

If of a ſpherical triangle two ſides containing any one knowne angle ſhalbe giuen, how to finde the other ſide and angles.

LEt there be propoſed the ſphericall triangle A B C, whoſe angle A, let it be knowne 58½ degrees togither with the two ſides containing it vz. A C 36. degrees, and A B 25.

Reckon the quantitie of the one ſide giuen, as of A B, 25. deg. on the *Limbe* of your *Iewel*, thereto lay the *Reules* Zenith fixed: then on the meridian diſtant from the *Limbe*, as much as the angle A propoſed commeth to videl. on the 58½ meridian, reckon from the pole the quantitie of the other ſide propoſed, videl. of A C 36. degrees, and looke what azimuth croſſeth the ſame 58½ meridian in the ſaide 36. deg. the ſame followed to the *Finitor*, ſheweth the degrees of one of the angles ſought for, videlicet of the angle B ſubtended by the ſide 36. which you ſhall finde 82½ degrees counted from the *Limbe*: then are the degrees of the ſame 82½ azimuth betweene the ſaide croſsing and the zenith, the quantitie of the third ſide ſought for, which you ſhall finde to be 30. deg. Thus doe you ſee according to your accuſtomed manner your whole triangle in his forme, his three ſides, & two angles knowne, wherefore by the 11. Chapter the third angle is eaſily had in the laſt part of which Chapter, this Chapter was performed ere I was ware, ſuch is the dexteritie of my *Iewel*, neither woulde I haue repeted it here againe, had it not beene to imitate the courſe of Regiomontanus and Copernicus.



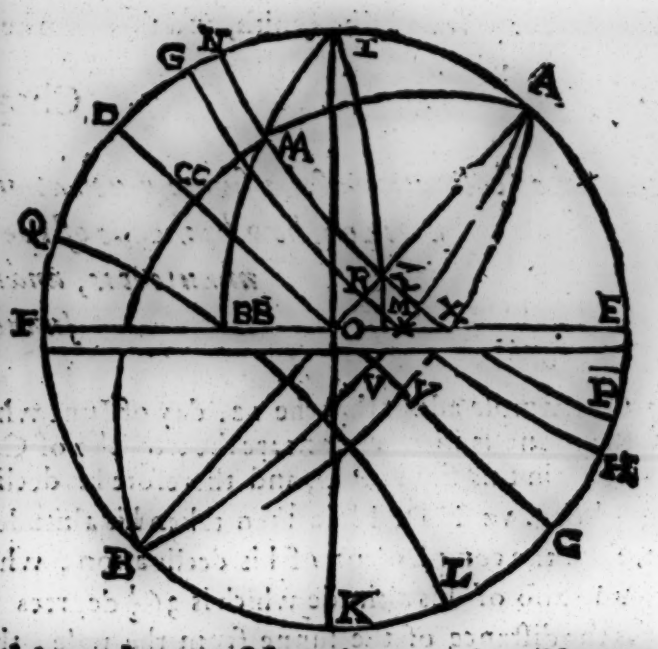
In briefe, lay the one ſide giuen betweene the pole and the zenith, the other reckon on the meridian as much diſtant from the *Limb*, as the angle giuen commeth to, there ſhal meete you the azimuth making vp your triangle.

Chapter 14.

How by helpe of the 13. Chapt. to know the height of the ſunne at any houre, and minute propoſed in the whole yeare, and alſo in what azimuth he is.

IN my thirde booke 31. Chapter is ſhewed the ſame by ordinary vſe of the *Iewel*, and that moſt exact and redily without any trouble, yet ſhall it not be amiſſe to know to do the ſame by the Art of ſphericall triangles, which performeth it as reddily as Gem. Friſius did on his Catholicon, though nothing ſo eaſie as my *Iewel* doth it in the ſame 31. Chapt.

For example the 12. of Iune, *at 10 of the clocke* the ſun then being in the 1. degree of ♋ his declination there 23½ deg. I woulde know of what height, and in what azimuth the ſun ſhall then be. The angle made betweene the 10. of the clocke houreline, and the meridian being in this figure, the angle A A, A I, muſt needs be 30. deg. for that euery houre is 15. degrees, then taking the ſunnes declination for that day vz. D N, or rather C C A A 23½ deg. out of 90. deg. videl. C C A, there reſteth the arch of the 10. of clocke houreline betweene the tropicke or ſunnes declination, and the pole vz. A A, A being 66½ degr. Then the portion of the meridian betweene the pole and the zenith, being alwayes the complement of the latit. vz. A I, is wel knowne 38½ deg. Here then haue you a ſphericall triangle A A, A I, whoſe two ſides A I, and A A A, are knowne, including the knowne angle A A, A I vz. 30. deg. wherefore by the 13. Chapt. the 3. ſide I A A ſhalbe knowne 36½ deg. and the angles A I, A A & A A, A I, ſhal alſo be knowne: Now then ſubduct the ſaid third ſide I A A vz. 36½ deg. being then the diſtance of the ſunne from the zenith out of I B B vz. 90. deg. ſo reſteth the arch A A, B B 53½ deg. the altitude of the ſunne ſought for, and the angle A I, A A, ſheweth the azimuth of the ſun in the very ſame manner, as in the 12. Chapt. hath beene ſaid.



Chapter

Chapter 15.

Ioannes Regiomontanus, Lib. 1. Cap. 51. and lib. 4. cap. 29. *proposeth and sayth that the knowing of two sides of a triangle not right angled, with one angle subtended by one of those sides, cannot serue to get the other side and angles.*

HEere is a wonderfull case, that by their whole art of spearicall triangles, with their signes, coardes, arches complements, proportions, calculations, tables, and such like, they cannot doe that which my poore *Iewel* will doe immediatly without any difficultie. Nay all the sporte is, Regiomontanus spendeth muche speache in the saide 51. and 29. chapter, to prooue that it is vnpossible to bee done, without letting downe a perpendiculer, respecting or subtending the angle proposed, which in the thirtieth chapter following, hee teacheth to doe. But we will doe it, add that without any perpendiculers, a little readier then he doth it.

The figure of the 13. Chap. serueth for this place.

Admit that of the triangle A B C, proposed in the thirteenth chapter, the two sides A B, and A C, wer knowen as there they were, together with one of the angles B or C, subtended by one of the sides giuen: for example, the angle B $82\frac{1}{2}$. degrees, subtended by A C, & that the angles A and C were vnknowen, with the side B C: Is it not possible thinke you by these three giuen, to reache to the rest, I hope it be, els I loose my labour.

Reck ō on the *limbe* of the *Iewell* frō the pole ♑ side giuen, not subtendyng the angle giuen, vz. ♑ side A B, being 25. degrees, thereto lay the zeneth point of the *reete* fixed, reckon also on the *Finster* from the *Limbe* inwards, the angle B giuē, vz. $82\frac{1}{2}$. de. there haue you the $82\frac{1}{2}$. azimuth, then reckon the other side giuen, vz. A C. 36. degrees among the parallels, counting from the pole, mark where this 36. parallel, and the said $82\frac{1}{2}$. azimuth doe crosse. For the portion of the ~~azimuth~~ meridian cutting there, counted too the ~~pole~~ zenith, is the thirde side, which you shall finde 30. degrees, ~~of the $82\frac{1}{2}$ meridian counted from the~~ *limbe*. The same $58\frac{1}{2}$. meridian followed also to the equinoct. sheweth $58\frac{1}{2}$. deg. one of the angles sought for, vz. A. The third angle C cannot be vnknowen by the 11. or 13. Cha. Thus you see this vnpossible matter now possible, then iudge whether my *Iewel* bee a *Iewell* or no.

Yet thus much will I foretell you, least happily you shoulde be amazed at any time. In some triangles you shall finde that the azimuth representing the angle giuen, will crosse the parallel that worketh this feate in two places, and then shall you not tell which place of crosing to take, yet commonly the crosing fartheſt within the *Limbe* is hit: in all the triangles that I haue had experience of, but you may trie by shifting of the sides, as you are wont to doe in finding the thirde angle which is it.

Chapter. 16.

The azimuth of the Sunne giuen or taken at any time by Instrument, with his declination, how by helpe of the fifteenth Chapter, to knowe what height the Sunne is of, and what it is a clocke.

IT is taught in the 12-chapter, spherically to get the houre & azimuth of the Sunne by his altitude, contrariwise, heere by the azimuth shall bee hadde the houre and the altitude.

The figure of the 14. chap. serueth for this place.

For example, the 12. of Iune, admit that before noone I had take by some Topographicall Instrumēt, or otherwise the azimuth or horrizontall distance of the Sunne 40. degrees from the East towardes the South, which in this figure is O B B, thereto I adde 90. degrees, videlicet, E O: so haue I the arche *EBB* 130. deg. the declination of the Sunne then is $23\frac{1}{2}$. degrees, by the 3. booke and 4. chapter. Nowe it is most manifest, and apparantly to bee seene on the *Iewel*, that any parallell of the sphoere doth cut off the meridians, or houre lines, all of like equall distance from the pole, as heere you shall see the arches of the houre lines A X. A Z. A AA, A N, are all complements of the Sunnes declination: each of them, and namely A AA, $66\frac{1}{2}$. beeing $66\frac{1}{2}$. degrees, the arche betweene the pole and the Zenith, vz. A I, is certaine $38\frac{1}{2}$. degrees. Here haue you nowe a spherical triangle A I, AA, whose two sides A I, and A AA, are knowen with the angle A I, A A, which angle is subtended by A, AA, one of the knowen sides, euen the like too that whiche the fifteene chapter proposeth. The third I, AA, and the angles AA, A I, and A, AA I, are vnknowen. VVherfore if the worlde were so harde, and the matter so vnpossible, as Regiomontanus proposeth it, we shoulde bee neuer the neere of our purpose: but working by the 15. chapter, notwithstanding his proofes too the contrary, you shall finde the angle *I A, AA*, 39. degrees, the Sunnes houre distaunce from the South, which must needes be two houres euen ten of the clocke: also you shall finde the thirde side, *I, AA*. $36\frac{1}{2}$, degrees, whiche taken out of 90. vz. *I, BB*, leaueth the arch *AA BB*, $53\frac{1}{2}$. degrees, the height of the Sun at that instant. As for the thirde angle, you shall finde him 32. degrees, but wee haue no vse of him here. 30.

Ioh. Regio. lib. 4. cap. 31. & 32. Copern. 12.

Chapter 17.

If of any ſpheerical triangle, not right angled, two angles ſhalbe giuen with the ſide lying betweene thoſe angles or els the ſide ſubtending one of them, how to get the other two ſides and thirde angle.

IF the ſide lying betweene thoſe angles ſhall bee giuen, it is moſt eaſie, for placing that ſide on the *limbe* betweene the pole and the Zenith as you are wont, & reckoning the one angle giuen among the azimuthes, the other among the meridians, where theſe croſſe, ther is your triangle made vp, in your wonted maner. For example, admit that of the triagle A B C, the two angles A, 58½. deg. & B 82½. degrees were giuē with the ſide AB, 25. deg. lying betweene them: reckon AB, vz. 25. deg. on the *Limbe* from the pole, thereto lay the Zenith of the *Reete* fixed: that done, reckon the one angle, vz. A 58½. deg. amongſt the meridianes, the other angle vz. B 44½. deg. amongſt the Azimuthes both from the *Limbe*, and where this 58½. meridian, and 44½. S2.½ azimuth, do croſſe, there ſhall you finde your triangle made vp after your accuſtomed maner, for on the 58½. meridian between the croſsing, and the pole, you ſhal find 36. deg. for the ſide A B, & on the 44½. azimuth to the Zenith 30. deg. for the ſide B C, theſe had, the third angle is to be had by the 11. or 13.

S2.½

S2.½

The figure of the 11. Chap. ſerueth for this place.

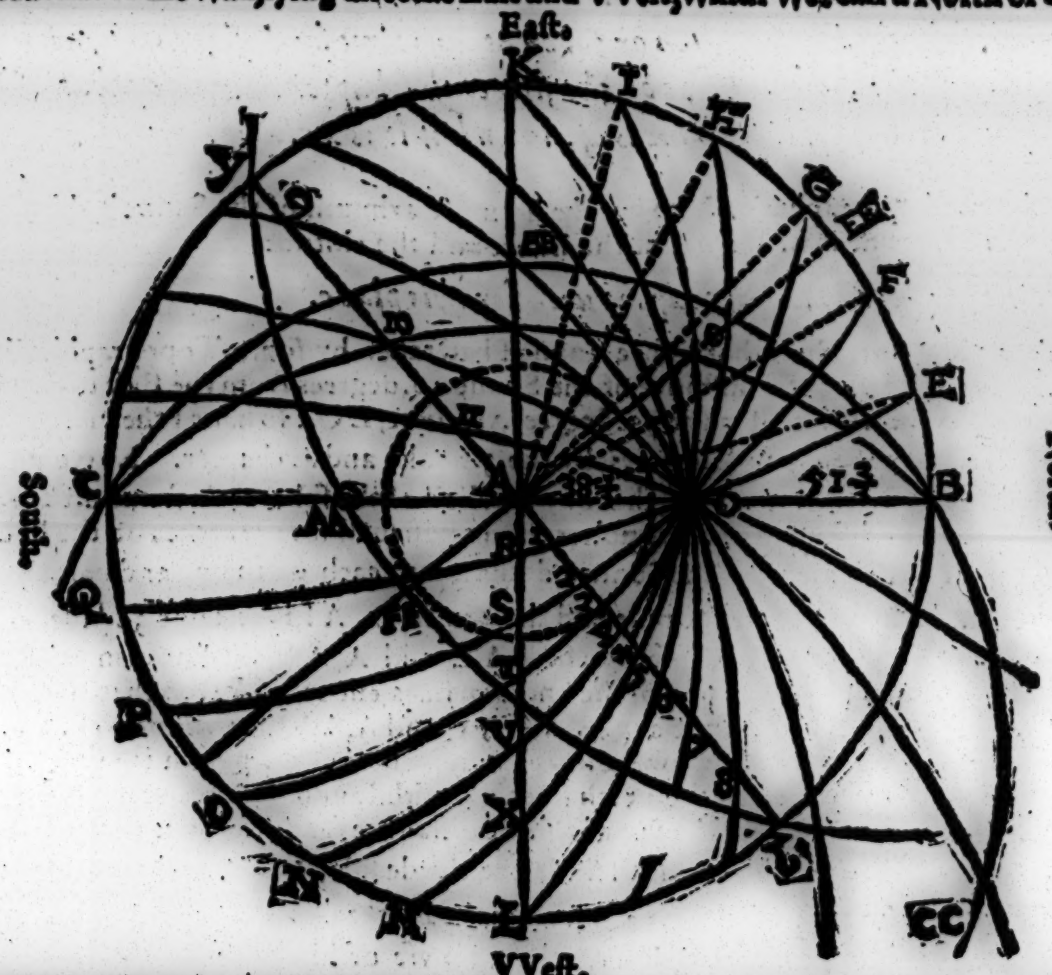
But admit, that one of the ſides A B, or B C, ſubtending one of the angles A or C, had beene giuen with the ſaide angles A and C: as for example, the ſide B C, ſubtending the angle A. Heere reckon the one angle giuen, vz. 58½. de. ſubtended by the ſide giuē as before among the meridians, the other, vz. 44½. deg. among the azimuthes, that done, number on the ſaid 44½. azimuth counting from the Zenith, the ſide B C, giuen, vid. 30. deg. and turne the *reet* til the ſame 30. deg. of the ſaid 44½. azimuth do cut the ſaid 58½ meridiane, and there is your triangle made vp as before for you ſhall finde the Zenith too cut in the *limbe* 36. deg. from the pole, as before he did, and the very ſame croſsing as before.

Chapter 18.

Howe by the helpe of the 17. chap. to make any declining diall, that is to ſay, a dial to any wall ſhouldring or bending from the full beholding of the South.

ALl walles are commonly made perpendiculer to the horrizon wheron they ſtand, and point directly to the Zenith. VVherefore of neceſſitie they muſt lie euen in the flat of ſome one of the azimuthes, and therefore euery one, ſome one horrizon in the worlde by the preambles to my 6. booke, where this matter is handled more at large.

But now to the concluſion of this chapter, where as I taught yoū in the 10. chap. ſphearically too make a diall to the North flat of the wall, lying directlie Eaſt and VVeſt, which wee call a North or South wall, vz.



K L, according to the plage that hee beholdeth. Heere in like maner wil I ſhew to make a diall to the North flat of a wall, declining from the former Eaſtwards, howe much ſoeuer, and for ezample, 40. degrees repreſented in this figure by the azimuth Y Z, where by the angle K A Y, ſhalbe 40. degrees, and as much is the angle

angle Z A L, by the 5. Theoreme, further the angle K A Y, vz. 40. taken out of K A C, videlicet, 90. leaueth his complement the angle Y A C, 50. degrees, to the which the angle B A Z, muſt needes bee equall by the ſaid 5. Theoreme. This circle Z A Y, ſince it repreſenteth here an oblique horriſon, therfore the 24. meridians, or rather houre lines iſſuing from the pole D, ſhall cut the ſame, Z A Y, into 24. vnequall partes, where you ſee 1. 2. 3. 4. &c ſeruing for the houres euen in the very like maner, as to the horriſon K A L, they did in the tenth chapter, and how much euery of theſe houre ſpaces containeth: I will nowe alſo teache to finde by helpe of the 17. chap. The firſt houre line from our meridian VVeſtwardes towardes L, videlicet, D Q doth cut this horizon: Z A Y, in the point I. VVherefore the quantitie of the arche, I A, ſhalbe the degrees of the firſt houre from our meridians VVeſtwardes, which in this diall is 11. of clock, as by the 6. booke will more at large appeare. Heere haue you nowe a ſphearicall triangle D A I, whoſe ſide D A, is knowen, as hath often been ſaide to bee $38\frac{1}{3}$. degrees. The angle A D I, beeing the quantitie of one houres ſpace, muſt needes bee 15. deg. the angle D A I, is as before is ſaide, knowen 50. deg. theſe had, videlicet, the angles A, D I and D, A I, with the ſide A D, lying betweene them; you ſhall moſt eaſily by the firſt part of the 17. chapter finde the ſide I A, to bee $12\frac{1}{2}$. degrees, ſo much is the ſpace of the firſt houre VVeſtwardes of the plum line in this diall, then the ſecond houre line, vz. D P, hee cutteth the horriz on Z A Y, at the note 2, there haue you another triangle D A, 2. whoſe angle A D, 2. is knowen to be 30. becauſe it is 2. houres ſpace, & the angle D A, 2. 50. as before with the ſide D A, lying betweene them, vz. $38\frac{1}{3}$. VVherefore by the 17. you eaſilye obtaine the arche D, 2. being the diſtance of the ſeconde houre line of your diall, weſtwards of the plumme line. For the third houre you haue the triangle D A, 3. and ſo cõſequently for all the reſt on that ſide of the meridian B C, where ſtill the ſide A D, and the angle D A Z, are common too them all. But nowe for the houres, one the Eaſt ſide betweene A and Y, the angle B A Y, muſt bee the common angle, whiche is eaſily had by adding the angle K A Y, videlicet, 40. to the B A K, vz. 90. thereof commeth the angle D A Y. 130. deg. ſo that for the firſt houre on the Eaſt ſide you haue the triangle D A 11. whoſe ſide D A, & the angles D A, 11. and A D, 11. are knowen: thereby get the quantitie of the ſide A, 11. for your firſt houres ſpace, & ſo for the reſt.

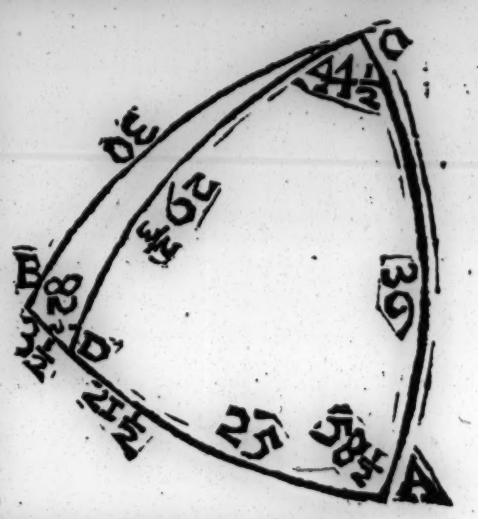
Heere note that as this figure ſheweth the maner of the cutting of the houre circles, to the North ſide or flat of this wall Z A Y, iſſuing from the North pole D. ſo if you would haue made the diall to the South flat of Z A Y, you muſt haue placed the South pole in your figure $38\frac{1}{3}$. deg. Southwardes in the line A C, vid. in A, from whence the houre lines iſſuyng will cut the South flat of Z A Y, in his right forme: but truly into the very ſelfe ſame diuiſions as before: yet in contrary quarters, for then the diuiſions, which are now in the quarter A Z, will be there in the quarter A Y, *& è conuerſo*: then alſo is the point A, no more the zenith but his Nadir. and A A A, the arch betwcen the Nadir, and the South pole. But what needs all this, when as one mould turned vpſide down, or in & out, will ſerue to diall both the flats, and two more of their couſins as in my 6. booke and 20. chap. is ſhewed, in which booke very many more things neceſſary to dials are ſhewed which to bring in here, wold bring me out of my ſpherical triangles, & that muſt not be. But thus much aſſure your ſelf, you hauing read & well vnderſtood my 6. booke of diallyng, you ſhall euer after by the preccpts of this, the 10. & 22. chap. make dials to any wall or flat whatſoeuer, for thoſe three include the whole art. Much more, and many wayes I could write, but for breuities ſake, which I wholy intende, theſe ſhal ſuffice.

Chapter 19.

Ioh. Regio. Lib. 4. Cap. 8. & 30.

How to let downe a perpendiculer arche from any angle of a knowen ſphearicall triangle vnto his baſe or ſubtending ſide, and to know the quantitie of the ſame perpendiculer, and of the parts whereinto hee deuideth the baſe.

LET there be propoſed the triangle A B C, of the 17. chapter, of knowen ſides and angles, from whoſe angle C, I woulde let fall a perpendiculer arch to his baſe, or ſubtending ſide A B. Firſt you know that euery perpendiculer arche muſt haue his originall from on of the poles of the ſide or circle, whereon it muſt light by the 3. diff. wherefore you ſhal alwayes place that ſide. vz. A B. 25. degrees on the *Limbe*, betweene the pole and the zenith as you are wont, then is the center of your *Iewell* the pole or poles of the circle, wherein the ſame ſide A B, conſiſteth by the 2. diff. Therefore make vp the other two ſides of your triangle as you are accuſtomed, either as in the eleuenth chapter, by the croſsing of the almicantare and parallel, equall to the other two ſides, or els of the meridian and azimuth, equall too the other two angles, as in the 17. at whiche croſsing, the angle C, muſt needs be included. VVell, then vpon the croſsing of the $82\frac{1}{2}$. azimuth, and the $58\frac{1}{2}$. meridian (if you worke by the 17.) or of the ſaide almicantare and parallel: if by the 11. laie the fiduciall or center line of the *labell*, which ſhall there deuide the 25. lying on the *limbe* betweeu the pole & ẏ zenith into $3\frac{1}{2}$. degrees, vid. the arche B D, and $21\frac{1}{2}$. degrees, the arch D A, the partes of the baſe ſought for. And the degrees of the *labell* betweene this croſsing, and the *Limbe* ſhall bee $29\frac{2}{3}$. degrees, the perpendiculer ſought for, vz. C D.



Chapter

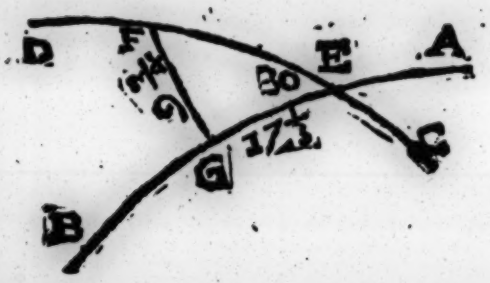
Chapter 20.

To doe the ſame after another manner of working, yea though one of the other two angles, and one of the other two ſides, of the triangle were vnknowen.

ADmit, that frō the angle C, of the triangle A B C of this laſt chapter, I woulde know the perpendiculer, and the partes he diuideth the baſe into, by another maner of working: to doe this, you ſhall reckon one of the other two angles B or A. For example A. 58½. deg. on the *limbe* from the Equinoctiall vpwardes, thereto lay the rule fixed, whereon you ſhall from the center reckon the ſide, which with the baſe includeth this angle A, vz. A C, 36. deg. & there ſhall croſſe you the 21½. meridian counted from the axtree, ſhewing on the Equinoctiall line 21½. degrees, the portion of the baſe cut of the next angle, A, videlicet. A D: and the portion of the ſame 21½. meridian betweene the croſsing of the rule and the Fquinoctial, you ſhall finde, 29¾. deg. being the perpendiculer C D. The other portion of the baſe, is had by ſubducting this 21½. out of the whole baſe giuen, videlicet, A B, 25. deg. and all this is done without knowing the angle B, or the ſide B C. This working differeth nothing from the 9 chapter, if you mark it: but only it bringeth you acquainted with perpendiculers: you might alſo haue done it in this maner by the 19. chapter, or partly by the 7. the skill is not great, ſo you vnderſtand well where about you goe.

Chapter 21.

If two great circles of the ſpheare croſſe one another, making a knowen angle howe to get the perpendiculer arche, falling from any poynt aſsigned of the one circle vnto the circumference of the other, together with the arche of the ſame circle, betweene the perpendiculer, and the croſsing.



LET there bee propoſed the two great circles A B, and C D, croſsing one another at E, making the knowen angle, D E B, 30. deg. Admit that from the point F, of the circle C D, beeing 20. deg. diſtaunt from the croſsing E, I woulde learne the perpendiculer arche falling on A B, videlicet, F G, together with the ſpace E G. This matter differeth nothing from the 20. cha. For if on the rule layed at the *Limbe*, to 30. deg. being the angle at the croſsing E, you count the diſtance E F. vid. 20. deg. frō the center, ther ſhal meet you the 17½. meridian whoſe portion frō the rule to the Equinoct. you ſhall finde 9¼. deg. your perpendiculer F G, and the ſaide 17½. is the arche E G. Or thus after the 19. chap. if on the 20. deg. of the 30. meridian counted from the pole, you lay the rule, his part there cut of next the *limb*, you ſhall finde 9¼. and on the *limbe* betweene the rule and the pole 17½. as before.

Chapter 22.

How by helpe of the 21. chapter to knowe both the height of the cocke, to any declining dial, and how much he muſt be placed byas from the perpendiculer which I call his deflection.

THeſe rules are general in all vpright walles, the plum line is our meridian, the line where the cock ſtandeth, is the meridian line of the horriſon repreſented by that wall. The meridian circle of euery horriſon, ſtandeth perpendiculer thereon, by the 1. booke, & 4. chap. Laſtly the eleuation of the pole aboue any horriſon, which is alwayes the height of the cocke, is the arche of the meridian, betweene the pole and the ſame horriſon, which arch muſt needes be perpendiculer to the horriſon, aſwell as the whole meridian is. Then to conclude in the figure of the 18. chap. you haue the two great circles B A C, and Z A Y, croſsing at A, making the knowen angle B A Z. 50. deg. then if from the point or pole D. of the circle B A C. being as in the 18. chap. is ſhewed 38½. deg. from the croſsing at A, you can deuiſe to know the perpendiculer arch falling to the horriſon Z A Y, vz. D, +, which by ꝑ 21 you ſhall find 28. de. the ſame is ꝑ height of the cock to that diall, and the ſpace + a, is the quantitie of the angle of deflection, vz. 27. by the 21. chap. It were great folly for me to ſtand longer of this matter, ſince the ſelfe ſame is done in the 6. booke, 16. 17. & 18. chapters, meaning here but a little to touch the vſe of the ſphearicall triangles for the encouragement of the young Reader.

Chapter 23.

In a right angled ſphearicall triangle where one of the angles ſhal chaunce too bee but certaine min. as it often hapneth, how to get the ſide ſubtending that angle preſiſely notwithſtāding.

FOr example, admit that of the right angled triangle A B C, the angle A, were but one deg. 10. minutes, & the ſide A B, ſubtending the right angle C were knowen to be 15. degrees, you might now by the 9.

chap.

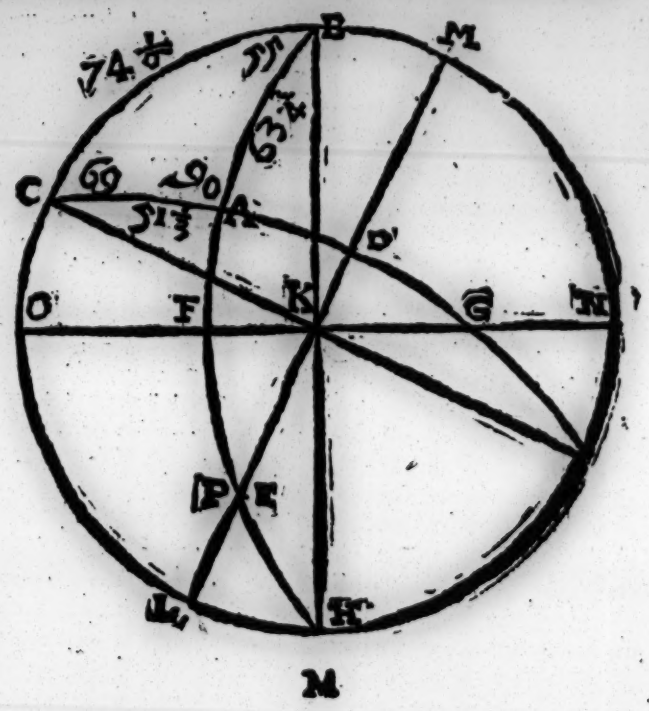
chapter, get the side B C; yet without any exact certaintie, because the angle A is so little, and that the side B C, wyll fall out to bee but certaine minutes, whiche except your *Iewell* be monstrous cannot be discerned any thing neere. To helpe this you shall insteed of one degree tenne minutes, whiche are 70. minutes in the whole, reckon on the *Limbe* of your *Iewel* from the Equinoctiall vpwardes 7. degrees, and thereto lay the rule as in the nienth chapter, for so doth euery degree of the *limb* represent 10. minutes. In this case then, as in the 9. reckon on the rule from the center, the side A B, vz. 15. degrees, and there shall you finde the $14\frac{6}{7}$. meridian to cut, whose degrees betweene the crosing of the rule, and the Equinoctiall are $2\frac{1}{5}$. degrees, the side B E, after the 9. chapter. But here you must allowe each degree, but 10. minutes, as you did at the *limbe*, so that this $2\frac{1}{5}$. deg. represent but 22. minutes, so much is the side B C. And in this working you must in no wise exceede 10 or 15. deg. in the *Limbe* at the most, yea and that where the angle proposed exceedeth not two degrees, for in these small or minute angles, as I may terme them, there is kept a Geometricall proportion betwene the parts of the triangle, whiche in greater, the nature of the spheare alloweth not: neither yet in these, but that the error is vnsensible, this helpe did Gemma Frisius inuent, as in my 3. booke 56. chapter appeareth, from whom I woulde be loth to derogate: but in my next edition I will supply it farre better, & more exact, by another deuise euen vnto 30. degrees, which heere you can scant extend to three degrees.

Ioh. Regio. lib. 4. ca. 26. Coperni. 5. triang. sphe.

Chapter 24.

If the three angles of a right angled sphearicall triangle shall bee giuen, howe to get the quantitie of the three sides vnknowen.

THis proposition had I once half giuē ouer as vnpossible on my *Iewell*, which made me greatly to mislike him: but in respect hee made mee amendes in performing another that Regiomont. left impossible, as by the fifteen chapter appeareth, I thought it a lesse fault, yet still wheresoeuer I went or sate alone, I coulde not choose but be deuising vppon it: and at last while I was abroad, in my eueninges walke, whiche I commonly vsed, it came into my head on the sodaine, but then was it no neede too bidde me trudge a pace home to my *Iewel* to see it in experience. All the skill thereof is too make the right angle too lye among the crosinges of the meridianes and azimuthes within the *limbe* in maner as the one angle of all triangles, not right angled doth, so that when the constitution of the triangle is founde on the *Iewell* his side, which lyeth on the *Limbe* betweene the Zenith, and the pole may be the side subtending the right angle giuen: which you shall thus do. Let there be proposed the right angled sphearicall triangle A B C, whose three angles are giuen vz. C. 69. degrees, B 55. and A 90. I reckon one of the angles not right, vz. 69. among the azimuthes of my *Iewell*, and the other vz. 55. among the meridianes alwayes counted from the *limbe*: this done, all the misterie of this matter is, how in turning about the *Reete* to know when this 69. azimuth, and 55. meridian doe crosse another square, which can neuer be by the 3. def & 3. theoreme except they cut one anothers poles, which poles I may easily know where to finde by the 12. theoreme. By which I say that the poles of euerie meridian of my *Iewell* are to be found in the Equinoctiall line, and not els where. For as it is apparaunt by the third def. and the third theoreme, that the pole or poles of the vttermost meridian, beeing the innermost circle of the *limbe*, lyeth in the Equinoctiall line, in the center of the *Iewel*: So looke how muche euery other meridian lyeth within the *limbe* on the one hand, euen as much doth one of his poles lye distaunte from the center on the other hande. As for example, the pole of the fifth meridian, next the *limbe* on the South part, is the 5. degree of the Equinoctial line from the center towardes the North parte, and so of all the rest, as in my fourth book and fourth chapter doth also appeare, yet thus much more plainely chose out any meridian whose pole you require, then reckon from him on the Equinoctiall line towardes the center, and beyonde till you come to 90. degrees, and there shall bee his pole: as for example, if from the fift meridian on the South part next the *Limbe*, you count on the Equinoctiall Northwardes 90. degrees, the same shal end at 5. de. beyond ♄ center, this matter now I hope is plain enough for you to gather that in the very like maner the poles of all the azimuthes are to bee founde in the *Finitor*, and not els where, wherefore to goe forwards with our triangle you shall by the rules heere last mentioned, reckon on the *Finitor* from the 69. Azimuth, whiche as before was appointed too containe the angle C, tyl you come too 90. degrees, and there shall bee his pole: wherfore if you turn about the *reete* tyll the same 90. degree of the *Finitor* doe touche the 55. meridian appointed before to contain the angle B, then whereseoeuer you shall finde the same 69. azimuth, and 55. meridian to crosse, there must needes be foure right angles by the thirde theoreme, and therefore no



no doubt but at the same crosing is made vp your triangle desired in his right constitution after the wonted maner, as in very deede it is: for from this crosing to the Zenith point of the *Reete*, you shall find on the 69. azimuth $51\frac{1}{8}$. deg. for the side A C, and thence to the pole on the 55. meridian $63\frac{1}{4}$. deg. the side A B, & on the *limb* between the pole and the Zenith $74\frac{1}{2}$. degrees, the side B C, subtending the right angle A. And now that you haue all, doe, if you list to followe the 69. azimuth vnto the Equinoctiall line, hee wyll there cutte the 55. deg counted from the center being in truth the pole of the saide 55. meridian by the forsaid computation, which is a great confirmation of this matter, els coulde not there haue been a right angle at the said crosing by the third theoreme, if each did not cut others poles. Thus haue you a most pleasaunt and profitable handling of the chapter, yeelding great light to many questions about the same sphear, which I thought good, the better to manifest by this figure.

B C H, the *limbe* of your *Iewell*, K the center.
B, the North pole of the *Iewel*: N O, the Equinoctiall line.
C, the Zenith of the *reete* placed, M L, the *Finitor*.
C D, the 69. azimuth.
E, the pole of the 69. azimuth, K E, being 69. degrees, or D E, 90.
B F, the 55. meridian, G, his pole, G K, beeing 55. degrees equall to F O, or F G, being 90.
P, the point of the 55. meridian, where E, the pole of the 69. azimuth doth touch in turning the *reete*.
A, the point of their crosing making vp the triangle A B C. desired.

In briefe, of the two angles not right, reckon the one amongest the azimuthes, the other among the meridianes both counted from the *Limbe*, then turne about the *reete* vntyll the 90. degree of the *Finitor* counted towardes the center, and beyonde from the same azimuth doe touch the same meridian, and then whersoeuer you finde that azimuth and meridian, to crosse, there is your triangle made vp in the wonted maner.

Chapter 25.

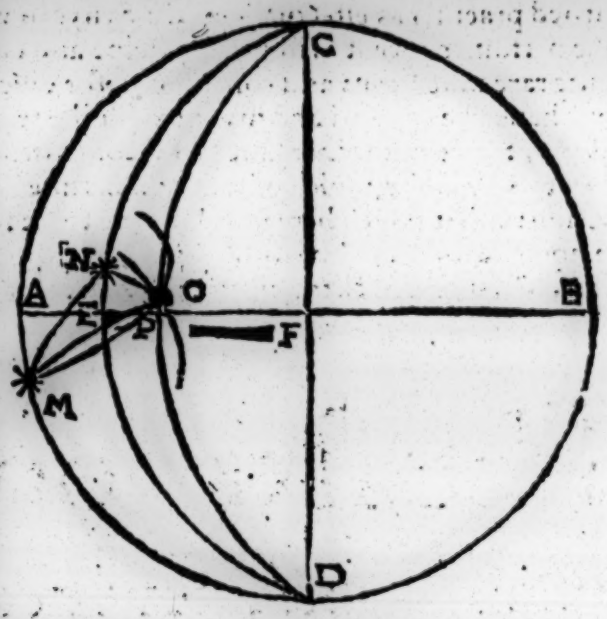
If by the crosse staffe or other Instrument, you shall take the true distance of any planet, comet, or starre vnknowen from two knowen starres, how by helpe of this my woorke of sphericall triangles, to get the longitude and latitude in the zodiacke of the same planet, comet, or starre, and also his declination and right ascension: and what you had need to do in al spheарical questions.

I Haue hytherto for the pleasure & delight of the learner, set downe diuers Astronomicall questions, and conclusions to be resolued by these triangles in sundrie chapters before: least otherwise, if I should haue giuen the bare preceptes, as Regiomontanus and others haue done, this work beyng of all the rest most profitble might happily haue been neglected, & thought on the first blush of small account: knowing that nothing draweth on mens minds more to any studie thē that they may at the first entrance perceiue their rules to serue to good purpose. And as I haue hitherto touched a fewe sleight questions: so might I haue witten a number, and therewith filled great volumes, the matter beeing in manner endles, were it not that I both hast the end of my trauaile too too long alreadie looked for of diuers of my familiars, and presume that by those, any man may performe all other that be of no greater waight. But now there are in like maner an infinite number of intricate questions more harder far then any yet propounded, which by Regiomont. Copernicus, and others doctrine, grow to great toile with their Synes, calculations, & proportions, wherin first they hunt about for one Syne, which they call *inuentum primum*, then for another, which they call *inuentum secundum*, & cōmonly *inuentum tertium*, & perhaps *quartum*, & so foorth. After all these are found, then they multiplie & deuide them, & compare their proportions: & when al is done, that they haue founde the Syne sought for: they yet are faine to goe to their tables for the arth correspondent. Those *inuenta* are not ouer easilye done by my *Iewel*: but yet an hundred fold easier then as they performe them: notwithstanding you muste haue all your wits about you, & after a sort seek a *primum, secundum, & tertium inuentum*, as they do, though with no trouble at all: when you haue once the conceite of it in your head. For in all these questions, it is more hard to conceiue how they should be resolued then to resolue them. Wherefore the best way is for any man that hath not well setled the circumstance of the spheare in his head: first by the helpe of his rule & compasse, or els grosly by ame to drawe an Idea, or shape of such circles as hee shall haue to deale withall in his purpose, as also before in my other slight questions I haue done: which done, hee shal by inspection therof very readily spie out which angles or sides be knowen, & thereby get the rest, or els see how to adde, inuent, or cast out other circles, arches, & triangles, which may help him to his intention, euen as the lande meater doth commonly resolue a cragged deformed ground into sundry squares & triangles, too compasse the contentes thereof, which to do, I hope by this one example following you will be sufficiently instructed for all.

The 14. day of October 1582. about 6. a clock in the morning, & 30. min. past, seeing the morning cleer & ♂, ♃, & the Moone all 3. aboue our horrizon, according to my accustomed maner, I could not choose but haue a cast at them for my recreation, wherefore with a large crosse staffe that I had made after my fashion, as in my next edition I will shew, I tooke the distance between Mars & the great Star in the right shoulder of Orion, & found it 31. deg. $37\frac{1}{2}$. min. Likewise the same instant I took the distance between ♂, & ♄ Northermost of the 2. stars of ♊, called Apollo, & found that 10. de. 25. mi. ♂ being orient. of thē both. Hereof I thē made a note in my book where at this time I mean it shal serue for an exāple to this chap. and by this distance of ♂, taken from those two knowen starres, I will shewe you by sphearicall triangles to get in what longitude, latitude, declination, and right ascēs. hee then was, by a knowen starre, I meane a star whose longitude and latitude is knowen eyther by tables or your Astrolabe. But now the better to conceaue howe to

bring

bring this matter to paſſe, my beſt way is firſt to make a ſlight pattern of the thing I pretende in this maner: I make a circle at aduenture, vz. A B, C D, whoſe diametre A B, ſhall repreſent the Ecliptike, the point C, beeing his North pole, and D the South: this done, I goe to Stadius Tables, and there I finde the longitude of the ſaid ſtar of Orion, 23.deg.0.min.in ♊, his latitude, 17.deg.0.min. South of the Ecliptick, and the longitude of Apollo 14.deg. 20.min.in ♍, his latitude 9.degrees 40.min. Northwards: this knowen, I appoint the arche C A D, to bee the circle of Orions longitude: that is to ſay, the great circle of the ſpheare, cutting the 23 0.of ♊ iſſuing from the Eclipticke poles, then taking 23.0 of ♊ out of 14.20.in ♍. there reſteth 21,deg. 20. min. for whiche I ame out a portion of the Eclipticke A B, containing by eſtimation $21\frac{1}{3}$.ſuch partes as A F, containeth 90.vz.A E.and then opening my cõpaſſe I draw the arch or ſemicircle C E D, which is the circle of Apolloes longitude, vz.of the 14. 20 of ♍. After this from A towardes D, I ame 17. ſuche partes as A E, containethe 21 $\frac{1}{3}$, or A F, 90, and there I ſet M, or make a ſtarre, for there is the place of Apollo, according to his longitude and latitude, then in the arch C E D, frõ E towards the North pole D.I ſet by ame $9\frac{2}{3}$.of ſuch like partes being the latitude of Orion, vz. in N: and ſo are theſe knowen ſtarres placed: which done, I open my compaſſe as neere as I can geſſe to 31.degrees $37\frac{1}{2}$.min.of ſuch like parts, vz.the diſtance taken by the croſſe ſtaffe betweene Orion and ♂, and ſet the one foote in Orion, vz. M and with the other make a blinde arche cutting the Eclipticke A B, towardes the center: that is to ſay, with the ſequele of the ſignes, for that ♂ was orientall. Againe I take $10\frac{51}{60}$.lyke partes by geſſe, alſo in my cõpaſſe: that is to ſay, the diſtance taken betwene Apollo & ♂, & ſetting the one foot in Apollo, vz.N, with the other I make a little blinde arche croſsing the firſt towards the center F alſo, becauſe ♂ was orientall of them both, vz. in D, where muſt needes haue been the place of ♂, if my figure had beene truly made which for this purpoſe needeth not but euen ſlight for a ſhew. Then with my compaſſe at the width of A F at the leaſt, I draw the arches M N, M O, and N O, but with no great curioſitie: laſtly I drawe the arche C O D, which is the circle of the longitude of ♂, and then is my platforme laid, whereupon I deſcant in this manner.



Firſt to begin with C A, the ſame is 90.de. and A M, 17.the latitude of Orion: therefore C A M is 107. alſo C E is 90.degrees by the ſecond def. and N E, but 9.degrees 40.min, the latitude of Apollo: therefore taking N E, out of C E, there reſteth C N, 79.deg.20.minutes. Alſo the angle M C N, being the difference of Orion and Apolloes longitude is knowen to bee 21.degrees 20.minutes by the 4.def. Therefore nowe hauing two ſides of the ſphearicall triangle M N C, vz. M C, 107. and N C, $79.\frac{20}{60}$.encluding the knowen angle M C N, $21\frac{20}{60}$.degrees, you ſhal eaſily by the 13. chap. find both the 3.ſide M N, to be $34\frac{2}{3}$.deg. and alſo the angle C M N, to be $39\frac{1}{3}$. Then haue you the triangle M N O, of all three ſides knowen. vz. M N. $34\frac{2}{3}$. N O, $10\frac{51}{60}$. and O M 31.degrees. $37\frac{1}{2}$.min. VVherefore by the the 11. chapter you ſhall eaſily finde the angle N M O to be $17\frac{1}{2}$.deg. which added to the angle C M N, vz. $39\frac{1}{3}$.maketh 57. almoſt the quantitie of the angle C M O, and yet you will ſay, what am I the neere for all this. Soft a while, not too faſt for falling: nowe are you paſt your *Inuentum primum & ſecundum*, which in Synicall workyng, perhaps would haue myred you twiſe ere this, & yet you ſcant half way ouer. VVel then to the *inuentum tertium*, you haue a third triangle vz. C M O, whoſe two ſides C M, 107. and M O, 31.deg. $37\frac{1}{2}$.min. are knowen, containing the knowen angle C M O, 57.degrees: wherefore by the 13.chapter you ſhall eaſily finde the ſide C O, $88\frac{3}{4}$. degrees, which take out of C P, vz. 90. leueth O P. $1\frac{1}{4}$.degrees, the latitude of ♂. You ſhall likewiſe by the 13.find the quantitie of the angle M C O, 26.degrees, which is the difference of longitude of ♂ from the ſaid great ſtar of Orion, and in reckoning forwardes according to the ſequele of the ſignes; that is ſay, from the 23. de. of ♊ being the longitude of Orion 26.degrees forwardes on the Eclipticke, falleth out too bee the 19, degrees of ♍, the longitude of ♂ deſired: and ſo is this buſie matter at the laſt brought too good paſſe by the ſinguler vſe & help of ſphearicall triangles, & thus ſhal you be able to performe any other matter of the ſphere whatſoeuer, for you ſhall not lightly finde any queſtion ſo harde, but if you once drawe your platfourme you ſhall deuiſe how to adde ſome arches or angles, yeelding ſufficient helpe too accompliſh your pretence, and many times after you are a little exerciſed, you ſhall ſet your platfourme more expreſly by the lineamẽts of your *Iewel*, then with your pen, *Vſus promptos facit, fœmina ludificantur viros*: but tarry, I had almoſt forgot the declination and right aſcenſion of ♂, for which you had neede make another platfourme, as heere you ſee A B C D, is the great circle or colure cutting the ecliptikes poles G H, and the poles of the worlde A B, C D: the Equinoctiall K L, the ecliptick R S, the circle of Mars his latitude, and G O PH, of his longitude: G A. is the diſtãce of the clip. poles frõ ♈ poles of ♈ world vz. $23\frac{1}{2}$.deg. A O ♈ the circle of ♂, right aſcenſion: T O of his declination. Now haue you the ſpherical triangle G A O, whoſe ſide G A, $23\frac{1}{2}$. and G O, $79\frac{1}{3}$.are knowen containing the angle A G O, being the difference of longitude of ♂ from the beginning of ♍, vz. 19. deg. wherefore by the 13. you may eaſily get the ſide A O, and the angle G A O, which had, take A O, out of A X, being 90. leaueth O X, the declination of ♂, then take the angle G A O. out of C A D, vz. 180. it leaueth the angle D A X, the difference of the right aſcenſion between ♂, and 0. in ♍, ſo that the right aſcenſion of ♍, being alwayes 90. of Mars, it was more at that time by the arch D X,

The concluſion of this worke of triangles.

I Am not ignorant that the Synycal working is moſt exact, & that this maner of working triãgles, though on a *Iewel* of two or three foot diametre made of purpoſe is to final effect, for the calculations of the motions of ſtarres, planets, comets, and ſuch like, *Phainomena* of parallaxe, or any other Aſtronomical or coſmographicall operations, requiring exactneſſe vnto minutes, ſeconds, &c. Yet by a great quadrant mixte of the lineaments of Gemma Friſius, and I. Roijas aſtrolabes, which I haue alreadie contriued with his furniture, and meane in my next edition to publiſh, I will ſerue that turne euen vnto minutes and ſecondes, very ſenſiblie and in ſmall angles almoſt vnto tenthes, if neede bee. VVhiche as I doubt not but it ſhall ſerue the vnlearned practiſer as effectually, as the Synycall woorking doth to the learned, ſo I hope by meanes of the eaſie perfourmance, it will greatly helpe and encourage many willing and forwarde wits hereafter to proſecute rare inuentions and concluſions, for diuers Aſtronomicall and Coſmographicall matters, who before tyme haue been ſtalled and myred with the tedious ſcience of Synes, and their appurtenances: beyng indeed as I may terme it, another kinde of Geometrie, and a darke and intricate art. In the meane ſeaſon let no man reiect this 5. booke, which of it ſelfe is highlie to be accounted of: as well in that it is a moſt needfull introduction to that which hereafter I meane to ſet foorth. As alſo for that all the concluſions which I haue written, and hereafter ſhall write of this my *Iewel*, may be wholy perfourmed by it: ſo that this firſt booke might be an abbreuiarie to them all after they are thorowly learned and graft in minde.

The ſixt book of the Mathematicall *Iewel*, newly compiled, deuiſed, and ſet foorth *by* Iohn Blagraue *of Reading, Gentleman: ſhewing the* Theoricall reaſon and grounde of all Dialles and Horizons, and the practicall making of all ſortes of horizontall, murall, declining, reclining, and inclining *Dials, by the onely helpe of this Mathematicall* Iewell, *and that moſt plainely*, readilie, and exactly, as hereafter ſhall followe: which worke, beſides the art of Diall making, bringeth a man acquainted with the full vnderſtanding of the ſpheare, and is the dore into the ſame.

Chapter. I.

Of certaine preambles to bee noted in vnderſtanding this worke following.

HOw profitable, how pleaſant, & how neceſſary dials are, there is none ſo ſimple a ſoule but ſeeth it, neither any I thinke ſo ſottiſh to ſay againſt it, ſince no order could be kept in things, if diſtinctiõ of times were not. For what is the cõſeruation, grace, & vpholding of all good ordinances, but euery thing to be done in his due houres. The trauailer by land is not a little cõforted, whẽ he may with a ſmall toy ſee how the day goeth away before him: and alſo may ſee to what place he may reach before nyght ouertake him. The Seamã without dials, or ſuch like inſtrumẽts to know their houres and times, coulde not trauaile at all. Chirſte himſelfe made often mention of houres and times. Therefore ſince their commodities are ſo manifeſt, no doubt but the inuentours were much to be accounted of, and the inuention highly to bee eſteemed. Some will ſay what then? VVhy ſhould wee nowe bee troubled with more bookes thereof, ſince ſo many are alreadie extant, a man may haue too much of his mothers bleſſing. But yet ſir, if you will follow mee, I may happily leade you a pleaſant path, where others are ſtumbled with many a ſtile, and were it not that my worke doth for eaſe ſpeede, and plaineſſe excell all the authours that euer wrote heereof, yet wonlde I not leaue it out, becauſe it wonderfully ſetteth foorth the ſinguler vertue and efficacie of my *Iewell*, which as I haue often ſayd, & manifeſtly ſhewed, will needes ſupply all vſes of Aſtronomie. For as hee hath in the laſt booke made ſmooth all the rockie intricacie of ſphearicall triangles: ſo heere with no leſſe ſingularitie he expreſſeth the making of thoſe Dials, which all other Authours haue left lyke a Labyrinth to the learner, as if you reade Munſter Orõtius and others, and eſpecially Andreas Sconerus, of declining or reclining Dials, you will confeſſe. VVherefore I doubt no but this woorke of myne ſhall bee both acceptable too the beſt, & longed for of the reſt, of which I am too prepoſe theſe preambles following.

1 The whole Globe of the earth in compariſon to the circuit of the ſpeere, wherein the ſunne is carried bea-

eareth a very ſmall bigneſſe, but a centre pricke to ſpeake of, and in reſpect of the other heigher, as the 8. 9 and 10. ſpeeres, it carieth no acount at all. For which cauſe all Dials are made, euen as though we dwelt in the centre of the earth. And betweene a flat heere on the circuit of the earth, and a flat cutting the centre of the earth, ſo they be parallel, there is no difference in this behalfe.

2 All plaine flats whatſoeuer and howſoeuer they decline, recline, or incline, doe repreſent ſome one horizon in the world.

3 Euerie Horizon whatſoeuer, repreſenteth a great circle of the ſphere by his definition.

4 Euerie great circle hath his two poles, euen as the equinoctiall hath by the 2. definition.

5 The poles of any horizon are his zenith, and his Nadir.

6 But to auoyd confuſion of vnderſtanding henceforth in this worke of Dyals, when I ſhall ſpeake of the pole or poles, I will meane onely the pole or poles of the world.

7 And when I ſpeake of the Zenith or Nadir, I will meane it ſimplie for the Horizon of the place whereof I ſpeake, as the common order of ſpeech is.

8 And for that the poles of any Horizon repreſented by any wall, bancke or flatte whatſoeuer, are in truth the Zenith and Nadir of the ſame horizon, therefore I will call the vppermoſt the pole Zenith, and the vndermoſt the pole Nadir of any ſuch horizon, wall, bancke or flatte.

9 Euery Flat whatſoeuer imagined hath two faces, as I may call them, on ech ſide one, as in a Counter, (which though it be a ſolide and ſorie *ſimile*) the one face is croſſe, the other pile.

10 In euery great circle there is a flat to be imagined of two faces, or two flats I might ſay, reſpecting ſeuerall parts of the heauens: as for example, If you imagine a flat in the equinoctiall circle, the ſame ſhall haue two faces, the one reſpecting the North Pole, the other the South Pole.

11 You ſhall finde no great circle of the Sphere howſoeuer imagined, but that looke how much the north pole is eleuated aboue his one flat, ſo much is the ſouth pole eleuated aboue his other flat, or rather depreſſed, to ſpeake it more aptly to my purpoſe, and ſo contrariwiſe, which on your *Iewel* you may moſt euidently ſee. For placing the *Finitor* to our latitude vz. $51\frac{2}{3}$ degrees fixed vnder the north pole, you ſhall ſtreight ſee the ſouth pole depreſſed $51\frac{2}{3}$ deg. vnder the *Finitor*, which is the eleuation of the ſouth pole to thoſe people which be Antypodes to vs, an ſo forth in any other latitude.

12 Sythens euery flat whatſoeuer repreſenteth ſome one horizon in the worlde, and euerie horizon ſome one great circle of the ſphere, as is before ſaide: therefore euerie flat not being a right horizon, lieth in ſome one great circle, aboue which one of the poles is eleuated, hauing alſo an other Antypodall flatte imagined on the backſide, vnto which the other pole is as much depreſſed, or to ſpeake it Antypodally I may ſay eleuated: for you muſt note that euery right horizon hath neither pole eleuated as ſhalbe ſhewed.

13 In euerie latitude you ſhall finde both the poles to be eleuated aboue the flats of the Verticall circle, or Eaſt Azimuth (which repreſenteth alwayes the North and South wals) as much as the latitude lacketh of 90. As for example, ſet the *Finitor* to our latitude videlicet $51\frac{2}{3}$: you ſhall ſee that the zenith line which alwayes repreſenteth the ſaid verticall circle hath the North pole eleuated $38\frac{1}{2}$ deg. aboue his one flatte, and the ſouth pole as much aboue his other flat.

14 Note that when you finde this word (Latitude) ſimplie in this 6. Booke, I meane the Latitude of the place where you remaine, and that to all other walles and flats, I will vſe this word Eleuation, or poles Eleuation for diſtinctions ſake.

15 Alſo I wil vſe this word Horizon ſimplie but to the place of your biding, the reſt I wil cal wals or flats when as I meane the Horizons or great circles repreſented by them.

Chapter 2.

Of the diſtinction on Dials into two kinds, Inſtrumentall and Sphericall.

THe ſhapes, formes, faſhions, and deuiſes for Dials are very many in generall: yet in particular they are but of two ſorts, for either they are made to deſcrie the houre by the altitude of the ſunne, of which ſort are Quadrants, Rings, Circles, Shippes, Cilinders, and ſuch like: and therefore rather to be called inſtruments then Dials: Or elſe they are to ſhew the houre by the ſhade of a cocke, ſtile, or Gnomon without reſpect of ſunnes altitude, of which ſort are your common Horizontall and Murall Dials of all ſorts: your portable Dials with needles: your Equinoctiall, your conuex, concaue, and ſuch like.

Of the firſt ſort the deuiſes are ſo manifolde, which euerie learned Aſtronomer may and hath deuiſed to his fancie, that a great volume would not contane them. VVherefore ſince they be trobleſome and buſie to make, and being made ſeldome ſerue but to one latitude, and withall cannot be vſed but of the learned ſort: I do not meane to entreate any whit of them, but will ſet ouer thoſe that are deſirous of ſuch fancies vnto Munſter, & Orontius *de horologiis*: to Apian, who hath made a buſie Horoſcope: to Sconer, not inferiour to any of them: to Stophler, and ſuch ancient writers, where they ſhall ſee deuiſe ynough to werie them. And the rather I omitte theſe inſtrumentall Dials, becauſe all their vſes are farre more redd ly performed by my *Iewel*: Yet knowne that the vnderſtanding of my *Iewel*, ſhall cauſe any man almoſt at the firſt inſpection to make any of them: for the onely and chiefeſt matter in their fabrication dependeth on the knowing of the Sunnes true altitude for euery houre through the yeare: which by my 3. Booke 31. Chap. is had for the looking on. And ſo I leaue all Dials of that ſort, to them that like better of them then euer I did. For I neuer loued to carie a Diall about me, whereon I muſt bee driuen to hunt after the houre when I had neede of him: which minde of mine, was the onely cauſe that made me to deuiſe the furniture of the *Iewel* as you ſee: For that as I haue often ſaid by Gemma Friſius, Regula, Curſor, and Brachiolum, I could get nothing without hunting after it, and ſomtimes the Curſor led me ſuch a courſe that I haue beene faine to giue ouer the chaſe, or not fully purſued it to the very point. Thus much of inſtrumentall Dials. The ſecond ſort which are more aptlie called Dials, I meane at large to proſecute in this volume, becauſe they depende altogither *Ex protractione Spherica*, as

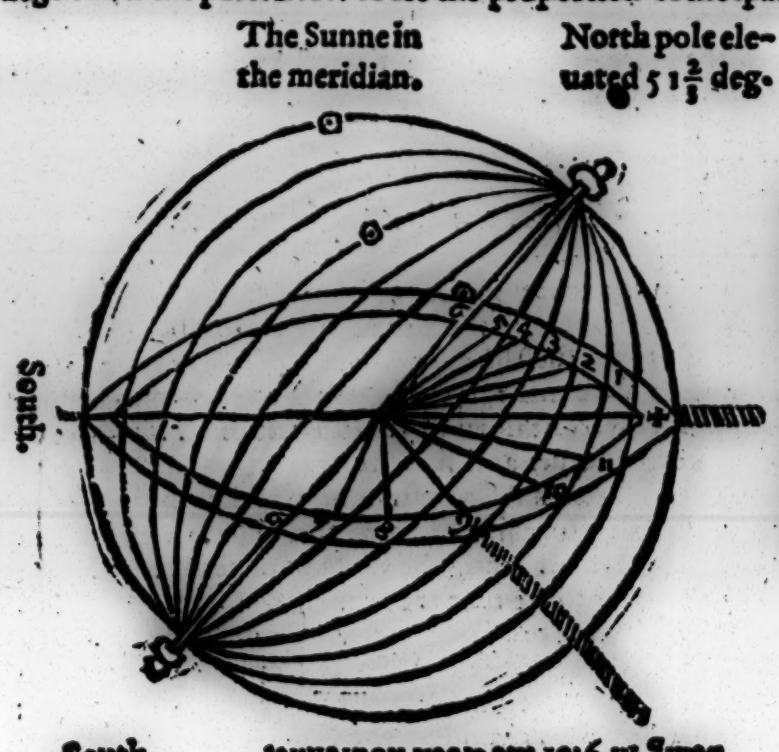
vses, as in the next Chapter appeareth, and make not a little to the liuely vnderſtanding and wonderful vſe of my *Iewel*. In latin they are called *Solaria* or *Inſtrumenta Scioterica*, alſo *Horologia* of the greeke word.

Chapter 3.

Of the definition and derivation of Dyals and the grounde of them all.

A Dyall in verie trueth, might be called a plaine ſphere: but onely that it wanteth parallels: for the lines drawne thereon to diſtinguiſh the houres, are the verie meridians or hourelines of the ſphere proſpectiuely proiected on a plaine *Superficies*, by the Sunnes acceſſe vnto them: as here following I wil as much as I can, make manifeſt, bicauſe the moſt part will hardly beleeue it. But firſt to define an oblique Dyall which chiefely commeth in queſtion more artifically: An oblique Dyall is moſt commonly a rounde *Superficies* or flatte, lying equiballance to ſome one horizon, concentricke to his circumference, from whoſe centre, iſſue out certaine ſtreight lines, vnto the interſections of euerie 15. meridian or houreline of the ſphere with the Horizon: cutting alſo the circuite of the ſame Dyall flatte, into like and proportionable parts vnto the ſaide interſections of the Horizon. For the plainer demonſtration of this definition, take my *Iewel* in your hand and ſet the *Finiter* to the equinoctiall, which repreſenteth an Horizon where one of the poles is Zenith: now if in this horizon you imagine a rounde plate to make a Dyall on: the ſame muſt needs be equiballance, and concentricke, both to the equinoctiall and *Finiter*, becauſe they are in this conſtitution of the ſphere all one circle: and for as much as the parallels grow leſſe and leſſe, euen vnto nothing at the pole, and that the pole is here the zenith, therefore it muſt needes be that ſome one of theſe parallels is equall and equidiſtant, to this round plate for your Dyall, ſo that a perpendiculer line let downe from the zenith or pole, ſhall cut the centres both of the Horizon, parallel, and plate. Then who knoweth not that the equinoctiall, and likewiſe all the parallels are deuided into 360. equall parts by the meridians of the ſphere, of which 360. the 24. part is 15. therefore euery 15. meridian are commonly called hourelines. Theſe hourelines (or rather ſpherically called houre circles) doe nowe deuide the *Finiter* in this conſtitution into 24. equall and euen parts: wherefore if you imagine 24. lines iſſuing from the centre of the *Finiter*, (which is alſo the centre of the Dyall plate, and repreſented here by the centre of the *Iewel*) vnto the 24. interſections of the ſaid houre circles with the *Finiter*: they ſhall deuide the Dyall plate into 24. equall partes, becauſe the interſections in this horizon are equall, euen as the parallel that ſtandeth directly ouer the plate is deuided: ſo that by this definition you may conclude that a Dyall, made to that Horizon, where the pole is zenith, is onely a circle deuided into 24. equall parts which is called an equinoctiall Dyall.

As this demonſtration is moſt manifeſt in the equinoctiall Horizon, where the ſaide 15. meridians cut the Horizon into 24. equall parts: ſo falleth it out plaine ynough in any other horizon, where one of the poles is eleuated aboue the ſame otherwiſe howſoeuer. For euery ſuch Horizon, is alſo deuided by thoſe ſelfeſame 24. meridians, into 24. partes, but vnequall: and by how much leſſe the pole is eleuated, the more vnequall they are. For demonſtration whereof, ſet the *Finiter* of the *Iewel*, vnto our latitude here at Reading, *vz.* $51\frac{2}{3}$ deg. vnder the pole. Now to ſee the proportion of the ſpaces for the hourelines of a Dyall to the ſame Horizon, marke how the 15. prickt meridians or houre circles of the *Iewel*, doe cut the *Finiter*, for if you tell the degrees on the *Finiter*, betweene ech cutting you ſhall haue thereon the degrees ſeruing to the ſeuerall diſtances of the hourelines of your Dyall, ſo that if as before, you ſituate in this horizon, a rounde plate for your Dyall, the ſame ſhall be concentricke thereto, and equall to one of the Almicantares, which grow leſſe and leſſe to nothing at the zenith, euen as the parallels do at the pole, wherefore if from the centre of your plate you imagine 24. lines iſſuing vnto the interſections at the worlds end, as they ſay, of the 24. houre circles with the *Finiter*, they ſhal cut the round plate into 24. like vnequall parts vnto thoſe, made on the horizon or *Finiter* in all reſpects proportionably an Symitrially agreeing, ſo that whither your Dyall plate be great or little, his diuiſions are proportionable.



For better explanation heereof vnto the young learners: admit that you had 24. wyers, of equal length, made iuſt ſemicirclewiſe, and all of them ioyned at two points or poles: theſe 24. wyers being ſet ech of like diſtance from other, would make 12. whole circles: whereof you might appoint any one of them for the meridian circle, then if you did prepare a round plate, or thinne boorde, with a hole in the centre, of equall circuit to one of theſe 12. wyer circles, and did put him within theſe wyers, like a bird in a kage, but ſo that the one pole where theſe wyers meete, might be iuſt $51\frac{2}{3}$ deg. reckoned on the meridian wyer aboue this plate, and the other pole as much vnder: and laſtly thruſt a ſtreight wyer through both the ſame poles, & the centre of the plate,

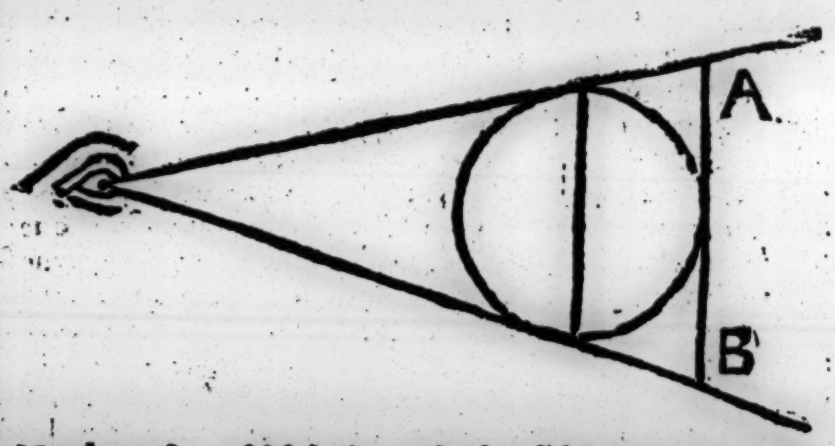
plate, as in this figure appeareth: you haue a pleine ocular demonstration of the grounde and reason of all Dyals, for if now you fasten a threede at the centre of the plate, to the axtree line: or else a ruler, and lay the same from place to place, where the 24. wyers do touch the round plate, you may thereby drawe the houre lines for your Dyall, as many as shall serue your turne, as I haue done in this figure, which hath but 12. houre circles, or wyers here represented, because the whole 24. could not wel be drawne, but the other 12. you must imagine to lie so opposite and directly one to another, that euery one hideth his match. Now if you place this plate within the wyers, neere or farther off from the pole, according to the latitude of any countrie whatsoeuer: you may thereby make a Dyall thereunto. So that a man might make a pretie instrument of 24. wyers, to make all manner of Dyals by, if there were no other shift, but you shall haue my *Iewel* about you when the wyers will be to seeke, neither can the wyers be exactly bent true compasse, nor being bent, keepe their iust places long.

Chapter 4.

What the cocke or Gnomon of euery Dyall is, and by what reason they giue the true shade to the Dyall.

FIrst you shall vnderstande that the cocke, Stile, or *Gnomon* of euery Dyall, I meane the line thereof that yeldeth the shade to know the houres by, lyeth directly in the axtreeline of the world, or at the least, parallel vnto him, which worketh the same effect.

Secondly I shewed in the last Chapt. that the sphere is set about with 360. houre circles, meeting at the poles: which maketh 24. times 15. whereby the space of time, wherein the ☉ commeth vnto euery of these 15. by the motion of the *Primum mobile*, or first mouer about the axtree of the worlde, maketh one houre: So that an houre is the 24. part of this reuolution once about. These 24. houre circles of the sphere are represented in the last Chapter, by 24. wyers, of which, euery one is so directly opposite to his match against him, that they both make one whole circle. Insomuch that of the 24. hourecircles, or rather semicircles, (as they are in deede) there are made 12. whole circles, meeting and crossing eche other in halfe at the poles: wherefore of necessitie the axtree must needes be dyametre to them all by the first definition, and first *Theoreme* of the fifte Booke. This knowne, I thinke there is no man so ignorant, but may conceiue without the 22. Theoreme of Euclides prospectiue: that if a circle with his diametre or axtree, be turned edgelong towards a mans eye, that then it appeareth to sight no circle. but a streight line like vnto A B in this figure. By this onely prospectiue reason it commeth to passe, that the axtree line giueth shade vnto the Dyal. For when the Sunne is in any houre circle, the shadowe of the axtreeline is extended thereby directlie on the houre circle opposite to the same, and by that meanes sheweth the houre. As for example, if you place the instrument of wyers mentioned in the last Chapt. so that the round plate encluded within them, may stande equiballance to the horizon, & the meridian wier, directly North and South, the pole aboue the plate Northwards, in order spherically: you shall then see that the Sunne being in the meridian circle Southwards, shall cast the shadow of the axtree line directly on the same meridian Northwards, which is the reason in all horizon Dyals that haue trianguler cockes, they giue no shade at 12. of the clocke. Likewise the sunne being in the 9. of clocke houreline, casteth the shade of the axtree line directly on his opposite, being the 3. of clocke circle: in so much that in your instrument of wyers, you shall perceiue that at euerie compleate houre, the same houre wyer with his opposite, and the axtree, will all three giue but one streight line for shade, which shall be directly on the houre line, drawne one the rounde plate for the same houre. Insomuch, that by one Dyall exactly made, a verie rude and vnskilfull man may make any like Dyall, if the cocke were set vp to his hande, for at euerie houre his shade will most truely expresse the houreline, and so for a shift he might make a Dyall in a day: if the Sunne chaunce not into a cloude: then he must tarie the Lords leisure. This matter is expressed in the figure of the last Chapter.



Chapter 5.

Of the diuision of Horizons, and especially into three sortes.

ALthough that euerie least declining of anie wall, flatte, bancke, or buttresse, is particulerly a diuerse horizon, by the first preamble: and hath a sundrie eleuation of one of the poles, or none at all, so that in this sense the horizons be infinite: yet by two other meanes they are brought to a farre lesse number: whereof the greatest is 90. and the lesser but two. For as in all Astronomicall workings 90. horizons serue through the world, euen as I shewed in the 2. Booke and 21. Chapter, the reason is, as I there saide, because he that coulde go continually vpon one of the poles, shall finde that when the pole is once 90. deg. eleuated, hee then is at the heighest, euen in the zenith: then let him go on streight he shall see him sinck againe on the other side of him. So also in another sense, they make but two differences videl. *Horizon rectus*, and *horizon obliquus*, yet some make three, which I for distinction of my Dyals thinke best of, vz. *Horizon rectus* or *Polaris*, *Aequinoctialis*, and *obliquus*. *Horizon rectus* or *Polaris* a right or polare Horizon, they will to be where the Aequinoctiall is zenith, and the poles leuell with the Horizon, neither of them appearing aboue: and

and for as much as all the meridians meete at the poles, therefore euerie right Horiz on muſt needes be, and lie in one of the meridians, for that there are no circles of the Speere, which haue their pole Zenith, and Nadir in the equinoctial, but onely the meridians as I partly ſhewed in my 24. Chapter of triang. Wherefore to conclude, euery Eaſt or VVeſt wall, are right horiz ons becauſe they lie directly with the meridian, and ſo are any other reclining or inclining wals or flats, cutting either poles, that is to ſay aboue which neither pole is eleuated.

Horiz on Equinoctialis the Equinoctiall horiz on, is where the pole is in the z enith, and the equinoctiall become all one with the horiz on, ſome write he is *Horiz on obliquiſſimus*, becauſe the ſignes and the ſtarres in this horizon do go all a ſlope as though they were round about the ſide of an hill. But if my opinion might ſerue for authoritie, I would call this Horiz on *Polaris*, for that the pole is z enith, and the other *Equinoctialis*, for that the equinoctiall is there z enith: my reaſon is becauſe the denomination of any thing ought to proceede from the worthieſt part of it, which muſt needes be the highermoſt, for the ſtarres and planets haue moſt power and force when they are in the heigheſt part of the heauen, alſo the Zones, Clymats, and Parallels take their names of thoſe parts of the Sphere, which are in the Zenith, for it were abſurde to ſay that a man were in the hot z one, when the equinoctiall is the Horizon, where he ſhould finde himſelfe in the coldeſt.

Notwithſtanding I will yeelde in this behalfe to the authoritie of all the learned men that haue written, rather then preſume to offer to ſtande in armes againſt them, knowing my validitie too weake to encounter them: yet hereafter in this worke, I muſt craue leaue of them to terme euerie wall and flatte, by the place where his pole z enith lieth as a ſouth wall, to be a flatte, whoſe pole zenith lieth in the ſouth point of the horizon, and ſo of the reſt.

Horiz on Obliquus, an oblique Horiz on is where one of the poles is eleuated how much or little ſoeuer, till it come to 90. degrees.

Chapter 6.

Of the diuiſion of all Dials into three ſorts, and whie they make ſhew of greater diuerſitie then there is.

IN the laſt Chapt. I diſtinguiſhed all horizons into three ſorts, and that for two cauſes: the one, for that the formes of making all manner of Dials is but of three ſorts, as ſhalbe ſhewed in the next three Chapt. therefore marke them well, make thoſe and make all the other: for that the dials to euery of thoſe Horizons require their cocke or *Gnomon* of a ſeuerall forme. For the dyals of all flats lying in any right horizon, haue their hourelines parallel one to another, and their cocke a long ſquare, although ſome vſe a ſtile or wyer, which is not ſo good nor ſo agreeable to the nature of the diall: becauſe the ſtile muſt be of a iuſt determinate length: and then can yelde ſhade to the Dyall but from the very toppe: whereas the long ſquare cocke yeldeth a long line for ſhade, and the ſame alway parallel to the hourelines, and like vnto them. The Dyals of all flats lying in the equin. are but a circle deuided into 24. equall parts, the hourelines meeting at the centre, whoſe cocke or *Gnomon*. moſt commonly is a ſtile or pin erected: neither is it materiall how long or ſhort hee bee. The Dyals to all flattes lying in any oblique or byas horizon, are a circle deuided into 24. vnequall parts, the hourelines concurring at the centre, whoſe cocke or *Gnomon* muſt needes be a triangle, his ſide yelding the ſhade to the Dyall, eleuated in height according to the poles eleuation: though ſome barbarouſly, as I may terme it, vſe a wyer bent a ſlope to the poles height, which euery bird almoſt with ſitting on it, may way from his pitch, and then all is marde till ſome ſkilfull man come to mende it, becauſe euery man knoweth not what the height of the pole meaneth. For I remember well I asked a cunning gardiner once, that had ſet vp a poſt aſlope in a curious Garden in the middeſt of a circle, what he would make: a Dyall (ſaid he) why? ſaid I, how heigh is the pole here, ſaide he ſixe foote heigh, pointing to the poſt that himſelfe had ſet vp: whereat I laughed hartely ſeeing ſo cunning a peice of worke towards: but now againe to my matter. As there are but three diuerſities of Dyals and horizons, ſo are they eaſily to be made in euery country, whervnto they ſerue without any difficultie, if a man were in euery ſuch place ſeuerally, by occaſion of trauaile or otherwiſe. But yet to make the Dyals, which of right ought to ſerue in other countries, to ſhew the houres and ſerue your turne in your owne countrie, maketh a great ſhew of skill, and of diuerſe ſorts of Dyals, to thoſe that vnderſtand not the cauſes of them. For the mural declining, reclining, inclyning orientall and occidentall Dyals and all others, the plaine horiz ontall excepted, are dyals which of right belong to other countries, and horizons, and are but wreſted as it were to ſerue altogether to one place: neither is there any of them but is one of the three ſortes before mentioned. There is none of them but of right ſerueth to a countrie, differing from yours, either in longitude, or latitude, or both: but eſpecially the difference of longitude maketh this great diuerſitie and difficultie, for that onely driueth vs to alter the true courſe of the hourelines, and their numbers to make them ſerue in our countrie. As for example, in an Eaſt Dyall, in the place where the cocke ſtandeth he ſheweth no ſhade, and then is it 6. of the clocke in our countrie. but to that countrie whoſe difference of longitude is 90. deg. Eaſtwards, or more plainely whoſe pole z enith is in the eaſt point of our horiz on, it is then 12. of the clocke, and ſo would it giue alſo in our countrie, if you laide the ſame Dyall ſo that his pole Zenith may be in our meridian. his flat cutting the poles.

Likewiſe in the declining Dyals here in our countrie, the perpendiculer is alwayes 12. of the clocke: but in the horizon or countrie whereto the Dyall ſhould ſerue, the 12. of clocke is iuſt where the cocke ſtandeth, ſo that it is a general rule if you ſee any Dyal, his cocke yeelding no ſhade at 12. of clocke: that the ſame is made to the longitude of the ſame place: wherefore to conclude my purpoſe, there are but three diuerſities of Dyals, onely the ſituation and change of the place maketh the alteration.

Chapter

Chapter 7.

How to make the first kinde of dial to the Equinoctial horizon, that is, where one of the poles is zenith.

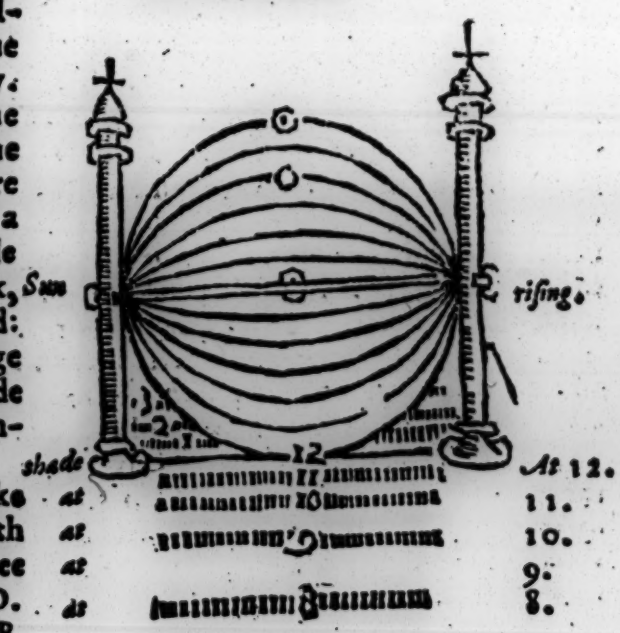
THe matter is short and already sufficiently to be had by the 3.chap.deuide a circle into 24. equall parts frō the centre, drawing the houre lines vnto them, then set to the figures frō 1.to 12, & thence from 1. to 12.again, to make vp 24.it is no matter where you begin to number, so you make an end wel: if you would needs make this dial by the *Iewel* in the 3.chap.you haue instructions sufficient, the cocke is but a stile or wyre erected as plum as may be from the flat, so that the top be no neerer to the one part of the circle then the other. This diall though he be sleight can least be spared, as the shipmen can witnesse, which with a quadrant annexed behinde him, serueth in euerye latitude where the ship goeth, a thing moste common, or els I would spend sometime about it, if you know it not, look *Orontius* & *Munster de horelegijs*, or in English the Art of nauigation and the regiment of the Sea.

Chapter 8.

How to make the second kinde of dial to a right horizon, that is, where the Equinoctiall cutteth the zenith, together with the ground and reason of al dials to any right horizon.

I shewed in the 3.and 4.chap the reason of making all dialles to any horizon, where any one of the poles is eleuated to come of the intersection of the houre lines, with the horizō. And by example of wyres the same conclusion, & withal how that the axtree did after a sort delineate out the lines of all dialles, by his shade. But now in a right horizon where neither of the poles are eleuated, because the Zenith is in the Equinoct.one of the meridians must needs be the horizon; & of the rest of the meridians, the one halfe be alwayes wholly aboue the horizon, & the other half vnder, they al cutting the horizō together at 2.points, which are the poles: which is the cause that they distinguish not the houre lines of the diall by their intersection of the horizon, as in the Equinoctial, and obliqne horizon, but yet by helpe of the axetree, they wil do it here as wel as in them, and that in this manner. Take your 24.wyres spherically conioyned, as in the 3.chap. and erect the meridian wyre perpendiculer on some plaine flat, so that the axetree wyre may be parallel to the flat as in this figure you see. Now if you were vnder the Equinoctiall, or if you erect one part of this flat, so that the poles & axetree of the wyres thus placed, may lye leuel with the poles of the world: that is to saye, in your meridian according to the poles eleuation, you should then see the Sunne at his rising, to cast the shade of the 6.a clocke wyre, and the axtree wyre directly on the other 6. a clocke wyre: that is to say, parallel to the flatte: so that the dialles vnder the Equinoctiall haue no houre line for 6.a clocke. But after that the Sun rising to 7. a clocke, doth cast the 7. a clocke wyre, the axetree wyre and the other against him, al three in one shade, making one streight line equal to the axetree wyre, & may serue for the 7. a clock houre line, if he be not before drawne. Likewise at 8.a clocke, the 8. a clocke houre wyre, the axetree and the other against him, yeelde al three but one streight line for shade: which sheweth 8.a clock, and so at 9.10.11.and then at 12.the Sun is directly ouerhead: which would cause the Gnomon or axetree if hee were a longe square, to yeeld no shade: then the Sun by his comming down describeth the after noon houres, as by this figure I hope you plainly conceiue.

A right Horizon.
Sun in the meridian.



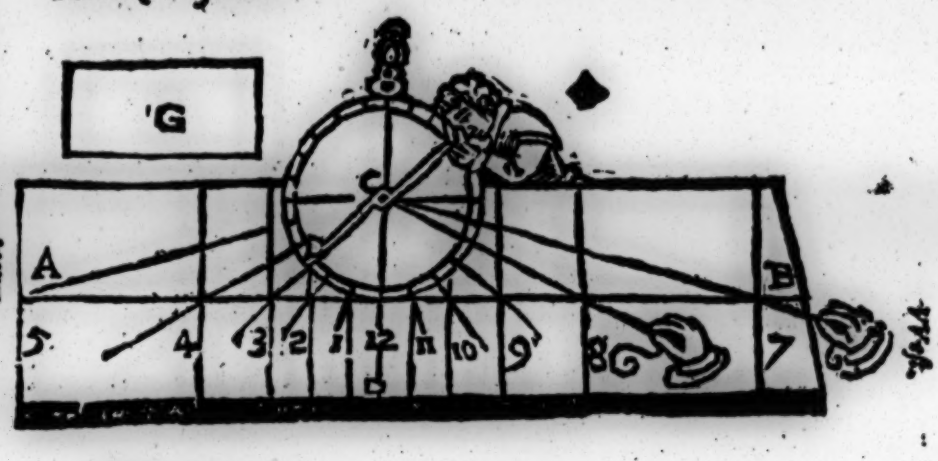
The shade of the axetree at 7. a clocke.

This wel vnderstood, you may make the dial easily thus: Take a long square stone or flat : and crosse him in the middest both wayes with two lines square ech to other; let the longer line bee East and VVest, videl. A B: the other North and South, vz. C D. Then erecte your *Iewell* perpendiculer on this East line A B. and directly with it : but so that the plum line of the *Iewell* maye stand perpendiculer thereon, in the crosing of these two lynes: and there deuise by some meanes to fixe him : that line of your *Iewel* so fyxed, set your rule with sightes to the line of *Leuel*, which is then parallel to the stone or flat, and representeth 6.a clocke in any countrey where the Equinoctial is Zenith, then lift vp the end of the ruler, vnto the next 15.deg.of the *Limbe*, and looking through the sights, mark with a pen (as you may easily doe) where your eye beame doth point on the said East line, there draw a lyne square ouer the stone for the 7.a clock houre line, that done, lift vp the ruler other 15.deg. and there marke with a pen where your eye pearcing the sightes, doeth pointe on the sayde lyne, drawing there also another crosse line, for the 8.a clocke houre line, and so doe with euery 15.degree of the *Limbe* of your *Iewel*, in the vppermost halfe, vntill you haue done, as in this Figure you may beholde. Also you may if you list lay the *Iewell* flat vpon the stone, so that his plumbe line lye direct with the crosse line C D, and his *Limbe* touching the East line A B and then fyxing a threed in the centre of your *Iewel*, leading about the same from 15. degr. to 15. deg. of the *Limbe* with your hande, and at euery place where this threed doth cut the East line, there drawe lines crosse the stone ech parallel to other, and square to the East line: then set numbers thereto as in this Figure

gure you may ſee. The cock or gnomon to this dial muſt be a lōg ſquare like vnto G, erected in the 12. a clock line, whoſe length is commonly vſed equall to the line C D, or it may bee lōger or ſhorter at your pleaſure, but the height muſt needs bee equal vnto the ſpace betweene the centre of the *Iewel*, & the line A B. Thus maye you note that whether your *Iewel* ſtande perpendiculer on the flat, or lie down theron the croſsings of the long line A B, which ſome call the touch lyne are al one: ſo that the leuell line of your *Iewel* be parall. to the touch line.

The Iewel either erected to the eye or depreſſed to the flat.

The cocke or Gnomon.

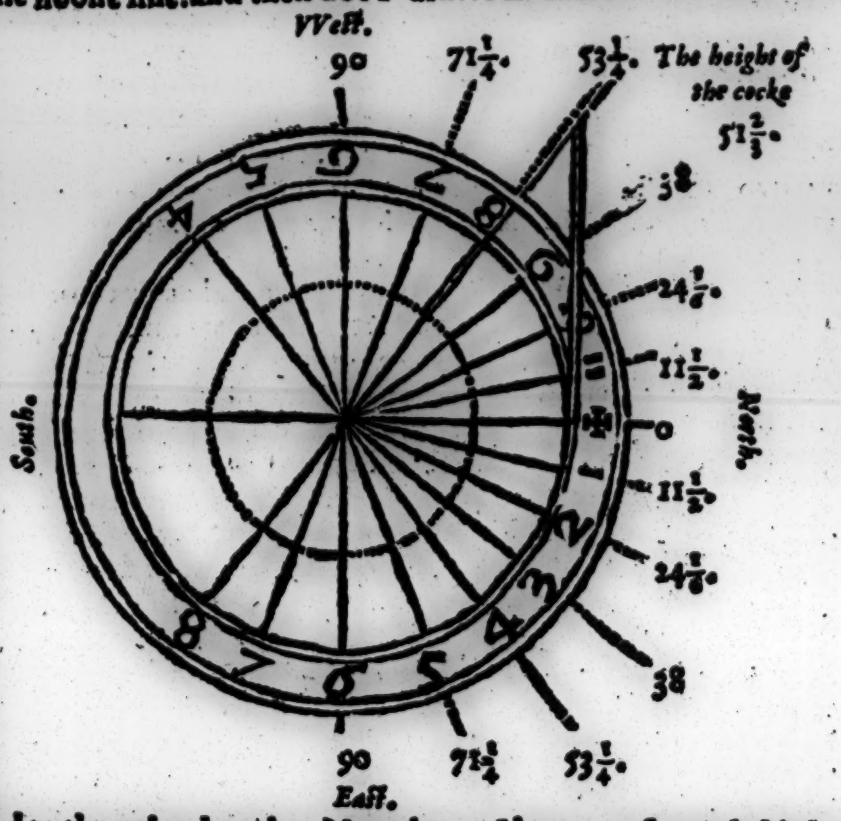


Chapter 9.

How to make the third kind of diall to an oblique horizon, which they commonly cal an horizon diall, and that very eaſily.

WHen you come into any countrey, where you would make a dial: firſt you muſt get the poles eleuation or latitude, there by the 3. book 5. & 8. chap. & if you fynd the pole eleuated any whit, then you may be ſure it is an oblique horizon: to which if you wil make a dial, ſet the *finitor* of your *iewel* vnto the ſame latit. found where you ſhal ſee the *Limbe* or vttermoſt meridian of your *iewel* and the axetree or Eaſt houre line, to cut the *finitor* 90. deg. or one quadrant a part: which is the cauſe that in an horizon diall, the firſt point is to deuide the circle into 4. quarters. Furthermore, if you behold your *iewel* wel in that ſcituation, you ſhall ſee, that the houre circles, (I meane euery 15. meridian) on the one ſide of the axetree line, doe cut like equall degrees on the *finitor*, vnto the houre circles on the other ſide, eche to his matche, beeing both reckoned eyther from the ſeueral vttermoſt partes of the *Limbe* or on ech ſide from the centre: which is the cauſe, that if the deg. of an oblique diall be knowne for one quarter, all the other 3. quarters are to bee drawne by that: ſo that by this meanes, the two diameters ſeruing for the noone line and Eaſtline being drawne, 5. ſpaces more knowne ſhall ſerue to make the whole dial by: which in my opinion can be no great toile.

Example: here at Reading the latitude is 51 $\frac{2}{3}$. deg. and therefore is an oblique horizon. To make a diall thereto, I get a flat ſtone or plate, and thereon make a circle, or rather a *Limbe* to the diall: croſsing the ſame ſquare vppon the centre, with 2. lines into 4. quarters, of which lines I appoint one to be the noone line, & at his North end write 12. the other the Eaſt line, & ech end write 6. Then for my further eaſe, becauſe I wold not deuide the *Limbe* of my dial into 360. deg. as *Cor. Gemma* teacheth, I make on the centre of this dial a blind circle, equall to the graduated circle on the *Limbe* of my *iewel*, al which done, I ſet the *finitor* of my *iewell* to the 51 $\frac{2}{3}$. latitude, and there doe I marke what deg. of the *Finitor*, the 1. 2. 3. 4. & 5. a clocke houre circles doe cut: I finde at 1. a clocke 11 $\frac{1}{2}$. deg. at 2. 24 $\frac{1}{6}$. at 3. 38. deg. almoſt, at 4. 53 $\frac{1}{4}$. at 5. 71 $\frac{1}{4}$. all of them reckoned on the *Finitor* from the *Limbe* of my *iewell* inwardes: all theſe ſeuerall ſpaces of degrees, I take with a payre of compaſſes orderly one after other from the *Limbe* of my *iewell*, and ſet them, or notes for them on ech ſide of the noon line in the ſaide blynd circle, beginning from the noone line: and then doe I drawe lines from the centre of the diall, through all the notes in this blind circle vnto the *Limb* of the dial: ſeruing to the houre lines, as by this figure you may more plainly perceiue, then by many words. To theſe lines I ſet theſe numbers, 1. 2. 3. 4. and 5. on the Northeaſt ſide, & on the Northweſt quarter: to them theſe numbers: 11. 10. 9. 8. 7. as you ſee: then if you draw forth the 4. and 5. houre lines vnto the Southweſt quarter: you haue the 4. and 5. houre lines whiche the Sun riſeth before 6. in the Sommer time. Likewiſe drawing foorth the 7. & 8. your dial is made: & for the cocke or gnomon to this diall, if you take the degrees of the latitude, vz. 51 $\frac{2}{3}$. deg. in your cōpaſſe & ſett the ſame in your blinde circle, drawing thereto a line from the centre, and erecting at the North end of the noon line a perpendiculer to meete him there, you ſhal haue the iuſt patterne for your cocke, as in this figure you may behold: for it is a generall rule that the cocke of a diall to an oblique Horizon, whether it decline, recline, incline, or howſoeuer, muſt be eleuated from the horizon, equal to the poles eleuation. Note that as I haue performed this by one quarter of the houre lines of the *iewel*, you may by the *iewel* perform the whole round: but al commeth to one effect, ſince as I ſaid the ſpaces of euery quarter are like, and containe like number of degrees, being taken equally on ech ſide of the noone line.

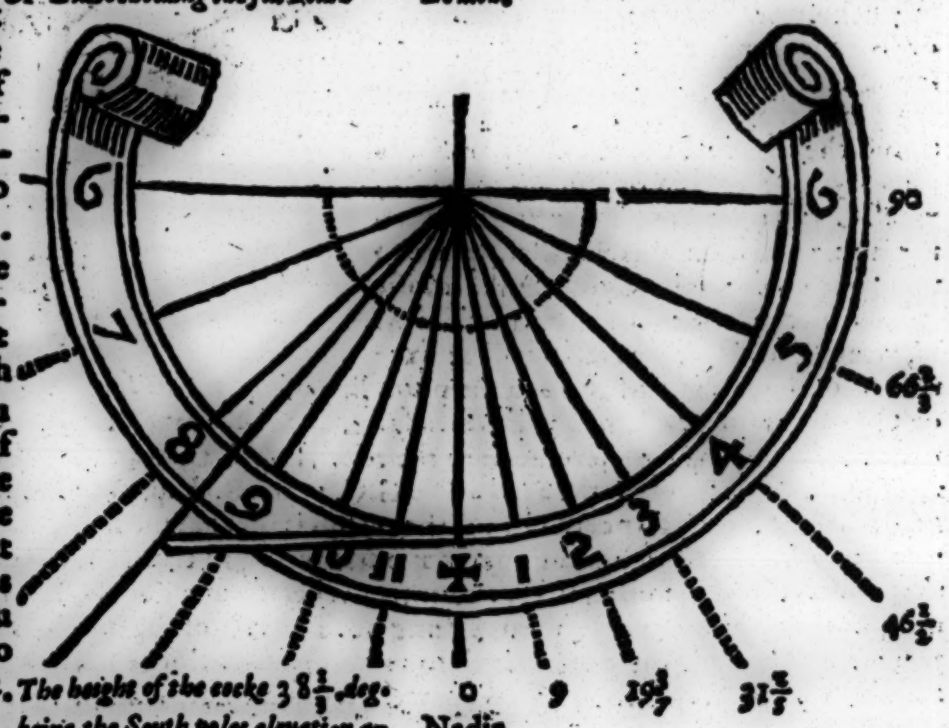


Chapter 10.

How to make a diall to a perpendiculer wall beholding the South fully, called of ſome Authors Horologium verticale.

THat wall which I call a South wall, is an horizon, or great circle, whoſe pole Zenith lyeth in the South point of the horizon of the countrey where it ſtandeth, and lieth euē with the Eaſt azimuth of the ſame place, being all one circle with him. Inſomuch that he is on my *Iewell* in euery latitude, repreſented by the line, which I call the Zenith line, or Eaſt azimuth. To this wall the South pole is eleuated alwaies as muche as the latitude of the countrey lacketh of 90. by the 13. preamble, which here at Reading is $38\frac{1}{3}$. degrees, and therefore is it an oblique horizon by the 5. chap. wherto you may eaſily make a dial by the laſt cha. only here in it altereth a little, you need make but bare 12. houres: for that no perpendiculer wall in theſe partes canne ſerue aboue that compaſſe, and that you muſt turne the dial and the cock downwards, that he may behold the South pole, prouiding that the 12. of clocke lyne be drawne very perpendiculer down the wal. For your better vnderſtanding, ſette the *Finitor* to the latitude of Reading, videl. $51\frac{2}{3}$. vnder the pole, then if you behold the Zenith line, which as I ſaid, repreſenteth the South wall, you ſhall finde at the Nadir of the zenith line, the ſouth pole to be eleuated $38\frac{1}{3}$. degr. if you reckon towardes the South parte of your *Iewell*. wherefore in reſpect the Zenith line is graduated as wel as the *Finitor*, you may thereuppon ſee what deg. euery houre requireth as wel as in the laſt cha. by the *Finitor*. &ſo you ſhall find the 1. a clocke houre line to cutte 9. deg. the 2. $19\frac{3}{7}$. the 3. $31\frac{2}{5}$. the 4. $46\frac{1}{2}$. & 5. $66\frac{2}{3}$. & the 6. alwaies 90. by which make your diall, as in the laſt chap. and as in this figure appeareth. Note that the eleuation of the wall knowne, you may ſet the *Finitor* to the ſame latitude vnder either of the poles, without reſpect, and make your diall therby, which indeed ie the beſt courſe, becauſe the degrees are farre more eaſily to be counted on the *Finitor*, then on the Zenith line.

A Dial beholding the ful South Zenith.



The height of the cocke $38\frac{1}{3}$. deg. being the South poles eleuation aboue this flat.

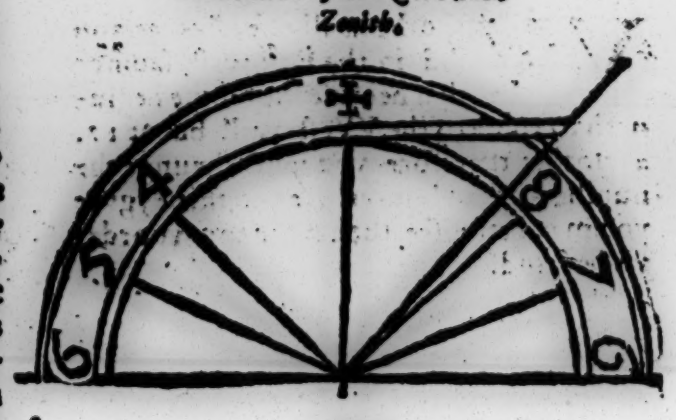
Nadir.

Chapter 11.

To make a diall to a ful North wall.

IHaue ſhewed in the 9. and 10. preamble, that euery perpendiculer wall or flat hath two faces: (that is to ſay) if a wal be in length Eaſt and VVeſt, then on one ſide the South beholdeth him full, on the other ſide the North, inſomuch that looke how muche the South pole is eleuated to the South ſide, ſo muche is the North pole eleuated to the North ſide by the 12. preāble, both which the zen. line repereſenteth. VVherefore you may conceyue that one diall ſerueth them both, onely they alter in this: that this diall to the North wall muſt be turned vpſide downe, that the cocke may beholde the North pole, and all the houres ſauing 4. 5. and 6. in the morning, and 6. 7. and 8. at Euening left quite out, becauſe the Sun commeth to a North wall, but before 6. in the morning, and after 6. at euening, as by this Figure you may perceiue.

A Diall to a full North wall. Zenith.



Nadir.

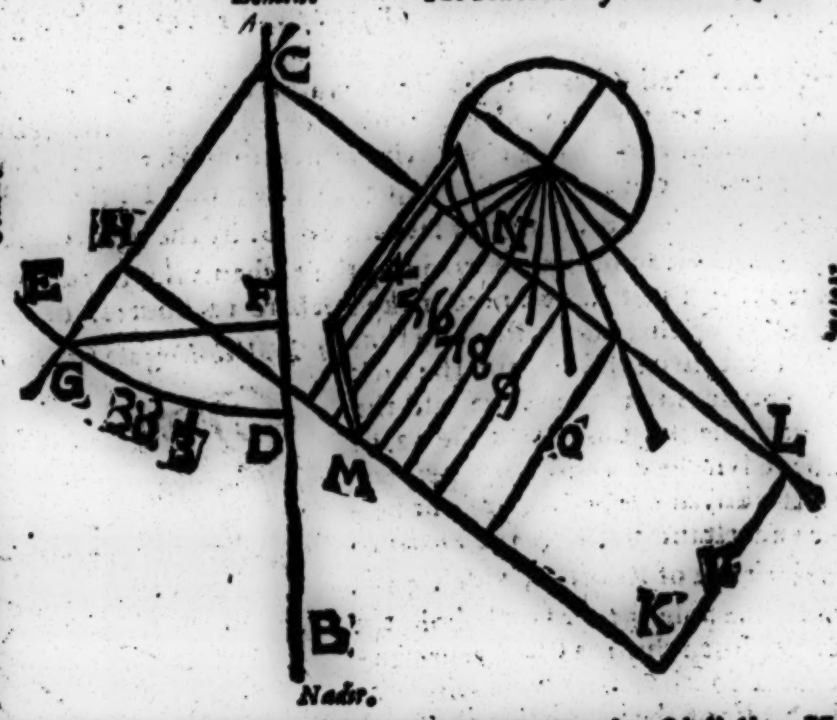
Chapter 12

How to make a diall to an Eaſt wall.

AN Eaſt wall is an horizon, whoſe pole Zenith lyeth in the Eaſt point of the horizon, whereon it ſtandeth, or more plainly an horizon lying euen with the meridian of the horizon whereon it ſtandeth, ſo that an Eaſt wall pointeth directly North and South, whereby you may gather by the 5. chap. that it is a right horizon: and therefore you may eaſily make a diall thereon, by the 8. chap. only the alteration is in the placing of the diall, and numbering the houres, which you ſhall thus doe. Firſt drawe a perpendiculer on the wall, videl. A B, then pitching your compaſſe in ſome one point thereof, as C. drawe a blind arch

arch Southwards of the plumme line videl. D B, whereon number by deg. the eleuation of the South pole about the perpendiculer or Zenith line, videl. D G, which as in the 10. here at Reading is $38\frac{1}{2}$. deg. the drawing a line from that deg. to the center of the arche, videl. G C, and another perpendiculer to the plum line, or parallell to the horizon, all is one, videl. G F, so haue you G F C the verye patterne of the cocke to the South wall diall, and that in his direct scituation to eyther pole, his Gnomon line issuing from the centre, videlicet G C, representing the axetree of the worlde, and directly beholding both the poles: wherefore drawe another line square vnto him longe inough for your diall, videl. H K, and the same shall represent the Equinoctiall flatt, to whiche drawe another line parallel as farre dystant as your dial shalbe in width, videl. C L, this done draw a line betweene the Equinoctial and his parallel, square to them both, euen where you thinke moste conuenient for the cocke to stand, videl. M N, which shall also be parallel to the axetree G C. In this line M N place, the plum line of your *Iewell*, his *Limbe* touching the Equinoctiall line H K or C L, all is one, and there your *Iewell* fastened with waxe, or staied with ones hand, laying a threed to euery 15. deg. of the *Limbe* extended to the touch line C L, draw your houre lines at ech crossing, so that euery of them be parallel to the axtree line G C, by the 8. chap. But now in an East diall you must number the line where the cock standeth with 6. the reason is for that the meridian of the horizon represented by an East wall is 90. deg. or 6. houres distant from ours: beeing in truth the 6. a clocke houre circle, in our countrey, so that when it is high noone to that countrey whereto our East wall is horizon, it is here but 6. a clock in the morning, the difference of longitude being 90. The rest of the numbers for the houres, sette in as in this figure you may see, because the Sunne riseth to vs neere at 4. a clocke in the morning: therefore I haue set no houres aboue that, and bicause the Sunne departeth from an East wall at 12. therefore past a 12. I cannot goe.

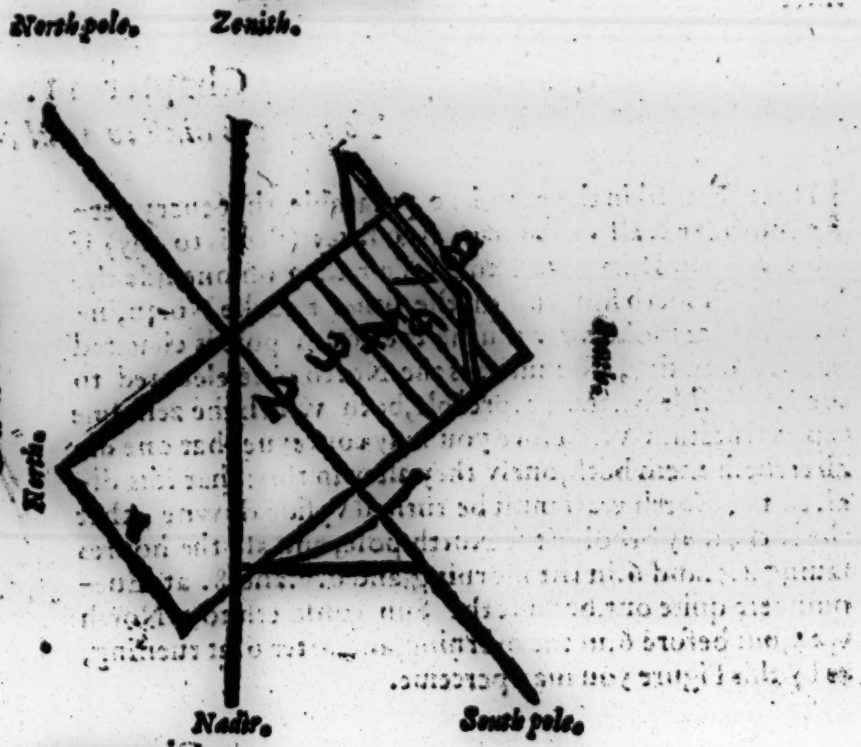
A Diall beholding the full East.



Chapter 13.

How to make a Diall to a West wall.

A West Diall.



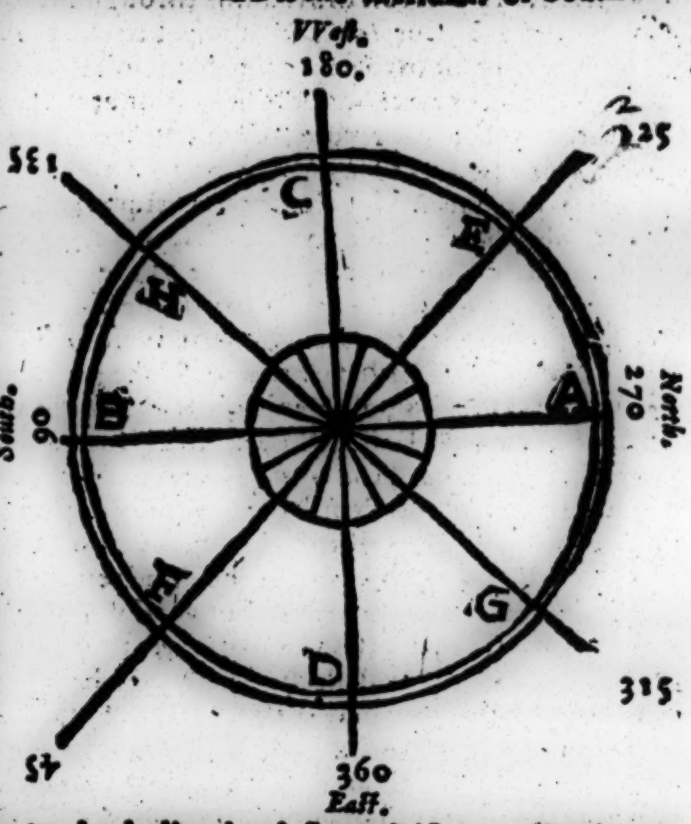
WOorke in all respectes as you did in the East wall, sauinge that because the Sunne beginneth to come to the VVest wall but at 12. a clocke: therefore you must chaunge the numbers of the houres, as in this Fygure you may better see, then by many woordes vnderstand.

Chapter 14.

Of declining walles, and what circles of the sphear they represent.

IT is a Maxim in philosophie, (as I haue already said) that *Omne graue tendit ad centrum*, insomuch that if you graue a circle round about the world with a threed and plummet in your hand, the plummet shal still point vnto the centre of the earth, all vpright walles are made perpendicular by a line and plummet: therefore the great circle of the sphear lying euen with any wall, must needes cut the centre of the earth, and be concentricke, &c a great circle by the 1. def. And because euery wall perpendicularly erected, cutteth or pointeth to the zenith, therefore eche of them must needes lye in one or other of the azimuthes: and do crosse or at the least vnyte one another vpon the centre of the earth, & in the zenith, if they were bigge inough to fill out a great circle

circle of the ſpheare, euen as you ſee the lines of this preſent figure, euery of which repreſenteth a perpendiculer wall, the circle A D, B C, is here the horizon, his centre is the Zenith: A B is the meridian or South azimuth, in which circle, the East and VVeſt wall lyeth, by the 12. and 13. chap. C D is the East azimuth, called of many authors ſimply, *Circulus verticalis*, the vertical circle, whereof the South diall is called of diuers, *Horologium verticale*: notwithſtanding al the azimuthes are vertical circles, ſo called, *quia tranſeunt per verticem*, I ſay in this East azimuth C D lieth the South and North wall by the 10. and 11. chap. the line E F repreſenteth a South wall, declining East, and a North wall declining VVeſt, lying in the 45. and 125. azimuth, for the azimuthes are counted from the East vnto 360. The line G H repreſenteth a South wall declining VVeſt, and a North wall declining East, lying in the 135. & 315. azimuth, and ſo of all the reſt, which you ſee in this figure drawn within the leſſer circle, repreſenting the globe of the earth, & are to be extended, or as many other as you can imagine. It is moſte certaine and manifeſt by the 2. def. that the poles zenith and Nadir of euery azimuth lie in the horizõ, as the poles of euery meridian lie in the Equinoct. VVherfore as the pole zenith of a ſouth wall, muſt needes lie by the ſayde definitiõ in the ſouth point of the horizõ: euẽ ſo the pole zenith of any declining wall reſpecting the South, departeth ſo much frõ the South point, as the declinatiõ cõmeth to: & ſo likewiſe of any wal, reſpecting the north: his pole Zenith departeth from the North point, according to the declination, inſomuch if you will aske mee what the declination of a wall is: I ſay it is the diſtance between his pole zenith, & the ſouth or north points by deg. of the horizon: & looke which way the pole zenith departeth, thence is the declination denominated or called, as if the pole Zenith of a wall lie betweene the South & East point, then is that wall called a South wal declining East: & ſo of the reſt. As for example, in this figure the pole zenith of the South wall C D is B, but the pole zenith of the North wall C D, is A. Likewiſe of the East wall A B is D, & of the VVeſt wal A B, is C, for euery circle hath 2. faces, as is ſaid in the 10. preamble. Alſo of the South wal declining VVeſt E F. the pole zenith is H, whoſe declination is the ſpace H B, but of the Northwall declining East, repreſented by E F, the pole Zenith is G, whoſe declination is A G, and ſo of the reſt. By all this you may well gather, that euery wall, flat, and horizon, ought to take his name of the place or point where his pole zenith lieth, and not of his owne ſituation: for if I ſhould ſay, make me a diall to the wal C D, who could tell, whether I meant a North or South diall, but if I ſay make me a diall to the South wall C D, it is eaſie to vnderſtand: wherfore me thinketh as I ſhewed in the 5. chap. they abuſe their tearmes who call that an Equinoctial dial, whoſe pole Zenith is the pole it ſelfe, mentioned in the 7. chap. which in my opinion ought to bee called a pole or polare diall. Laſtly, al declining walles are deuided into 2. ſorts, & eche of thoſe againe ſubdiuided into other two: the diuiſion is into North declining, and South declining: of which the firſt haue their pole zenithes in the North ſemicircle C A D, the 2. in the South ſemicircle C B D, the ſubdiuiſion of the North declining is into North, declining East: whoſe pole Zenithes lie in the northeaſt quadrãt A G D, & North declining VVeſt, whoſe pole zenithes lie in the northweſt quadrant A E C. The ſubdiuiſiõ of the South declining, is into ſouth, declining East, whoſe pole zeniths are in the Southeaſt quadrant B F D, & into Southweſt, whoſe poles are in the quadrant C H B.



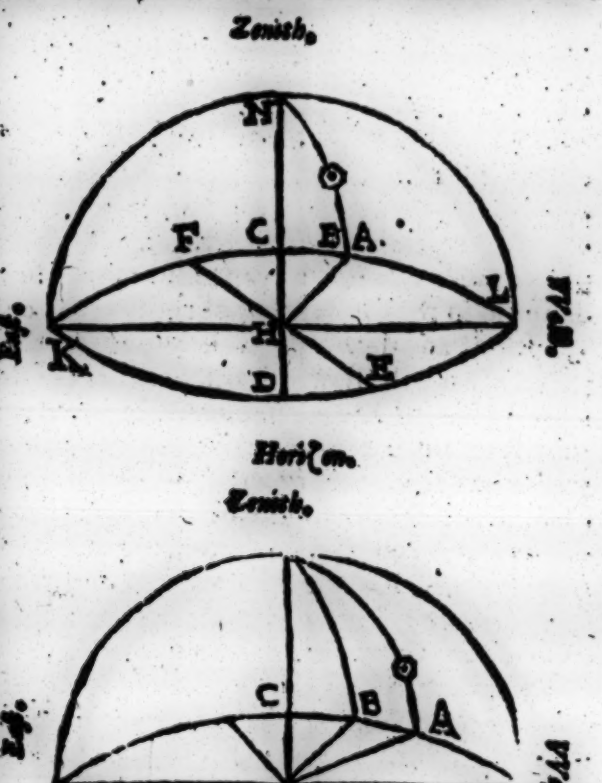
Chapter 15.

How to know the ſcituation of any wall, and to finde how much he declineth.

VVHat the declination of walles are with the diuerſities of them, is ſhewed in the laſt chap. In this I wil ſhew how to take the declinations of al walles reſpecting the South: by which any man may do the like for walles, reſpecting the North, the woorking being al one. By a wall reſpecting the South, I mean a South declining wall, & it is eaſily to be known, becauſe the Sun being in the South, yet ſhineth on him though he decline neuer ſo farre East or VVeſt, by which you may alwaies knowe a South declining wall: wherfore when you come to any ſuch wal: Firſt deuiſe to place your *Iewel* vpon ſome ſtoole or otherwiſe *Leuel* with the horizon: but ſo that the line of *Leuel* may directly ſtand perpendiculer or ſquare from the wall: neither is it material whether he ſtande cloſe to the wal or not: which to do one way is thus: If the wal haue any returning ſquare neere the place, then chooſe your ſtanding, ſo that laying the rule on the ſaid *leuel* line, you may through the ſights looke directly with the end of the wal, as if you would make a dyal on the ende of a houſe, commonly the ende of the houſe ſhutteth ſquare vpon it, ſo that if you lay the *Leuel* line directly with the end, then he ſtandeth ſquare to the ſide. Another way is thus: get a large ſquire, ſet one ſide euen to the wall, and parallel to the horizon, and hang at the other ſide 2. plum lines, at eche end one, there let one hold it til you haue choſe you a ſtãding: ſo that the rule lying on the *Leuel*, you may ſee through the ſightes, the one plum line hiding the other, & there fixe your *Iewel* on the ſtoole.

The

The third and beſt way as I think is, either to faſten your *Iewel* with waxe or pitch on the ſide of a ſquire ſo that his line of *Lewel* lye euē with it, or els to get a little ſquare boord or block: or for a ſhift, a ſquare trēcher may ſerue, and thereon draw a line through the middeſt, perpendiculer from one of the ſides: in which place the *Lewel* line of your *Iewel*, and apply it to the wall. VVell, to come to the matter, when you haue thus placed your *Iewel*, then doth the *Lewel* line poynt directly to the pole Zenith of the wall: therefore turning about the rule till the ſun pearce both the ſightes, or at the leaſt erecting a wyre in the one ſight, the ſhade thereof beyng proiected on the other: you haue in the *Limb* of your *Iewel*, the horizōtal diſtāce between the Sun, & the walls pole ☇enith (that is to ſay) betweene the ☊imuth, wherein the Sunne is then, and the ☊imuth cutting the walles pole ☇enith: wherefore you muſt preſently take vp your *Iewel*, and take the altitude of the Sun, by whiche & the 3. booke 6. chap. you ſhall know in what ☊imuth reckoned from the South, the Sun is then, either Eaſtwards or VVeſtwardes. Nowe by the Sunnes ☊imuth knowne, and his diſtaunce from the pole ☇enith of the wall, you ſhal alwayes know the declination of the wall by theſe rules following: therefore marke thē well, together with theſe 4. figures, in which the circuite C L D K repreſenteth the horizon D C, the meridian C being the South part thereof, N the ☇enith, N A the ☊imuth wherein the Sun is: E F the baſe of the declining wall, B his pole ☇enith, B H the line in which the line of *Lewel*, muſt lie, K Eaſt, L VVeſt. Firſt, if the Sunne be directly with the *Lewell* line of the *Iewell*, that is to ſaye, hauing no diſtaunce from the pole ☇enith of the wall, as in this figure, then is the Sunnes ☊imuth reckoned from the South, equall to the walles declination, that way as the Sun lyeth, Eaſt or VVeſt: but as this ſeldom happeneth, or not at all, if you chooſe the beſt times: ſo in all other caſes you haue three thinges to bee noted, that is, whether the wall decline Eaſt, which you may bee ſure of in the morning: for in al walls declining Eaſt, the Sun commeth to them before 6. & departeth at night before 6. In VVeſt declining he commeth after 6. and ſetteth after 6. then muſt you note whether the horizontall diſtaunce of the Sunne from the pole ☇enith of the wall be on the Eaſt or VVeſt part of the *Lewel* line. Laſtly, whether the Sunnes ☊imuth found, bee Eaſt or VVeſt from the meridian: al which you muſt be ſure to note downe, which done, then in all Eaſt declining walles, if the diſtaunce of the Sunnes ☊imuth from the *Lewel* line, chance to be Eaſt from the *Lewel* line, as A is from B, and the Sunne alſo Eaſt from the meridian as A is from C, in this 2. fygure, then take the deg. of the diſtaunce, videl. A B out of the degr. of the ☊imuth, videl, A C, the remainder vz, B C is your deſire, if the diſtaunce be VVeſt, as A is from B, and the Sun VVeſt, as A from C, in this 3. fygure, then take the ☊imuth, videl. A C, out of the diſtance videl. A B, ſo reſteth B C your deſire: but if the diſtance be VVeſt, as A is from B, and the Sun Eaſt as A is from C, in this fourth fygure, then adde the diſtaunce videl. A B to the ☊imuth, videl. A C, and ſo euery way haue you B C the declination of the wall if hee be a South wall declining Eaſt.



But if the wall decline VVeſtwards, then if the diſtaunce bee VVeſt, and the Sun VVeſt, take the diſtance from the ☊imuth, if the diſtance be Eaſt, and the Sun Eaſt, take the ☊im. out of the diſtance. But if the diſtance be Eaſt, & the Sun VVeſt: adde the diſtaunce to the ☊imuth: and ſo haue you the declination of any South wall bearing VVeſt. All which you may perceiue by the laſt 3. figures, ſo that the words Eaſt and VVeſt change places. Theſe rules ſhew harde, but you ſhall little neede them when you are working: for the very ſituation of the diſtaunce and the Sunne, will almoſt alone direct you. Onely I ſet down theſe, leaſt you ſhoulde doube in any thing. Beſides that, you may eaſe the matter thus: if the wall decline Eaſt, come betimes in the morning, and the Sunne and diſtance ſhall be both Eaſt: if VVeſt, towards the euening, & they ſhall both be VVeſt: ſo that in both theſe you need but take the diſtaunce out of the ☊imuth, according to the ſeconde figure: whiche tymes are more conuenient then about the middeſt of the daye to gette the Sunnes exact ☊imuth by his altitude, as by proofe you ſhall finde.

For example, the 26. of Iulie 1581. the ☉ then beeing $13\frac{1}{2}$. in ♌, I being requeſted to make a diall on the Church wall of Sooning beſides Reading, to ſet the clocke by, did place the *Lewell* line of my *Iewell* ſquare from the wall by helpe of a ſquare boorde: then turning about the rule to the Sunne, it cut in the *Limbe* $36\frac{1}{2}$. deg. Eaſtwards of the *Lewell* line, which I noted in a paper, and quickly tooke vp my *Iewell* and tooke the altitude of the Sun 45. deg. the ſame inſtant, by which & the 3. booke 6. chap. I found the Sun to bee in the 53. ☊imuth reckoned from the South Eaſtwards: wherfore according to the 2. figure, taking the diſtance $36.\frac{1}{2}$. out of the ☊imuth 53. there remained $16\frac{1}{2}$. the true declination of the wall. For taking the declinations of North walles the diffe-

difference is nothing but that you muſt reckon the Sunnes azimuth from the North, as here is done from the South.

Another example, the 14. of Auguſt 1581. the ☉ being in ♍ $1\frac{2}{3}$. de. I took the declinatiõ of another wal about the ſame church, declining VVeſt, to ſerue the after noone: wherefore comming thither towards the euening, I found the diſtance VVeſt 20. deg. and the azimuth VVeſt $73\frac{1}{3}$. degrees, then taking as before the diſtaunce, vz. 20. out of the azimuth $73\frac{1}{3}$. there reſted $53\frac{1}{3}$. deg. the wals true declination VVeſtwards. Note that if the ruler of your *Iewell* want neceſſary ſights for this purpoſe, you may hold a threed, and plummet in your hand, ſo that the ſhade of the threed cut the centre of the *Iewell*, and then placing the edge of the rule euen with the ſame ſhade, you haue your deſire.

Chapter 16.

How by the declination of any wall knowne, to finde his eleuation with his angle of deflexion.

THe eleuation of a wal is alwaies the height of the cock, for as I haue often ſaid, the cock of any oblique diall, muſt be eleuated in his due place, as much as the pole is eleuated aboue the wal wheron it ſtãdeth. The angle of deflexion, I cal the angle made between the 12. a clock or plũb line of the wal, & the line where the cock muſt ſtand: which line is alſo the meridian of the horiz. repreſented by the wal. VVherefore reckon on the *limbe* of your *Iewel* the walles declination learned by the laſt chap. and that from the pole rightwards, if the wal declining Eaſt, or leftwards, if VVeſt (though it be not material which way but only for the demonſtrations ſake following,) and thereto lay the Zenith of your *Reete*, then reckon on the axetree line frõ the centre the eleuation of the South wall or the cõplement of the latitude, al is one, & mark what azimuth of your *Reete*, doth cut the axtree line in that deg. for if on the ſame azimuth you reckon thence to the *Finitor*, you haue the poles eleuation to that wall, and the degrees of the *Finitor* between the ſame azimuth & the centre is the 4. angle of deflexion.

Example, the declination of the firſt wall mentioned in the laſt chap. was there found $16\frac{1}{2}$. degrees Eaſtwards, which $16\frac{1}{2}$. deg. I reckon on the *limb* of my *Iewel* frõ the pole rightwards, thereto I lay the zenith of my *Reete*, which done, I number on the axetree line from the centre, the South walles eleuation, vz. $38\frac{1}{3}$. degrees & there do I ſee to cut the $12\frac{1}{2}$. azith. reckoned frõ the zen. line, on which $12\frac{1}{2}$. azi. cõfinting frõ the ſaid $38\frac{1}{3}$. de. to the *fin.* I find $36\frac{1}{3}$. deg. which is the eleuation of the South pole to that declining wall, then on the *finitor* betweene the ſayd azimuth, and the centre I find $12\frac{1}{2}$. deg. which is the angle of deflexion of the ſame wall: & ſo much muſt the cock ſtand from the plumbe line, which in all perpendiculer walles repreſenteth our meridian. This truely is a ſinguler concluſion which you will confeſſe, if you conceiue the reaſon of it, which is thus. The *limb* of the *Iewel* repreſenteth our horizon the axtree line our meridiã or Eaſt wall: the Equinoctial the Eaſt azimuth or South wall: the end of the Equinoctial line rightwardes, the Eaſt point of our horizon, the other end the VVeſt: the centre the Zenith or rather his Nadir: the *Finitor* the declining wall, and the Zenith of the *Reete*, the pole Zenith of the ſame wall. All this ſo determined, and the diſtaunce of the South pole from our Nadir, vz. $38\frac{1}{3}$. deg. reckoned on the axtree line frõ the centre, there muſt needs bee the ſouth pole. Therefore the $12\frac{1}{2}$. azimuth cutting the ſame $38\frac{1}{3}$. deg. muſt needes repreſent the meridian of the wall by the definition of a meridian which is to cut the poles and paſſe by the Zenith of his horizon, and becauſe the height of the pole is alwaies the arch of the meridian betweene the pole eleuated and the horizon: therefore the ſaide $36\frac{1}{3}$. deg. muſt needs be it. But what may you ſee more, O notable ſingularity of an inſtrumeng that is, inaſmuch as you ſee that the angle made betweene the ſaid $12\frac{1}{2}$ azimuth. and the axetree at their croſſing in the ſaid $38\frac{1}{3}$. deg. to be VVeſtwards on the *Finitor*: therfore you might be ſure if I had not tolde you, that the angle of deflexion muſt be made on your dial VVeſtwards from the plumbe line, for it is a generall rule, if the diall decline Eaſt, then muſt the cocke ſtand on the VVeſt ſide of the plumbe line, if VVeſt then on the Eaſt ſide.

Chapter 17.

To perfourme the laſt chap. a ſecond way by the 6. chap. of ſpherical triangles.

THough I tooke ſome more paines then needes in the laſte chap. to ſhewe you ſome pleaſure, I meane to take the more eaſe in this: for in truth the laſt chap. is onely performed by the 7. chap. of ſpherical triangles, inaſmuch as the queſtion demanded fel out to be the 2. ſides vnknown of a right angled triangle, whoſe one ſide $38\frac{1}{3}$ was knowne, and one of the angles not right, which was the complement of the declition. that is to ſay, the angle betweene the axetree and the *Finitor*: and therefore the laſt chap. and the ſayd 7. chap. of trian. wel vnderſtood, it is as eaſie to doe it by the 6. chap. thus: the declination of the wal giuen, videl. $16\frac{1}{2}$. deg. take that from 90. ſo haue you the complement thereof, vz. $73\frac{1}{2}$. whiche number from the *Limbe* among the meridians in the ſame $73\frac{1}{2}$. meridiã fond, reckon ſide giuẽ being the ſouth poles eleuatiõ, vz. $38\frac{1}{3}$. deg. and thereto lay the rule, & ſo are the degr. of the rule betweene the ſaide $73\frac{1}{2}$. & the *Limbe* the poles eleuation, which you ſhall find, as in the laſt chap. $36\frac{1}{3}$. deg. the deg. of the *limbe* between the rule and the pole, vz. $12\frac{1}{2}$. as before are the deg. of the angle of deflexion: now iudge whether my worke of ſpher. trian. be not profitable, the ſtudy of which brought me to al this, and wil bring any man to much more that will conſider wel of it.

Chap.

Chapter 18.

To performe the ſame a third way.

GOod God ſo infinite is the vſe of this *Iewell*, that I ſhould not know how to finiſh, but that want of time will driue me to haſten one end or other: here haue you now a third way to fynd your angles of eleuation, and deflexion, more apparant without wreſting of the ſpheare then either of the other: I ſhewed before in the 15. chap. that the *Finitor* beyng ſette to the latitude, the Zenith line repreſenteth the South wall, and the azimuthes the declining walles, whoſe pole Zenithes are all ſcituate in the *Finitor*: wherefore now to come to the matter, ſet the *Finitor* to the latitude, and reckon among the azimuthes from the Zenith line towards the pole, the walles declination, and the azimuth there repreſenteth the declining wall: then to finde his eleuation, you muſt ſeeke his meridian or ſome portion thereof, which is alwayes a great circle paſsing by the pole zenith of this wall, and one or both of the poles of the worlde. As for the poles you ſee where they are, and where the pole Zenith of this wall is, cannot be farre to ſeeke: for looke howe much the wall or azimuthe is declined from the Zenith line Southwardes: ſo muche his pole zenith is departed from the South point of the *Finitor*, towardes the centre. VVherefore if you reckon thence inwards the degrees of the walles declination, there is, no doubt, his pole zenith: which being founde, looke what meridian circle you ſee to goe from thence ſtreight to the pole, muſt needes be the meridian of the wall by definition of a meridian, and therefore marke where he cutteth the azimuth of the wal, for his degrees thence reckoned to the pole, is the eleuation no doubt, and the portion of the azimuth betweene this hys meridian, and the Zenith poynte of the *Recto*, is the angle of deflection. This chapter doth depende on the reaſon of the 24. chapter of ſpherical tryangles.

For example, in the wal declining VVeſt $53\frac{1}{3}$. deg. mentioned in the 15. chap. the *Finitor* ſet to the latitude, reckon the $53\frac{1}{3}$. azimuth from the Zenith line ſouthwards, which azim. repreſenteth the ſaid wal: thē to fynd the walles pole Zenith, reckon on the *Finitor* $53\frac{1}{3}$. deg. inwards from the South, & there ſhal you fynd the 60. meridian almoſt counted from the *Limbe*, thence taking his way to the pole, which muſt needs be the meridian of the horizon repreſented by this wall: therefore marke where he cutteth the $53\frac{1}{3}$. azimuth, the diſtaunce on him thence to the pole, you ſhal fynd $21\frac{3}{4}$. deg. the walles eleuation, and the ſpace betweene the ſaid croſſing and the zenith, counted on the $53\frac{1}{3}$. azimuth, you ſhal fynd 32. deg. $\frac{1}{3}$. the angle of deflexiō. But ſee more the forwardneſſe of your *Iewell*, which ſheweth vnasked the difference of longitude of this country from the countrey where to the wall is horizon: vz. 60. deg. almoſt the diſtaunce betweene the meridians, cutting the pole Zeniths of eyther place.

Chapter 19.

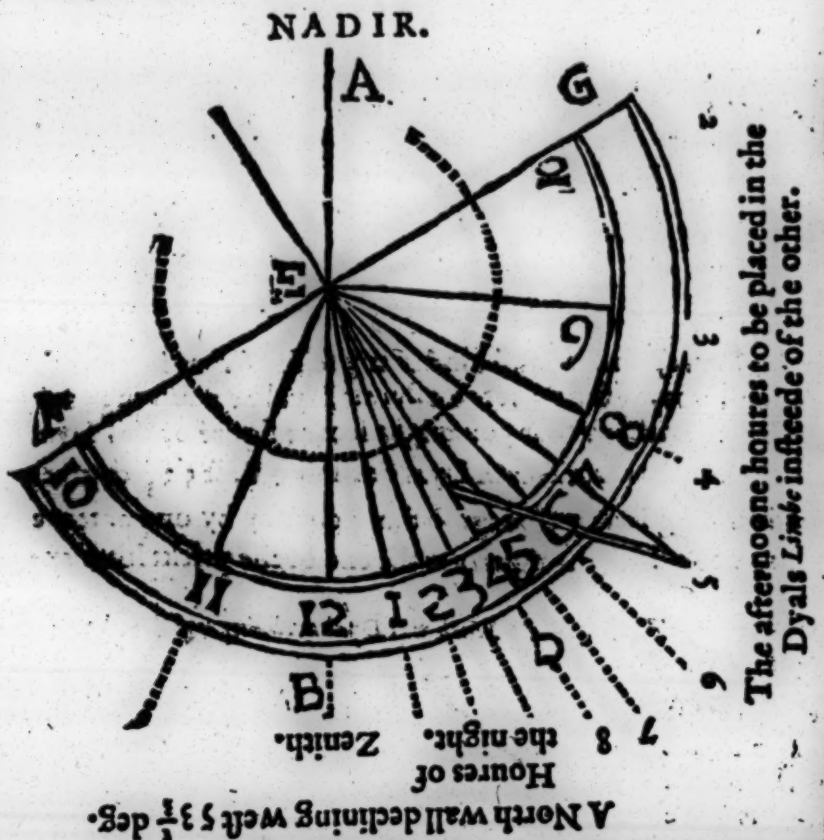
How to make a diall to any declining wall, reſpecting the South.

THe making of declining dialles before now that this my *Iewel* wil eaſe it, hath byn very troubleſome & paineful with a number of lineaments to be put out after the dial ended, of which the Theorical demōſtration was not ſhewed, as in *Munſter* & *Orontius de horologijs*, you may ſee, whereby you had verye fewe but went blindly to woorke on them. I haue ſeene inſtructions in many bookes, how to make the horizon & South wall dialles, as namely in *Albertus Durerus, Daniel Sambecke, Gemma Friſius, Curteiſe*, and diuers other: but whē t hath come to the declining dials, they haue giuen it ouer in the plaine field.

But to the matter, you may note thus much of your ſelfe, that euery declining wal is an oblique horiz. becauſe you are driuen to ſeek his eleuation, wherby you wold think that the dial ſhould differ nothing in making from the oblique diall in the 9. cha. as in effect it doth not, after you haue placed the ſaid angles, & foūd where you muſt begin to reckon your houres. VVherfore in Gods name, firſt make a plumb line on the wal, wherein appoint the centre of your diall: theron making a blind circle equal to your *Iewels Limbe*, as in the oblique diall you made a quadrant: in this blynd circle ſet the deg. of deflexion, (takē from your *Iewels Limbe*) as you are wont, and that below the centre VVeſtwards of the plumbe line, if the wall decline Eaſt: or Eaſtwardes, if VVeſt: thereto from the centre draw a long line where the cock ſhal ſtand, & croſſe the ſame lyne on the centre with another, ſufficiēt for the diameter of your dial: vpō which diameter, make the circle or *limb* of your dial, in which write 12. at the plumb line, for that is alwaies the 12. a clock line: but now to proceede further, for the deg. of the other houre lines, ſet the *Finitor* ſo many deg. vnder one of the poles (the matter is not great which) as the wals eleuation cōmeth to: and ſo doth the *Finitor* repreſent the wall, & the *Limbe* the meridiā, to the wal or to that countrey whoſe horiz. it is. So that if you were in that countrey, you might proceed to make this dial as in the 9. chap. But now to place the houre lines in him to ſerue our coūtrey, you muſt ſeek out on your *Iewel* or meridian: for the *limbe* is here the walles meridian, and from our meridian of neceſsity we muſt haue al the houres reckoned: which to find nūber on the *finitor* placed to ꝑ eleuatiō as before, the angle of deflexion from the *limbe* inwards, marke what meridian cutteth the *finitor*, there the ſame is the meridian of our coūtrey, which when you haue, marke him wel that you may know him: for it is he that leadeth this daunce, & is already repreſented on the wall by the plumb line. VVherefore if beginning from this meridian, you diſtinguiſh on your *Iewel* euery 15. going thence in order from 15. meridian to 15. quite roūd, returning at the *limbe* til your compaſſe be vp: theſe meridians or rather houre circles thus diſtinguiſhed, ſhall cut the deg. on the *finitor* ſo ſtanding to the walles eleuation anſwearable to euerye houre in all reſpectes, as in the oblique dial in the 9. chap. which deg. by help of the blind circle made before, you may place in your dial, remembring that you muſt here begin from our new found meridian, & not frō the *Limbe* as there: & the ſpaces, deg. or houres from this meridiā towards the *limbe* of your new *iewel*, muſt be ſet in your diall towards the line where there the cock ſhal ſtand and ſo round: the reaſon is for that the *limbe* is now the meridiā of the wal, & on the dial is the line of the cock, wherby you may ſee that the cock will not ſtand out of the meridian of his horizon.

For

For example I will take in hand to make a Dyall to the wall, mentioned in the 15. Chapt. declining VVest 53$\frac{1}{3}$ deg. therfore the cocke muſt ſtande on theaſt ſide of the plumbeline by the ende of the 16. Chapt. Firſt I draw a perpendiculer on this wall vz. A B by helpe of a plumbe rule, therein I pitch the centre of my Dyall vz. E. whereon I make a blinde circle equall to the graduated circle on my *Iewels Limbe*, as in the 9. Chapt. whenſe I take in my compaſſe the quantitie or degrees of the angle of deflexion founde by the 16. 17. or 18. Chapt. vz. 32$\frac{1}{3}$ and ſet the ſame in the blinde circle Eaſt from the plumbeline at C, becauſe the wall declineth VVeſt by which and the centre E. I draw the line E C D, and croſſe the ſame ſquare on the centre with the line F E G, which ſhall be the dyametre of my Dyall, to which on the centre E, I draw two ſemicircles or more, to ſerue for the *Limbe* of my Dyall as you ſee, wherein at the plumbline, I write 12. This prepared, I ſet the *Finitor* of my *Iewel* to the wals eleuation, founde in the laſt Chapter 21$\frac{3}{4}$ deg. whereon I reckon inwards the angle of deflexion 32$\frac{1}{3}$ deg. and there do I ſee to croſſe the *Finitor*, the 60. meridian almoſt counted from the *Limbe*, which is now become our meridian, and is he that we muſt truſt to, for he is alreadie repreſented on the wal by the plumbeline. Now becauſe this 60. meridian is an houre circle alredie marked and diſtinguiſhed on the *Iewel*, I neede not diſtinguiſh him a newe, nor any of the reſt, for that if I reckon from 15. to 15. from him they all fall out in the ordinarie houre circles, which for the moſt part will not ſo fall out though it chaunced ſo here. But now being as it is, marke what degrees of the *Finitor*, the next fifteene houre circle from this 60. meridian towards the *Limbe* doth cutte: you ſhal finde 12$\frac{1}{3}$ degrees, which place in your blinde circle from the plumbeline towards the cocke, like as in the oblique Dyal, thereby draw an houreline from the centre of your Dyall, the ſeconde houre circle towards the *Limbe*, cutteth in the *Finitor* 20$\frac{1}{3}$ deg. counted ſtill from the interſection of our newe found 60. meridian, the thirde towards the *Limbe* cutteth 27. deg. on the *Finitor*, theſe two notes ſet alſo in the blinde circle, and by them drawe two houre lines more: well, the fourth houre circle is the *Limbe* it ſelfe cutting on the *Finitor* 32$\frac{1}{3}$ deg. being the angle of deflexion which is alreadie placed in your Dyall where the cocke muſt ſtande. But nowe for the fift houre you muſt returne backe againe on the *Finitor* from the *Limbe* where you ſhall finde the 5. houre circle to cut in the *Finitor* 38. degrees counted from the ſaide 60. meridian to the *Limbe*, and thenſe backe againe: the 6. houre circle cutting 44$\frac{1}{3}$ deg. the 7. 52$\frac{1}{3}$ degrees, the 8. 66. deg. all which placed in the Dyall as before, you neede go no further that way, for that the ſunne at the moſt in our country yealdeth but 8. houres before or after 12. Now for the hourelines on the otherſide of the plumbline, begin againe from the ſaid 60. meridian, but the contrarie way towards the centre. and you ſhall finde the firſt houre circle to cutte 24. degrees: the ſeconde 57$\frac{2}{3}$ deg. which, place in your Dyall, and you haue done: For beyonde the dyametre you neede neuer go in a perpendiculer wall. Laſtly the hourelines thus drawne, their numbers a childe almoſt may ſet in by the 12. of clocke, or by this figure. And as for the cocke let him be erected verie plumbe from the wall in the line of deflexion drawne for him: his length may be equall to the Dyals ſemidiametre, or ſhorter if you will, but his height muſt needes be to the walles eleuation vz. 31$\frac{1}{4}$ and thus haue you a ſinguler peice of skill, which being wel vnderſtoode is more eaſie then to do it Geometricallie after Orontius or Munſter, but ere I finiſh my *Iewel* I will teach to do it Geometrically more eaſie then theirs by farre, hauing but a modicum of my *Iewels* helpe.



The afternoone houres to be placed in the Dyals *Limbe* inſteede of the other.

Chapter 20.

How to make a Dyall to any declining Wall reſpecting the North.

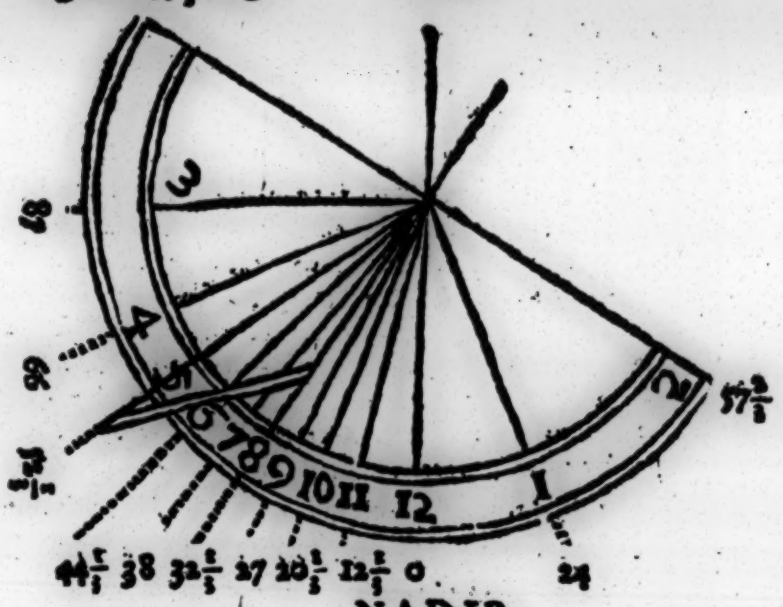
THere is no diuerſitie of making a North declining Dyall, from the South declining, mentioned in the laſt Chapter, but that where as there you did drawe the circumference downewards vnder the centre towards the South pole: heere you muſt make him vpwards towards the Zenith, reſpecting the North pole, and then following on the rule in all reſpectes as in the laſt Chapt. Namely if the wall decline Eaſt, ſet the cocke on the VVeſt ſide of the plumbeline and ſo contrariwiſe: But now is the plumbline 12. of clocke at midnight, whereby you may number the reſt of the houres, the cocke alſo muſt be ſet vpwards, according as I tolde you in the 11. Chapt. and the ſuperfluous houres of the night left out. There is ſo little diuerſitie in the making, that looke what Dyall you make to a fourth wal declining weſt the very ſame Dyall will ſerue to a North wall declining weſt, like much, if you turne him vpſidedowne and alter the numbers: and ſo like wiſe the South declining Eaſt, ſerueth the northeaſt as by this figure you may plainely ſee, being but the Dyal of the laſt Chapter, turned topſitoruie and the numbers of the houres altered vnto thoſe which I haue plaſed

at the endes of the prickt lines as you see, and nowe serueth to a North wall declining VVest 53 1/3 degrees, whereto the sun commeth not before 2. of clock afternoone, and there continuing til eight of clocke at night, all the rest of the houres be superfluous.

A South wall declining East 53 1/3 deg.

Zenith.

A North wall declining East 53 1/3 deg.



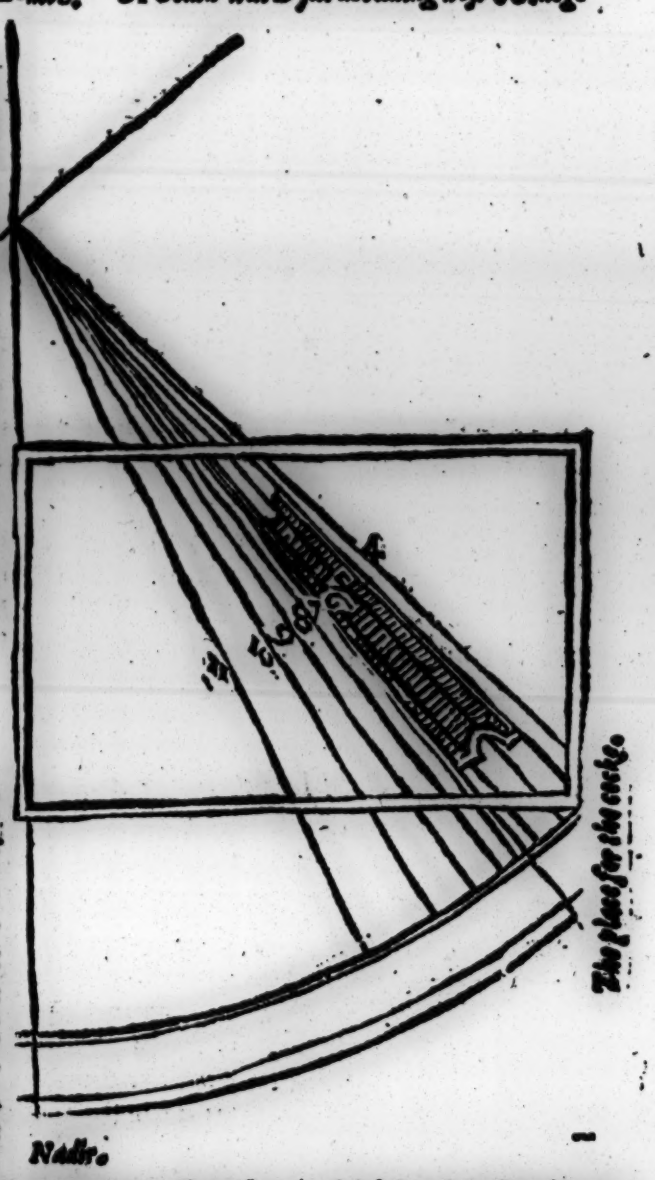
NADIR.

And heere if I should conceale one thing, I should offer you iniurie and hinder my selfe of that I pretende which is breuitie and facillitie: I will therefore that you know if the patterne of your Dyal be made in paper first, as I commonly vsed, and that the paper were oyled or prickt so that you might see the figure of this Dyal on the backside, the same doth represent the cocke turned vpwards, a North Dyal declining East 5 3 1/3 degrees, & the cocke turned downwards a south wall Dyal declining East 5 3 1/3 deg. so that you may by one paterne make 4. seuerall Dyals as by these figures you may see, vnto which the figures at the end of the prickt lines expresse the houres.

Chapter 21.

How to order Dyals that decline very farre East and west.

Zenith. A South wal Dyal declining west 80. deg.



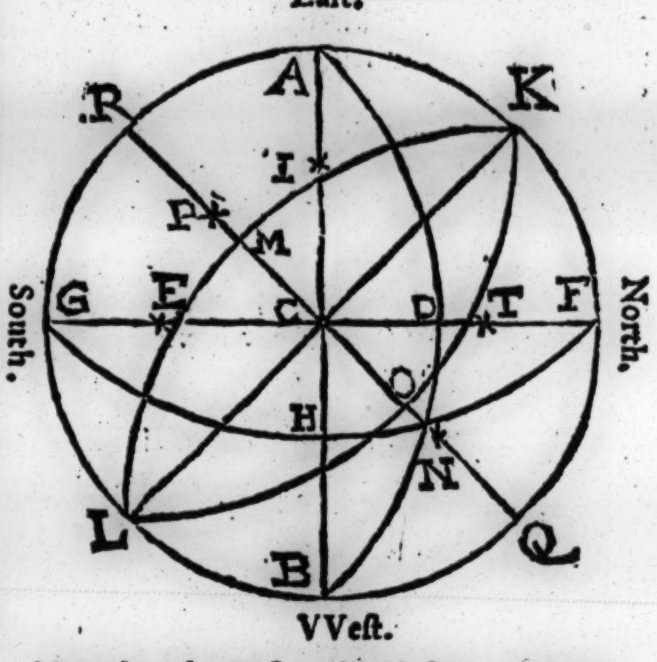
I Shewed you before in the 12. Cap. that euery east & west wall is a right horizon, hauing the hourelines of the Dyall parallel, & the pole nothing eleuated to that wall, because he standeth in the meridian & cutteth the pole: wherefore it must needs appeare that the more any wal declineth towards the East or VVest, the lesse the pole is eleuated, euen vnto nothing at the last, whereby the hourelines neere the cock or meridian of the wall, grow so neere togither that towards the centre they thrust one vpon another, & farther off they shew almost parallel: for in a wal declining 80. deg. east or west, some of the houre spaces are not aboue 2. deg. or little more, which space except the dial be monstrous big, cannot be sufficiently discerned, wherefore you may cut of the dyal, cock and all, with a fashionable square like an east Dyal, as you see done in this figure, which the sheweth almost like vnto it, you may make him first in paper, & then hauing cut off all superfluitie, set the 12. of clock line euen with the plumbline, and so by the paper prick the other houres.

Chapter 22.

Of wals reclining and inclining, and of the diuersitie of them.

I Shewed in the 14. Cap. the diuersities of declining wals and that euery perpendiculer wal declining or not declining lay in one or other of the azim. their pole zenithes being all of them in the horizon or *Finitor*. In like manner reclining wals are flats, whose bases lie leuell with the diametres of one or other of the azim. crossing all vpon the centre of the earth, but their flats recline or beare backe from the zenith, & their pole zenithes are eleuated aboue the horizon in the azim. standing square to their base as much as the reclination commeth to. The inclining wall or flat, for his base he is like the reclining, lying in the diametre of one of the azim. but his face or flatte is inclined forwards, lying groueling from the zenith: in so much that his pole zenith is sunck so much vnder the hor. as the inclination cõmeth to, which by this figure you may conceiue, wherin the circle A F B G is the horiz. the centre C the zenith, the straight lines crossing at the centre

centre repreſent both azimuthes and their diametres, the arch A D B is a reclining wall, whoſe baſe is the line A B, being the dyametre of the azimuth A C B, this wall A D B reclineth from the zenith as manie degrees as C D containeth, wherefore his pole zenith ſhalbe eleuated in the azimuth ſquare to his baſe vz. F C G, as much as C D commeth to, and is the point E, ſo that G E is equall to C D. Likewiſe the arch A D B repreſenteth an inclining wall, whoſe baſe is A B alſo, bending forwards towards the point F from the zenith C, according to the quantitie of C D, whoſe pole zenith is the point T, which you muſt imagine depreſſed or ſunke vnder the horizon A F B, in the ſame azimuth F C G, as much as D inclineth from C, or as the point E is eleuated aboue the horizon, and ſo of all the reſt in this figure. And thus vpon the diametre of any azimuth whatſoeuer, there may be imagined both a reclining and inclining wall, nay rather a number of both kindes according to the reclination of the point D from the centre C. Then to proceed further, know that as the diuerſitie of perpendiculer wals are eaſt, weſt, north, ſouth, and declining, ſo is there the like of theſe, for either the baſe of reclining, or inclining walles lie in the dyametre of the Eaſt azimuth, as A B, and then are they called North or South, reclining or inclining, or in the diametre of the meridian or ſouth azimuth, as F G, and then are they Eaſt or VVeſt, reclining or inclining, or elſe in the diametre of ſome other of the azimuthes, as K L, or Q R, or any other, and then are they called reclining walles declining and inclining wals declining: which you may deuide and ſubdeuide, I meane their denomination as in the perpendiculer wals in the 14. Chapt. as thus, K O L is a ſouth reclining wall, declining Eaſt, becauſe his pole zenith is the point P, his reclination is the ſpace C O, his declination R G, alſo K L O is a north inclining wall, declining weſt, whoſe pole zenith is the point N vnder the horizon, & thus you may ſpell them all, and as many other as you liſt to imagine, *tu ipſe annitere potis.*

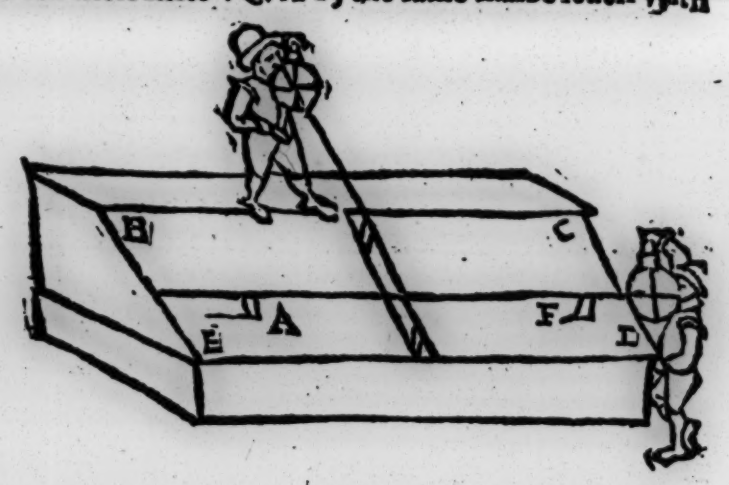


Note that the Eaſt and VVeſt are miſtaken in making this figure.

Chapter 23.

How to take the reclination, or inclination of any wall or flat, togither with his declination if any be.

VVHen you come to any reclining flatte, firſt by helpe of a Maſons leuell or ſome other deuiſe, draw thereon a line parallel to the horizon, which by helpe of your *Iewel* you may alſo thus do, ſticke a long ſtile or wyer vz. A plumb for the flat B C D E, and then ſetting your *Iewel* to your eye, the rule with ſights lying on the line of leuell, go euen with the ſide of the flat through the ſight, beholding the ſtile, and let ſome other bodie ſticke vp another ſtile vz. F, directly betweene your eye and the firſt ſtile, plume alſo from the flat: then drawing a line by both theſe ſtiles vz. A F, the ſame ſhalbe leuell with the horizon: this horizontall line I will call the baſe line of the reclining flat, to which I drawe another croſsing him ſquare, which I will call the baſes perpendiculer, both which drawne, I then applie my *Iewel* by helpe of the ſquare boorde or ſquire, mentioned in the 15. Chapt. vnto this baſe line leuell with him horizontalwiſe, and ſo by the doctrine of the 15. Chapt. take the declination of this reclining flat in his baſe line, euen as though it were a perpendiculer wall: which I note downe in a paper or tables, but if this baſe line haue no declination, that is to ſay, be full Eaſt, VVeſt, North, or South, then is the ſame flat barely reclining, and not reclining declining, which reclination to take do thus: erect two ſhort ſtiles in the baſes perpendiculer, both of equal height aboue the flat and plumbe thereto: then if the flat be a ſlope wall, as of a buttreſſe or ſuch like, hang your *Iewel* to your eye, remooving your rule with ſightes till you may through them ſee the very tops of the two ſtiles, and ſo are the deg. of your *Iewels Limbe* counted from the line of leuell, the mounting of this flatte aboue the horizon, the complement wherof being the deg. betweene the rule and the zenith or handle of the *Iewel* is the flats reclination, but if it chaunce to be the bancke of a garden, or any other that you may go aboue it, then is your beſt way to place your ſelfe aboue the ſame, and to looke downewards through your ſights by the ends of the two ſtiles.



For example Anno Dom. 1581. A Gentleman of auncient houſe and great worſhippe, Humphrie Foſter Eſquire by name, being elected heigh ſhiriffe of the countie of Berkes, beſtowed the ſame yeare ſome coſt about his houſe at Aldermarſton, where amongeſt other thinges, hee trimmed vp two Gardens with walkes vppon heigh ſlope banckes as the order is: and for becauſe one of the banckes lay verie open to the ſunne, and to the proſpect of moſt of the Chambers and roomes of the houſe, it pleaſed him to vſe my ſimple helpe, being his verie neere kinſeman, and ſuch a one as not without greate cauſe hee might commaunde, to make a greate Dyall at the whole bredth of the bancke being almoſt three yeardes deepe: wherefore I laide aſtreight boorde on the toppe of the bancke directly with the edge thereof, which

ſerued me for my baſe line, becauſe the banck was leueled, of which I tooke the declination by the 15. Chap. and founde it 78½ degrees, then did I by the firſt concluſion direct a perpendiculer from this baſe line, vnto the foote of the bancke, wherein inſteede of ſtiles I ſtucke two kniues of equall height aboue the banck: and then chooſing my ſtanding on the heade of the bancke, ſo that I might through the ſights of my *Iewel* ſee the verie toppes of both knyues, I found the end of the rule next my eye to cut on the *Limbe* 34. deg. which counted from the line of leuel which was the mounting of this bancke aboue the horizon, the complement whereof vz. 56. degrees, was his reclination from the zenith, being the degrees betweene the rule and the handle, or toppe of the *Iewel*, ſo that I was to conclude that this bancke was a South reclining flatte 56. deg. declining Weſt 78½ deg. whereto I will ſhew to make the Dyall in the 28. Chapt. But now for to take the declination and inclination of any flatte, the working is all one, as by experience you ſhall better finde then by manie words.

Chap. 24.

How to make a Dyall to a ſouth or north reclining or inclining wall, bancke or flatte.

A South or North reclining or inclining flatte is ſuch a one by the 21. Chapt. whoſe baſe lieth in the Eaſt line of the horizon, and his pole zenith in the meridian circle, which well vnderſtoode, and the reclination taken and knowne by the laſt Chapt. the making of the Dyall thereto is moſt eaſie. Firſt ſet the *Fmiter* of your *Iewel* to the latitude: then if the flatte recline or incline Northwards from the ζenith, remooue the zenith line of your *Reete* ſo many degrees towards the North pole, as the reclination commeth to, or towards the ſouth pole, if ſouthwards: and you ſhall thereon ſee preſently which pole, and howe many degrees the ſame is eleuated aboue your flatte, to which eleuation make your Dyall, either by the 9. or 10. Chapt. placing the cocke and 12. a clocke in the baſes perpendiculer ſouthwards, if on your *Iewel* you finde the ſouth pole eleuated: or northwards, if the north pole.

As for example, admit that in our latitude being 51⅔ deg. by the 23. Chapt. I founde a ſouth flatte reclining 20. deg. Firſt I ſet the *Finiter* to our latitude by the 2. booke 19. Chapt. there I finde the zenith point of the *Reete* to ſhew 38⅓ deg. counted from the north pole. Thence I remoue him according to the reclination found vζ. 20. deg. more from the ſouthwards, where I finde the North pole diſtant, or rather depreſſed 18⅓ de. vnder this flat, being the zenith line: but looking towards the other end therof, there I finde the ſouth pole 18⅓ degrees eleuated aboue this flat by the 10. preamble. For this is generall that the zenith line, I meane drawne forth vnto his Nadir, ſupplieth al flats, whoſe poles zenith & poles Nadir lye in your meridian circle. VVherefore now by the 9. or 10. Chapt. making an oblique Dyall to 18⅓ eleuation, his cocke and 12. of clocke ſtanding in the baſes perpendiculer turned towards the ſouth, you haue done. And the ſame Dyall the 12. and cocke turned north, ſerueth the north flat in our latitude inclining 20. deg. But if a South wall be reclining, or a North wall inclining according to the Equinoctials height aboue our horizon vz. 38⅓ degrees. then will the zenith line one your *Iewel* lie in the two poles, ſo that neither is eleuated. Wherefore by the 8. Chapt. you may know the ſame flat to be a right horizon, in which Chap. the Dyall is alredie made thereto. If a South wall recline or a North wall incline more then the Equinoctials height in your latitude: as admit with vs 60. deg. your *Iewel* firſt placed to the latitude, remoue the ζenith line towards the north pole and beyond, till he come to 60. deg. from his right place, and you ſhall there finde the north pole diſtant, and eleuated aboue the ſaid ſouth reclining flat 21⅔ deg. and the ſouth pole eleuated or rather depreſſed vnder the ſaid North reclining flat as much vz. 21⅔ deg. for the zenith line repreſenteth them both: to which eleuation you may make their Dyals by the 9. & 10. Chap. as is aboue ſaid. In the very like manner is it with north reclining and ſouth inclining flats *vice verſa*.

Chapter 25.

How to make a Dyall to an Eaſt and Weſt wall, bancke or flat reclining or inclining.

THe baſe line of euery Eaſt and Weſt reclining and inclining flat lieth directly in the meridian line, as in the 21. Chapt. and in truth euery of themſelues lyeth in the flat of one or other of the circles of poſition: (that is to ſay) they all croſſe vpon the interſection of the meridian with the Horizon, as the declining wals do in the zenith and Nadir, their poles zenith and Nadir lying all in the circle of the Eaſt, called of ſome the verticall circle. What a circle of poſition is, looke the 3. Booke 43. Chapt. The Dyall to euerie of theſe is made in all reſpects as the declining wall dyall. And to make the matter more plaine to the learner, whom I would as willingly helpe as I onſe could haue beene contented to be holpen, know that all reclining and inclining Dyals of this ſort are the very declining wal Dyals to that country which hath a ſouth latitude equall to the complement of your latitude being heere at Reading 38⅓ degrees. Wherefore if you did but imagine your ſelfe in a South latitude, of 38⅓ deg. & turne your *Iewel* vpſidedown accordingly as you do here in your owne, you may eaſily perceiue the zenith line to be there your verticall circle, and all the Azimuthes your declining wals, which are heere circles of poſition, and may make your Dyals to them by the 19. Chapter. Nowe notwithſtanding I thinke it plaine ynough to ſome alredie, yet I will not ſticke for other ſome to proſecute this Chapter demonſtratiuely, the rather becauſe I haue ſet but one example of declining VVall Dyalles. Admitte therefore heere in our latitude being 51⅔ at Reading: by the 23. Chapter, I founde an Eaſt VVall reclining from our zenith 50. degrees, I call that an Eaſt reclining

wall

wall or flat, whose pole Zenith is in the East quadrant of the verticall circle. To make a diall to this flat, you shall in steede of the *finitor*, set the Zenith line to the latitude, vz. $51\frac{2}{3}$. degree, vnder the North pole, so shall the azimuthes represent all the East and VVest, reclining or inclining flats, and the *finitor* line now supplieth the Zenith line for this turne. Then since my wall proposed, reclineth 50. degrees from the zenith, I reckon so many degrees on the *finitor* from the *Limbe* inwardes, and there doe I finde the 50. Azimuth representing my flat sought for: aboue which also I may see the North pole eleuated: but howe much, since it appeareth not, you may thus finde. Marke which of the almicantares cutteth the pole, and where he crosseth this 50. azimuth: if thereto you set the *label* fixed: the degrees thereof betweene this crosing and the *limbe*, you shall finde $36\frac{3}{4}$. which is the poles eleuation aboue that flat, and the degrees of the *limbe* betweene the *label*, and the pole, $39\frac{2}{3}$. the angle of deflection. Thus may you finde the eleuation and deflexion of any of the azimuths either in this constitution or any other like, as declining walles, &c. For it is but the rule of the 17. chapter, somewhat more plainely set downe for this purpose. These had, you shall draw through the middest of your reclining flat, his base line parallel to the horizon, vz. A B, which, as is said, in all these dialles, lyeth directly North and South, euen in the meridian line. In this base line A B, pitch the center of your diall, vz. D, and thereon describe a *Limbe* for your diall to write the numbers in, and within it a blind circle equall to your *Iewels limbe*, wherein set the note C, $39\frac{2}{3}$. de. (being the angle of deflectiō) aboue the base line Northwards: because the North pole is eleuated, drawing foorth the line D C, where your cocke must stande in all respects, as in the 19. chap. you did in the declining wall, from which this little had differed, if in steed of the reclination, I had taken this complement being 40. degrees, vz. the mounting of this flat aboue the horizon: and reckoned this 40. from the Zenith line, where here I counted 50. from the *limbe*, all commeth too bee one azimuth. But to proceede, now you must remoue the *finitor* of your *Iewel* to the walles eleuation founde) that is to say) set him $39\frac{3}{4}$. deg. vnder the pole, wheron from the *limbe* inwards, reckon the angle of deflection, vz. $39\frac{2}{3}$. deg. which angle alwayes hunteth out our meridian, for there shal you see to cut the *finitor*, the 54. meridian counted from the *limb*, which is he frō whence we count all our houres: being in this figure, the line A B, the *limbe* of the *Iewell* is as hath been saide in the 19. chapter, the meridian of the flatte, and representeth the line D C, where the cock stādeth. In this figure frō the crosing of this 54. merid. with the *finitor*, reckō to the next 15. meridian towardes the *limbe*, which you shall see to cut in the *finitor* $13\frac{1}{3}$. deg. frō the crosing: those deg. I set in the blinde circle from the base line A B, towardes D C, the line of deflection, and thereby draw a line thorow the same to the dials *limbe* for my first houre on that side: then look I on my *Iewell* to the next 15. meridian, being 30. meridians from the said crosing he cutteth on the *finitor* $24\frac{1}{2}$. deg. counted from the crosing for the second houre line. Then the next 15. meridian beyonde that, he cutteth 34. deg. of the *finitor* for the third houre line: these two houres I set likewise in my diall: then haue I but 9. meridianes more to the *limbe*, therfore I must reckon thence backe againe tyll 15. be vp, which are 6. deg. backe, reckoning on the *finitor* from the said crosing to the *limbe*, and thence backe againe to the same 6. meridian I finde so 43. de. for the fourth houre: which being set in my diall, falleth out beyond the line of deflection, so that heere the cocke standeth betweene two houres lines. And so proceeding for the rest, as in the declining diall; you shall finde 53. deg. for the 5. houre line, $63\frac{1}{2}$. for the 6. 76. for the 7. $93\frac{1}{2}$. for the 8. and so foorth if neede were. But in our countrey 8. houres from the 12. any way is enough at the most. In like maner for the houres vnder the base line, count on the *finitor* from the said crosing towardes the center of the *Iewel*, and you shall find $17\frac{1}{2}$. deg. for the first, 40. for the second, 65. for the third, $86\frac{1}{2}$. for the 4. and so foorth if neede bee. All these severall notes made in the blind circle, and houre lines by them drawen to the *limbe* of your diall, and your cocke set Northwardes plumbe from the flat in the line of deflection C D, equall in height to the walles eleuation, vz, $36\frac{3}{4}$. deg. hygh, your dyall is made. The base line A B, beeing that it lyeth in the meridian, must needes be 12. of clock. The rest of the numbers you may by reason set to the other houres. All which by the figure

$36\frac{3}{4}$

An East flat reclining 50. deg.

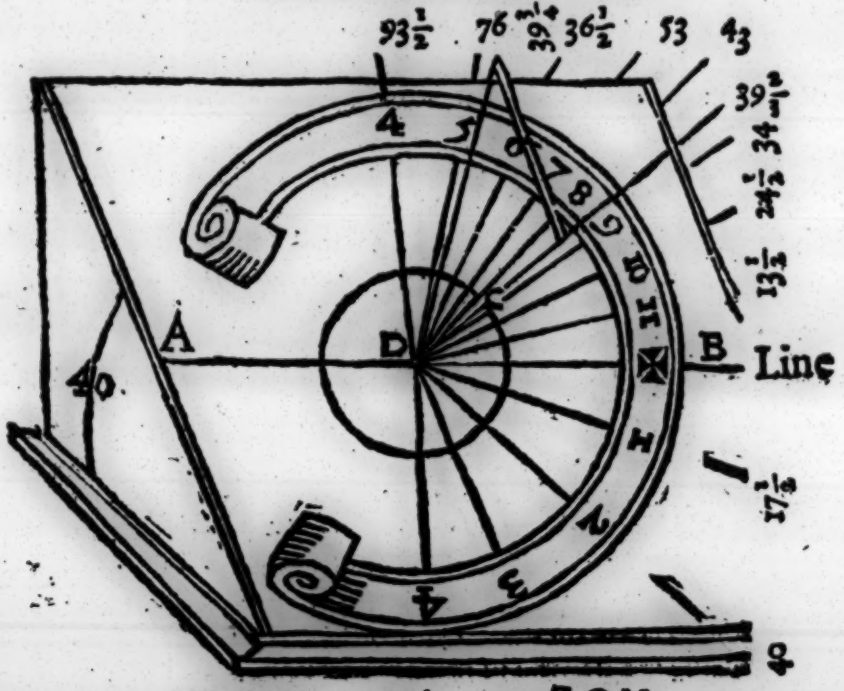


figure before you may moſt plainely ſee.

Then for Eaſt and VEeſt inclining flats, the dials are made in very like maner, ſauing that the line of deflection lieth Southwardes, & vnder the baſe line the cocke beholding the South pole, ſo that this diall would ſerue to a VVeſt inclining flat, 50. deg. being turned vpſide downe, & the numbers altered: of all which my minde giueth me that my *Iewel* ſheweth the reaſon ſo theorically that I neede ſpend no more ſpeech thereof.

Chapter 26.

How to finde the angles of the poles eleuation, dials deflection, and meridians aſcenſion, to any reclining or inclining wall declining.

BY the title of this chap. you may ſee that there belongeth to the making of a reclining diall many angles which muſt firſt ſphearically be gotten before the diall can bee made: namely his reclinatiõ frõ the Zenith, his declination from the North or South in his baſe line, his deflection from your meridian, and nowe laſtly another angle lept in, which is the aſcenſion of your meridian aboue the baſe line, which in the laſt diall were both in one ſelfe ſame line: therefore what did I ſay in the 19. chap. that the declining dialles were harde to make before time? It is not ſo perhaps, if that be true, then were inclining and reclining dials more harde, nay moſt hard, yea and ſo intricate, that I haue not ſeene any to haue perfourmed them, but only Andreas Sconerus, yet hee without any theoricall demonſtration. But it is ſuch an intricate peece of work and the worſe by miſprinting that he himſelfe had neede come againe to vnderſtand it. There muſt be ſuch a ſort of circles, lineaments, ſuch a number of quadrants and deuiſes drawen and diuided vpon the wall or paper in ſuche abundance ere the diall can beee made that a man would de thinke it a wilderneſſe of lines rather then any reguler peece of worke. Tell me then, if the diamond be ſet by, in that he is rare and ſhineth in the darkeſt places, and cutteth deepe where nothing els wyll ſcratche, who dare cauill that I call my inſtrument a *Iewell*, that ſheweth the moſt hidden & difficult matters of the ſpeare as broad as day, and performeth the hardeſt with plaineſſe & eaſe. But now to come to this defuſe matter which hath balked a great many who haue been very buſie to ſhew their skill in other ſlighter dials.

VVhen you haue the reclination of a wall, flat, or banke, together with the declination of his baſe line, as in the 33. chap. is ſhewed: Firſt place the *finitor* in the axtree line of your *Iewel*, which here ſhall be your meridian, thence remoue the *finitor* frõ the South pole towards the left hãd, if it be of a South flat reclining, &, his declination VVeſt or rightwards, if Eaſt according to the deg. of declination, and ſo ſhall your *Iewell* remayne in an excellent conſtitution for this purpoſe As in example of the banck mentioned in the 23. chap. the declination of his baſe line was $78\frac{1}{2}$. from the South VVeſtwards, the reclination 56, deg. VVherefore the *finitor*, firſt laid in the axtree, which heere I meane ſhall repreſent our meridian: remoue the South end thereof leftwards $78\frac{1}{2}$. deg. which done, you ſhall finde the Zenith point of the *Reete*, ſtanding $11\frac{1}{2}$. degr. from the North pole. Now vnderſtand what your *Iewell* doth here repreſent. Firſt the *limbe* is our horiz on, the axtree line is the Eaſt wall or meridian in whiche the North pole muſt needes be found, the Equinoctiall is the South wall or Eaſt azimuth, the center is here the Zenith of our horiz on, the right hand at this worde *meridies* is Eaſt, and the left hand at *media nox* VVeſt, in reſpect the *Iewell* is heere imagined to lie flatte, the Zenith line repreſenteth the ſide or baſe line of the banck, the *finitor* the baſes perpendiculer, wherein the reclination is taken. Laſtly, if on the *finitor* from the center you reckon the reclination of the banck, vz. 56. deg. there haue you the azimuth repreſenting the bancke or rather the horizon, wherein the flat of the banck lyeth, whereto our dyal muſt be made. Now ſir that you ſee the ſtanding of all the circles of the ſpheare needfull for thys purpoſe in theyr due conſtitutiõ, it is ſome cunning yet to get thoſe angles, which this chap. propoſeth, notwithſtanding one of them is there at hande: for if you reckon from the Zenith poynt of the *reete*, the deg. of the ſaid 56. azimuth vnto his croſsing with the axtree line, you ſhal finde them $13\frac{2}{3}$. deg. which is the angle of our meridians aſcenſion, that is to ſay, our meridian croſſeth the baſe line of this bancke making an angle about it of $13\frac{2}{3}$. deg. for the other angles, firſt reckon in the axtree line, which now is our meridian as in the 16. chap. you did the poles eleuation, vz. $51\frac{2}{3}$. deg. from the *limbe* beeing nowe the horiz on, there make a prick with inke for the pole: for now if you can deuiſe with all your skil where ỳ pole Zenith of the bank lyeth on your *Iewel*, and from him to lett downe an arch of a great circle, cutting th e ſaide pole or pricke of inke in the axtree line, hee muſt needes repreſent the meridian of the banckes horiz on, by the 1. booke and 4. chap. and ſtande perdendiculer thereon, wherefore his portion or arch betweene the 56, azimuth, and the ſayd pole or pricke muſt needes bee the poles eleuation & the deg. of the 56. azimuth betweene it and the axtree ſhalbe the angle of deflexion. Now to make this matter plaine by demonſtration, reaſon thus. The Zenith line repreſenteth a wall declining VVeſt $78\frac{1}{2}$. deg, that is to ſay, a wal perpendiculer on the baſe line of this bancke, therefore his poles Zenith & Nadir muſt needes bee in the endes of the *finitor*: but ſince this bancke reclineth from the Zenith beyng the center of the *Iewel* 56. deg. rightwardes: therefore his pole Zenith doth come in as much on the other ſide from the *limbe* towardes the center in the *finitor* line, as in the 4. booke and 4. chapter, and 5. booke and 24. chap. hath been ſaide. This 56. deg. on the other ſide of the *finitor* founde, you ſhall nowe for neceſities ſake lay the *label* on the axtree line, and therein make the foreſaid prick for the pole, and there the *label* fixed to the *reete*, you ſhall turne about the *reete* and *label* ſo fixed tyll you find ſome of the meridians of the *mater* too cut both the ſaide pole zenith founde in the *finitor*, and the ſaide pole pricke in the *label*, which you ſhall finde to bee done by the $65\frac{1}{2}$. meridian counted from the *limbe* there ſtay your *reet*, and reckon on the ſame $65\frac{1}{2}$. meridian the degrees betweene the ſaide pricke in the *labell*, and the ſaide 56. az imuth which you ſhall finde $35\frac{1}{2}$. deg. the ſame is the eleuation of the pole to this bancke. Then reckoning on the 56. azimuth from his croſsing with this $65\frac{1}{2}$. meridian vnto the fiduciall line of the *labell*, which nowe repreſenteth our meridian, you ſhall finde 28. deg. the angle of deflexion, and ſo much muſt the

cocke

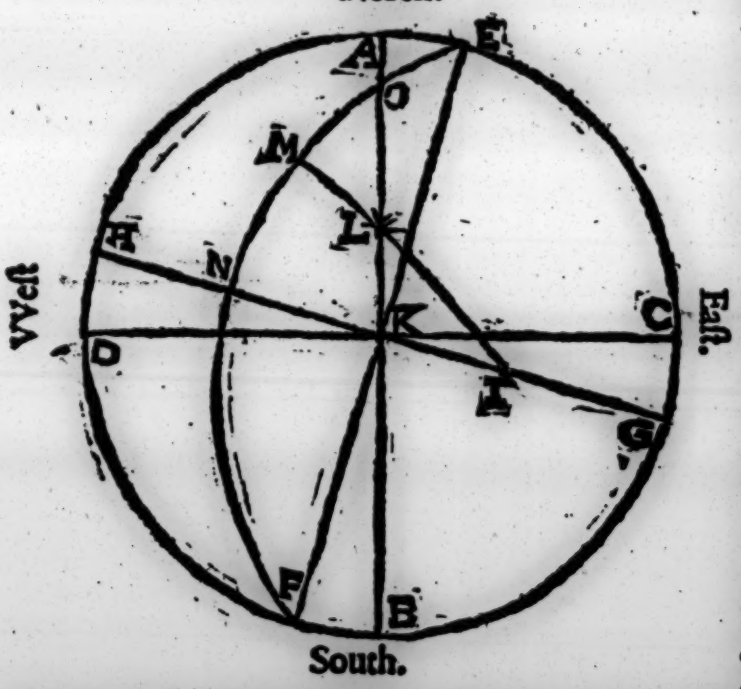
cocke ſtande aboue the meridian: and thus haue you gotten all thoſe angles, beholde the figure of the nexte chapter, which if you marke well doth after a ſort repreſent your *Iewell*, in the conſtitution of this chapter.

Chapter 27.

How to perfourme the laſt chapter by ſphearicall triangles very eaſily, and with great pleaſure and profite.

SOme perhaps will ſcant giue credite to the finding of the angles in the laſt chapter, perhaps miſtruſting my skill, ſince ſo many learned men haue been contented to paſſe by them: ſome againe may happily not well vnderſtande that hath been ſaide, and other ſome, I preſume loue more wayes to the wood then one who knowe well that change of paſture maketh fat calues, and therefore to all ſortes I thought thys chap. very conuenient. You ſhall as I taught you in my booke of triangles, drawe ina figure ſuche circles of the ſpheare, as you are to deale with in this queſtion, as here I haue done it to your hande, partly by ame, whiche for this turne ſerueth as well as the trueſt draught, where in the circle A B C D, repreſenteth our horizon A B. our meridian, C D, our Eaſt line, E F, the baſe line of the banck propoſed, G K H, the azimuth croſsing hym ſquare, in which the banckes pole zenith muſt needes be by the 22. chapter E N F, is the great circle of the ſpheare wherein the flat of the banke lyeth: A, the North poynt of our horizon B, the South, D Eaſt, C VVeſt, the center K, our zenith K N, the reclination of the banck, vz. 56. deg. I, the pole zenith of the bancke eleuated aboue G, as muche as N reclineth from K, by the 4. booke and 4. chap. G, the pole zenith of hys baſe line E F, or rather of his perpendiculer flat E K F, declined from the South pointe B, according too the quantitie of G B, founde in the 23. chapter to be $78\frac{1}{2}$. degr. the pointe L is the North pole eleuated in our meridian circle A K B, aboue hys North point A, accordyng to the quantitie of A L, being $51\frac{2}{3}$. degrees.

Your plate being thus laide and well vnderſtood, then begin to deſcant on it thus. And firſt too get the eleuation of this bancke ſince the angle G K B, is knowen to be $78\frac{1}{2}$. deg. his complement B K F, ſhal then be $11\frac{1}{2}$. deg. by the 5. booke, and firſt chap. to which the angle E K A, is equall by the 5. theoreme: E G is a quadrant or 90. deg. to which adding E A, being $11\frac{1}{2}$. deg. it maketh $101\frac{1}{2}$. the angle A K C, then A L, beeing as is ſaide $51\frac{2}{3}$. leaueth his complement L K, $38\frac{1}{3}$. deg. likewyſe G I beyng 55. deg. leaueth hys complement K I, being the diſtaunce of the bancke pole zenith, from our zenith 34. degrees. Heere then haue you a ſphericall triangle, videlicet, I K L, whoſe two ſides I K, 34. deg. and L K, $38\frac{1}{3}$. deg. togeather wyth the angle I K L, $101\frac{1}{2}$. deg. are knowen, wherfore by my 13. chap. of triangles, the ſide I L ſhalbe founde $54\frac{1}{2}$. deg. which is the diſtance betweene I. the pole zenith of the banke, and L the North pole, the complement whereof beeing L M, muſt needes be the eleuation ſought for, vz. $35\frac{1}{2}$ degr. Then for the angle of deflexion, you ſhall vnderſtand that the ſame is the arch of any flats horizon comprehended betweene the meridian of the place, and the meridiã of the flat: as in this figure A K B, is our meridian, becauſe he paſſeth by the pole L, and our zenith K, and I L M, is the meridian of the bancke, becauſe it paſſeth by the pole L, and the banks zenith I, by the definition in the firſt booke & fourth chapter: and therefore the arche O M ſhalbe the degrees of deflexion, which to finde deſcant thus: ſince by the 5. booke & 13. chap. you haue founde the ſide I L, of the triangle I K L, and alſo the angle I L K, or at leaſt if you haue not, you may: the angle O L M, is equal to it by the 5. theoreme: the angle O M L, is a right angle by the 3. def. and 3. theoreme. VVherfore to be ſhort, here you haue the right angled triangle L M O, whoſe one ſide L M, $35\frac{1}{2}$ deg. & his one angle not right O L M, are knowen: wherfore by the 5. bo. 9. cap. the ſide O M, ſhalbe found 28. de. Now laſtly to the angle of the meridiãs aſcen. which is the arch of any flats horiz. cõprehended between the meridiã of your place, & his baſe line, which in this figure is the arch E O, for which you haue the right angled triangle E O A, of which the angle A is a right angle by the 3. def. & 3. theor. & the angle A E O, is knowen, becauſe by the 4. def. it is denominated by N H, being 34. deg. the cõplement of ♑ recli. the ſide E A, is ſaid before to be $11\frac{1}{2}$ deg. wherefore by the 5. b. 9. c. you ſhal find the ſide E O, $13\frac{1}{2}$ deg. which is the angle of aſcenſ. ſought for, & ſo is ♑ ſide O A, $7\frac{1}{2}$. de. which is the mounting of our merid. aboue our hor. in this bank or flat, if we had need therof.



Many wayes might you deſcant vpon this figure, to come by the premiſes, as by a little exerciſe you ſhall finde: as for the purpoſe another way, and that a ſhorter way, is thus: if you will begin with the meridians aſcenſion, you may firſt finde all the ſides and angles of the right angled triangle E A O, as I haue laſt before ſhewed. And after that haue you the right angled triangle L M O, whoſe one angle not right L O M, by the 5. theoreme is equall to the angle E O A, of the former triangle knowen, and the ſide O L, is knowē by ſubducting the ſide of the former triangle A O, out of A L, the $51\frac{2}{3}$. latitude. Therefore againe by the ſaide 9. chap. you ſhall know the ſide O M, which is the quantitie of the deflexion, and the ſide M L, the poles eleuation aboue the bank ſought for: & ſo I cõclude this chap. leauing the reſt to your diligent indeuour.

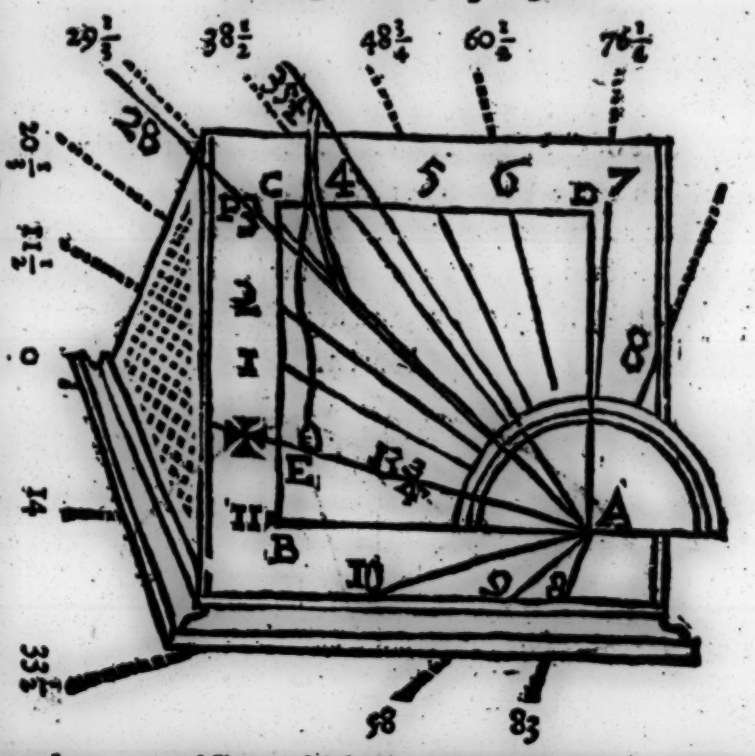
Chap.

Chapter 28.

The angles of reclination, delination, elenation, deflection, and aſcenſion had as before, how to make the dial to a reclining wall, bancke, or flat declining.

HAuing ſpent ouermuch ſpeech about dialles alredie I feare mee, and yet like to ſpende more, the matter yeelding it ſelf in ſuch abundance, I wil for breuities ſake proceed ſtreight with an example, which ſhall bee the diall I made to the bancke in my good couſins garden at Aldermarſton, mentioned in he 23. chapter, where becauſe I wold haue my work laſt, and be ſurely placed, I cauſed a ſquare to be made of foure peeces of timber, eche twelue inches broade, and in length three yardes, equall too the depth of the bancke, as in this figure you ſee A B C D, and to be ſet in euen with the turfe: ſo that of the lower peece, the vpper edge A B, did ſerue for the horizontall or baſe line of the banke. Then did I appoint the center of this diall in the lower angle Southwardes, vz. in A: In which point A, I placed the center of a woodden ſemicircle, which I had alwayes readie for ſuche like purpoſes, his diameter being 3. foot in lēgth, which I laid euen in the baſe line A B, as in this figure you ſee: for on the graſſe I could make no blinde circle: whichdon, faſtening a threed in the center A, I extended the ſame ſo many deg. aboue the baſe line A B, in this wooden ſemicirele, as our meridianes aſcenſion founde in the 25. or 26. chapter came to, vz. $13\frac{1}{4}$. deg. whiche threede or line croſſeth the ſide B C, in E, this line A E, is the meridian too vs here at Reading or Aldermarſton all is one, ſo little differeth not being but 8. miles betweene: wherefore being ſo, it muſt needs be alſo the 12. a clock line, for in all dialles the 12. is in the meridian of the place: that done, I extende the threede from A, to ſo many degrees in the ſemicircle aboue the meridian A E, as the angle of deflection founde in the 25. or 26. commeth to, vz. 28. and it croſſeth the ſide B C, in P, this lne A P, is the place for the cock, and ſo is this bancke prepared for this diall. Therefore nowe am I too betake mee too my *Iewell*, whence I muſt fetch all the houres ſpaces. VVherefore I ſet the *Finitor* vnder the pole according to the eleuatiō of the bank found in the two laſt chapters, wherevpon then I reckon the degrees of deflection, vid. 28. degrees from the *limb* inwardes, there I finde to cut the 43. meridian, which as I told you in the 19. chap. becommeth our meridian, and repreſenteth the line A E, in the bancke, the *limb* being in this caſe the banckes meridian, vz. the line A P, wherfore you may proceed euen as you did in the declining diall imagining the meridian A E. to bee the plumbe line on the wall: as for example, I reckon on my *Iewel* too the 15. meridian from the ſame 43. meridiā towards the *limbe*, which I find cutting in the *Finitor* $11\frac{1}{2}$. degr. for the firſt houre, wherefore extending the threed from A to $11\frac{1}{2}$. deg. in the ſaid ſemicircle aboue the line A E, I draw thereby the firſt houre for one of the clocke: and ſo proceeding in all reſpectes with the reſt, euen as is done in the diall of the 24. chapter, as by this preſent figure doth more euidently appeare, then by manifolde words: wherein I haue ſet to the degrees of euery houres ſpace, for your better ſatisfaction. Ther is no difference but that there I began the houres from the baſe line, the reaſon was, for that the ſame was there the meridian. The diall being made in maner of this figure, I made the cocke of a barre of Iron iſſuing a ſlope out of the center A, according to the eleuation of the bancke being $35\frac{1}{2}$. deg. ſtanding plumbe ouer the line of deflection A P. This barre was ſupported with two other, on ech ſide one: but in placing of the firſt barre the pretieſt matter I would note vnto you is, that when as by a quadrant his center fixed in A, you lift vppe the ſlope barre too this $35\frac{1}{2}$. eleuation if you let downe a plumbe line from the vpper edge thereof hee then ſhall be truly placed when as this plumbeline hangeth euen on the 12. a clocke line, vz. A E.

To a South flat reclining 56. degrees, declining VVeſt $78\frac{1}{2}$. degrees.



Chapter 29.

Of inclining dials declining.

THE reaſon, order, and making of theſe is all one with the reclining, but moſt commonly *vice verſa*, as it was in the 19. & 20. chapter betweene North and South declining dials, the one vpwardes, the other downwardes: In which the very conſtitution of the *Iewell* vpon any cituation propoſed, as hath been ſhewed will inſtruct any man of iudgement more by the bare beholding of it then I ſhould doe if I wonlde contend to write great volumes of all the diuerſities which are very many. For as well of the reclining as inclining, ſome will fall out to bee right horizons, ſome oblique, ſome haue the South pole eleuated, ſome the North pole: in ſome our meridian lighteth aboue the baſe line, as all doe yet that I haue ſhewed, in ſome againe our meridian wyll light vnder the baſe line eſpecially in inclining dials, and then may it bee called the meridians deſcenſion. Al theſe thinges the *Iewel* ſheweth for the aſking, of which he that well vnderſtandeth

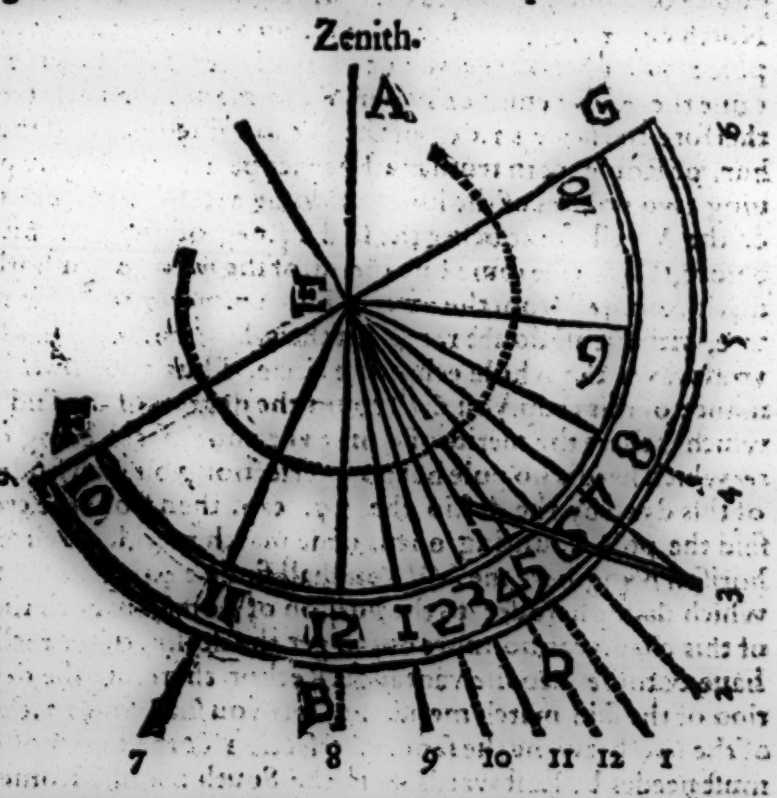
all

all this sixt booke cannot bee to seeke, and for him that vnderstandeth not the same, it were great pitie too trouble him with more. I presume that I haue so theorically, or rather vranically shewed the making of dials in this booke, that I may turne you loose to the rest: and therefore if you giue it ouer for once reading you learne not that of me.

Chapter 30.

Howe to knowe in what countrie any declining, reclining, or inclining diall, woulde or of right shoulde serue as an horizon diall, and also to make him shewe the houres of the same place together with yours.

EVery man that hath yet taken vpon him to be the authour of diall making hath delighted too adde some noueltie of deuise with them: as in Sconer, Orontius, & Munster you may see: of which the putting in of the signes with the planetarie houres, and such like hath beene most taken vp and accounted of. Therefore I am to bee borne wythall, if I augment them with a deuise or two of mine: since then there are no declining, reclining, inclining, or East or VVest dials (the North and South reclining & inclining only excepted) but in very deede are dyals of other countries, wrested from their owne deliniation, to serue our turne heere, as in the sixt chapter I haue saide, and that the true meridian of the diall to his owne countrie, is the line where the cock stãdeth: therfore to place other houre lines in any declining or reclining diall to serue his owne longitude shall bee more easie to doe then the declining houres. For when by the 19. 24. and 27. you haue placed the houres of your countrie by helpe of your newe found meridian, as there hath been shewed, you shall againe (your *Iewell* standing as there too the wall or banckes eleuation) count these newe houres from the *limbe* in very like manner as in the oblique and South wall diall you did: & place them in your dyall on each side of the line of deflexion like distaunce as in the nienth & tenth chapter, they were on eyther side of the meridian, that is to say: as you finde the houre circles distaunt from the *limbe* of your *Iewell* by degrees of the *finitor*, so place the houre lines of your diall like distant from the line of deflexion by degrees of the blind circle, among the other before made, but with red or some coloured lines and figures for distinction. This line of deflexion being as is saide the meridian of the other place must needes bee the 12. of clocke line to these new houres, from the which the other houres may easily be numbred accordingly, as by this figure you may better see then by much speech, which is the diall made, in and by the nienteenth cha. in respect of the blacke lines and figures: and the prickte houre lines and figures are drawen and placed by the 9. or rather 10. chapter, according to the walles eleuation being $21\frac{3}{4}$. deg. euen as though he did not decline: that is to say, the *finitor* being set as in the nienteenth chapter $21\frac{3}{4}$. degr. vnder the pole all those prickte houre lines are counted on the *Finitor* from the *limbe*, and not from the 60. meridian, as there the other were, and so set on each side of the line of deflexion. Then at the last to conclude our purpose, this rule is generall, that the degrees counted on your *Iewel* among the meridians betweene the *Limbe* & that meridian from whence the houres are counted by the 19. 25. and 28. which in this diall are 60. is the differen. of longitude of such countries frõ ours, whereto the prickt houre lines serue, or more plainely, the diall of right ought to serue, which if the diall decline East must bee added, but if VVest as this doth, then subducted from yours, vz. 60 out of 14. our longitude here at Reading: which cannot be taken: therefore adde (as in all suche cases you must) 360. vnto 14. it maketh 374. then take 60. out of that, you haue left 314 the longitude sought for: of all such places to which the shade of the cock among the prickt houre lines yeeldeth the houres. Nowe to know what countries lie vnder that longitude, you must repaire to Ptolomees, Apians, or any other mans tables (you think best) of regions, cities, prouinces, &c. and seek howe many places you can finde of the same longitude, write them in some voide roome of your diall, for to all those doe the prickt houre lines serue. I haue sought in Apians tables called *Abacus regionum* & can find very few of or neere this 314. long. whereby it should seem that either the world is not much discouered on that coast, or els that the Sea is large that way: only these I noted out of *Carthago Insula* lying in *America*, whose longi. I find 315. deg. 30. min. which is very neere. Also another Iland in *America* called *Spagnola*, where is greenstone of *Guacum*, whose middest is in longitude 315. deg. Also two other partes of *America* in the land of *Peru*, called *Mons Auri* & *Aluisme*, in longi. 315. & *Caput de stado* 317. which are neere enough to they exceed not 3. deg. all these I write in this diall. Too bee there, you see that when the Sun giueth 12. of clock to vs it sheweth so any of those places, but 8. of clocke among the prickt houres, & this is that which in my 1. book 12. cha. I mentioned. You will say perhaps that all these differ in lat. That is not here materiall for the long. only maketh the difference of houres. VVel the latit. may cause the Sun to rise & set sooner or later in one of these places then an other, yet keepe they one sight of their houres. But if you will needes knowe to what countrie this diall serueth, both in respect of longitude and latitude, seeke in the tables whether anye of these places before founde, bee in latitude equall



to the walles elevation or height of the cock, vz. $21\frac{3}{4}$.deg.& the same is he who you look for: you shal find *Carthage insula* in lat.22.deg.15.min.which is neere enough to serue your turne, & so I leaue you. Note that in this figure because the line of deflex.fell out iust in one of the houre lines,therefore all the prickt houre-lines of necessitie fall in the rest of them,by which meanes I neede but set too new figures seruing the other countries,and it is done.

Chapter 31.

How to place a flat representing the horizon of any countrie,citie or place in the worlde both according to the longitude & latitude of the same.

AS it is a pleasant thing by the last chap.to set the houres in your dials too the longit. of other countries whereto they serue: So no doubt,it is as pleasant,or rather more pleasant to know how to make a body,which shal haue diuers flats standing both to the longi.& lati.of what places you wil nominate.For if you make a body repleniſhed round with sundrie flats as well reclining inclining,&c. vniformlie as the most do, it is a chaunce if any of them answeare any speciall place you desire either in long.or latit.To performe this matter is harder then a man would take it for: it spent me some studie at the first to compasse it: but now I hope to lay it open enough to the learner.Admit therfore that I were to make a bodie of sundry flats looking every way some vpwards,some downwards,some this way, some that way, as in times past I haue done with great labor of seeking the angles: which now by help of my *Iewel* I could do with more ease, and that in the same body I would place one flat exactly lying parallell & leuell with the horiz. of Rome, that it might be seen how that horiz.lyeth vnto ours: the first thing I do,I go to Ptolomes or Apiãs tables where I find the long.of Rome 37.deg.almost the lat.41.de.40.mi.Our longi.here at Reading is 14. well neer, our lat.$51\frac{2}{3}$.deg. then taking 14 out of 37.there resteth 23.deg.the difference of long. of Rome frõ Reading,& that Eastwards,bicause that lõg. was greatest.These had,you shal with waxe fixe the *label* $51\frac{2}{3}$.deg.vnder the zenith point of the *veete* leftwards,which point representeth the pole, & the *label* the *Finite*, set to our lat.that done,count among the azi.from that part of the *limb* whereto the *label* is fixed the differ. of long.vz.23.deg.& on this 23. azi count from the zenith point downwards,the lat.of Rome,vz.41.40. there make a sure prick with inke,for in that that the said 23.azi.standeth for the meridian or circle of longitude of Rome,therefore that prick being that it boundeth the poles elevation ,there must needes bee the North point of the horiz.of Rome.Now for that this 23.azi.commeth in towards the center, therefore hys pole zenith or Nadir must needes bee the 23.deg.of the *finit*.beyond the center by the 5.bo.24.ca.And because the poles zenith & Nadir of the merid.of any horiz.lye in the East & VVest points of the same hor. therfore the same 23.deg.of the *finit*.so coũted beyond the center shall here be the East or VVest point of the hor. of Rome, & in truth must needes be here the VVest point if you conceiue it as wel as I do: wel then,hauing two pointes of the horiz.of Rome,vid.the North point being the pricke made with inke in the 23 azi. & the VVest point being the said 23.deg.of the *finit*.you shall turne about the *veet* & *label* fixed together as before,till you see some one merid.of the *mater* to cut both those points which you shall find too bee the 17. merid.counted from the axtree. This 17.merid.with his match,vz.the 17.merid.on the other side of the axtree,without all doubt represent the whole horiz.circle of Rome euen as in the 4.bo.9,ca,two merid. made vp alwayes the whole ecliptick circle. Thus your *Iewel* beieng in an excellent constitution for performance of this chap.you shall count the degrees of the said 17.merid.from the said pricke in the 23.azimuth, which is there the merid.of Rome vnto our merid.being the *limb*,which deg.end in the pole point of the *mater* which here is no pole but an intersection,you shall finde them 16.which are the quantitie of the deflexion of this dyall by the def.in the 27. cap. then from the crosing of this 17.merid.with our merid.being as I said the pole counting on his side match merid. videlicet, the 17.merid.vnder the axtree till he crosse our horison being here the *label*,you shall finde 24.degr. & better, which is our meridians ascension in that flat which shalbe here set to the horison of Rome,but both these angles are more then bargaine: for the drifte of this chap.was no more but to get the declinatiõ & reclin. of this proposed flat,which you shall also now haue,& those into the vantange.Reckon therefore the deg.of the *label* betweẽ the *limbe* & the last intersection of the said match merid.the same you shall find 23.deg.the complement whereof,vid.67. being the de. of the *label* betweene the same intersection & the center of the *Iewel*,is the declination of the base line whiche must needes be Eastwards from the South,because Rome lyeth that way from vs.Now for the reclination of this flat you shall from the said last intersection in the *label* count on the said match merid.to the pole being as is said 24.de. and so round thence by the deg.of the other 17.merid.tyll you come to 90.which endeth at 66.deg.thereof counted from the pole: directly ouer that 66. degr. make a pricke with inke on the *Veete*, then turne about the *veete* & *label* ſtil fixed till the *label* lye euen with the Equinoctiall line,& there you shall finde the $22.\frac{3}{4}$ merid.counted from the axtree to cut the said prick last made in the *veet* ouer the said 66 or rather 90.degr.whose deg.from the same prick counted to the pole,are $71\frac{1}{2}$.the reclination of this flat sought for: the complement wherof being the deg.of the same $22\frac{3}{4}$.from the prick to the *label*,vz.$18\frac{1}{2}$.deg. is the mounting of the same flat aboue our horiz.in his bases perpendiculer. Thus in end (since the declinat. is found 67.deg.& the reclination $71\frac{1}{2}$.deg.) I conclude that to place a flat here at Reading, leuel or parallel with the horiz.at Rome,that is to say, to agree with it both in longi.& lat.the same must be a South flat declining East 67.deg.reclining $71\frac{1}{2}$.deg. which by the 22.& 23.chap. you may easily knowe too set,and by the 28.make the dial thereto,and by the last chapter place the houres of Rome.

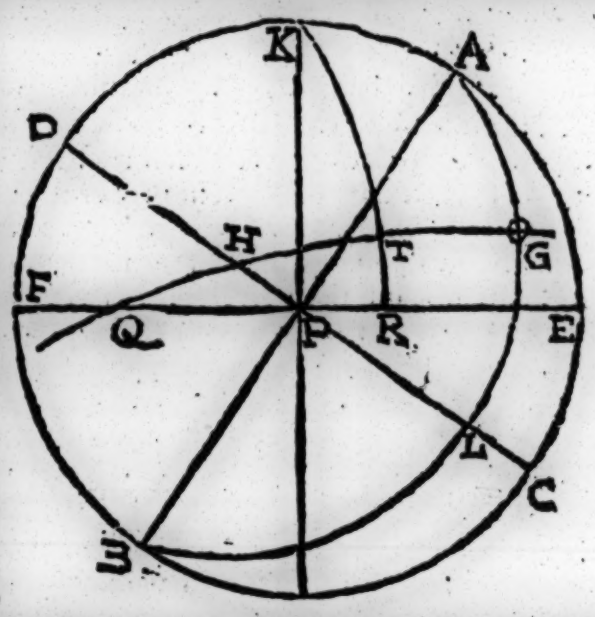
By the way note thus much that though in the constitution of the *Iewel* by this chap. the pole point of the *mater* chaunced aboue the *label*,yet you shall in the like working manie times finde the pole fall out to bee vnder the *label*.In euery such case that which here being aboue the *label* was your meridians ascension,shall then bee your meridians descension being vnder the *label*:many suche like notes you shall finde by your own practise and industrie.

Chapter

Chapter 32.

To performe the same by sphearicall triangles.

TO bring you in vse with sphearicall working by triangles, I take this needles paines, first make your platform as here you see, A B C D, is our merid. A B the axtree, A the North pole, C D the Equi. E F our horizon, K our zenith, E North, F South, the center P East and V West, E A our eleuation, vz. 51-$\frac{2}{3}$.deg. C L the difference of longitude, vz. 23. deg. A G the eleuation at Rome, vz. 40. degr. 40. min. G the North point of that horizon P H equall too C L, and therefore H the pole Nadir of A L B by the 5. booke 25. chap. being also the V West point of the horizon of Rome: I drawe therefore a circuler line by ame from G to H, and beyonde letting him to crosse E F, some where at aduenture in Q: This circle G H Q, if hee were truly drawen representeth the horizon of Rome in his right situation to ours Now then I reason thus G L, being the complement of A G must needes be 48$\frac{1}{3}$. deg. which by the 4. def. denominateth the angle G H L, to the which angle D H Q is equall by the 5. Theoreme: then the angle D H Q, vz. 48$\frac{1}{3}$. taken out of a semicircle, vz. D B C leaueth 131.$\frac{2}{3}$. degr. the angle P H Q, & angle D P F is 38$\frac{1}{3}$. de. the Equinoctials height aboue E F, our horizon P H is 23. deg. as before. Heere therefore haue you a sphearicall triangle P H Q, whose two angles H P Q 38$\frac{2}{3}$. deg. and P H Q 131$\frac{2}{3}$. deg. are knowen with the side lying between them, vz. P H, 23. deg. wherfore by the 5. bo. 17. ca. you shall find the side P Q to be the 67. deg. which is the declination of the base line of this flat proposed, and the angle H Q P (which by the 4. def. is denominated at 90. deg. frō Q by R T to be 18$\frac{1}{2}$. deg the flats mounting aboue our horizon E F, whose complement T K, is his reclinat. sought for. I might by this figure get the deflexion & ascension, but they are easie enough by the 26. or 27. ca. now the declination and reclination are had.



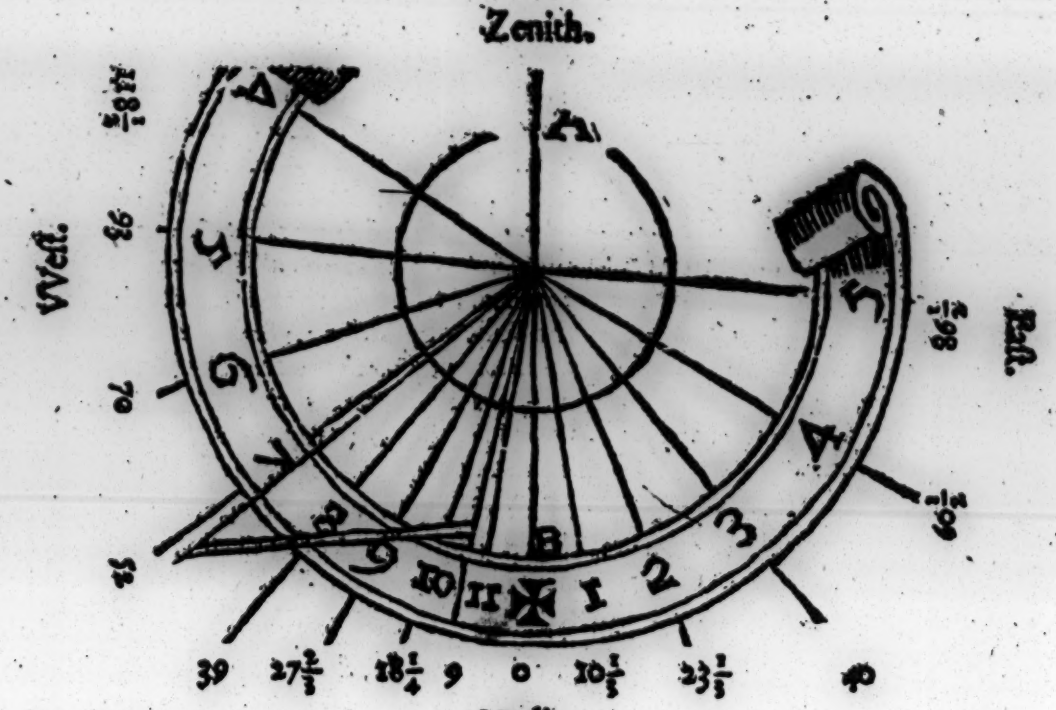
Chapter 33.

To describe dials to al maner of declining walles, and also to all East and West reclining and inclining flats more at plaesure then yet hath been shewed without foreknowing the angles of eleuation or deflexion.

SO wonderfull is the copie of operation by this instrument that it is strange so much matter & so great varietie should be cōprehēded in so simple a thing as it is to behold. Truly I am so drencht in the bowels of this *Iewel* that I know not almost which way to turn me, or whē to determine an end. For being now redie to send these 6. bo. to the presse: there sodainely commeth into my head a farre easier course for all declining reclining & inclining dials of what sort soeuer: which for the most part are very troublesom by the 19. 20. & 28, c. in numbring the houres from the new found meridian there. Verily were it not that I challenge the first inuention of the former way, I might here euen blush to think what a compasse, I haue fetcht what extreeme paines & studie it hath cost mee too atchieue the same euen by the racking of the sphere as I may terme it. But it is written of a great clerke, *facilius est addere inuentis quā de nouo componere*, which shall be a sufficiēt tutele for me in this behalf. Neither am I to be more ashamed of this then all other writers before me euen the princes of Astronomie as som of them haue been termed, are to be reproued in that all they haue not taught to make these bufie kind of dials by the vse of either the Spheare, Globe, Astrolabe or any other instrument as wel as they haue the other 2. easie & ordinarie dials. But now to the matter, first you shal vnderstand that all declining dials, whose making are taught in the 19. and 20. chapters, and all East and V West reclining or inclining taught in the 25. cap. are in maner all of one sort, that is to say, their flats represent circles of posirion to some one lati. or other: as for example, all our declining walles represent circles of position to the 38$\frac{1}{3}$. South latit. euen as our East & V West reclining flats are decli. walles to the same 38$\frac{1}{3}$ lati. as in the 25. cha. I shewed: to al flats of both these sorts you shal thus presently make the diall without respect either of eleuation or deflexion, which in all my other cap. must be foreknowen. And for your better instruction I wil here shew to make in this maner a diall to the South decli. wall Eastwards 16$\frac{1}{2}$. de. mentioned in the 15. c. V Which to do I place the *finis*, to our lat. vz. 51$\frac{2}{3}$. deg. vnder the pole, then from the zenith line towardes the *limbe* I reckon on the *finitor* 16$\frac{1}{2}$. deg. rightwards, there do I finde the 16$\frac{1}{2}$. azimuth, which representeth this wall proposed, or more plainely, the great circle or horizon in whose flat this wall lyeth, & is the 16$\frac{1}{2}$. circle of position, to those that haue the South pole 38$\frac{1}{2}$. de. eleuated, which being found let him there rest awhile. And now I must haue you (since I can assure you that euery of the dials of this sort are oblique dials) too, call to remembrance the def. of an oblique diall mentioned in the 3. chap. & the grounde & reason whēce his lineaments proceed there also shewed: & then if you returne again with this imagination to the saide 16$\frac{1}{2}$. azi. where we left, you wil be redier to tel mee then I haue been hitherto to tel you, that the verye houre lines alreadie on your *Iewel* distinguished by fifteenes, must needes deuide the same 16$\frac{1}{2}$. azimuth into too hys horizontall spaces seruing this diall: since the *limbe* is still in this constitution our merid. from whence the

houre ſpaces in all dyals muſt be counted. VVherefore to go on: becauſe in South declining dials the South pole is euer eleuated, and in North declining, the North pole, therefore it ſhall not bee amiſſe too turne about the *reete* that this ∠enith and Nadir may chaunge places to the intent you may as well reape the reaſon as driue on your diall. For there belongeth yet more thynges too bee obſerued and foreknowen: of whiche this is one generall. You muſt knowe although I heere teach too deſcribe your dials without firſte ſeeking the angles of eleuation and deflexion: yet for the height of the cocke and his place you muſt, the diall finiſhed, craue their acquaintance, and in the meane time haue a great regarde of them, of which too bee fully inſtructed, firſt you muſt note that on your *Iewell* any two azimuthes lying on eache ſide of the Zenith line, like diſtaunt from it, doe by proſpectiue reaſon make vppe and repreſent one whole great circle of the ſpheare, as in my ſeconde booke and the one and twentie chapter, I haue ſomewhat largely touched, whether reſort, ſo that this ſaid $16\frac{1}{2}$. azimuth with his matche beyng the $16\frac{1}{2}$. ∠imuth counted from the zenith line on the other ſide doe make vp the whole hori∠on or great circle, wherein this wall lyeth. Next to this, ſince you know that the *limbe* repreſenteth the meridian of our hori∠on, & that, as I haue often before ſaid, euery wall or flat hath a meridian proper to himſelf lying among the reſt of the meridians, you ſhall know that the *Iewell* being ſet to the conſtitution of this chapter the ſame his meridian as wel in this wall as any other flat, which you wyll make any dial to, by the inſtruction of this chapter lyeth alſo betweene the zenith point of the *reet*, and the necreſt croſsing of the axtree line, with your propoſed a∠imuth, for whiche cauſe all the houre ſpaces gathered on the ſame a∠imuth from the ∠enith point of the *reete* towardes the axtree, of neceſsitie muſt in your dial be appointed and ſet from the 12. a clocke line, towardes the line where the cocke ſhall ſtande, that is to ſay, VVeſtwardes of the plumbe line in this diall, becauſe he declineth Eaſt as before is ſaide, or Eaſtwardes if hee had declined VVeſt. Theſe circumſtances ſettled in your minde, then come wee neere to the matter, firſt drawe on this wall a plumbe line, vz. A B, in which pitch the center of your diall vz. C, wheron deſcribe a rounde *limbe* bigge enough for your diall, and within it a blinde circle, equall to your *Iewels limb* as you are wont: This done, ſet the *finitor* to our latitude, vz. $51\frac{1}{2}$. his Zenithe poynte downewardes towardes the South pole, as before is ſaide: and from the Zenith poynt towardes the axtree looke where the next 15. houre line (counted from that part of the *limbe* where the Zenith point ſtandeth) doth cut the ſaide $16\frac{1}{2}$. aZimuth, and you ſhal find theron 9. degrees for your firſt houres ſpace towardes the cocks place, which by help of your blind cirele you ſhal ſet in this diall VVeſtwards of the plumb line: That done, looke againe howe many degrees of the ſame $16\frac{1}{2}$. azimuth are cut by the nexte diſtinguiſhed houre line of the *mater* you ſhall finde $18\frac{1}{4}$. for the ſecond houres ſpace: likewiſe for the third $27\frac{2}{3}$. for the 4. 39. for the 5. 52. for the 6. 70. then can you go no further, except you turne againe your *reete*, that hys Zenith and Nadir may chaunge places: In ſo doing you ſhall get for the 7. houre 93. deg. for the 8. 118$\frac{1}{2}$ beyonde eight neuer goe in our countrie: All theſe ſpaces you ſhall by helpe of the blinde circle ſet on the VVeſt ſide

To a South wall declining Eaſtwardes $16\frac{1}{2}$. degrees.



of the plumbe line of this diall. Then for the houres on the other ſide, you muſt looke to the ſaide matche azimuth that is the $16\frac{1}{2}$. azimuth on the other ſide of the Zenith line, & you ſhall find the ſaid firſt houre line to cut on him $10\frac{1}{3}$. deg. counted frō the Zenith point: the ſeconde houre line cutteth $23\frac{1}{3}$. degrees, the thirde 40. degrees, the fourth $60\frac{1}{2}$. degrees, the fifth $86\frac{1}{2}$. degrees, and ſo foorth if neede were. But ſince all walles ſerue but to 12. houres, therfore howe many houres of the one ſide of the plumbe line needeth aboue 6. the other ſide hath ſo many leſſe, notwithſtanding you may thus put thē in round if you thinke ſtore is no ſore. Theſe laſt houres being put in by help of ỳ blind circle on the other ſide of the plumb line ỳ deſcription of this diall is finiſhed. For the height of the cock & his line of deſie. look the 16. or 17. ca. & perform that, as hath

hath beene before ſhewed, or rather if you will to the 18. Chapt. whereby you ſhall needs go no further then this preſent conſtitution of your *Iewel* for your whole Dyall *cum pertinentiis*. For if according to the inſtruction of the ſame 18. Chapt. you reckon on the *Finitor* from the left part of the *Limbe* inwards $16\frac{1}{2}$ deg. there ſhalbe the pole Nadir of the ſaid $16\frac{1}{2}$ azimuth lying rightwards of the zenith line, I ſay his pole Nadir in reſpect of his face beholding the ſouth pole, and at this ſaide $16\frac{1}{2}$ deg. of the *Finitor*, you ſhall finde to cut the $20\frac{2}{3}$ meridian counted from the *Limbe*, which in truth is the meridian of this declining wall wherefore following him towards the ſouth pole you ſhall finde him to croſſe the ſaid $16\frac{1}{2}$ azimuth in the $12\frac{1}{2}$ deg. from the zenith point of the *Reete*, which by the definition in the 27. Chapt. is the quantitie of the angle of deflexion. And the deg. of the ſame $20\frac{2}{3}$ meridian betweene this croſsing and the pole being there $36\frac{1}{4}$ deg. is the eleuation of the ſouth pole aboue this wall, and is the height of the cocke. All the premiſes you ſee performed in this preſent figure.

In the verie like manner ſhall you worke, for all Eaſt and Weſt flats reclining or inclining, mentioned in the 25. Chapt. but that inſteede of the *Finitor* there, you muſt here place the zenith line to the latitude. And ſo doe the azimuthes ſerue your turne in the very ſelfe ſame ſort, as by this Chapt. compared with the 25. you ſhall moſt plainely vnderſtand, the baſe line of the bancke or flat, being 12. of clocke: note that the ſaide azimuth with his match muſt for Eaſt and VVeſt reclining Dyals be counted from the *Limbe*.

Chapter 34.

To reduce all reclining and inclining flats declining vnto Eaſt and Weſt, reclining and inclining flattes to ſome one latitude or other, and thereby moſt eaſily to make the Dyall.

THe reclination and declination of any flat being propoſed or elſe taken by the 23. Chap. you ſhal thereby get the angles of eleuation and deflexion as is taught in the 26. & 27. Chapt. which had, if you finde there the North pole eleuated then by the 25. Chapt. if ſouth pole, then by the 16. 17. or 18. Chap. you may as eaſily by the ſaid angles of eleuation and deflexion get the latitude and reclination, as in the 25. chap. by the latitude and reclination, you got the deflexion and eleuation, and that in this manner. Reckon the angle of deflexion giuen on the *Limbe* of the *Reete* from the zenith point, thereto lay the *Label* whereon from the *Limbe* inwards count the eleuation giuen, and the azimuth there croſsing the *Label* counted from the *Limbe* is the circle of reclination, whoſe degrees to the zenith is the latitude, wherein that reclination is to be taken that is to ſay the latitude of that countrie whereto the flatte propoſed ſhalbe an Eaſt or Weſt, reclining or inclining flat, or more plainely where the ſame flat lyeth in one of the circles of poſition.

For Example the bancke at Aldermarſton his reclination found by the 23. Chap. to be 56. deg. and his declination in his baſe line $78\frac{1}{2}$ deg. whereby in the 26. and 27. Chapt. his eleuation was founde $35\frac{1}{2}$ deg. and his deflexion 28. deg. By this eleuation and deflexion, you ſhal finde in what latitude this bancke were a bare Eaſt or Weſt reclining flat, and how much that reclination there ſhalbe. Reckon firſt the deflexion vz. 28. degrees on the *Reetes Limbe* from the zenith point, thereto lay the *Label*, whereon count the eleuation vz. $35\frac{1}{2}$ deg. inwards, and there ſhall you finde to croſſe the *Label* the $56\frac{2}{3}$ azimuth counted from the *Limbe*, which I ſay is this newe reclination ſought for, and the degrees of this $56\frac{2}{3}$ azimuth from this croſsing with the *Label* to the zenith you ſhall finde $44\frac{1}{6}$ deg. the ſame is this new latitude ſought for wherein the ſaid bancke is a weſt flatte barely reclining $56\frac{2}{3}$ deg. which being had after the line of our meridians aſcenſion and the line of deflexion are placed on the bancke as in the ſaid 28. Chapt. is ſhewed: you ſhall inſteede of the *Finitor* euen as was done in the 25. Chapt. ſet the zenith line to this new latitude, that is to ſay $44\frac{1}{6}$ deg. vnder the north pole leftwards. And thereby the interſections of the 24. diſtinguiſhed houre circles of the *Iewel* with the ſame $56\frac{2}{3}$ azimuth, and his match azimuth perſue the making of this Dyal in all reſpects as in the laſt Chapt. you did, placing the houres in this Dyall from the line of aſcention being the 12. of clocke line, as in this laſt chap. you did from the plumbline, and it commeth to the very ſame effect with the 28. Chapt. but with more eaſe a great deale, as by experience you ſhall trie.

In the verie like manner may you performe inclining Dyals declining, taking a ſpeciall regard by the conſtitution of the 26. Chapt. which pole it eleuated.

Chapter 35.

To do the ſame ſomewhat eaſier.

BEcauſe in the working of the laſt Chap. I referre you to the 26. and 27. Chap. to get the eleuation & deflection of any reclining flatte declining, before you can worke your feate, and knowing the working of the 26. Chapter to be in ſome reſpect troubleſome, I thought good to beate about for ſome better way for you, and thus much haue I founde: After you haue taken the declination and reclination of any ſuch flat by the 23. and then ſet your *Iewel* to the firſt conſtitution of the 26. Chap. you ſhall eaſily thereby ſtreight finde your meridians aſcention, and your meridians mounting, which two note in minde, and then ſet the *Label* to your latitude fixed, and aboue it towards the pole place the zenith point of the *Reete* as much as the meridians mounting commeth to, and there marke where the almicantare counted from the zenith equall to the meridians aſcention doth cut the *Label*, for the azimuth meeting him there counted from the *Limbe* ſheweth the new reclination ſought, which with his match repreſenteth your deſired horiſon. And the zenith

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point ſtandeth there vnder the pole to your new latitude by which meanes you may proceede as in the 33. and 34. Chapt. is ſhewed. As for example in the ſame bancke.

By his reclination being 56. deg. and the declination of his baſe line being $78\frac{1}{2}$ deg. found in the 23. Cap. I placed my *Iewel* in the firſt part of the 26. Chapt. to a moſt excellent conſtitution repreſenting the ſame and euery neceſſarie circle thereto belonging whither you muſt repaire: and there ſhall you finde that the zenith point was $11\frac{1}{2}$ deg. from the pole, where we finde the 56. azimuth repreſenting this bancke, whoſe degrees betweene his croſsing with the axtree and the zenith of the *Reete* which I called the meridians aſcention, were $13\frac{2}{3}$ deg. and the degrees of the axtreeline betweene the ſame croſsing and the *Limbe* or pole point of the *Mater* being in truth our meridians mounting were, $7\frac{1}{2}$ deg. if I had made accoumpt of them there, but I then had no vſe for them. Theſe had which you may alwayes haue without any paines for the looking on, I firſt ſet the *Label* to our latitude vz. 51 $\frac{2}{3}$ deg. vnder the pole, then aboue the *Label* I ſet the zenith of the *Reete* equal to our ſaide meridians mounting vz. $7\frac{1}{2}$ deg. towards the pole, ſo is the pole eleuated aboue it $44\frac{1}{6}$ degrees, and is our new latitude deſired. Then counting from the zenith of the *Reete* among the Almicantares the quantitie of our meridians aſcention vz. $13\frac{2}{3}$ deg. I finde the ſame $13\frac{2}{3}$ almicant. to cut the *Label* in the $11\frac{1}{2}$ deg. from the *Limbe*, which in truth is the complement of the declination as in the 31. Chapter it was, from which this conſtitution now differeth not: but now in the ſame croſsing of the ſaide almicant. with the *Label* you ſhall finde the $56\frac{2}{3}$ azimuth to cut being the reclination deſired: So that now if you keepe the zenith of the *Reete* fixed, you haue him readie placed at the new latitude euen as he was in the laſt Chapt. and the $56\frac{2}{3}$ azimuth your deſired horizon, wherefore let the *Label* now go his way, for the reſt is to be done without him by the 33. Chapt.

Note that if in ſteede of the meridians aſcention among the almicantares you had here numbred on the *Label* the baſe lines declination vz. $78\frac{1}{2}$ deg. from the centre, or his complement vz. $11\frac{1}{2}$ deg. from the *Limbe*, there had you found the ſaid $56\frac{2}{3}$ azimuth without reſpecting the aſcention.

Note alſo when by the ſaid conſtitution of the 26. Chapt. you ſhal at any time conceiue that your meridian is depreſſed vnder your horizon as in diuerſe caſes it happeneth, then that which heere is the aſcention is called the deſcention, and that which here is the mounting is then the depreſsion. And in euery ſuch caſe the zenith of the *Reete* is to be placed vnder the *Label* as here it was aboue.

Chapter 36.

To performe the 31. Chapter ſomwhat more eaſily, or at the leaſt more pleaſingly.

FIrſt ſet the *Finitor* to the latitude of the place propoſed, whoſe horizontall flat you deſire to ſcituate, then reckon inwards among the meridians, the difference of longitude, and the meridian ſo found ſhal cut on the *Finitor* the angle of deflexion of the ſame flatte in this our countrie, euen as in the 19. Chapt. the deflexion found the difference of longitude, which had I reckon this deflexion on the *Limb* of the *Reete* from the zenith point, thereto I lay the *Label* whereon I count againe the latitude propoſed inwards, and there croſſeth the azimuth ſhewing the poſition reclination as I may terme it. And the degrees of the ſame azimuth betweene this croſsing and the zenith point ſheweth the poſition latitude, that is to ſay, the latitude wherein the horizon of the place propoſed is an Eaſt or Weſt reclining flatte equall to this reclination founde or more plainely wherein he lyeth in that circle of poſition, ſuch a one as in the laſt two Chapters are mentioned. VVherefore taking this latitude out of yours, there reſteth the mounting of your meridian if your latitude be the greater, or the depreſsion if the leſſe, according to which if the *Label* and zenith point be ſet in manner of the laſt Chapt. the ſame azimuth at his croſsing with the *Label*, if you count thenſe to the centre, findeth out the baſe lines declination of your propoſed flat, and from the ſame croſsing to the zenith point your meridians aſcention. And hauing nowe theſe, then if by the firſt conſtitution of the 26. Chapter you place the zenith line of the *Reete* according to this declination as there is taught: and then reckoning from the *Limbe* on the axtree line, the mounting of your meridian found, there ſtreight meeteth you the azimuth ſhewing the true reclination in his baſe lines perpendiculer ſought for. By example this will be more plaine which ſhalbe the example in the 31. Chap. of the ſcituation or placing of the horizon of Roome, the latitude thereof being $41\frac{2}{3}$ deg. and the difference of longitude from vs here at Reading 23. deg. both propoſed: Firſt I ſet the *Finitor* to the $41\frac{2}{3}$ latitude vz. $41\frac{2}{3}$ vnder the pole, then from the *Limbe* among the meridians I count the difference of longitude vz. 23. this 23. meridian cutteth in the *Finitor* 16. deg. which is the angle of deflexion of the meridian of Rome from ours here in our countrie. Theſe had I then reckon this angle of deflexion vz. 16. deg. on the *Limbe* of the *Reete* from the zenith point, thereto I lay the *Label*, nowe on the *Label* inwards I reckon the latitude of Roome vz. $41\frac{2}{3}$ degrees, and there do I finde to cut the 73. azimuth counted from the *Limbe* which I call the poſition reclination: And on the ſame 73. Azimuth from the cutting with the *Label* vnto the zenith of my *Reete* I finde $44\frac{1}{4}$ degrees, which I call the poſition latitude, that is to ſay, in that countrie where the latit. is $44\frac{1}{4}$ degrees. The horizon of Rome is an Eaſt or VVeſt reclining or inclining flatte lying in one of the circles of poſition there: therefore taking that latitude vz. $44\frac{1}{4}$ out of our latitude vz. $51\frac{2}{3}$ there reſteth $7\frac{5}{12}$ deg. the difference, which becauſe our latitude is the greater, is the mounting of our meridian in this flatte rightly placed. Theſe knowne you ſhall firſt ſet the *Label* to our latitude vz. $51\frac{2}{3}$ deg. vnder the pole, then place the zenith of the *Reete* according to the other latitude vz. $44\frac{1}{4}$ degrees vnder the

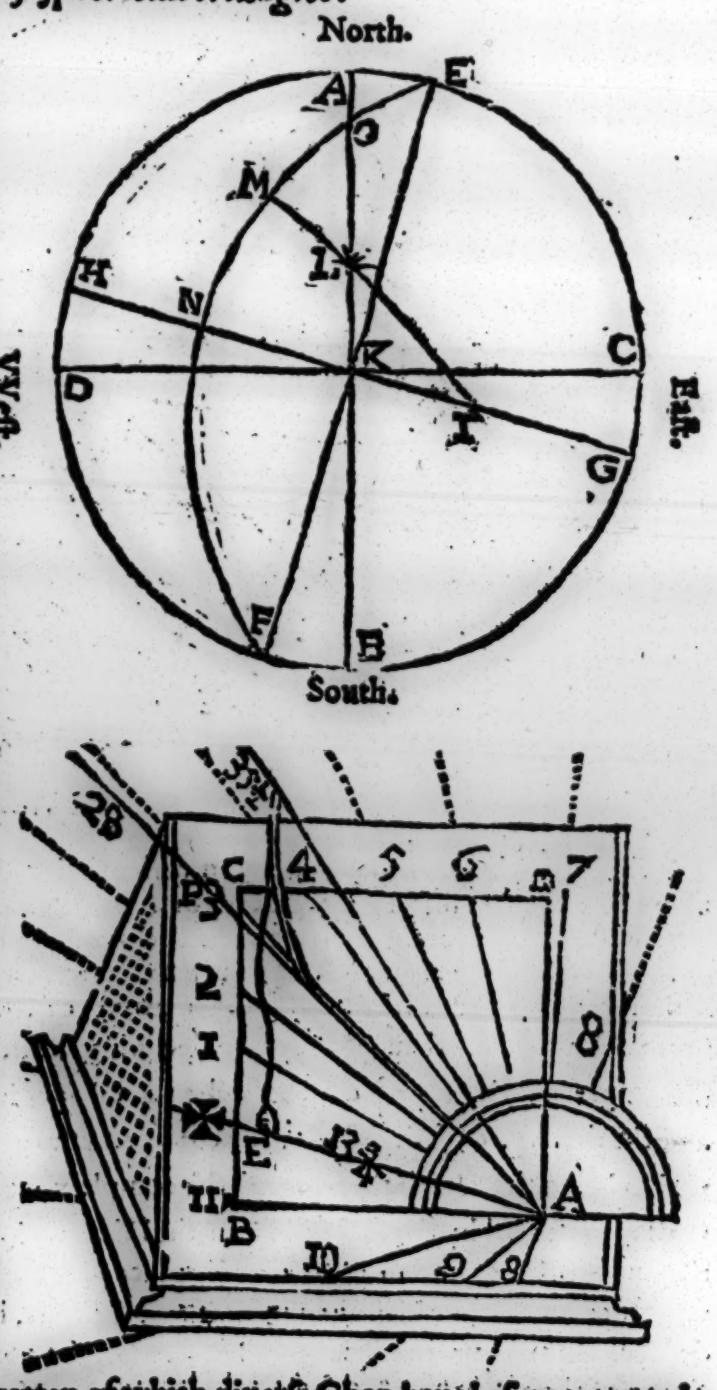
ſame pole on the ſame ſide, or according to the difference of theſe latitudes vz. $7\frac{1}{2}$ degrees aboue the *Label* becauſe the meridian mounteth, all commeth to one, and then marke where the ſaid 73. azimuth cutteth the *Label* and you ſhall finde betweene it and the centre of the *Iewel* 67. deg. the declination of the baſe line of this flatte ſought for, and from the ſame cutting to the *Limbe* is our meridians aſcention aboue the baſe line, ſo you count it on the ſame 73. Azimuth which to our purpoſe is little pertinent heere but that the *Iewel* will needs do vs the curteſie. The declination of the baſe line thus knowne, you ſhall as in the firſt part of the 26. Chapt. place the zenith point of the *Reete* accordingly, and according to the inſtruction there it ſhall winde from the Equinoctiall 67. deg. towards the pole, where the *Reete* fixed I counte on the axtree line from the ſame pole, the foreſaid mounting of our meridian here found vz. $7\frac{1}{2}$ deg. and there I finde to cut the $71\frac{1}{2}$ azimuth counted from the zenith line which is the circle repreſenting the horizon of Rome in his right ſituation to ours, and his true reclination: but if by the 33. Chapt. you woulde make a Dyall thereto, then muſt you (the zenith point being as before at the $44\frac{1}{4}$ latitude) take the ſaide 73. azimuth and his match to performe your purpoſe. Otherwiſe you may proceede by the 28. Chapter, or rather now by the 46.

Note that when you haue the poſition reclination and latitude, and thereby the meridians mounting as before: you may ſomwhat eaſier get the true reclination and declination by a Sphericall triangle imagined ſuch a one as A E O is in the 27. Chap. For being that as I haue ſinſe ſhewed in the 37. Chap. the angle E O A is equall alwayes to the poſition reclination: therefore hauing of the triangle A E O the ſide A O knowne with the angle not right E O A you ſhall eaſily by the 5. Booke 9. Chapt. get the other ſides and angle, of which the ſide E O ſhalbe the meridians aſcenſion, and E A the declinations complement, and the angle A E O the complement of the true reclination: hereof looke more in the 37.

Chapter 37.

A moſt eaſie and briefe way to reduce all reclining and inclining flats declining, vnto Eaſt and Weſt reclining flats to a new latitude, otherwiſe then in the 34. & 35. Chap. by helpe of ſphericall triangles.

BEhold yet againe in theſe 4. ſhort Chapters following the intricateſt part of the whole Art of Dyalling moſt plainely performed, not thought of when I wrote the 33. 34. & 35. Chapt. but freſhly came vnto my minde as I was buſilie occupied in cutting the prints for this 6. booke. Better it is that it come too late then neuer: I were beſt now to ſay that there be two courſes in making of Dyals, Longation and Curtation, as the prieſt ſaide to Baluinus in Eraſmus Dialogue called *Alcumiſtica*, and that it was my chance to hit firſt vpon Longation, as in my 19. 20. 25. & 28. Chapters full well appeareth: and yet truely they are very pleſant and profitable in diuerſe reſpectes though not ſo redie for Dyalling as theſe & others, & therfore not to be reiected, yet let no mā ſtand longer on them then to conceiue the reaſon and manner of them. Wel then, now you ſhall haue Curtation, which becauſe I will make ſhort of, behold this firſt figure whoſe lineaments are ſufficiently declared in the 27. Cap. & therefore I neede not repeat them againe repreſenting the firſt conſtitution of the *Iewel* in the 26. Chap. wherein I am in this Chapt to let you vnderſtand that the arch L O is our new latitude ſought for, & the angle L O M our newe reclination mentioned in the 34. and 35. Chap. which are moſt eaſily to be had becauſe the triangle E A O hath all three ſides knowne either by the verie conſtitution of the 26. Chap. or by the 27. wherfore the angle E O A is eaſily had by the 5. Booke 4. Chap. vnto which the angle L O M is equall by the 5. theoreme and is our new reclination deſired, called in the 34. and 35. the poſition reclination. Then taking the arch A O out of A L, our latitude there reſteth L O this newe latitude deſired called in the 34. and 35. the poſition latitude. Laſtly you haue the right angled triangle O L M whoſe one angle not right L O M with the ſubtending ſide O L are knowne, wherefore by the 5. booke 9. Chapt. the other ſides & angle ſhall be knowne, of which the ſide O M is the angle of declexion and L M the eleuation as in the 27. is ſhewed.



Now do you ſee how eaſily theſe arches and angles are gotten, of which diuerſe Chap. haue before entreated: And therfore me thinks the very lineamēts of this figure ſhould eaſily ſhew him that hath read the reſt of my book of dyalling with any iudgmēt, how to place theſe angles on ỳ dyal euen without ỳ figure of the 28. ca. which notwithſtanding I haue here ſet for your better conceiuing, for who ſeing the circle E F M to repreſent

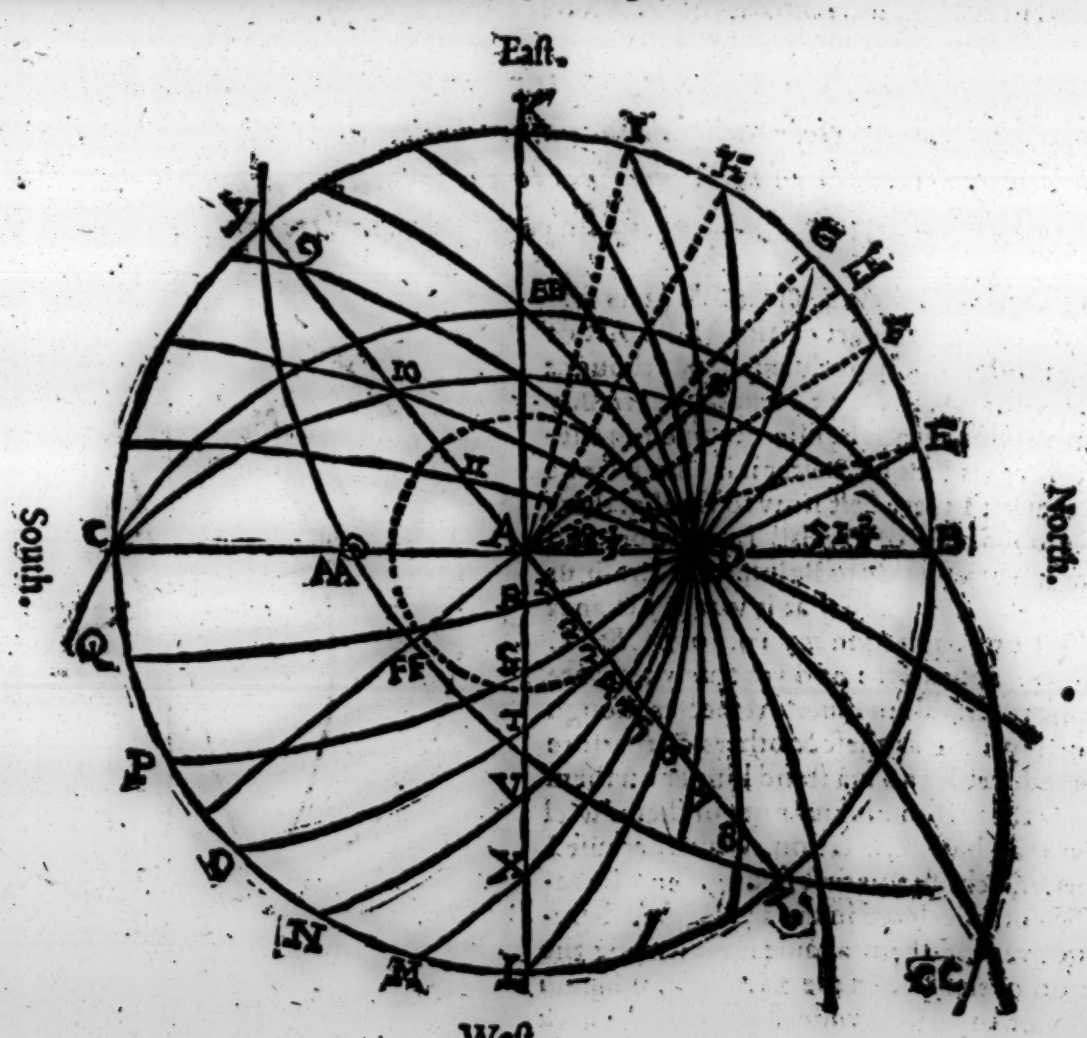
the

the flat of the bancke propoſed, and therein how the notes E O M do ſtande, will not eaſily imagine that the degrees of the arch E O being our meridians aſcention are to be placed aboue the baſe line of the bancke, as in this ſecond figure you ſee E A, and aboue that againe O M to be placed for the angle of deflexion or Gnomons place as in this Dyall you ſee A P. But now for L M you may conceiue that muſt ſtand perpendiculer on the flat, being indeede the height of the cocke as in this Dyall you ſee.

Chapter 38.

A briefe note how Eaſt and Weſt reclining or inclining Dyals are to be made two ſeuerall wayes.

HAuing firſt placed all the angles on your Dyall, as in the laſt Chapter, you ſhall forget that E N F is a South declining flat reclining VVeſt, as in the 23. Chapt. he was found to be: and that A L is your latitude, and K E N your angle of reclination as it was in the 27. and 28. Chapter. But you ſhall nowe take O L to be your latitude and the flat O N P to be a bare weſt reclining flat according to the angle K O N or K P N. wherefore if nowe you take your *Iewel* in hande, and count on the *Limbe* thereof the latitude O L from the pole towards the equinoctiall, and thereto lay the zenith point of your *Reete* fixed, and then chooſing out the azimuth counted from the *Limbe* equall to the angle K O N your new reclination, worke your Dyall by him and his match azimuth in all reſpects as in the 33. Chapt. is ſhewed.



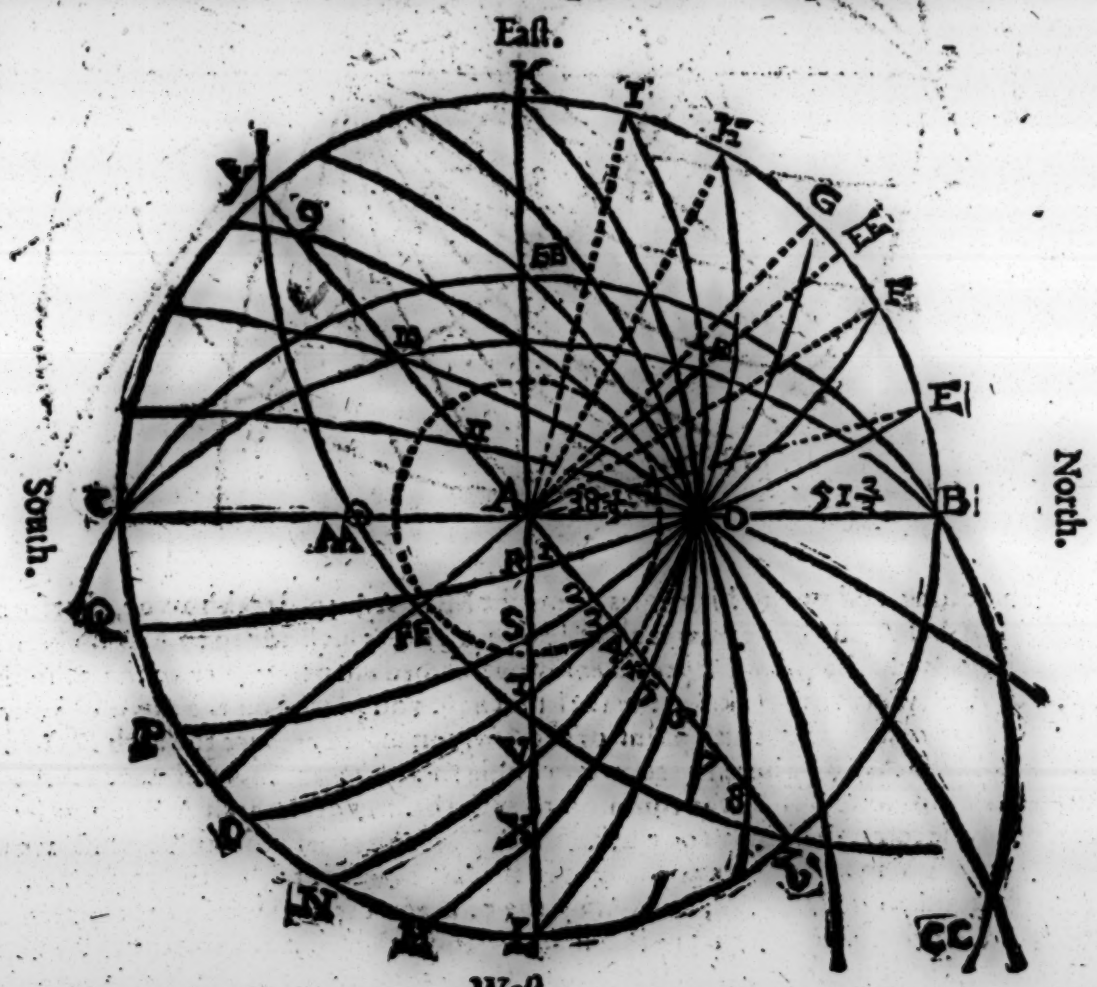
If this way like you not, or that you deſire variety, I wil here ſhew you to make this dyal by the 5. Booke 18. Cap. after another ſort. Firſt therfore for as much as all Eaſt or VVeſt reclining or inclining flats in any latit. howſoeuer ſeene or imagined, are verie declining walls to that countrie whoſe lat. is equal to the complement of the ſame latitude as I haue ſaid in the 25. & 32. Chap. and by the 13. preamble is partly manifeſt, therfore if you conceiue not or like not of the making of the Dyals to theſe Eaſt and VVeſt reclining or inclining flats you may turne them plainly into declining walles. As for example: ſinſe the latitude O L is $44\frac{1}{2}$ and the angle K O N $56\frac{2}{3}$ deg. as in the 34. Chapt. is ſhewed, you ſhall take the complements of them both, and the firſt vz. $45\frac{5}{6}$ deg. being the complement of $44\frac{1}{2}$ ſhall be the latitude wherein O N P is a declining wall o being the zenith there. And the complement of $56\frac{2}{3}$ degrees being $33\frac{1}{3}$ deg. is the declination of the wall repreſented by the ſaid O N P in the ſaid $45\frac{5}{6}$ latitude, and that weſtwards if it were material. But ſinſe the angles of E O and O M be al redie placed on the flat, therefore it needeth not to ſeeke ſo carefully which way ſuch a wall ſhould decline in that $45\frac{5}{6}$ latitude, becauſe which of the 4. wayes ſo euer hee declineth the degrees of the houres will ſerue as I ſhewed in the 20. Chap. And now laſtly if you do but lay before you the figure of the 5. Booke 18. Chapt. here preſent, and imagine the arch B D to be this foreſaide latitude vz. $45\frac{5}{6}$ and the circles Z A Y to be this wall declining $33\frac{1}{3}$ deg. you may by the ſaide 5. Booke 18. Chapt. make the Dyall without more teaching in all reſpectes as there was ſhewed.

Chapter

Chapter 39.

How to make a diall to any East or West, reclining or inclining flat, by helpe of spherical triangles most easily.

FOr as much as in the last chap. to make the East or VVest reclining diall, I referred you to the 33. chapt. which doubtlesse is singuler good, but that my example there is a declining dial, or els to reduce the same to a declining wall by the complements, it may be that first you vnderstand not, and the second you vtterly mislike. Therefore I wil here briefly teach you to performe the East and VVest reclining after the fyfte booke 18. chap. in other manner then in the last chap. I shewed, whereby you shall not choose, but fully e to vnderstand the 33. chap. Therefore behold this Figure and imagine B D to be your new position latitude mẽtioned in the 34. 35. and 37. chap. videl. $44\frac{1}{6}$. deg. and let the circle B BB C represent the new position reclynation mentioned in the same chapters, reclyning from the Zenith A, according to the arche A BB, beeyng $56\frac{2}{3}$. deg. Here may you beholde on this Figure, being but a poore platforme of your pretence, in what manner the houre circles of the spheare doe cut your proposed flat B BB C, The distances between euery intersection are to be had in all respectes, as was done in the saide 5. booke 18. chap. the case being but a very little altered. For in steed of the arch D A, which in working the same 18. chapt. lieth on the *Limbe* of your *Iewel*, betweene the pole point of the matter, and the Zenith point of the *Reete*, heere you must place the arche B D, and in steede of the angle D A Z, which in the same 18. chap. was common to the finding of euery houres space on the arche A Z, so heere the angle D B *BB* is common to the synding of euerye houres space on the arche B BB and beyondeif you list vnto C. And euen as the angle B A Y was there the common angle to gett the houres of the arch A Y, so here the angle D B CC is the common angle to get the houres of the arch B CC, and beyond as farre as you list: which angle D B CC is easily had, seing CC B Z is equal to K B B B, the complement of the reclination proposed, vz. $33\frac{1}{3}$. deg. and thereto adding the angle D B Z or A B L videl. 90. deg. there commeth $123\frac{1}{3}$. deg. the angle D B CC. And when you haue all done, you shall find that the working it spherically in this manner, shall differ no iot from the working of the 33. chap. remembring this rule in general, looke whereabouts on your platforme, or on your *Iewel* you see the houre circles to cut your flat or horizon proposed thickest or neerest together, thereabouts bee sure the cocke or Gnomon must stand as here you see they doe betweene B and BB, which I note because you may be sure that those houres from B towardes *BB* are in your diall to be placed from our meridian, being the 12. a clocke line towards the line where the cocke must stand.

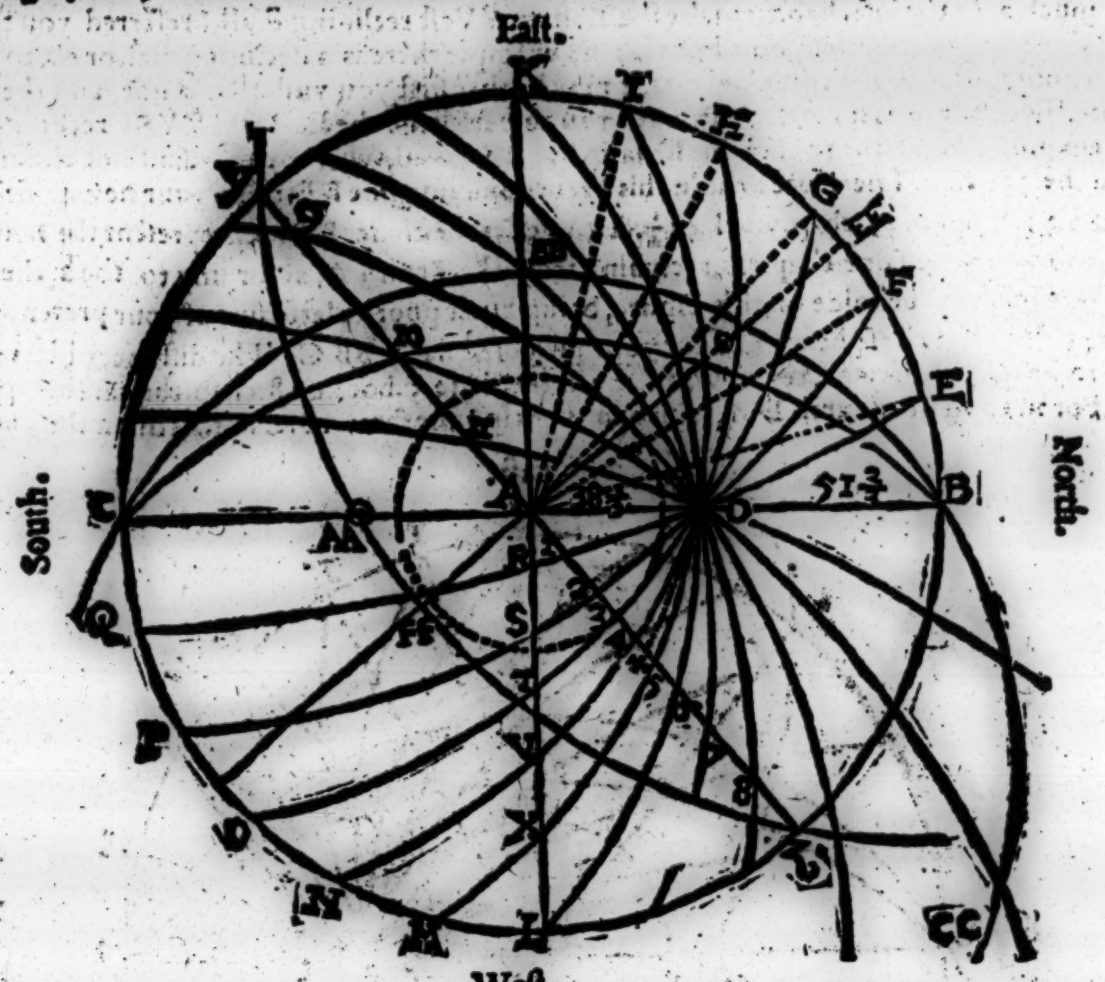


Chapter 40.

Most excellently and easily to make dialles to al maner of reclining or inclining flats declining, by the Iewell and helpe of spherical triangles.

I Knowe some will thinke it a great toyle when he hath reclining or inclining flats declining, to seeke newe position latitudes and reclinations by the 34. 35. or 37. chap. and so to goe to worke, the next way about by Logation lane: to satisfie them, I suppose the flat Y FF Z in this figure to be a North flat reclining Southwards

wardes according to the quantity of the arch A FF, vz. 30. deg. declining Eaſtwardes as much as the arche B EE is, vz. 40. deg. Y Z being the baſe line of this flat, and E E the pole ꝛenith of the baſe. The firſt thing you doe you ſhall ſeeke the angle D A A Z, and the arch A A D: both which you ſhall thus eaſily find. The arch A FF is perpendiculer on the circle Y FF Z, by the 3. def. the angle FF A C is equal to E E A B, being the declination, vz. 40. deg. by the 5. theor. Here now haue you the right angled triangle A FF A A, whoſe one ſide A



FF is known to be 30. deg. and one of his angles not right FF A A A 40. deg. knowne, therfore by the 5. book 9. chap. the angle A A A Z ſhalbe knowne, and alſo the ſide A A A which added to A D maketh the whole arch A A D knowne: both which had, this chapter is eaſily performed euen in maner of the 5. booke 18. chap. For inſteed of the arch A D, which in working, the ſaid 18. chap. you placed on the *Limbe* of your *Iewell* between the pole point of the *mate*, and the ꝛenith point of the *Reet*, you ſhal here place the arch D A A. And there the *Reete* fixed let the angle D A A Z be here the common angle to all the houres of the arch A A Z, euen as in the ſaid 5. book 18. chap. the angle B A Z was common to al the houres of the arch A Z. Likewiſe let D A A Y be here the common angle to all the houres of the arch A A Y, euen as there the angle B A Y was common to al the houres of the arch A Y in all reſpects as I taught in the laſt chap. for taking the angle Z A A D out of 180. there reſteth the foreſaid angle D A A Y, known by the 5. theoreme. you might haue gotten the arch A A A, and the angle D A A Z. otherwiſe, and that in this maner, and I chinke it the ſurer way, you haue the ſpherical triangle A Y A A, whoſe angle Y A A A is equall to the declinations complement, vz. 50. deg. and the angle A Y A A 30. deg. propoſed, the ſide lying betweene theſe angles, vz. A Y is 90. deg. known, therefore by the 5. booke 17. chap. the angle A A A Y ſhalbe found 124. degr. whiche taken out of the ſemicircle Z A A Y being 180. leaueth the angle A A A Z 56. deg. and by the ſame 17. cha. the arch A A A ſhalbe found 37. deg. whereto adding the arch D A, vz. 38 1/2. ſo haue you the whole arch D A A 75 1/2. deg.

And here I thought good to note thus much for the credit of my 33. chap. that if you reckon on the *Limbe* of your *iewel* the ſaid arch D A A, vz. 75 1/2. deg. from the pole, and thereto ſet the zenith poynt of the *Reet* fixed, and there chooſing ourth azimuth counted from the *Limbe* equal to the angle A A A Z, viz. the 37. azim. you may by the ſame 37. azim. and his match worke this diall, as in the 33. chap. in all reſpectes, and yet in concluſion wil differ nothing from the premiſſes.

Chapter 41.

To know in what Longitude and Latitude our horizon or any other ſhall repreſent any declining wall propoſed, or els any circle of poſition propoſed.

GOod God if I ſhould bend my ſelfe to inuent but ſo many propoſitions to be performed on my [illegible] my courſe capacity could accompliſh, I thinke I ſhould ſpend al my dayes in writing. For whiles I think to make haſt to an end, I fare like a dog that goeth in a wheele hauing an endleſſe path to tread, let him make neuer ſo much haſte it helpeth him not til he fynd a meane to get himſelfe quite out. So I haue no other meane to come to an end, but euen here to ſtop and lay a ſtraw as they ſay for this time, this chapter being ended, which cannot but be ſomewhat pleaſurable. As if a man would know in what longitude and latitude

our horizon here at Reading were a south wall declining East 30. deg. or els were the 30. circle of position: euery man perhaps could not performe this, notwithstanding all that which hath byn said. You shall reckon our latitude on the *Label* from the *Limbe* inwards, and lay the same deg. on the 30. azimuth counted from the Zenith line and the degrees of the same 30. azimuth counted thence to the Zenith point shall bee the latitude wherein our horizon is the 30. circle of position, and the complement thereof shall be the latitude wherein it is a wall declining 30. deg. And the deg. of the *Rester Limbe* betweene the *Label* and the ∠enith point shall bee the deflexion of the diall to that declination and circle of position. Lastly, set the *Finitor* vnder the pole according to our latitude, and thereon reckon inwardes the deflexion before founde, and the meridian cutting there the *Finitor* counted from the *Limbe* sheweth the the difference of longit. which being added to ours sheweth the longit. desired.

As for example, our horizon here at Reading is in longit. 14. in latitude $51\frac{2}{3}$. deg. First I recko our latitud. videl. $51\frac{2}{3}$. deg. on the *Labell* inwardes and apply the same degree of the *Label* to the 30. azim. counted from the ∠enith line, which cutteth off on the said 30. azim. $64\frac{2}{3}$. deg. counted thence to the zenith pointe of the *Rester*, and in the *Limbe* the *Label* cutteth $46.\frac{2}{3}$. deg. counted also from the zenith point, which are the deg. of deflexion of which I will speake anon. But before I proceede further, I conclude that the said $64\frac{2}{3}$. degr. are the deg. of latitude in that countrey where our horizon shall lye in the 30. circle of position counted there towardes the ∠enith as the circles of position are alwayes reckoned: then if you take the complement of the said $64\frac{2}{3}$. deg. vz. $25\frac{1}{3}$. deg. the same shalbe the latitude wherein our horizon shall represent a wall declining 30. deg. yet by your leaue you may be in doubt whether this be a South latitude or a North, but you shall haue this rule general, if you once knowe whiche pole is eleuated aboue anye South or South declining wall, you may be sure the contrary pole is eleuated in the latitude whereon that wal standeth, as here seeing the North pole is eleuated $51\frac{2}{3}$. deg. you may be sure where this our horizon shall be a south or south declining wal, that countrey hath a South latitude: and therefore this $25\frac{1}{3}$. is a South eleuation. But in what longitude withal he shal be a South wal, declining VVestwards 30. deg. that shall the deflexion helpe vs too, being $46\frac{2}{3}$. as I before saide. Therefore you shall set the *Finitor* according to the sayde South latitude videl. $25\frac{1}{3}$. deg. vnder eyther pole, and thereon reckon inwardes the saide angle of deflexion vz. $46\frac{2}{3}$. deg. where you shall finde the said 68. meridian counted from the *Limbe* to crosse the *Finitor* being the sayde difference of longit. from ours. Now because I propose our horizon to decline VVestwards from the meridian of that $25\frac{1}{3}$. latitude, therfore I adde those 68. deg. to our longi. being 14. it maketh 82. deg. the long. desired. So that hereby I conclude my purpose, affirming that our horizon here at Reading shal represent a South wall declining VVest 30. degr. in that countrey where as the longit. is 82. deg. and the South pole eleuated $25\frac{1}{3}$. deg. & you do not beleue me me, goe thither and trye it. It is a prety walke for a recreation: wel, yet if you subduct 68. out of 14. (whiche without adding first 360. thereto you cannot doe) so is it 374. there resteth then 306. the longi. of that countrey where our said horizon is a South wall declining East 30. deg. the same latit. vz. $25\frac{1}{3}$. remayning stil. In the very same maner by hauing regard of the longitude, you may knowe where it is a North wall declining East or VVest 30. deg. since those two are but oppoite to these.

Note that the said 30. circle of position is the 60. reclining or inclining flat in the said $64\frac{2}{3}$. latit. but whether you will haue him East or VVest reclining or inclining, that shal you euen as easily doe by taking regard of the differences of longitude in maner as before.

VVel then though in the 37. I likened my 19. 20. 25. and 28. chapters vnto longation, hauing since in some respectes lighted on Curtation, yet commeth this fruite (if it be ought worth) out of them: and they are introductions to diuers such like, for which cause I thought good not to omit this chapter although the 30. 36. and diuers other chapters came wholly of them, and therefore I repent not much my paynes.

The conclusion.

THus gentle Reader, an hundred times heartily farewel, beeing sorye that my other 6. litle bookes of the *Iewel* be not ready to go to the presse with these: but I hope by that time these shalbe throughly perused to send the rest after somwhat more industriously performed, I feare some will take it vnkindly that those bookes to come, being chiefly for the bent of this present time to be desired should bee kepte in. But truely I could not doe withall, except I should haue set the cart before the horse: such hath beene my want of leysure through the malice of fortune. And therefore considering I haue kept a needful order in this my woorke by passing from the lowest matters to the highest, the one to explane the other: & that I haue done my vttermost (as is wel knowne) that my leysure would permit, I hope I shalbe the lesse blamed.

The order of the fixe bookes behinde.

THe 7. booke of the Mathematical *Iewel* of singuler and perfect meanes as well to describe as to deuyde the lineaments of my *Iewel*, as of my great quadrant to come: to performe the crosse staffe or any prospective proiection of the globe howsoeuer, and of certaine vses of the crosse staffe.

The

books of [illegible] of working both rectilineall and Spherical trian-

[illegible] of Planes, or of the manage [illegible]

[illegible] and such like

[illegible] and finding [illegible] of the longitude of regions, &c

[illegible] Geometrically for Mariners or any other that woulde not enter

[illegible] case, that the like hath not (I presume) beene written in any lan-

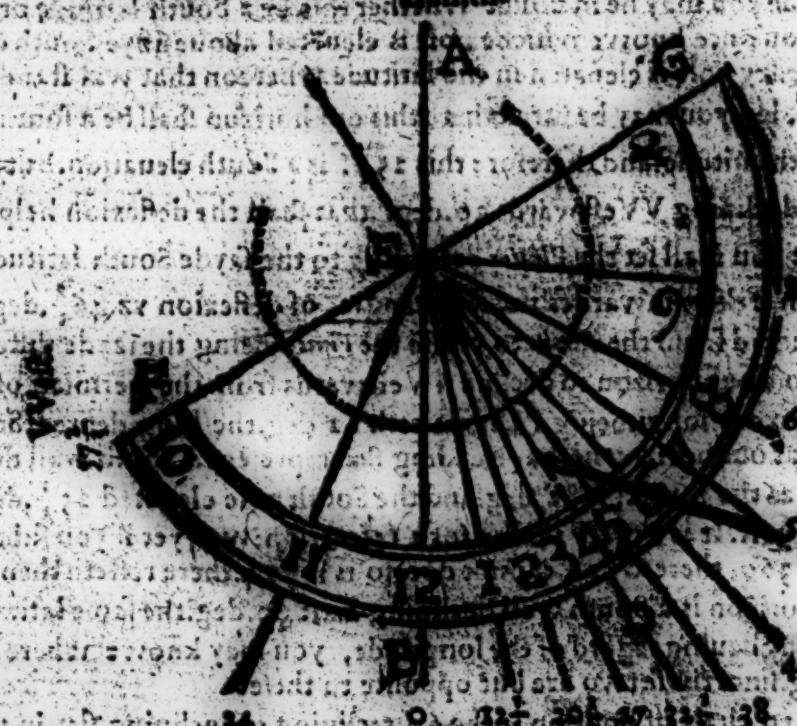
guage.

[illegible] all these will fall [illegible] to the vse of my *Iewel*, except it be where I must perforce vse my great qua-

[illegible] cronicle staffe, to proue vnto the exact proofe of things

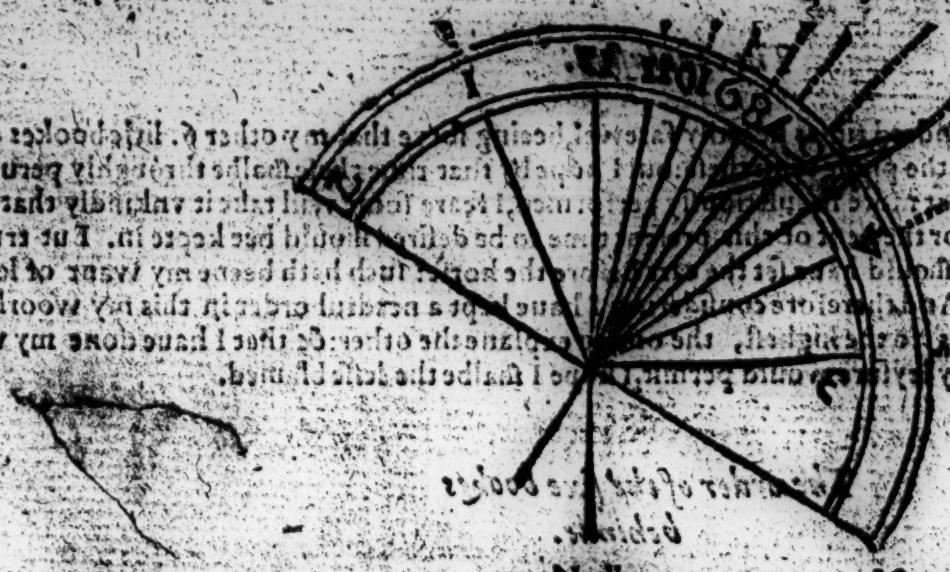
This figure must stand in the 6. booke 19. chap. and the figure which you see placed in the same 19. chap. must be placed to the eye according to the wordes zenith and Nadir in the beginning of the 20. chap.

To a South wall declining West 5 3/4 deg.



This figure must stand in the 6. booke 20. chap. in the voyd place there left for him.

A North wall declining half 5 3/4 deg.



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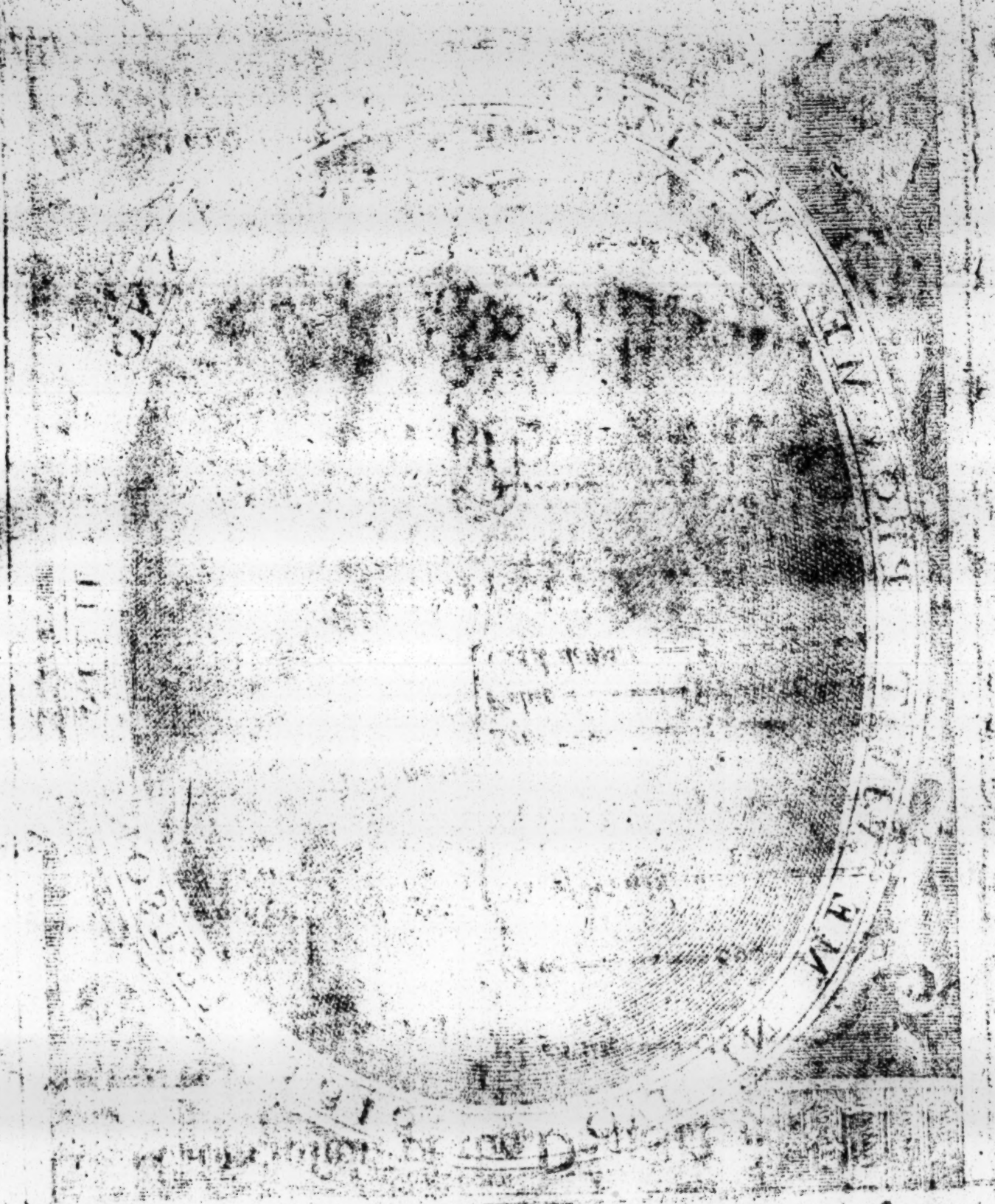
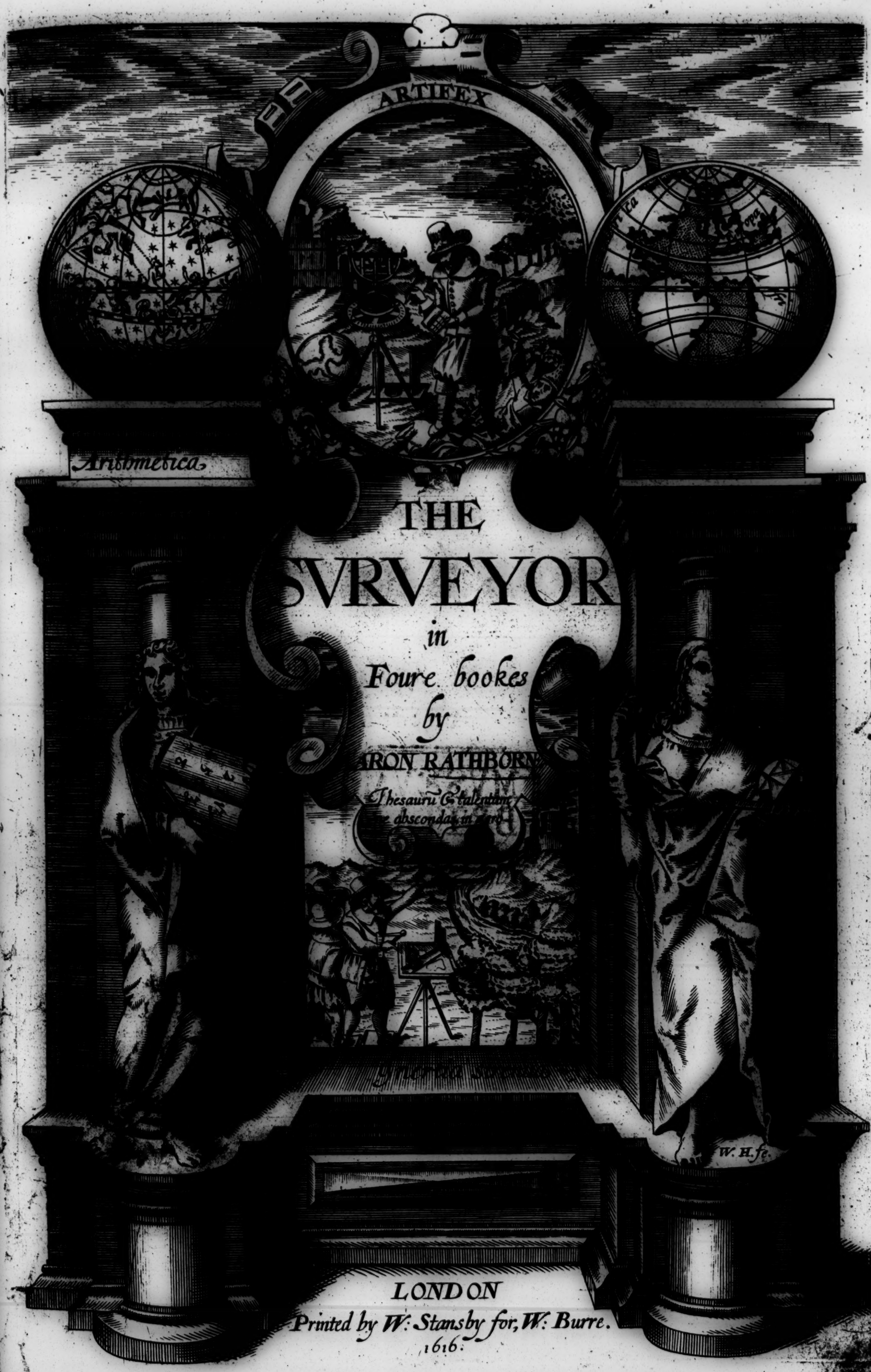


Z. V.

PRO ME MORTUUS EST ANNO SALUTIS NOSTRÆ
1616 ÆTATIS SUÆ IN MEMORIAM

Rl: 15: 3

7.F.7



LONDON

Printed by W: Stansby for, W: Burre.

1616.

MVSEVM
BRITAN
NICVM

TO THE HIGH AND MIGHTY PRINCE *CHARLES*,

Prince of WALES, Duke of CORNWALL, YORKE, ALBANY and ROTHSAY: Marques of ORMONT, *Earle of* ROSSE; *and Baron of* ARMANOCH: High Seneſchall of SCOTLAND: Lord of the ILES; and Knight of the *moſt noble Order of the* GARTER.

IF in former ages (moſt noble Prince) the ſtudies Mathematicall were held meet for *Princes*; I doubt not but in theſe, they may implore (by your fauourable admittance) the approbation and defence of your gracious patronage; which emboldens me the rather, to preſume the dedication of theſe my labours to your Highneſſe protection. Wiſedome is defined by CICERO to be *diuinarum atq; humanarum rerum ſcientia*, of the former part of which de-

finition (being the moſt abſolute) I will leaue to ſpeake vnto thoſe who can better write; yet will thus much auerre, that no man ſhall obtayne the abſolute perfection thereof, being abſolutely ignorant of the rules, rudiments and principles of Mathematicall diſcipline, as the due conſideration of that ſacred and myſticall *Vnitie* and *Trinitie*, may well approue. And how auaileable and important they are, for the attayning to that *humanarum rerum ſcientiam*, in Peace or Warre, is worthily witneſſed by PLATO, VEGETIVS, LIVIVS, and other Authors; who teſtifie of LYCVRGVS, and that famous *Syracuſean* ARCHIMEDES; by the one, what excellent Lawes and Ordinances were eſtabliſhed and miniſtred, in the time of peace; and by the other, what more then wonderfull deuiſes and ſtratagems were wrought againſt the inuincible forces of MARCELLVS, in the time of Warre, which they worthily impute to this their *ſcientiæ mathematicarum*. But ſhould my weakeneſſe here vndertake to illuſtrate the excellency of that worth, which all worthy men admire; and that to your Highneſſe, whoſe iudgement is beſt able to diſcerne; were but to depraue the one and the other, and reſt in mine owne reproch. Wherefore, aſſuring my ſelfe, of your Highneſſe loue and affection to theſe Artes, and your reſpect to the Profeſſors thereof; with your power and abilitie of defence, againſt the malignant courſes of malicious detractors, I preſume in all humilitie to intreate your Patronage of theſe my labours, which in all dutie I proſtrate at your Highneſſe feet; with continuall inuocation to the Prince of Princes, euer to preſerue your Princely dignitie.

Your Highneſſe moſt humble

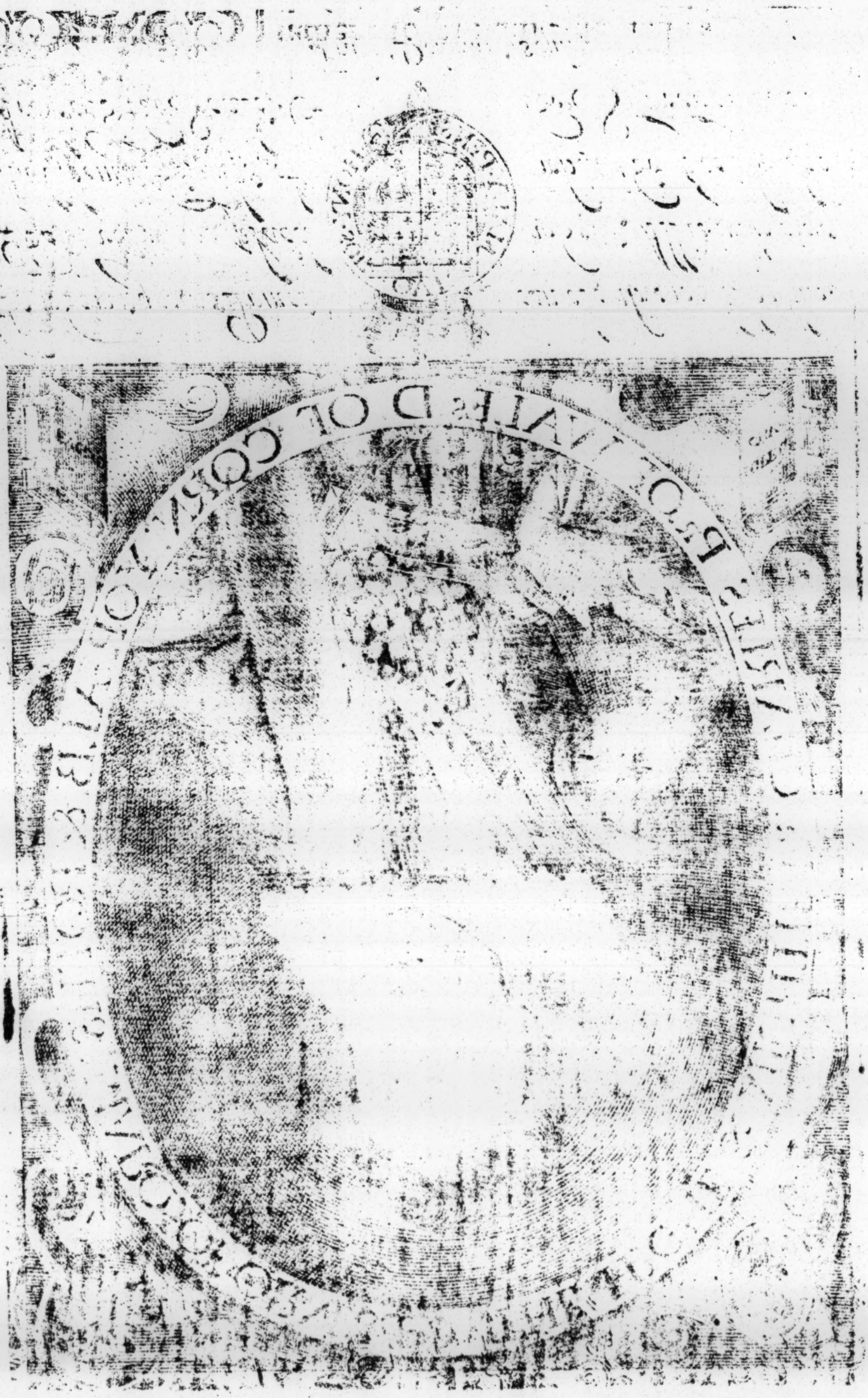
and deuoted

AARON RATHBURNE.

To w Some greate Prince
Sen you nowe plac'd
And, being the rule of
Are bots the Scale, &

can cls this work be duc.
w dire. All is in yo view
what the people doo,
the Surveyor too.

Fra Delaram sculp.



The Preface.

I Am not ignorant (friendly Reader) that hitherto, in writing, neuer any man pleaſed all; nor will I expect to be the firſt. To perſwade the courteous, were cauſeleſſe, for they are naturally kind; and to diſſwade the captious, were bootleſſe, for they will not be diuerted: Let the firſt make true vſe of theſe my Labours, and they ſhall find much pleaſure and profit therein; let the laſt (if they like not) leaue it, and it ſhall not offend them. To make apologie or exornation, in defence or commendation of the ſubiect whereof I treat, were needleſſe; it being alreadie, in the worlds opinion, of ſufficient habilitie and reputation, both to defend and commend it ſelfe: Onely of my manner and order in handling the ſame, I will ſay ſomewhat, as briefely as I may, for thy better inſtruction and vnderſtanding thereof.

In generall, I haue diſpoſed and digeſted the ſame into foure Bookes; whereof, the two former tend ſpecially to the principles and rules of Geometrie, with performance thereby of many vſefull and neceſſarie concluſions; and the two later, to matter of ſurney, with many inſtrumentall concluſions, tending as well to that, as diuers other purpoſes.

More particularly, in my firſt Booke, I begin with the Matters, Grounds, and Elements of GEOMETRIE, *as the definitions and termes of art belonging thereunto; moſt fitting firſt by the Practicioner to be learned and well vnderſtood: then next haue I placed therein diuers Geometricall* THEOREMES, *as the foundations, grounds, and reaſons whereon the practike part dependeth.*

In the ſecond (hauing formerly laid the foundation) I ſhew the meanes and practike operation of manie neceſſarie concluſions and Geometricall PROBLEMES, *as the diſtinction, application, and diuiſion of Lines and Angles; and the deſcription, reduction, addition, inſcription, tranſmutation, diuiſion, and ſeparation of all formes and kinds of ſuperficiall Figures, with their ſeuerall dimenſions. And conſidering, that as well in the* THEOREMES *of the firſt Booke, as the* PROBLEMES *in the ſecond, I wholly omit (for breuities ſake, and auoiding confuſion to*

the

the learner) their ſeuerall demonſtrations, vſing onely explication and conſtruction; I therefore expreſſe in the Margent againſt thoſe THEOREMES *and* PROBLEMES, *where, how, and in what place of* EVCLID, RAMVS, *and other Authors, to find their ſeuerall demonſtrations at large: and likewiſe, at the end of each conſtruction, I haue inſerted the like numbers and notes of reference, from the* THEOREMES *in the firſt Booke, to the* PROBLEMES *in the ſecond; and the contrarie: whereby moſt plainely and readily is found, and had, as well the reaſon and ground of any* PROBLEME *propoſed, as the effect and operation of any* THEOREME *deliuered.*

In the third Booke I begin with the deſcription of the ſeuerall Inſtruments fit and vſuall for Suruey; and of their ſeuerall vſes: wherein ſomewhat haue I ſpoken (though too ſparingly) concerning their abuſe; being now growne ſhamefully generall, by the multitude of ſimple and ignorant perſons (vſing, or rather abuſing, that good plaine Inſtrument, called the Plaine Table) *who hauing but once obſerued a Surueyor, by looking ouer his ſhoulder, how and in what manner he directs his ſights, and drawes his lines thereon; they preſently apprehend the buſineſſe, prouide them of ſome caſt* Plaine Table, *and within ſmall time after, you ſhall heare them tell you wonders, and what rare feats they can performe; yea, and will vndertake (or I will for them) that for tenne groats a day, and their charges defrayed, they ſhall be able to vndoe any man they deale with; or at leaſtwiſe, to doe him ſuch wrong and preiudice, as perhaps he might, with more eaſe, and leſſe loſſe, haue giuen ten pounds a day to one that would haue ſpoken leſſe, and performed more. But what ſhould I ſay more of them then thus,* Monoculi inter cæcos oculiſſimi ſunt, *and ſo will I leaue the blind, with tumbling the blind into the myre.*

I further deſcribe in this third Booke, the compoſition and vſe of an Inſtrument of mine owne, which I call the Peractor, *and of a Chayne, which I call the* Decimall *Chayne, with the diuiſions and parts thereof: which rightly vnderſtood and practiſed (I dare boldly ſay without oſtentation) is farre more vſefull and abſolute for ſpeed and exactneſſe, then any yet euer vſed: And I will maintaine by ſufficient demonſtration, that no man (vſing not the ſame, or the like) ſhall attaine to the ſame or the like perfection, for preciſe exactneſſe in any dimenſion, as I will thereby performe.*

And I further ſhew therein, the beſt, ſpeedieſt, and exacteſt meanes, for the ſuruey and inſtrumentall menſuration of a Mannor, or any other ſuperficiall content whatſoeuer, by diuers and ſeuerall meanes; with

manie

manie extraordinarie obſeruations and courſes, therein to be had and taken, not vſually knowne or practiſed; as by the argument of that Booke more particularly appeareth.

Wherein by the way I would aduiſe the Reader, who deſireth to make vſe thereof, and to profit himſelfe thereby, in reading and practiſing; to take the Chapters before him as they lye in order; for that I haue ſtriued to place them in ſuch an orderly and methodicall forme, as the one neceſſarily follow the other in vſe and practiſe; well knowing that diſorder and irregularitie in this kinde, breedeth not a little trouble and confuſion to the weake practicioner.

The fourth and laſt Booke, conſiſteth of the legall part *of Suruey; wherein I firſt ſhew what a Mannor is and the ſeuerall parts thereof, with the appendants thereunto; how the ſame is created and maintayned, and how and by what meanes deſtroyed and diſcontinued: alſo the ſeuerall ſorts and kindes of eſtates whereby any lands or tenements may be holden, and the ſeuerall tenures, rents and ſeruices depending on thoſe eſtates: I further ſhew therein the order and manner of keeping Courts of Suruey, with the entrie of the tenants euidence and eſtates; and the orderly and artificiall manner of ingroſſing the ſame, with many other neceſſary rules and obſeruations tending to thoſe purpoſes, as more at large alſo appeareth by the argument of the ſame Booke.*

And here, as before, would I aduiſe the practicioner, to obſerue the like courſe in reading and practiſing the rules and inſtructions of this Booke, as I haue formerly directed for the third, for that I haue ſtrictly obſerued the like decorum in placing the Chapters each after other, as of neceſſity they are to be vſed and practiſed.

Now might I here much inlarge and protend this Preface, in explicating the wonderfull vſe of the two former Bookes, in the performance of infinite concluſions Geometricall, ſo farre paſsing this ſubiect of Suruey, as it in it ſelfe exceedeth the meaneſt matter of diſpoſe, which (to auoide prolixitie) I will here forbeare, leauing the conſideration thereof to thine owne iudgement, when thou ſhalt finde therein by thy diligent practiſe the ſweet and pleaſing taſte of ſuch ſenſe-beguiling fruit.

And further might I amplifie the ſame, not only in declaring the great and infinite pleaſure, with no leſſe profit, which the true knowledge, vſe, and vnderſtanding of the two later Books may bring (aſwell to Surueyors, as all owners and occupyers of land in generall) but alſo of the antiquitie and neceſsitie of Suruey; howſoeuer ſleighted by many, who will not beſtow a penny in points, or two pence in tape, or the like, but they will number the one, and meaſure the other, before they pay for

either,

either, and yet will disburse manie thousands in a purchase, without the certaine knowledge of either quantitie, qualitie, or value thereof: (and these are those which are called penny-wise, &c.) whereby it often happens (as I haue often knowne) that a valuable purchase being made, within few weekes after, the money hath beene raised of the woods, and the lands perhaps immediately sold for much more then the money disbursed; and the same againe vented at the third hand, hath yeelded a double value: and all this vnseene, and vnsurueyed, with what disaduantage to the first vendor, I will leaue to the consideration of my young Master, who hath thus offended in selling all, and resteth now in repentance, with full resolution not to offend in the like. And the like haue I knowne of a purchase made, when a moitie of the charge could scarcely be raised.

But to spend time to this purpose, were to little end, and therefore will I end this purpose; only intreating thee gentle Reader, that as I haue thus imployed mine idle houres, to find thee houres of imployment; if thou reape either pleasure or profit by these my paines, to afford me thy good opinion (for Virtus laudata crescit, & honos alit artes*) which is all I craue.*

AARON RATHBORNE.

From my Lodging, at the house of M. ROGER BVRGIS, *against Salisburie-house-gate in the Strand, this sixt of Nouember,* 1616.

THE SURVEYOR.

The first Booke.

THE ARGVMENT OF THIS BOOKE.

THis first Booke consisteth of two parts; the former whereof intreateth onely of the first Matters, Grounds, and Elements of GEOMETRIE, *as the distinction of Lines, Angles, Triangles, and other Figures, with their* DEFINITIONS, *shewing what they are: The second part containeth diuerse Geometricall* THEOREMES, *tending chiefely and most fitly to the subiect and matter prosecuted in the subsequent Bookes, whereby the ingenious Practitioner may readily conceiue and apprehend the ground and reason of the Precepts, Rules, and Problemes therein deliuered.*

THE FIRST PART.

DEFINITION I.

A Point is that which is the least of all Materialls, hauing neither part nor quantitie.

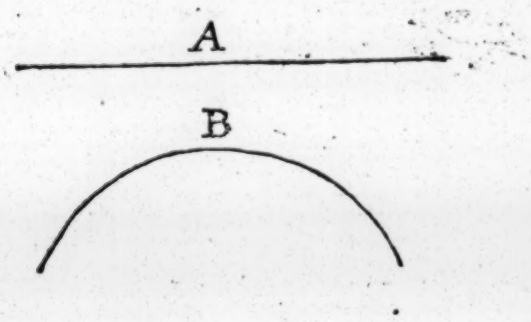
BEtweene *Vnitie* in ARITHMETIKE, and this *Point* in GEOMETRIE, there is a neere resemblance: but that, more simple and pure; this, materiall; and (although the least that can be imagined) requireth position and place, as this *Point* A. *Euc.* 1. *Def.* 1.

DEFINITION II.

A Line is length, without breadth or thickneſſe.

Euc. 1. *Def.* 2.

THis is the firſt quantitie in GEOMETRIE, and may be diuided into parts, in reſpect of his length, but admitteth no other diuiſion or dimenſion; and hath for his termes and limits that Geometricall point formerly ſpoken of. And of theſe lines are there two ſorts; namely right, as the line A. and crooked, or ſphericall, as the line B.

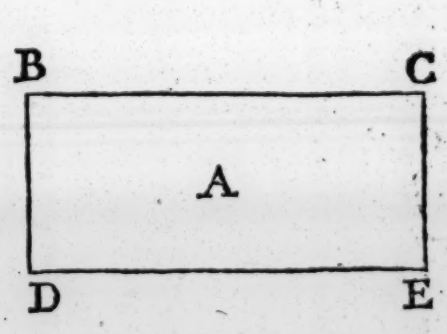


DEFINITION III.

A Superficies is that which hath onely length and breadth.

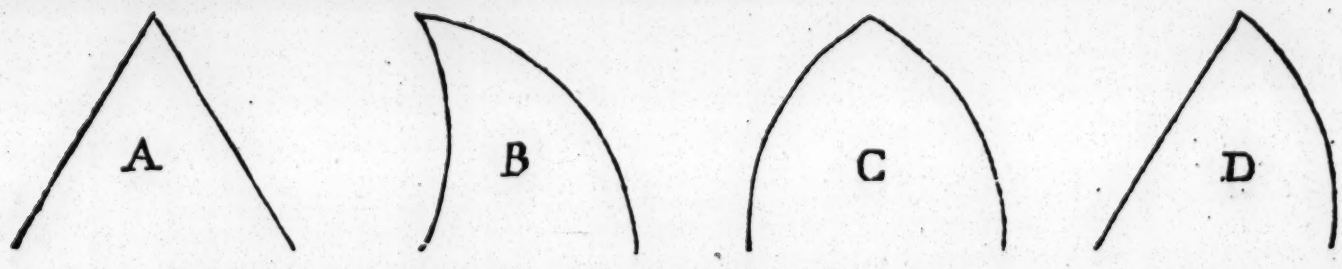
Euc. 1. *Def.* 5.

THis is the ſecond quantitie in GEOMETRIE, hauing two ſeuerall dimenſions, namely, length and breadth, without depth or thickneſſe (for that belongs to a ſolid or bodie, being the third quantitie in GEOMETRIE, and impertinent to this place, whereunto is attributed all three dimenſions, as length, bredth, and thickneſſe.) And as a line is limited with points, ſo is a Superficies with lines, and a Solid or Bodie with Superficies. As the figure A. being a Superficies, hath for his length B. C. or D. E. and for his breadth B. D. or C. E. which foure lines are the bounds, limits, and termes of the ſame Superficies.



DEFINITION IIII.

An Angle is the congreſsion or meeting of two lines in any ſort, ſo as both make not one line.



Euc. 1. *Def.* 8.

GEnerally of Angles (in reſpect of their lines) there are three ſorts; namely, right-lined, ſphericall, and mixed. As the Angle A. is called a right-lined Angle, being compoſed of two right lines; the Angles B. and C. are ſphericall, or crooked Angles; and the Angle D. a mixed Angle, being cauſed of both.

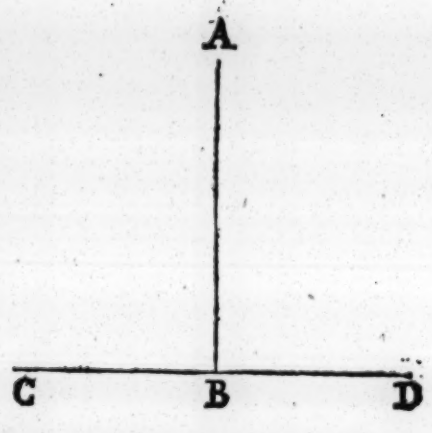
DEFI-

DEFINITION V.

If a right line fall on a right line, making the Angles on either side equall, each of those Angles are called right Angles: And the line erected is called a Perpendicular line vnto the other.

AS in generall there are three sorts of Angles, in respect of their lines, as aforesaid; so in particular, of right-lined Angles there are three sorts, in respect of their quantities; namely, an *Orthogonall*, or right Angle; an *Obtuse*, or blunt Angle; and an *Acute*, or sharpe Angle: whereof, according to this definition, the line A. B. falling on the line C. D. and making the Angles on either side equall, namely, the Angle A. B. C. on the one side equall to the Angle A. B. D. on the other side; those two Angles are called right Angles: And the line A.B. erected on C.D. without inclination to either side, is called a *Perpendicular*, or Plumbe line. *And here note further, that vsually an Angle is described by three letters; of which, the second or middle letter representeth alwayes the Angle intended.* PROB. 4. 5. 6. 9.

Euc. 1. *Def.* 10.

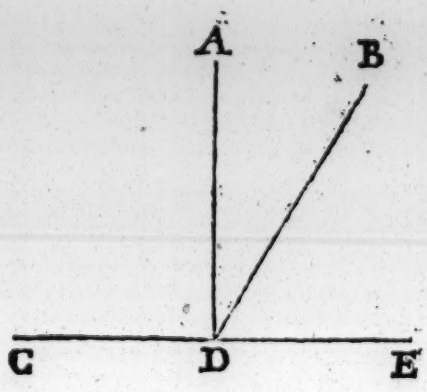


DEFINITION VI.

An Angle which is greater then a right Angle, is an obtuse Angle.

EVerie Angle in generall (not being a right Angle) whether greater or lesser, is called an *Oblique* Angle: but particularly, if greater then a right Angle, it is called an *Obtuse* Angle; if lesser, an *Acute* Angle: As the Angle B. D. C. (being greater then the right Angle A. D. C.) is an *Obtuse* Angle; for it containeth it, and also the Angle A. D. B.

Euc. 1. *Def.* 11.



DEFINITION VII.

An Acute Angle is that which is lesse then a right Angle.

THis Definition is manifest by the former Diagram, wherein the Angle A. D. B. is an *Acute* Angle, being lesse then the right Angle A. D. E. for the same right Angle containeth it, and also the Angle B. D. E. being likewise an *Acute* Angle.

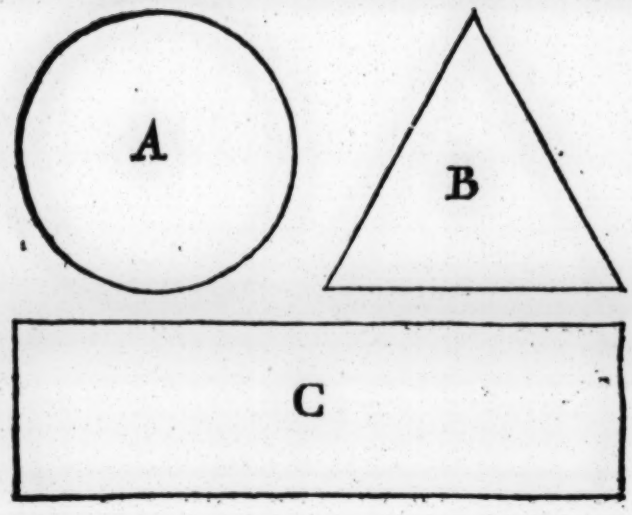
Euc. 1. *Def.* 12.

DEFINITION VIII.

A Figure is that which is contained vnder one & many limits.

Euc. 1. *Def.*14.

AS theſe three figures, A. B. and C. whereof the firſt is contained vnder one limit; the ſecond, vnder three; and the third, vnder foure; and the like of others. Wherein is to be noted, that of two right lines, no Figure can be contained.



DEFINITION IX.

A Circle is a plaine Figure, and contained vnder one line, which is called the Circumference thereof.

Euc. 1. *Def.*15. *Ram.* 1. 15.

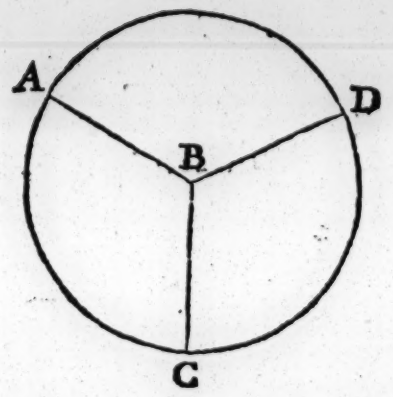
A Circle, of all other Figures, hath the prioritie, being of all moſt perfect and abſolute; and therefore moſt fitting firſt to be defined: as the Figure A. in the laſt Diagram.

DEFINITION X.

The Centre of a Circle is that point which is in the middeſt thereof: from which point, all right lines, drawne to the Circumference, are equall.

Euc. 1. *Def.*16.

AS the point B. in this Figure is the Centre thereof, from whence the lines B. A. B. C. and B. D. being drawne to the Circumference (and as manie others as are ſo drawne) are all equall, and are ſemidiameters to the ſame Circle. PROB. 34. 1.

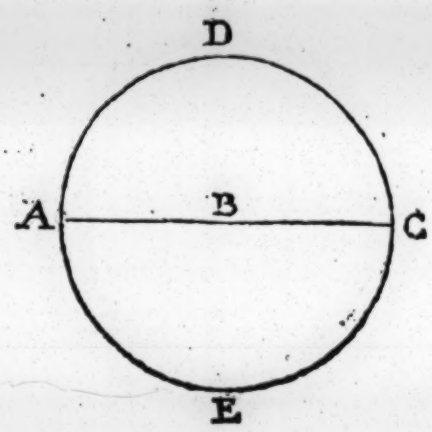


DEFINITION XI.

The Diameter of a Circle is a right line, paſsing by the Centre through the whole Circle, and diuideth the ſame into two equall parts: Either halfe

halfe of which Diameter, is called the Semidiameter of the same Circle.

AS the line A. B. C. in this circle is the diameter thereof, for that it passeth by the centre B. through the whole circle, as from A. to C. and also diuideth the circle into two equall parts, the one halfe towards D. the other towards E. Either halfe of which Diameter, as A. B. or B. C. is called the Semidiameter of the same Circle. PROB. 34.



Euc. 1. *Def.* 17.

DEFIN. XII.

A Semicircle is a Figure contained vnder the Diameter of a Circle, and the Semicircumference of the same Circle.

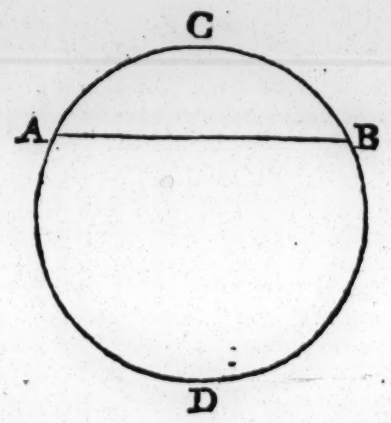
AS supposing the circle A. B. C. the Diagram of the last DEFINITION, to be diuided into two equall parts by the diameter thereof A. C. whereby two Figures are proiected, namely, A. D. C. and A. E. C. Now by this DEFINITION, each of these Figures are *Semicircles*, for that the one of them is contained vnder the diameter of the same circle A. C. and the semicircumference A. D. C. and the other vnder the same diameter, and the semicircumference A. E. C.

Euc. 1. *Def.* 18.

DEFIN. XIII.

A Segment, Section, or Portion of a Circle, is a Figure contained vnder a right line, and part of the circumference, either greater or lesse then the Semicircle.

AS the diameter of a circle passing by the centre thereof, diuideth the same into two equall parts; so any right line, drawne from any one part of the circumference to any other part thereof (and not passing by the centre) diuideth the circle into two vnequall parts, which are called *Segments*, *Sections*, or *Portions* of a circle: As in this circle, the figure A. C. B. because it is contained vnder the right line A. B. and the lesser part of the circumference A. C. B. is called a lesser *Segment*, *Section*, or *Portion* of a circle; and the figure A. D. B. because it is contained vnder the right line A. B. and the greater part of the circumference A. D. B. is called a greater *Segment*, *Section*, or *Portion* of a circle. *Here note also, that these parts and such like of the circumference so diuided, are commonly called Arches, or arch lines. And all lines (lesse then the diameter) drawne*



Euc. 1. *Def.* 19. 3. *Def.* 5.

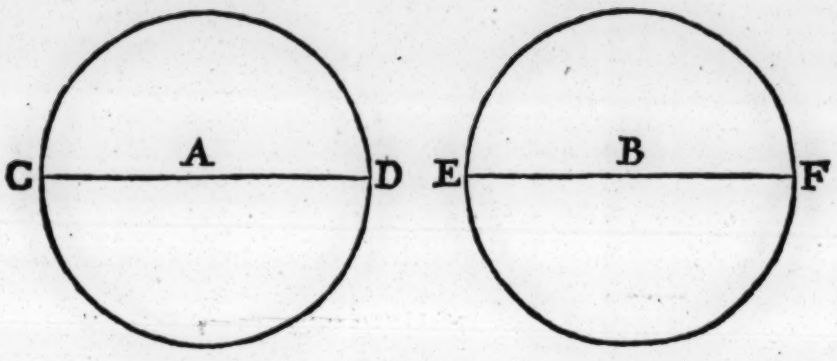
and applied as the line A. B. are called Cordes, or cord lines, of thoſe Arches which they ſo ſubtend; or Subtenſes, *becauſe they ſubtend both ſegments.* THEOR. 73. PROB. 34.

DEFIN. XIIII.

Equall Circles are ſuch as haue equall Diameters, or whoſe lines, drawne from their Centres, are equall.

Euc. 3. *Def.* 1.

AS theſe two circles A. and B. are equall, their diameters being equall, namely, C. D. and E. F. or their Semidiameters, (which, according vnto this DEFINITION, are lines drawne from the centres vnto their circumference) as A. C. or A. D. and B. E. or B. F.

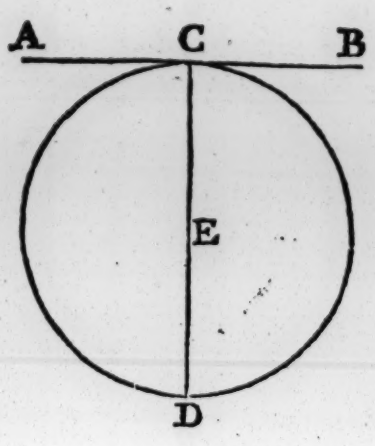


DEFIN. XV.

A right line is ſaid to touch a circle, which touching it, and being extended or produced, doth not cut the circumference thereof.

Euc. 3. *Def.* 2.

AS the right line A. B. being drawne by the point C. doth there onely touch the circle, and being produced vnto B. cutteth not the circumference thereof. This line is commonly called a *Tangent*, or *Contingent* line: whereof there is great and infinite vſe, in manie Concluſions, Geometricall and Aſtronomicall, eſpecially in the menſuration and reſolution of Triangles, as well right-lined, as ſphericall, by the Canons and Tables of Synes, Tangents, and Secants, in that behalfe calculated.



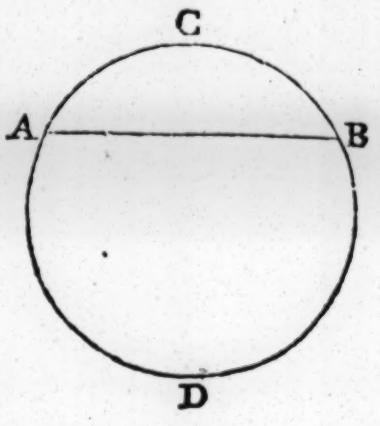
DEFIN. XVI.

An angle of a Section, or Segment, is that which is contained vnder a cord line, and the arch line of the ſame Section.

Euc. 3. *Def.* 6.

AS the angles A. B. C. and B. A. C. in the leſſer ſegment of this circle, are angles of a Section, becauſe they are contained vnder the cord line A. B. and the arch line A. C. B. Alſo the angles D. B. A. and

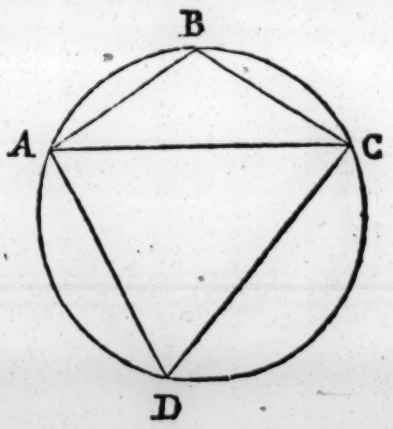
and D. A. B. in the greater Segment are angles of the ſame Segment, by the like reaſon. And all angles of this kind are called mixed angles, becauſe they are contayned vnder a right line and a crooked. Of which two Segments, the leſſer hath alwaies the leſſer angle; and the greater, the greater angle.



Defin. XVII.

An angle in a Section, or Segment, is when two right lines are drawne from any point in the arch line, to the ends or extreames of the cord line; the angle in that point of the arch line is called an angle in a Section or Segment.

AS the angle A. B. C. in the leſſer Segment is an angle in a Section, or Segment, by reaſon that the two right lines B. A. and B. C. are drawne from the point B. in the arch line to the ends or extreames of the cord line A. C. And alſo the angle A. D. C. in the greater Segment is an angle in a Section, or Segment, becauſe the two right lines D. A. and D. C. are drawne from the point D. in the arch line to the ends or extreames of the cord line A. C. *And here note, the greater Section hath in it the leſſer angle, and the leſſer Section the greater angle, contrarie to the mixt angles in the precedent* Definition *mentioned. And here alſo is to be noted, by the declaration of this and the former* Definition, *the difference betweene an angle of a Segment, and an angle in a Segment; the firſt being called a mixt angle, and this a right lined angle.*



Euc. 3. Def. 7.

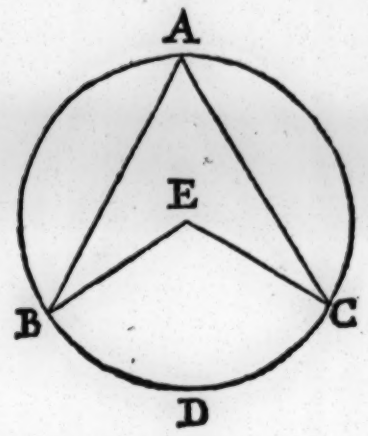
Defin. XVIII.

If two right lines be drawne from any one point in the circumference of a Circle, and receiue any part of the ſame circumference, the angle contained vnder thoſe two lines is ſaid to belong and to be correſpondent to that part of the circumference ſo receiued.

AS the angle B. A. C. contained vnder the right lines A. B. and A. C. drawne from the point A. and receiuing the circumference B. D. C. by this Definition is ſaid to belong, ſubtend, and pertaine vnto the circumference B. D. C. And if right lines be drawne from the centre

Euc. 3. Def. 8.

centre to the former points B. and C. then is that angle ſaid to be in the centre of a circle, as the angle B. E. C. Which angle likewiſe ſubtendeth the ſame circumference B. D. C. and is alwayes double in quantitie to the former angle, drawne from the circumference. And the circumference B. D. C. is alſo the meaſure of the quantitie or greatneſſe of the angle B. E. C. in the centre.



DEFIN. XIX.

A Sector of a Circle is a figure contained vnder two right lines, drawne from the centre of a Circle, and vnder part of the circumference receiued of them.

Euc. 3. Def. 9. Ram. 16. 3.

AS in the laſt DEFINITION, the figure B. E. C. is the Sector of a circle, becauſe it is contained vnder the two right lines E. B. and E. C. drawne from the centre E. and vnder part of the circumference, namely, B. D. C. receiued of them.

DEFIN. XX.

Right lined figures are ſuch, as are contained vnder right lines, of what number ſoeuer, aboue two.

Euc. 1. Def. 20.

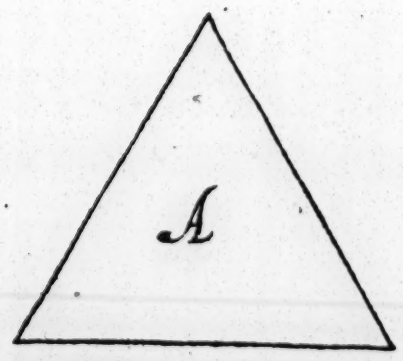
AS thoſe which follow, being contained vnder three, foure, fiue, or more ſides, who take their denominations, as well of the number of their angles, as of their ſides; ſo a figure contained vnder three lines, in reſpect of his ſides, is called a three ſided figure; and in reſpect of his three angles is called a Triangle: and ſo of the reſt. *Where is to be noted, that euery right lined figure hath as many angles as it hath ſides.*

DEFIN. XXI.

An Equilater Triangle is that, which hath three equall ſides.

Euc. 1. Def. 24. Ram. 8. 8.

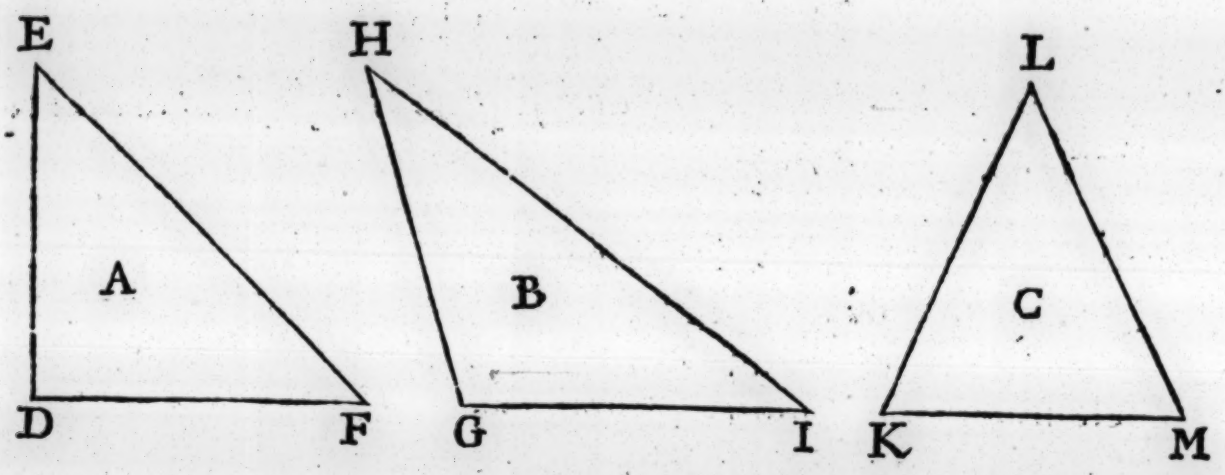
AS a Triangle is the firſt of all right lined figures (for vnder leſſe then three right lines can no figure be contained:) So of all Triangles, the Equilaterall Triangle is moſt ſimple and abſolute, hauing equall lines and equall angles; containing euery of them a Sextans of a Circle, which is 60 degrees. Triangles haue their denominations, differences, and appellations, as well of their angles as their ſides: As this Triangle A. in reſpect of his



his three equall ſides, is called an Equilater Triangle; and in reſpect of his ſharpe angles, is tearmed an *Oxigonium*, or acute angled Triangle. But this appellation (in mine opinion) to this kind of Triangle, is needleſſe; for that the name of an Equilaterall Triangle doth imply the ſame, ſeeing it can containe no other but acute angles: for if in any Triangle there be either a right or an obtuſe angle, the ſides cannot be all equall, and therefore by this DEFINITION no Equilater.

DEFIN. XXII.

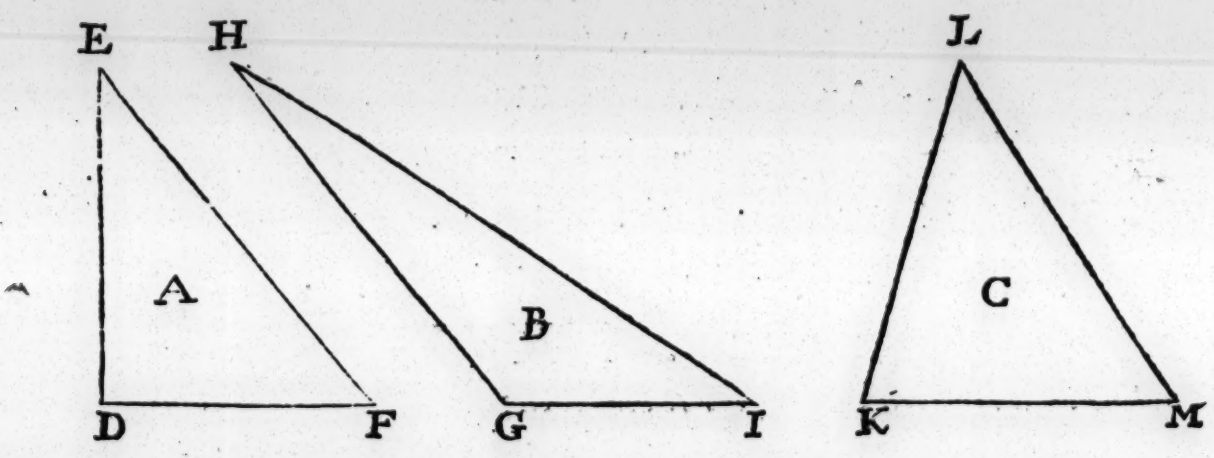
An Iſoſceles is a Triangle, which hath onely two equall ſides.



THis is the ſecond ſort of Triangles, and hath two ſides onely of one length, the third being either longer or ſhorter: As theſe three Triangles A.B. and C. haue euerie of them two equall ſides, and are therefore called *Iſoſceles*. But in reſpect of their angles, the Triangle A. is called an *Orthigonium*, or right angled *Iſoſceles*, for that his angle at the point D. is a right angle. Alſo the Triangle B is called an *Ambligonium*, or an obtuſe angled *Iſoſceles*, for that his angle at the point G. is a blunt or obtuſe angle. And likewiſe the Triangle C. is called an *Oxigonium*, or acute angled *Iſoſceles*, becauſe all his angles are acute or ſharpe. Alſo this Triangle is called an *Equicrurall* Triangle, in reſpect of his two equall ſides. *Euc.* 1. *Def.* 25.

DEFIN. XXIII.

A Scalenum is a Triangle which hath all his ſides vnequall.



THis is the third kind of Triangles, and hath all his ſides of ſeuerall lengths: As theſe three Triangles A.B.C. haue euerie of them all their ſides vnequall, and therefore called *Scalena*. But in reſpect of their *Euc.* 1. *Def.* 26.

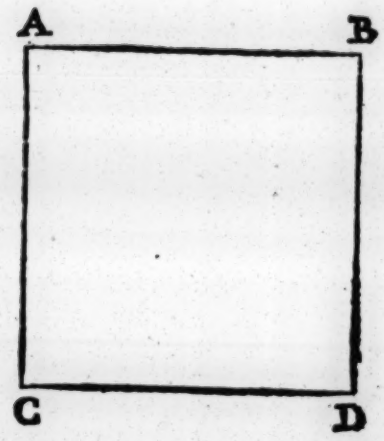
their angles, the triangle A. is called an *Orthigonium*, or right angled *Scalenon*, for that his angle at the point D. is a right angle. Alſo the triangle B. is called an *Ambligonium*, or an obtuſe angled *Scalenon*, for that his angle, at the point I. is a blunt or obtuſe angle: And laſtly, the triangle C. is called an *Oxigonium*, or an acute angled *Scalenon*, becauſe all his angles are acute or ſharpe. *It is to be noted generally in all Triangles, that in compariſon of any two ſides of a Triangle, the third ſide is called the Baſe; as of the Triangle A. in reſpect of the two lines E.D. and E.F. the line D.F. is the baſe: In regard of the two lines F.D. and F.E. the line E.D. is the baſe; and in reſpect of the two lines D.E. and D.F. the line E.F. is the baſe.*

DEFIN. XXIIII.

A Square, or Quadrat, is a foure ſided figure, whoſe ſides are all equall, and angles all right angles.

Euc. 1. *Def.* 30. *Ram.* 12. 2. 2. *Con.* 1.

AS the figure A.B.C.D. is a Square, or Quadrat, becauſe all the lines thereof are equall, and all the angles right angles. Theſe foure ſided figures likewiſe, as well as Triangles, take their appellation partly of their ſides, and partly of their angles; as by their ſeuerall DEFINITIONS hereafter appeareth.

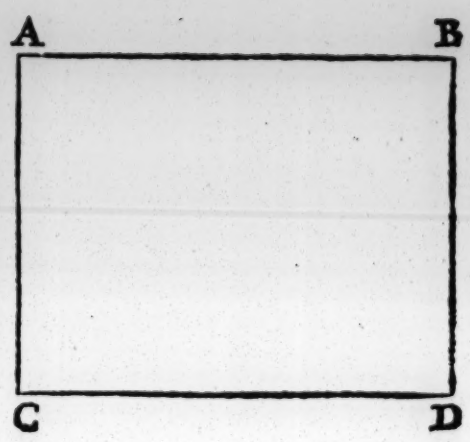


DEFIN. XXV.

A long Square is that whoſe angles are all right angles, and whoſe oppoſite ſides onely are equall.

Euc. 1. *Def.* 31. *Ram.* 1. 13.

THis figure differeth little from the Square, or Quadrat, laſt defined, hauing all equall angles like vnto it; but the ſides are vnequall. As in this figure A.B.C.D. all the angles are right angles, and the oppoſite ſides onely are equall, as the length A.B. is equall to the length C.D. and the breadth A.C. to the breadth B.D. but compare them otherwiſe, and they are vnequall.



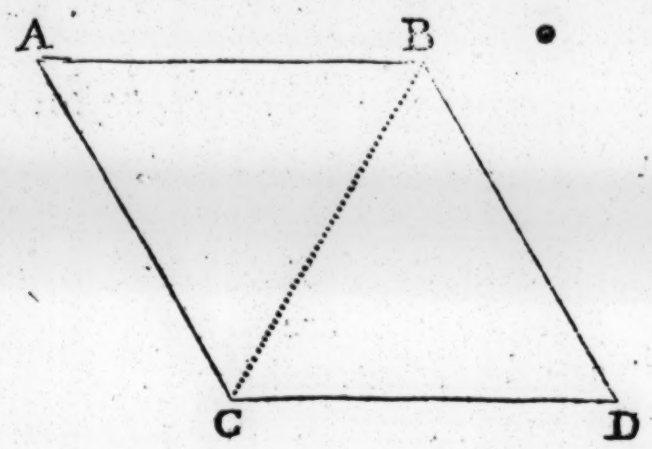
DEFIN. XXVI.

A Rhombus (or Diamond) is a figure with foure equall ſides, but no right angle.

Euc. 1. *Def.* 32. *Ram.* 8. 14.

AS this figure A.B.C.D. is a Rhombus, hauing all his ſides equall, and likewiſe the oppoſite angles; but the angles at A. and D. are acute angles, and thoſe at B. and C. obtuſe. Betweene a Square, or Quadrat, and

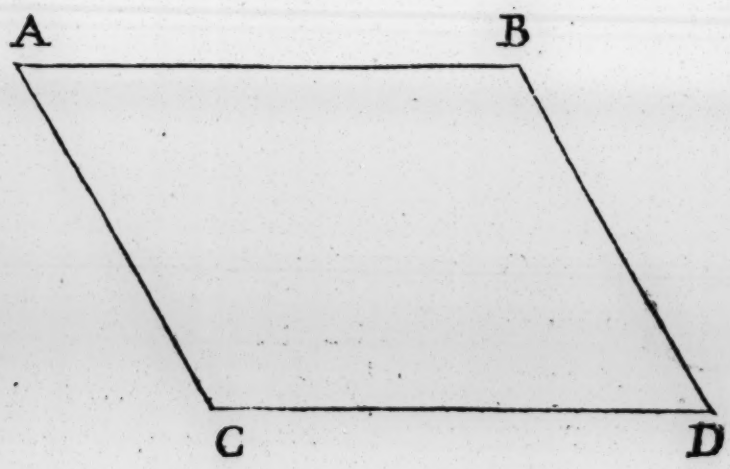
and this figure, is much reſemblance, either kind hauing all ſides equall; and likewiſe their angles in generall quantitie; but different in particular qualitie; that hauing foure right angles, this two obtuſe; and two acute angles; yet are they in generall quantitie equall to foure right angles: for by how much the two acute angles are defectiue or wanting of two right angles, by ſo much are the obtuſe angles abounding or exceeding. *This figure is deſcribed by the connexion of two Equilater Triangles, by any two of their ſides, as appeareth by the prickt Diagonall line B. C. which being omitted and left out, this figure remaineth perfect, and hath his acute angles equall to thoſe of an Equilater, namely, 60. degrees, and the obtuſe angles double thereunto.* PRO. 57.



DEFIN. XXVII.

A Rhomboydes (or Diamond-like) is a figure, whoſe oppoſite ſides and oppoſite angles are onely equall, and hath no right angles.

AS this figure A. B. C. D. is a Rhomboydes, and hath his ſides A. B. and C. D. oppoſite and equall, and likewiſe A. C. and B. D. but hath no right angle: For the angles at the points A. and D. are acute, oppoſite, & equall; and likewiſe the angles, at the points B. and C. are obtuſe, oppoſite, and equall.



Euc. 1. Def. 33. Ram. 9. 14.

Note here, that the foure figures laſt before defined, namely, a Square, a figure of one ſide longer, a Rhombus, and a Rhomboydes, are commonly called Parallelograms; of which foure, the two former are called right angled Parallelograms. PROB. 90.

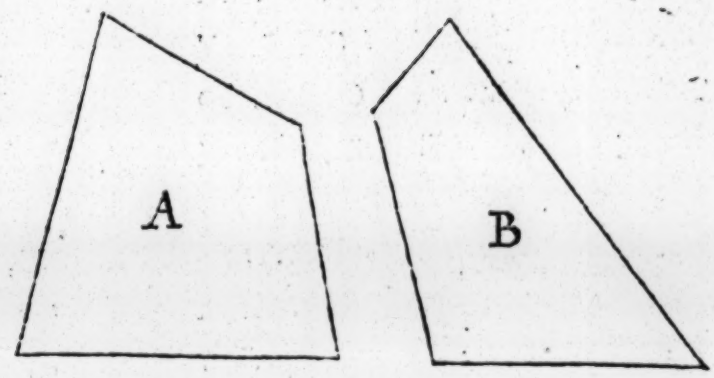
DEFI-

DEFIN. XXVIII.

All other foure lined figures, beſides thoſe formerly defined, are called TRAPEZIA, *or Tables.*

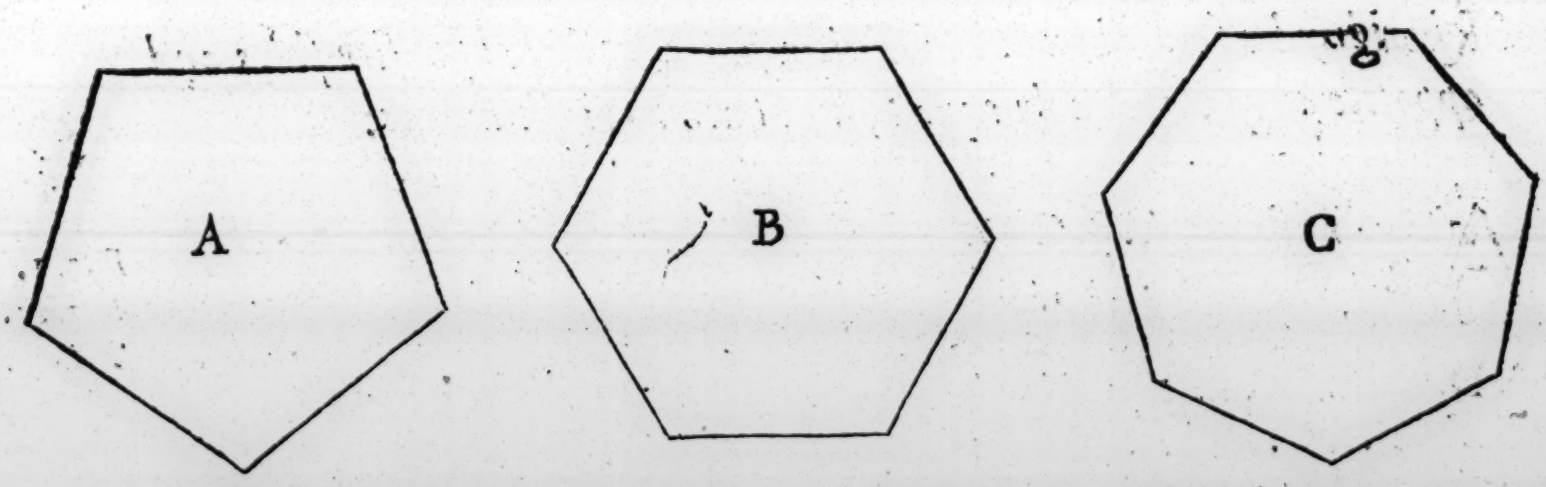
Euc. 1. *Def.* 34. *Ram.* 10. 14.

AS all figures, of foure ſides, which are made at aduenture, without reſpect or regard of equalitie, or inequalitie, or obſeruation of order, either in their lines or angles; which are therefore called irregular figures: as theſe figures A. and B. are.



DEFIN. XXIX.

Manie ſided figures are thoſe which haue more ſides then foure.



Euc. 1. *Def.* 23. *Ram.* 11. 14.

OF theſe, may infinite ſorts be deſcribed, by addition of lines: but if they containe aboue foure ſides, they are generally called *Polygona*, but particularly according to the number of their ſides: As the figure A. is called a *Pentagon*, becauſe it is contained of fiue ſides; the figure B. a *Sexagon*, being contained vnder ſix lines; and C. is called a *Septagon*, becauſe it is contained vnder ſeuen ſides. And the like of others. PROB. 62. 95.

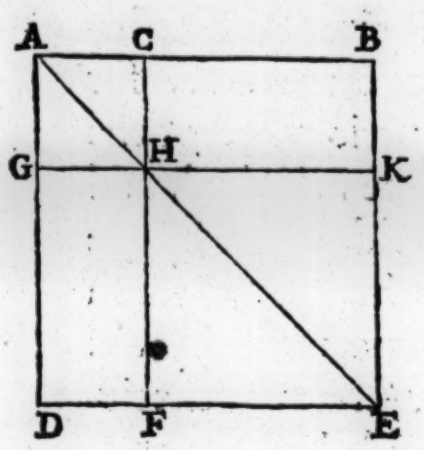
DEFIN. XXX.

Either of thoſe Parallelograms, which are about the diameter of a Parallelogram, together with the two ſupplements, is called a Gnomon.

Euc. 2. *Def.* 2.

RIghtly to conceiue this DEFINITION, it is requiſite firſt to vnderſtand, what thoſe Parallelograms are which are ſaid to be about the diameter of a Parallelogram; and likewiſe, what ſupplements are.

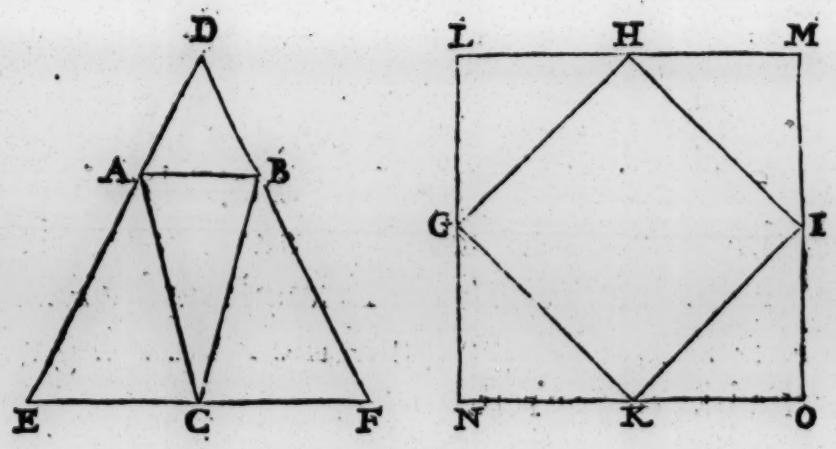
are. For the first, those are said to be parallelograms about the diameter, which haue for their particular diameters part of that which the whole parallelogram hath: And supplements are such, as are without the whole diameter, the diameter passing betweene them, and cutting them not. As in the parallelogram A. B. E. D. the particular parallelograms H.K. E. F. and A.C. H.G. are said to be about the diameter, because they haue for their particular diameters part of the whole diameter A.E. as A.H. and H. E. And the supplements are the two parallelograms C.B.K.H. and G.D.F.H. because they are without the whole diameter A. E. which passeth betweene them, and cutteth them not. Now take away either of those particular parallelograms, which soeuer it be, and the other remaining, together with the two supplements, is that which by this DEFINITION is called a *Gnomon*. PROB. 101. THEOR. 5.



DEFIN. XXXI.

That right lined figure is said to be inscribed in another right lined figure, which hath euerie angle touching euerie side of the figure wherein it is inscribed.

AS in these two figures, the Triangle A.B.C. is sayd to bee inscribed within the Triangle D. E. F. because euerie of his angles A. B. and C. doth touch euerie side of the Triangle D.E.F. Likewise, the square G. H.I.K. is said to be inscribed within the greater square L.M.N.O. because euery of his angles G. H.I.K. toucheth euerie side of the same greater square. The like consideration is to be had of circumscribing one right lined figure about another.



Euc. 4. *Def.* 1.

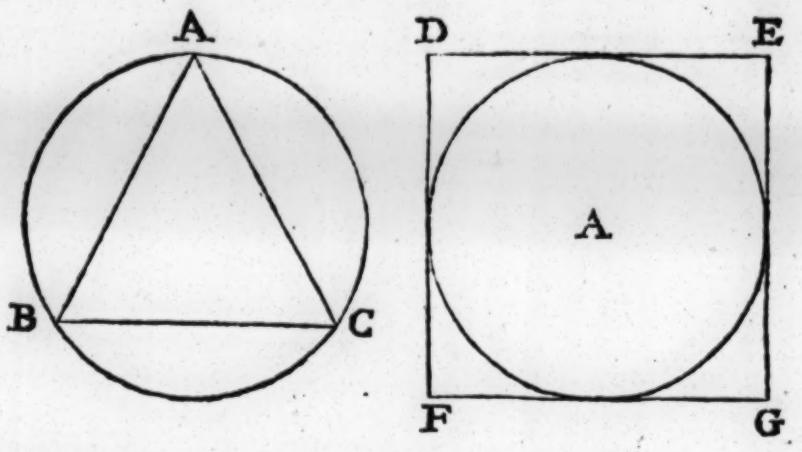
Hæc definitio de rectilineis homogeneis, siue æquali laterum numero terminatis intelligenda est.

DEFIN. XXXII.

A right lined figure is inscribed within a circle, when euerie angle of the inscribed figure toucheth some part of the circles circumference.

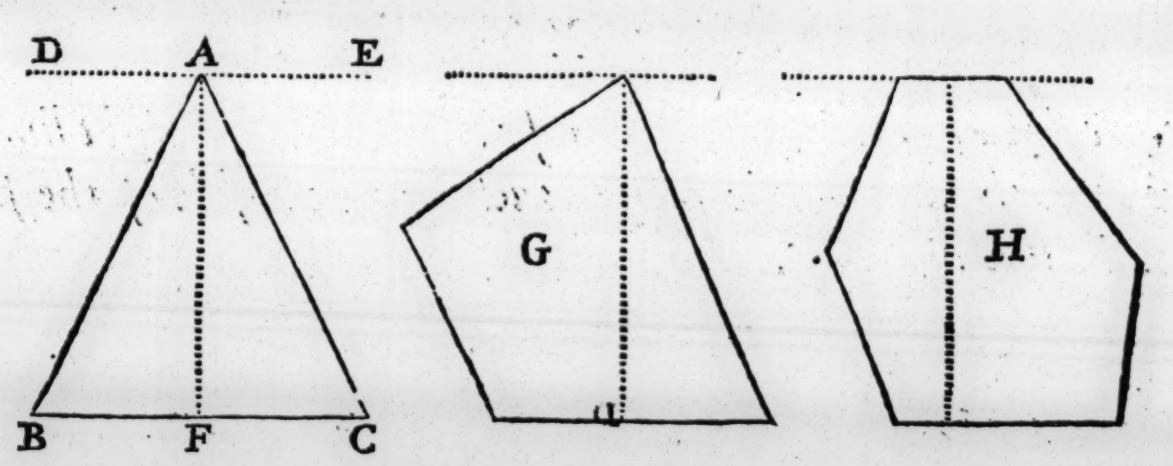
ALl the angles of a regular right lined figure, inscribed in a circle, or the sides of the like figure circumscribed about a circle, may easily touch the circumference thereof, by reason of the perfection and vniformitie of a circle. As the Triangle A.B.C. is inscribed in the circle A.B.C. *Euc.* 4. *Def.* 3.

and alſo the ſquare D.E. F.G. is circumſcribed about the circle A. for that euerie angle of the Triangle inſcribed, and euerie ſide of the ſquare circumſcribed, toucheth ſome one point of the circumference of the circle. And the like conſideration is to be had of circles inſcribed or circumſcribed within or about any right lined figure. PROB. 112. 113.



DEFIN. XXXIII.

The altitude of a figure, is the parallell diſtance betweene the top of a figure and the Baſe.

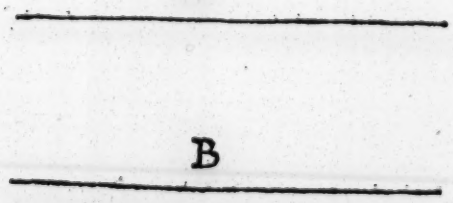


Euc. 6. *Def.* 4. AS the height or altitude of this Triangle A.B.C. is the ſpace or diſtance betweene the line D.E. drawne parallell to the Baſe B.C. by the higheſt point of the ſame Triangle, as by A. and the ſame Baſe B.C. which parallell diſtance is equall, and the ſame thing with the perpendicular A.F. And the like of the reſt.

DEFIN. XXXIIII.

Parallell lines are ſuch, as being drawne on any plaine Superficies, and produced either way infinitely, doe neuer meet or concurre.

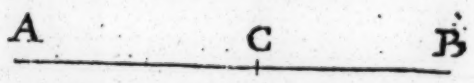
Euc. 1. 35. *Ram.* 2. 11. 3. 11. AS theſe right lines A. and B. which being produced and drawne forth infinitely, by reaſon of their equall and parallell diſtance the one from the other, will neuer meete or concurre; and therefore are called parallell lines. PROB. 2. 3.



DEFI-

DEFIN. XXXV.

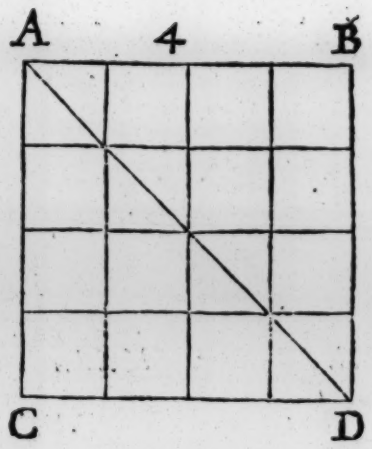
A right line is said to be diuided by extreame and meane proportion, when the lesser part, or segment thereof, is to the greater, as the greater is to the whole line.



AS the line A. B. being so diuided in the point C. that the lesser part, or segment, C. B. hath the same proportion to the greater part, or segment, A. C. as the same greater part hath to the whole line A.B. then is the same line A. B. diuided by an extreame and meane proportion. The meanes how to diuide a line in such sort, is hereafter taught by the 20. PROB. of the next booke. This line is of wonderfull and infinite vse in manie Geometricall operations, as appeareth manifestly almost through the whole thirteenth booke of *Euclid.* PROB. 20.

DEFIN. XXXVI.

The power of a line is the square of the same line, or any plaine figure equall to the square thereof.

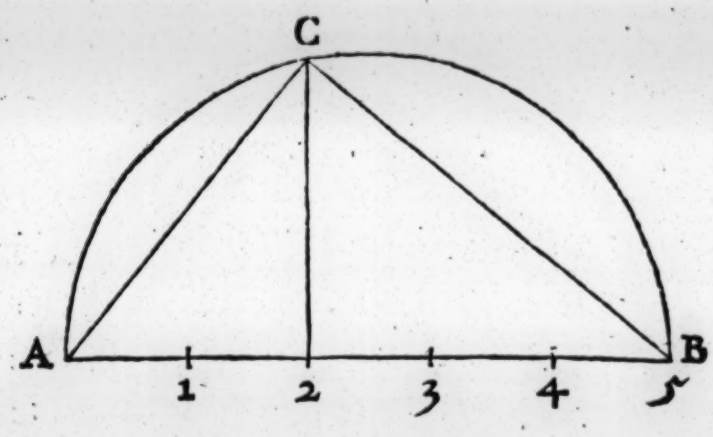


AS the power of the line A. B. is the square of the same line, namely, the figure A. B. C. D. or any other plaine figure equall thereunto. And so great power and abilitie is a line said to haue, as the quantitie of the square it makes: As this line A. B. containing 4. the power thereof is 16. In this kind is the Diagonall or diameter of a square (as the line A. D.) said to be double in power to the side of the same square, for that a square made of the Diagonall, is double in quantitie to the square made of the side. And likewise the line which subtendeth the right angle in an Orthigonall Triangle, is said to be equall in power to both the containing sides: as the line A. D. which subtendeth the right angle A. C. D. in the Triangle A. C. D. is equall to both the squares made of the two containing sides, namely of A. C. and C. D. PROB. 23.

DEFIN. XXXVII.

To diuide a giuen line in power, is to finde two other lines, whose squares together shall be equall to the square of the giuen line,

but the ſquare of the one to the ſquare of the other, to be in any proportion required.



AS if A. B. were a line giuen, and it were required to diuide the ſame line in power, according to the proportion of 2. to 3. It is hereby intended to finde two other lines, as C. A. and C. B. whoſe ſquares together are equall to the ſquare of the giuen line A.B. but the ſquare of the one, namely C. A. is to the ſquare of the other C. B. in ſuch proportion, as 2. to 3. that is, the ſquare of C. B. containeth the ſquare of C. A. once and a halfe. The meanes how to performe the ſame, is hereafter taught in the 23. PROB. of the 2. Booke. PROB. 23.

DEFIN. XXXVIII.

To inlarge a line in power, is to find another line, whoſe ſquare ſhall haue any proportion required (of the greater inequalitie) to the ſquare of the giuen line.

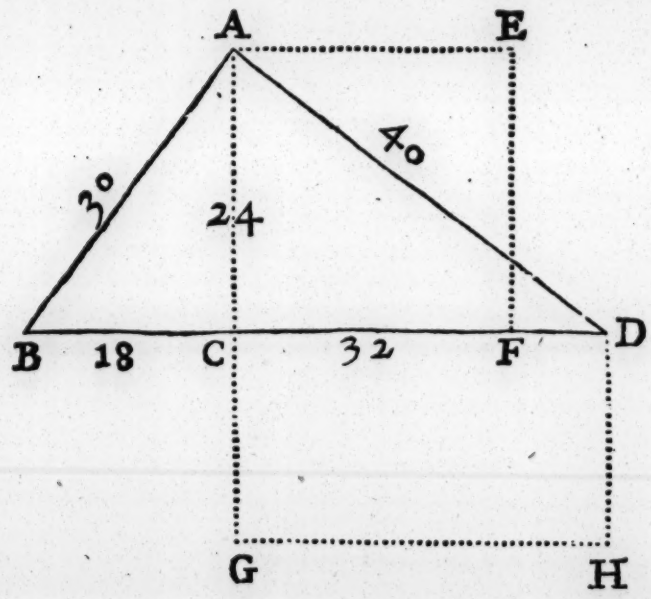
AS ſuppoſe the line 2 B. in the former Diagram were giuen to be inlarged in power, as 3. to 5. It is hereby intended to find out another line, as the line C. B. whoſe ſquare ſhall beare ſuch proportion to the ſquare of the giuen line 2 B. as 5 to 3. which is a proportion of the greater inequalitie, and is called *Superbipartiens tertias:* that is, as 5 containes 3. ſo the ſquare of C. B. the line ſought for, containeth the ſquare of 2 B. $1\frac{2}{3}$. The working whereof is taught in the 24. PROB. of the 2. booke.

Theſe lines are of infinite vſe in many Geometricall Concluſions.

DEFIN. XXXIX.

A meane proportionall line is that, whoſe ſquare is equall to the right angled Parallelogram, or long Square, contained vnder his two extreames.

A Meane proportionall line is ſo tearmed, in reſpect of the relation it hath to two other lines, which are called his extreames; for of a meane without extreames, or extreams without a meane, there is no compariſon. As in this Diagram, the perpendicular A.C. of the right angled Triangle A. B. D. is a meane proportionall line betweene the two ſegments of the

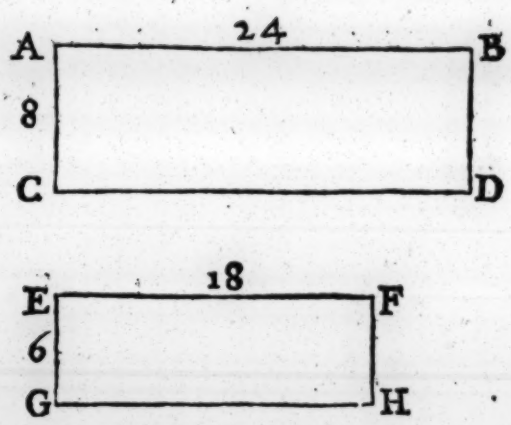


the Baſe B.C. and C.D. his extreames, becauſe the ſquare of the ſame line A.C. namely, A.E.F.C. is equall to the long ſquare, contained vnder the lines B.C. and C.D. (for the line C.G. is equall to the line B.C.) for as B.C. is to A.C. ſo is A.C. to C.D. Alſo the line A.B. is a meane proportionall betweene B.C. the ſegment of the Baſe lying next vnto it, and B.D. the whole Baſe; for as B.C. the leſſer ſegment of the Baſe, is to B.A. ſo is B.A. to B.D. the whole Baſe. And laſtly, the line A.D. is a meane proportionall, betweene C.D. the ſegment of the Baſe, lying next vnto it, and B.D. the whole Baſe; for as C.D. the greater ſegment of the Baſe, is to A.D. ſo is A.D. to B.D. the whole Baſe.

DEFIN. XL.

Like right lined figures are thoſe, which haue equall angles, and proportionall ſides about thoſe equall angles.

AS in theſe two right angled Parallelograms, the angle in the point A. of the greater, is equall to the angle E. of the leſſer; likewiſe the angle B. to the angle F. and C. to G. and D. to H. And moreouer, the ſide A.B. hath that proportion to the ſide A.C. as E.F. hath to E.G. and A.C. to C.D. as E.G. to G.H. and ſo of the reſt: Wherefore theſe two Parallelograms are called like right lined figures: and ſo of Triangles and all other figures, of what kind ſoeuer. PROB. 45.

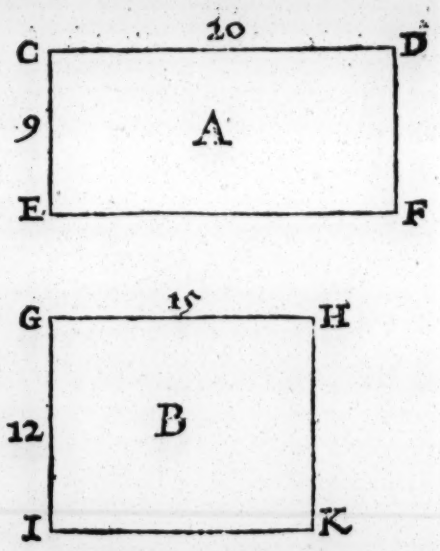


Euc. 6. Def. 1.

DEFIN. XLI.

Reciprocall figures are ſuch, as haue the ſides of either to other mutually proportionall.

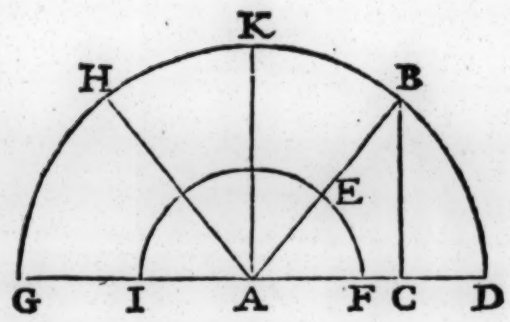
AS the Parallelograms A. and B. haue their ſides mutually proportionall; that is, as the ſide C.D. of A. is to the ſide G.H. of B. ſo is the ſide G.I. of B. to the ſide C.E. of A. and therefore are they called Reciprocalls: for as 20. is to 15. an antecedent of A. to a conſequent of B. ſo is 12. to 9. an antecedent of B. to a conſequent of A.



Euc. 6. Def. 2.

DEFIN. XLII.

The quantitie or meaſure of an angle, is the arch of a circle, deſcribed from the point of the ſame angle, and intercepted betweene the two ſides of that angle.



AS in the Triangle A. B. C. the meaſure or quantitie of the angle B. A. C. is the arch B.D. or E. F. for the circumference of euerie circle (whether greater, or leſſer) is diuided into 360 equall parts, which are called degrees, and euerie degree into 60 ſcruples or minutes, and euerie minute into ſo manie ſeconds, &c. Which parts or degrees are greater or leſſer, as the circles, whoſe parts they are, are greater or leſſer; and thoſe arches which containe the ſame number of parts or degrees in equall circles, are equall; and in vnequall circles, they are called like arches; as the arches B. D. and H. G. are equall; but the arches B. D. and E. F. are like arches: for as B.D. is 50 degrees in the greater circle, ſo is E. F. 50 degrees in the leſſer circle. And the like of others. PROB. 8.

DEFIN. XLIII.

The Quadrant of a circle is the fourth part thereof, or an arch containing 90 degrees.

AS the arch K. B. D. in the former Semicircle is a Quadrant of that whole circle, or a fourth part thereof, and containeth 90 degrees.

DEFIN. XLIIII.

The Complement of an arch, leſſe then a Quadrant, is ſo much as that arch wanteth of 90 degrees.

AS the Complement of the arch B. D. 50 degrees in the former Semicircle, is the arch K. B. 40 degrees.

DEFIN. XLV.

The Exceſſe of an arch, greater then a Quadrant, is ſo much as the ſaid arch is more then 90 degrees.

AS the Exceſſe of the arch H. K. B. D. 130 degrees, is the arch H. K. 40 degrees more then a Quadrant, that is more then K. D.

DEFIN.

DEFIN. XLVI.

The Complement of an arch, leſſe then a Semicircle, is ſo much as that arch wanteth of a Semicircle, or of 180 *degrees.*

AS the arch H.K.B.D. is an arch leſſe then a Semicircle, and containeth 130 degrees, and the Complement thereof to a Semicircle, is the arch H.G. 50 degrees, which is ſo much as the arch H. K. B. D. wanteth of a Semicircle, or of 180 degrees.

DEFIN. XLVII.

The Complements of Angles are as the Complements of Arches.

AS the arch K. B. is the complement of the arch B. D. to a Quadrant, and the arch B. D. of the arch K. B. So the angle K.A.B. 40 degrees is the complement to a right angle of the Angle B.A.D. 50 degrees; and likewiſe the ſame angle B. A. D. of the ſame angle K.A.B. And in this ſence is the third angle of any Triangle ſaid to be the complement of the other two, to two right angles, or a Semicircle: For the three angles of any Triangle are equall to two right angles; as is here-after declared.

* *
*

THE

The ſecond Part.

Inſtructions concerning this Part.

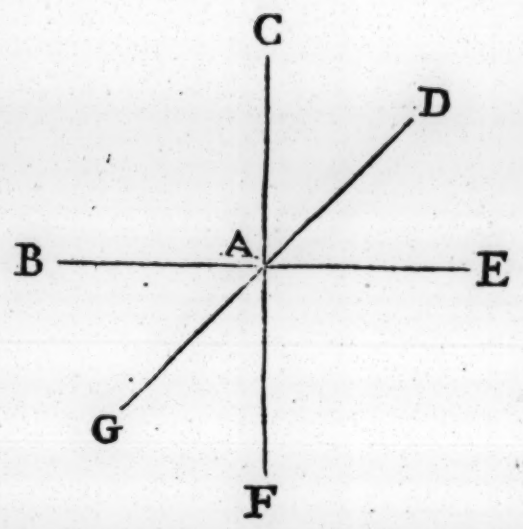
THis ſecond Part conſiſteth of diuerſe Geometricall THEOREMES, or approued Truths; which are the Foundations, Grounds, and Reaſons, whereon the Practike part dependeth. For as in the generall courſe and tract of all deſignes, before the vndertaking or execution of whatſoeuer action, the fitteſt meanes for an orderly performance, is iudicially to conſider; firſt, the Propertie, Paſsion, Nature, and kind of the intended enterpriſe; then, the beſt and moſt immediate meanes how to effect the ſame, and the Cauſes, Grounds, and Reaſons, why, by thoſe meanes ſuch effects may be wrought; and afterwards, to put in execution: So before we enter into the Practike part, I will firſt here premiſe diuerſe THEOREMES concerning this ſubiect, whereby the ingenious practitioner may moſt euidently conceiue and vnderſtand the ground and reaſon of all the Rules and Problemes in the following Bookes contained. Wherein I vſe onely *Explication* and *Conſtruction*, omitting (for breuitie ſake, and auoiding confuſion to the Learner) their ſeuerall *Demonſtrations*; yet with ſuch ample notes of direction in the Margent, as the Reader may readily find in EVCLID, RAMVS, and other Authors, their Demonſtrations at large. And for their further eaſe and helpe, I haue at the end of euery *Conſtruction* inſerted the like notes of Reference from theſe THEOREMES to the following PROBLEMES, and the like from thoſe to theſe; that hauing here the reaſon or cauſe, hee ſhall there moſt readily find the effect; or ſeeing there the effect, hee may as ſpeedily vnderſtand the cauſe or reaſon thereof, *Scire enim, proprie eſt, rem per cauſam cognoſcere.*

THEO-

THEOREME I.

If any two right lines cut the one the other, the opposite or verticall angles are euer equall; and both the angles, on one and the same side of either line, are either of them right angles, or (being both taken together) are equall to two right angles.

SVppose that B. E. and C. F. are two right lines, which cut the one the other in the point A. Then I say, first, that the opposite or verticall angles are equall, namely, the angle B. A. C. to the angle F. A. E. and the angle B. A. F. to the angle C. A. E. for they are euerie of them right angles: and let the right line D. G. be likewise drawne, cutting the line B. E. in A. Then I further say, that both the angles, taken together on one and the same side of either line, is equall to two right angles, as the angles B. A. D. and D. A. E. on the vpper side of the line B. E. and also the angles B. A. G. and G. A. E. on the nether side of the same line, are respectiuely equall to two right angles; for they consist of the right angles formerly mentioned. And the like of the angles on either side of the line D. G. PROB. 114, 117, 118.



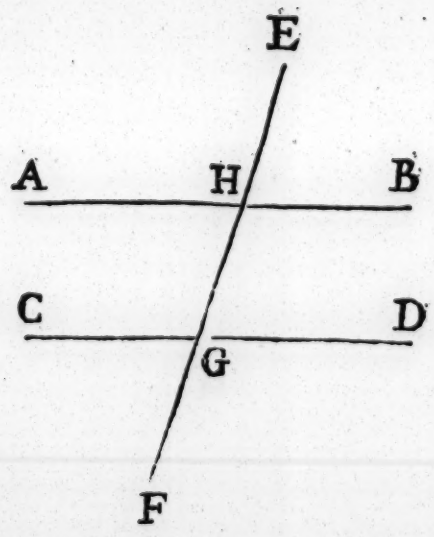
Euc. l. 1. p. 13. 15. Ram. 5. 8. Con. 2. Ceul. 2. 9.

Si quotcunque rectæ in eodem puncto mutuo sese intersecent omnes in communi sectione quatuor rectis æquabuntur.

THEOREME II.

A right line, falling on two parallell right lines, maketh the outward angles on contrarie sides of the falling line equall; and likewise the inward and opposite angles on the contrarie sides of the same line; and also the outward angle, equall to the inward and opposite angle on one and the same side of the falling line; and the inward angles on one and the same side equall to two right angles.

LEt the right line E. F. fall on the two parallell right lines A. B. and C. D. Then sayth this THEOREME, first, that it maketh the outward angles on contrarie sides of the falling line, namely, the angles A. H. E. and F. G. D. to be equall; and likewise the inward and opposite angles on the contrarie sides of the same line, as the angles A. H. F. and E. G. D. And also, that the outward angle, as A. H. E. is equall to the inward and opposite angle, on



Euc. 1. p. 29. Ram. 7. 9. Pit. 1. 38.

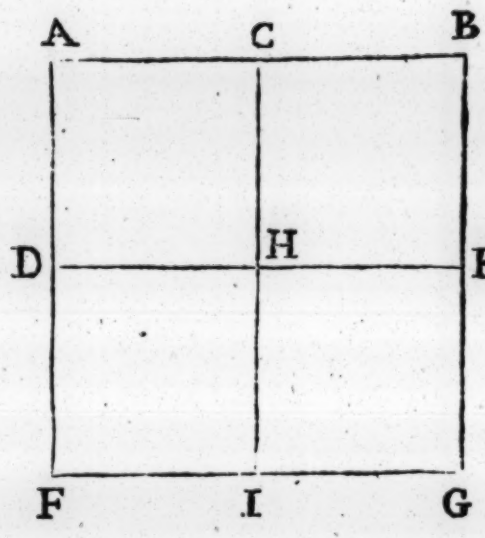
Lineæ eidem parallelæ inter se sunt parallelæ.

on one and the ſame ſide of the falling line, namely, to E. G. C. And laſtly, that the inward angles on one and the ſame ſide, as the angles E. G. C. and A.H.F. are equall to two right angles. PROB. 50.

THEOREME III.

If a right line be diuided into two equall parts, halfe the ſquare of that whole line is double to the whole ſquare of halfe the ſame line.

SVppoſe A.B. to be a right line, and let the ſame be diuided into two equall parts in the point C. Then I ſay, that halfe the ſquare of that whole line, namely, the Parallelogram A.B. D. E. (for the whole ſquare is A. B. F. G.) is double to the whole ſquare of halfe the ſame line, namely, to the ſquare A. C. D. H. as is manifeſt by the Diagram.

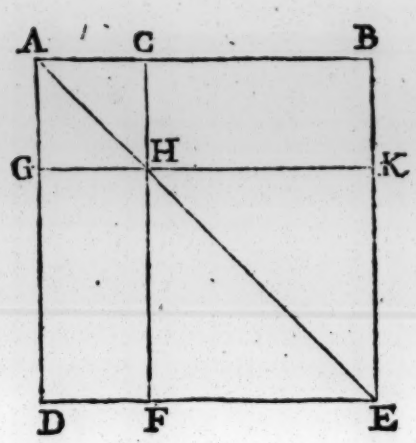


THEOREME IIII.

A right line being diuided by chance, the ſquare of the whole line is equall to both the ſquares made of the parts, and alſo to two rectangle figures, comprehended vnder the ſame parts.

EUC. 2. 4.

LEt the right line A. B. be diuided by chance in the point C. Then I ſay, that the ſquare of the whole line, namely, A.B.D.E. is equall to both the ſquares made of the parts, namely, to the ſquares A.C.G.H. and H.K.F.E. (for H. K. is equall to C. B.) and alſo to the two rectangle figures, comprehended vnder the ſame parts, namely, to the rectangle figures C. B. H.K. and G. H. D.F.



THEOREME V.

The Supplements of thoſe Parallelograms which are about the diameter in euery Parallelogram, are alwaies equall the one to the other.

Complementa ſunt æqualia. Euc. 1. 43. Ceul. 2. 81.

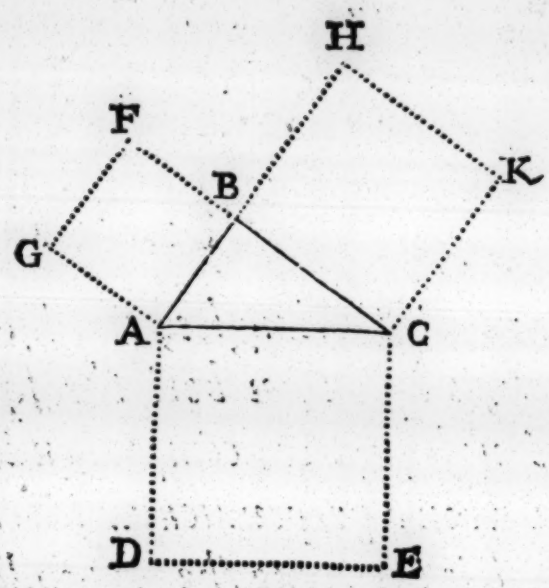
SVppoſe the figure A. B. D. E. in the former THEOREME, be a Parallelogram, whereof the diameter is A.E. and let the Parallelograms about the ſame diameter (according to the declaration of the 30. DEFINITION)

FINITION) be A. C. G. H. and H. K. F. E. Then I ſay, that the ſupplements of thoſe Parallelograms, namely, the ſupplements C. B. H. K. and G. H. D. F. are equall the one to the other. PROB. 18. 87. 88. 105.

THEOREME VI.

In right angled Triangles, the ſquare of the ſide ſubtending the right angle, is equall to both the ſquares of his containing ſides.

LEt the Triangle A. B. C. be a right angled Triangle, whoſe angle, at the point B. is a right angle; and let the line A. C. be the ſide ſubtending the ſame right angle, and B. A. and B. C. his containing ſides. Then I ſay, that the ſquare of the ſide ſubtending the right angle, which is the ſquare A. C. D. E. is equall to both the ſquares of his containing ſides, namely, to the ſquares A. G. F. B. and B. H. K. C. PROB. 23. 24. 25. 30. 36. 38. 52. 65. 99. 100. 101. 102. 104. 106.



In triangulo rectangulo figura ad baſin deſcripta æquatur figuris ad crura ſimilibus ſimiliterque ſitis.

Euc. 1. 47. *Pit.* 1. 50.

This former THEOREME, *and the two next following, are of infinite and wonderfull vſe in moſt Geometricall Concluſions; eſpecially in* TRIGONOMETRIE, *or the ſupputation of Triangles, by the Canons thereof; as thoſe excellent Tables of Logarithmes, or thoſe of Synes, Tangents, and Secants, in that behalfe calculated; and therefore eſpecially to be regarded; and the moſt excellent properties and paſſions thereof to be well underſtood and practiſed.* Nota.

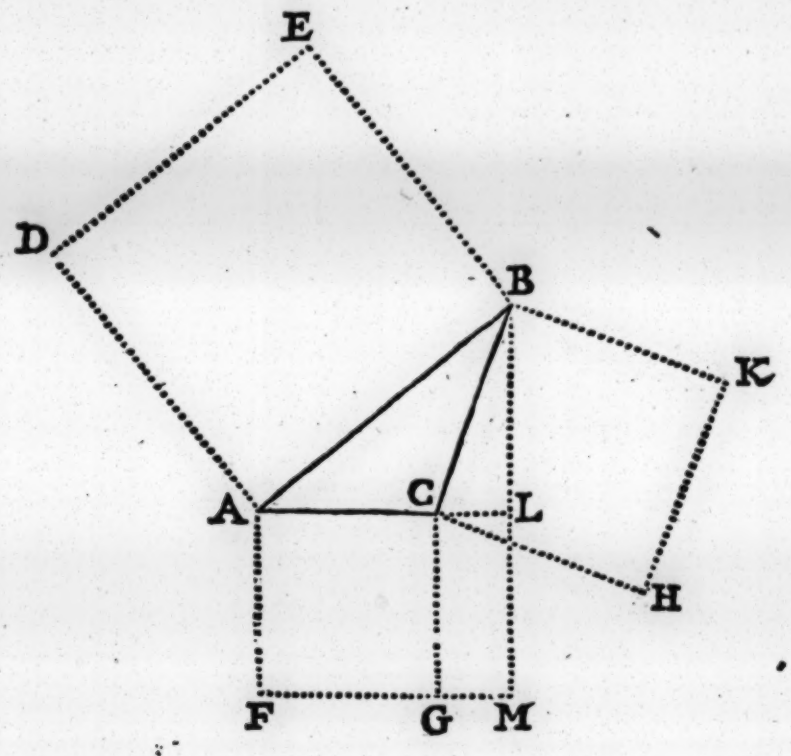
THEOREME VII.

In obtuſe angled Triangles, the ſquare of the ſide ſubtending the obtuſe angle, is greater then both the ſquares of the containing ſides, by two rectangled figures, comprehended vnder one of the containing ſides (being continued) and the line of continuation, from the obtuſe angle to a perpendicular let fall thereon.

LEt the Triangle A. B. C. be an obtuſe angled Triangle, whoſe angle at the point C. is obtuſe; and let the line A. B. be the ſide ſubtending the ſame obtuſe angle, and A. C. and C. B. his containing ſides; whereof, let A. C. be the ſide continued, and C. L. the line of continuation from the obtuſe angle at the point C. to the perpendicular let fall thereon B. L. Now I ſay,

In triangulis obtuſangulis baſis plus poteſt cruribus duplici rectangulo ex altero crure & eius continuatione ad verticis perpendicularem E. 2. p. 12.

I ſay, that the ſquare of the ſide, ſubtending the obtuſe angle, namely, A.D.E.B. is greater then both the ſquares of the containing ſides, namely, B.K.C.H. and A.C.F.G. by two rectangle figures, (which is all one, with one twice taken) comprehended vnder one of the containing ſides (being continued) and the line of Continuation, namely, C. L. G. M. being twice taken. PROB. 44.



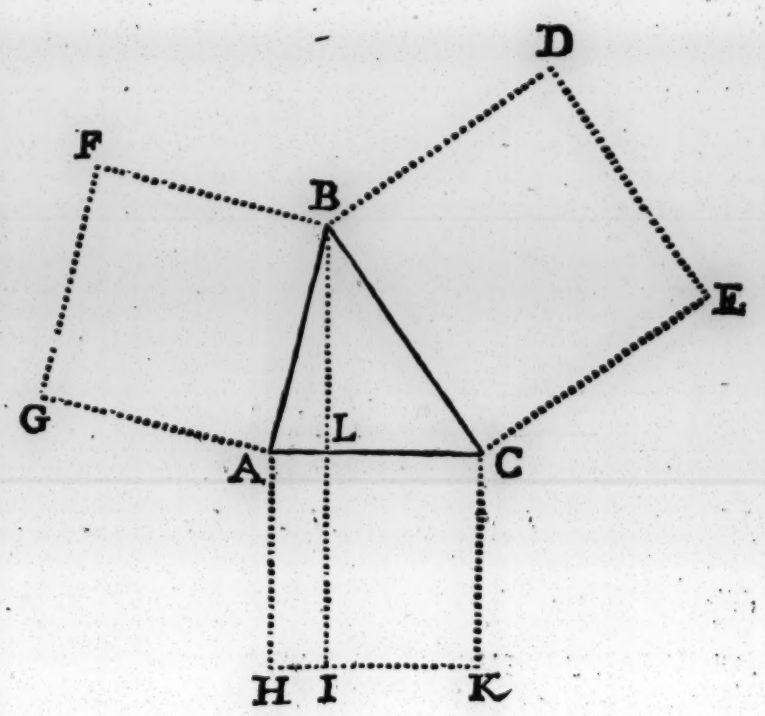
THEOREME VIII.

In acute angled Triangles, the ſquare of the ſide ſubtending the acute angle, is leſſe then both the ſquares of the containing ſides by two rectangle figures, comprehended vnder one of the containing ſides (whereon a perpendicular falleth) and that ſegment of the ſame ſide which is betweene the perpendicular and the acute angle.

Theorema generale eſt ad indeſtigationem perpendicularis intra triangulum cadentis data trium laterum quantitate.

Euc. 2. 15. *Ceul.* 2. 84.

LEt A.B.C. be an acute angled Triangle, hauing the angle at the point A. acute; let B. C. be the ſide ſubtending the ſame angle, and A.B. and A.C. the containing ſides: alſo let B.L. be the perpendicular, A.C. the ſide whereon it falleth, and A. L. the ſegment thereof betweene the perpendicular and the acute angle A. Now I ſay, that the ſquare of the ſide ſubtending the acute angle, namely, B. D. C. E. is leſſe then both the ſquares of the containing ſides, which are F.B.G.A. and A.C.H.K. by two rectangle figures (being all one, with one twice taken) comprehended vnder one of the containing ſides A. C. (wherennto A. H. is equall) and the ſegment A. L. namely, A.L. H.I. twice taken. PROB. 41.



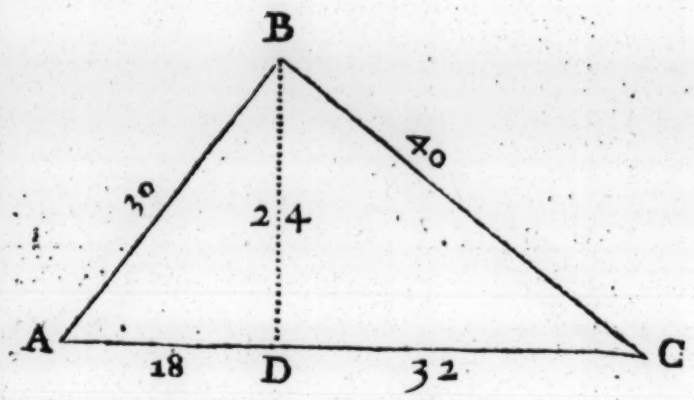
THEO-

THEOREME IX.

In rectangle Triangles, if from the right angle a perpendicular be let fall vnto the Base, it shall diuide the Triangle into two Triangles, like vnto the whole, and also the one like vnto the other.

Euc.6.8. Ram.5.12. 13.

LEt A. B. C. be a recte angle Triangle, whose angle at the point B. is a right angle; from whence, let the perpendicular B. D. be let fall to the Base A. C. Then I say, the perpendicular so falling, shall diuide the Triangle into two Triangles, that is, A.B.D. and B. C. D. like vnto the whole Triangle A. B. C. and also the one Triangle like vnto the other; which is (according to the 40. DEFINITION) with equall angles, and proportionall sides about those equall angles. PROB. 19. 23. 24. 25. 30. 38.

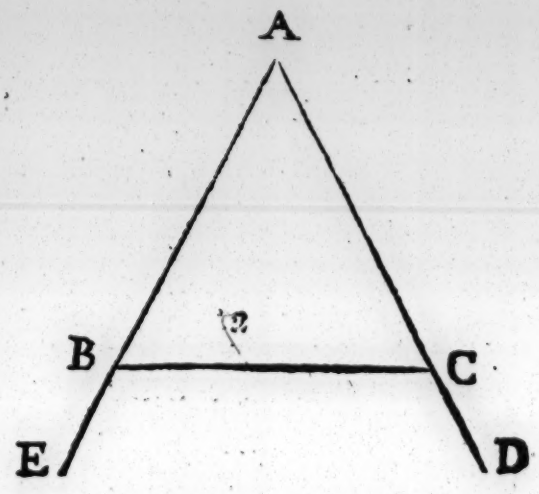


THEOREME X.

An Isosceles, or a Triangle of two equall sides, hath his angles at the Base equall; and the equall sides being produced, the angles vnder the Base are also equall.

LEt A. B. C. be an Isosceles, or a Triangle, whose two sides A. C. and A. B. are equall, and let A. C. be produced to D. and A. B. to E. I say then, that his angles at the Base, namely, A. B. C. and A. C. B. are equall; and that the angles vnder the Base, as E. B. C. and D. C. B. are also equall, the one vnto the other. PROB. 40.

Euc. 1. 5. Cenl. 2. 3. 4.



THEOREME XI.

All equiangle Triangles haue their sides, containing equall angles pro-

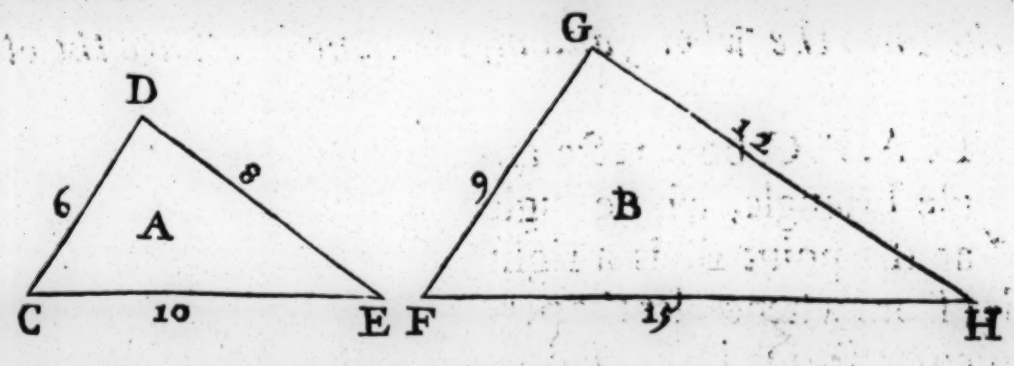
portionall,

portionall, and their ſides ſubtending equall angles, are of like proportion.

Hoc Theorema præcipuum eſt totius Trigonometriæ fundamentum.

Euc. 6. 4.
Ram. 5. 12.
7. 9.
Pit. 1. 46.
Ceul. 2. 62.

SVppoſe A. and B. to be two equiangle triangles, that is, hauing the angle D. equal to the angle G. and C. to F. and E. to H.



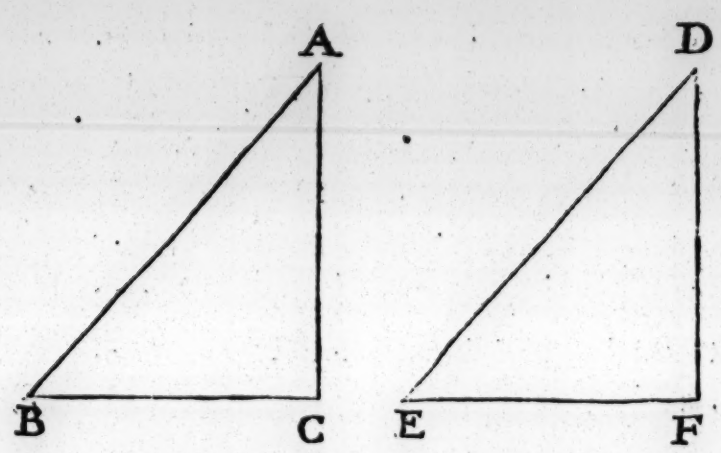
Then I ſay, they haue their ſides, which containe thoſe equall angles proportionall; as D. C. and D. E. in the Triangle A. are proportionall to G. F. and G. H. in the Triangle B. becauſe they containe equall angles, namely, D. and G. for as D. C. is to D. E. ſo is G. F. to G. H. and the like of the reſt: alſo their ſides ſubtending equall angles, are of like proportion, as D. C. and G. F. ſubtending equall angles E. and H. and C. E. and F. H. ſubtending equall angles D. and G. are of like proportion: for as D. C. is to C. E. ſo is G. F. to F. H. And the like of the other ſides and angles. PROB. 30. 38. 45. 65.

THEOREME XII.

In any two Triangles compared, if two ſides of the one be equall to two ſides of the other, and the Baſe of the one to the Baſe of the other; they ſhall alſo haue the angles contained vnder their anſwerable equall ſides, the one equall to the other in either Triangle.

Euc. 1. 8.

LEt A. B. C. and D. E. F. be two Triangles compared, hauing two ſides of the one equall to two ſides of the other, as A. B. and A. C. of the one, equall to D. E. and D. F. of the other, and alſo the Baſe B. C. of the one, equall to the Baſe E. F. of the other. Now I ſay, they ſhall haue their angles contained vnder anſwerable equall ſides (as the angle A. contained vnder A. B. and A. C. equall to the angle D. contained vnder the anſwerable equall ſides D. E. and D. F.) to be equall the one to the other. And the like of the reſt.

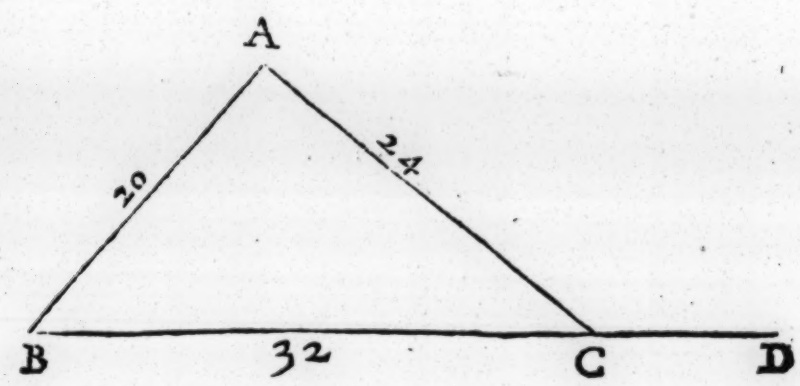


THEO-

THEOREME XIII.

If any side of a Triangle be continued, the outward angle made by that continuation, is equall to the two inward and oppoſite angles: And the three inward angles of any Triangle are equall to two right angles.

LEt A.B.C. be a Triangle, whereof let any of the ſides be produced, as B.C. to D. Then I ſay, that the outward angle, made by that production or continuation, as the angle A. C. D. is equall to the two inward and oppoſite angles, namely, the angles C.A.B. and C. B. A. And alſo, that the three inward angles of any Triangle, as C.B.A. B.A.C. and A.C.B. are equall to two right angles. PROB. 111.114.117.118.



Euc. 1. 32.
Ram. 6. 9.
Pit. 1. 48. 49.
Ceul. 2. 20.

THEOREME XIIII.

In euerie Triangle, two of his angles, which two ſoeuer be taken, are leſſe then two right angles.

AS in the Diagram of the former THEOREME, take any two angles, as thoſe at the points A. and C. or C. and B. or B. and A. and they are leſſe then two right angles; for by the ſame former THEOREME all three of them are equall to two right angles. *Euc.* 1. 17.

THEOREME XV.

In euerie Triangle, two ſides thereof (which two ſoeuer be taken) are greater (being ioyned together as one line) then the third ſide remaining.

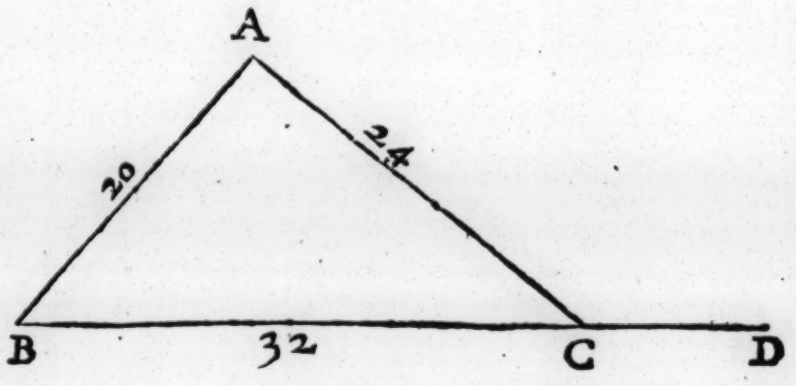
LEt A.B.C. (the Diagram of the 13. THEOREME) be a Triangle, whereof take any two of the ſides, as A.B. and A.C. I ſay, thoſe two ſides being taken and ioyned together as one line, are greater then the third ſide remaining, namely, B.C. And the like of any other two, taken together. Whereby it is manifeſt, that vnder all three lines (without reſpect of quantitie) a Triangle cannot be contained. PROB. 42. *Euc.* 1. 20.

THEOREME XVI.

In all Triangles, the greater ſide ſubtendeth the greater angle, and the leſſer ſide ſubtendeth the leſſer angle.

Trianguli maius latus ſubtendit maiorem angulum.
Euc. 1. 18. 19.
Ram. 6. 11.
Euc. 1. 47. 48.
Pit. 1. 5.
Ceul. 2. 19.

LEt A. B. C. be a Triangle, hauing the ſide A.C. greater then the ſide A. B. and leſſe then the ſide B. C. Then I ſay, that the angle A. B. C. being ſubtended by the greater ſide A. C. is greater then the angle A. C. B. being ſubtended by the leſſer ſide A. B. And alſo, that the angle A. B. C. being ſubtended by the leſſer ſide A. C. is leſſer then the angle B. A. C. ſubtended by the greater ſide B. C.

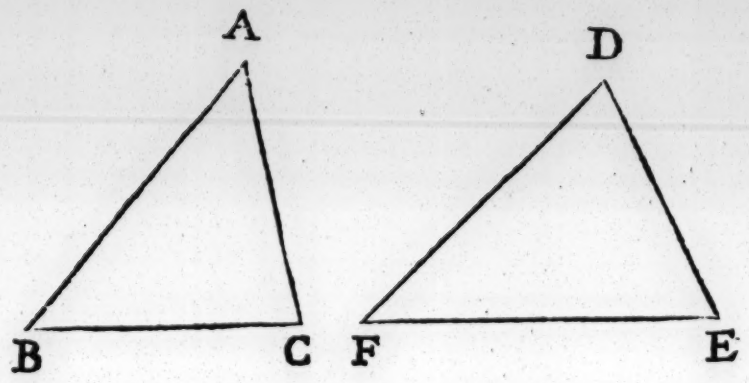


THEOREME XVII.

If two ſides of one Triangle be equall to two ſides of another Triangle, and the angle contained vnder the equall ſides of the one, be greater then the angle contained vnder the equall ſides of the other; then the Baſe alſo of the one (namely, of that which hath the greater angle) ſhall be greater then the Baſe of the other.

LEt there be two Triangles, A. B. C. and D. E. F. which haue two ſides of the one Triangle, as A. B. and A. C. equall to two ſides of the other Triangle D. F. and D. E. and let the angle F. D. E. contained vnder the equall ſides of the one be greater then the angle B. A. C. contained vnder the equall ſides of the other. Then I ſay, that the Baſe F. E. of the one (namely, of that which hath the greater angle) is greater then B. C. the Baſe of the other.

Euc. 1. 24.



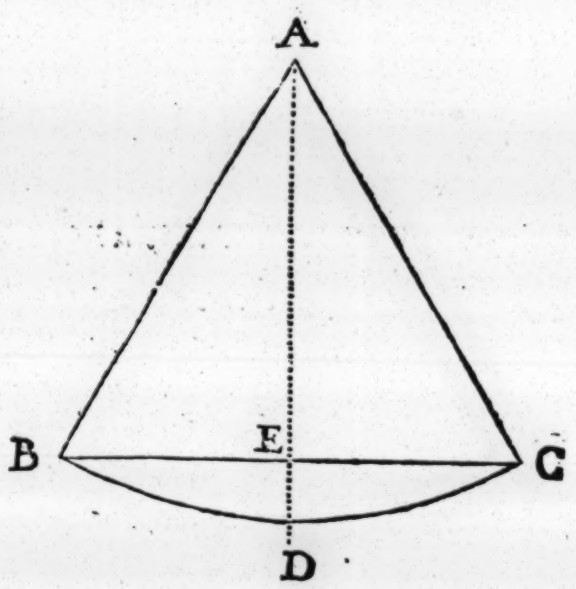
THEO-

THEOREME XVIII.

If a Triangle be equicrurall, or hauing two equall ſides; a perpendicular let fall from the angle contained vnder thoſe equall ſides to the Baſe, and continued, ſhall diuide as well the ſame Baſe and angle, as alſo the meaſure of that angle, into two equall parts: Et contra.

LEt A. B. C. be a Triangle, whoſe ſides A. B. and A. C. are equall, and let fall a perpendicular from the angle, included by thoſe equall ſides, as A. E. to the Baſe B. C. and let the ſame be continued to D. Now I ſay, that a perpendicular ſo let fall, ſhall diuide as well the ſame Baſe B. C. and angle B. A. C. as alſo the meaſure thereof, namely, the arch line B. D. C. into two equall parts. PROB. 10. 11. 40.

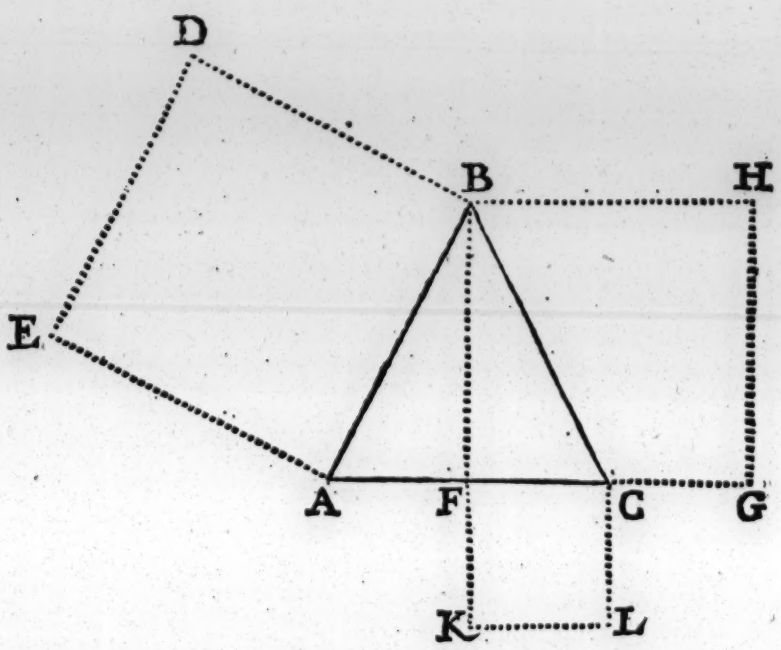
Pit. 1. 23.



THEOREME XIX.

If a Triangle hath two equall ſides, the power of one of thoſe equall ſides exceedeth the power of the perpendicular let fall on the Baſe from the angle it ſubtendeth, by the power of halfe the Baſe.

LEt A. B. C. be a Triangle, hauing two equall ſides, B. A. and B. C. and let B. F. be a perpendicular let fall to the Baſe A. C. from the angle it ſubtendeth A. B. C. Then I ſay, that the power of one of thoſe equal ſides, namely, the ſquare A. B. D. E. exceedeth the power of the perpendicular, namely the ſquare B. H. F. G. by the power of halfe the Baſe, namely, the ſquare F. C. K. L. PROB. 36. 40. 41. 64.



THEOREME XX.

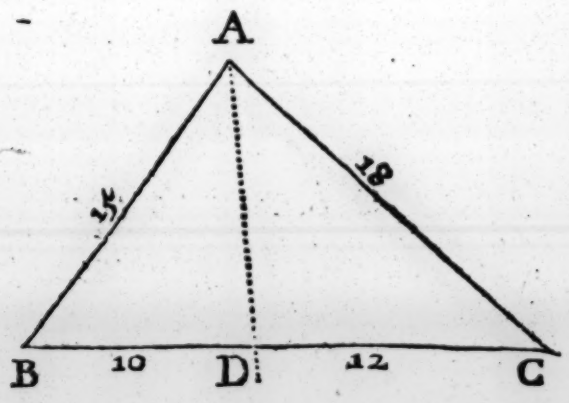
If the power of one ſide of any Triangle be equall to both the powers of the other two ſides, the angle contained vnder thoſe two other ſides, is a right angle.

Euc. 1.48. THis THEOREME is the conuerſe of the 6. THEOREME, and therefore the explication and conſtruction thereof ſerueth here.

THEOREME XXI.

If a right line diuide any angle of a Triangle into two equall parts, and if alſo the ſame line diuide the Baſe, the ſegments of the Baſe ſhall haue ſuch proportion the one to the other, as the other ſides of the Triangle haue: Et contra.

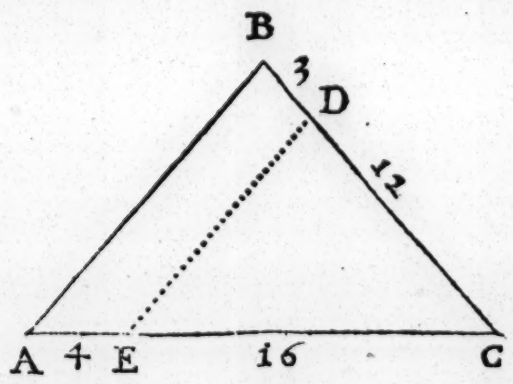
*Euc.*6.3. *Ceul.*2.61. LEt A.B.C. be a Triangle, and let the right line A. D. diuide the angle B. A. C. of the ſame Triangle into two equall parts; and alſo let the ſame line diuide the Baſe B. C. Then I ſay, the ſegments of the Baſe, namely, B.D. and D.C. ſhall haue ſuch proportion the one to the other, as the other ſides of the Triangle haue, namely, A.B. and A.C. for ſuch proportion as B. D. hath to D. C. the ſame hath A.B. to A.C.



THEOREME XXII.

If a right line be drawne parallell to any ſide of a Triangle, the ſame line ſhall cut the ſides of that Triangle proportionally.

*Euc.*6.2. *Ram.l.*6. *p.*9.5.*p.*13. *Con.* 1. 2. & 3. *Pit.*1.47.45. *Ceul.*2.27. LEt A. B. C. be a Triangle, vnto one of the ſides whereof A. B. is drawne a parallell line D.E. Wherefore the ſame line doth cut the ſides of that Triangle A. C. and B.C. proportionally: for firſt, as A.E. is to E.C. ſo is B.D. to D. C. alſo, as A.E. is to B.D. ſo is E.C. to D.C. and as A. C. is to A.E. ſo is B.C. to B.D. PROB. 12. 13. 14. 15. 16. 22. 98.

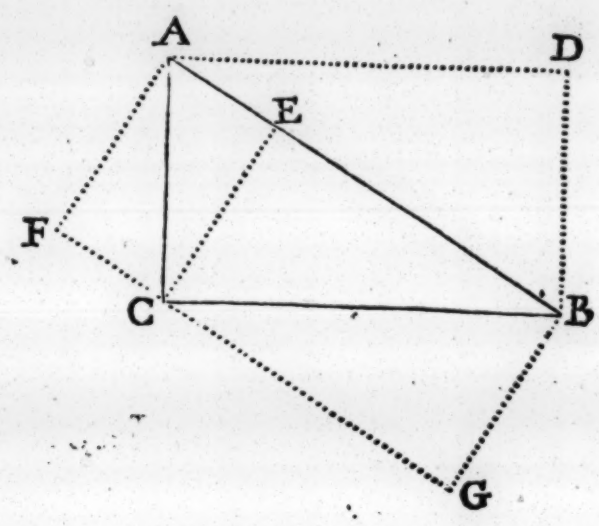


THEO-

THEOREME XXIII.

The ſuperficiall content of euery right angled Triangle, is equall to halfe that right angled Parallelogram, which hath his length and breadth equall to the containing ſides of the right angle; or whoſe length is equall to the ſubtending ſide, and breadth to the perpendicular, drawne from the right angle to the ſame ſide.

LEt A.B.C. be a rectangle Triangle, whoſe angle at the point C. is a right angle, whereof the containing ſides are A. C. and C. B. the ſubtending ſide A. B. and the perpendicular drawne from the right angle to the ſame ſide, is C.E. Now I ſay, the ſuperficiall content or *Area* of this right angled Triangle is equall vnto halfe that right angled Parallelogram (namely, A.D.C.B.) which hath his length C. B. and breadth A. C. equall to the containing ſides of the right angle; or whoſe length A.B. is equall to the ſubtending ſide, & breadth A.F. to the perpendicular line, drawne from the right angle to the ſame ſide, as the Parallelogram A.B. F.G. PROB. 39. 52. 92. 102. 106.

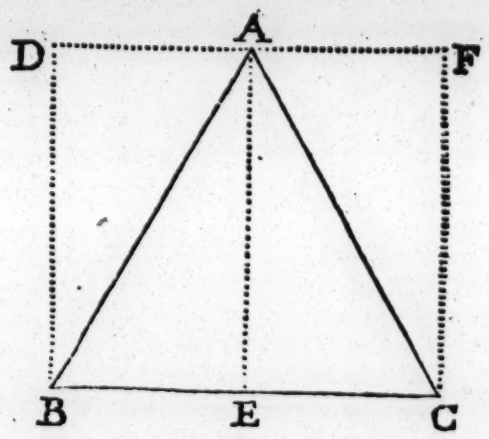


Euc. 1. *Def.* 27. *Ram.* 8. 2.

THEOREME XXIIII.

The Area or ſuperficiall content of euery Equilater Triangle, is equall to halfe that long ſquare, whoſe length and breadth is equall to one of the ſides and the perpendicular.

LEt A.B.C. be an Equilater Triangle, and A. E. the perpendicular thereof. Now I ſay, that the ſuperficiall content thereof is equall to halfe that long ſquare D. F. B. C. whoſe length B.C. and breadth B. D. is equall to one of the ſides, and the perpendicular. PROB. 37.

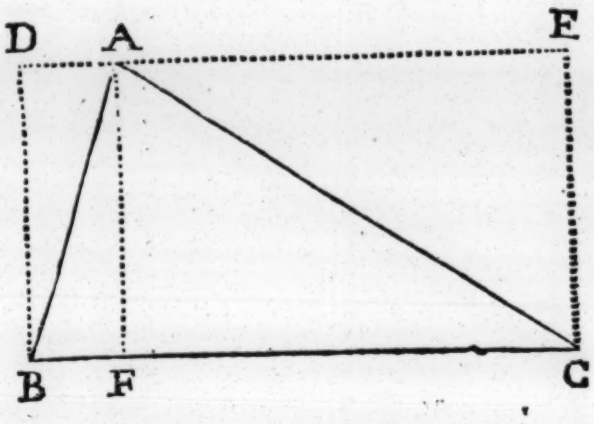


THEO-

THEOREME XXV.

All Triangles, of what kind ſoeuer, are equall in their ſuperficiall content vnto halfe that right angled Parallelogram, whoſe length and breadth is equall to the perpendicular, and the ſide whereon it falleth.

LEt A.B.C. be a Triangle, whoſe perpendicular is A. F. and the ſide whereon it falleth B.C. I ſay, that this Triangle A. B. C. is equall in his ſuperficiall content vnto halfe the right angled Parallelogram D. E. B. C. whoſe length B. C. and breadth B. D. are equall to the perpendicular, and the ſide whereon it falleth. PROB. 39. 41. 44. 72. 76. 77. 99.



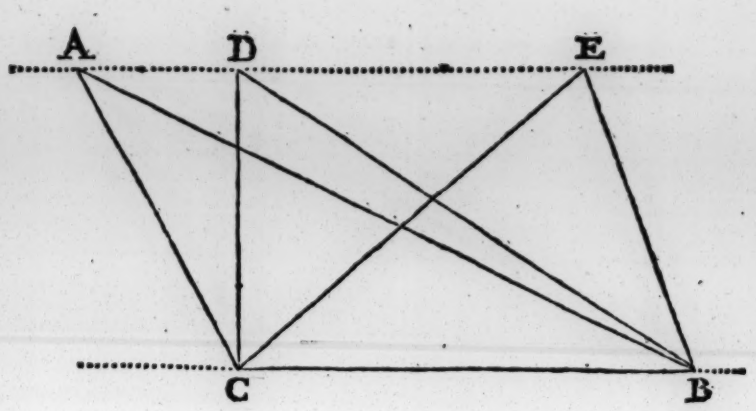
THEOREME XXVI.

Triangles which conſiſt on one and the ſame Baſe, or on equall Baſes, and in the ſame parallell lines are equall the one to the other.

Triangula in æquali baſi & intra eaſdem parallelas ſunt æqualia.

EUC. 1. 38.

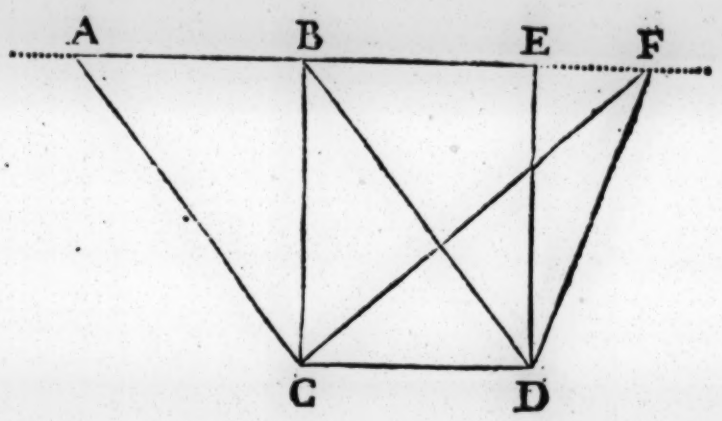
LEt A.B.C. D.B.C. and E.B.C. be three Triangles, conſiſting on one and the ſelfe-ſame Baſe B. C. (or on equall Baſes, which is all one thing) and in the ſame parallell lines A. E. and C.B. Now I ſay, that all thoſe three Triangles, A.B.C. D.B.C. and E.B.C. and as manie moe as may be drawne on the ſame Baſe, or a Baſe equall thereunto, and in the ſame parallell lines, are all equall the one to the other. PROB. 26. 27. 28. 29. 46. 47. 73. 74. 75. 79. 80. 81. 93. 103. 107. 110.



THEOREME XXVII.

If Triangles aud Parallelograms haue one and the ſame Baſe, or equall

equall Baſes, and be in the ſame parallell lines, the Parallelograms ſhall be double to the Triangles.



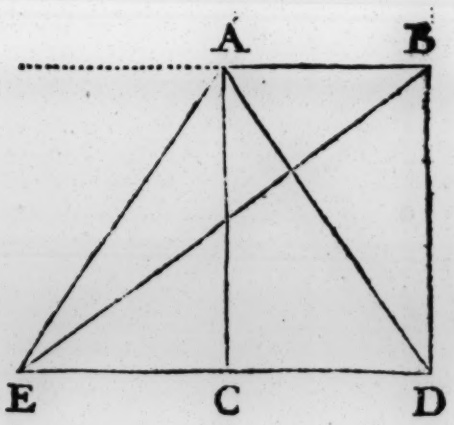
Euc. 1. 41. *Ceul.* 2. 25.

LEt B. C. D. and F. C. D. be two Triangles, and let A.B.D.C. and B. E. D. C. in this ſame Diagram be two Parallelograms, which Triangles and Parallelograms haue one and the ſame Baſe C. D. and are in the ſame parallell lines A. F. and C. D. Now I ſay, that either of thoſe two Parallelograms are double to either of thoſe two Triangles. PROB. 76. 77. 92. 110.

THEOREME XXVIII.

If a Triangle hath his Baſe double to the Baſe of a Parallelogram, and that they are both in the ſame parallell lines, then are they both equall the one to the other.

LEt A. B. C. D. be a Parallelogram, whoſe Baſe is C.D. and let A. E. D. and B. E. D. be two ſeuerall Triangles, whoſe Baſes E. D. are double to the Baſe of the Parallelogram (for E. C. and C. D. are equall) and who are within the ſame parallell lines with the Parallelogram A. B. C. D. Then I ſay, that either of thoſe Triangles are equall to the ſame Parallelogram. PROB. 29. 78. 91. 110.

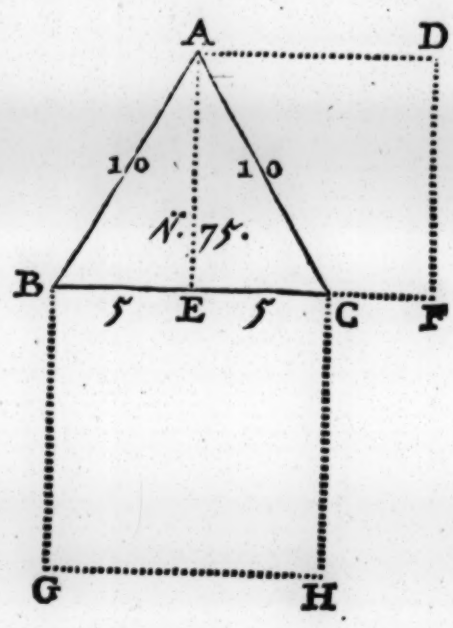


THEOREME XXIX.

The power of the ſide of an Equilater Triangle, is to the power of the perpendicular thereof let fall from any angle to the ſubtendent ſide, in proportion Seſquitertia, or as 4. to 3.

LEt A. B. C. be an Equilater Triangle, whoſe perpendicular is A. E. let fall from the angle B. A. C. to the ſubtendent ſide B. C. Now I ſay, that the power of the ſide of the ſame Triangle, namely, B. C. G. H. which is the power or ſquare of the ſide B. C. is to the power of the perpendicular thereof, namely, A. D. E. F. (which is the power or ſquare *Euc.* 13. 12.

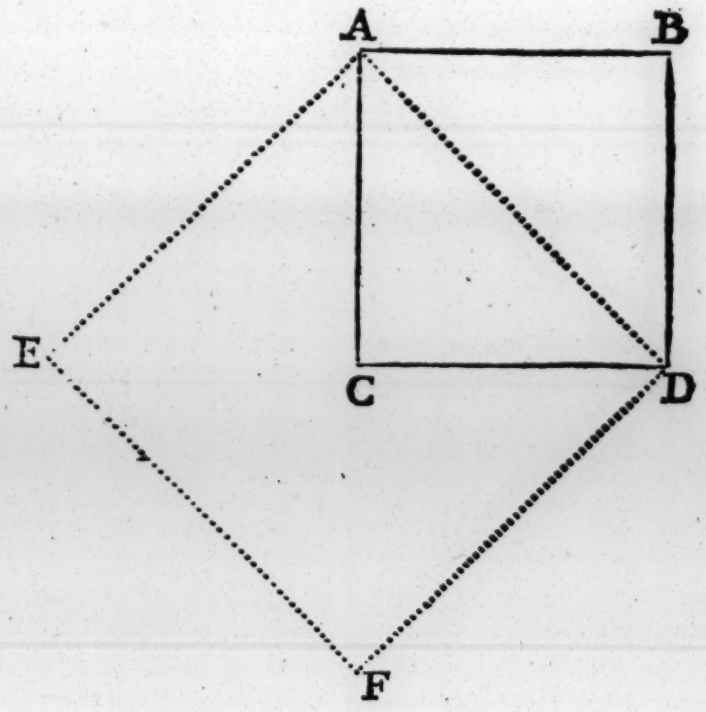
ſquare of the perpendicular A. E.) in proportion Seſquitertia, or as 4. to 3. For of what parts the line B. C. or B. A. containeth in power 8. of ſuch parts B.E. (which is the halfe of B. C.) containeth in power 2. Wherefore the perpendicular A.E. being the reſidue, containeth in power of ſuch parts 6. (for the ſquares of the lines A. E. and B. E. are by the 6. THEOREME equall vnto the ſquare of the line A. B. whereunto B. C. is equall.) Now 8. to 6. is Seſquitertia : wherefore the power of the line B. C. is to the power of the line A. E. in Seſquitertia proportion. So is the ſquare A. D. E. F. $\frac{1}{4}$. of the ſquare B. C. G. H. PROB. 36.



THEOREME XXX.

The Diagonall line, or Diameter of any Square, is double in power to the ſide of the ſame Square.

LEt A. B. C. D. be a Square, whoſe Diagonall line, or Diameter, is the line A. D. Now I ſay, that the ſame line A. D. is double in power to the ſide of the ſame Square, that is, the Square A. D. E. F. is double to the Square A. B. C. D. PROB. 99. 102. 106.



THEOREME XXXI.

A Square, whoſe ſide is equall to the Diameter of any other Square, is double in content or ſuperficiall quantitie to that other Square.

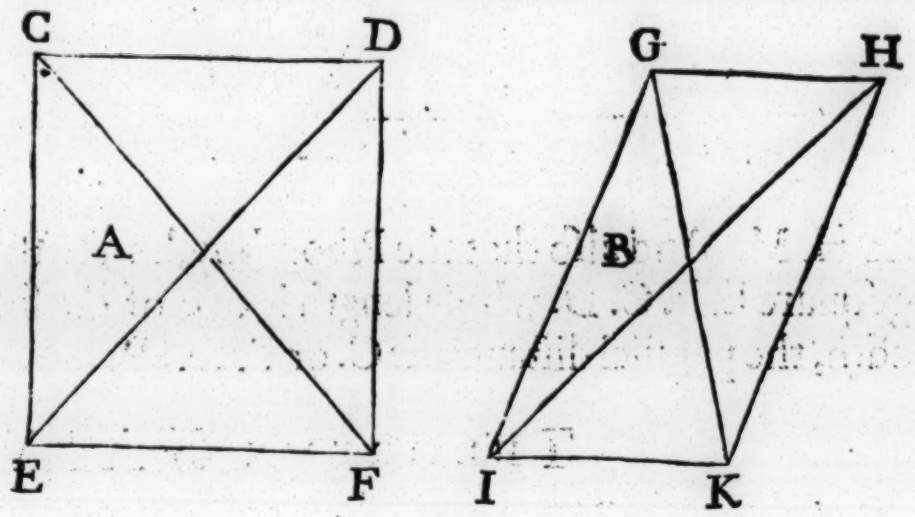
THe explication hereof, is manifeſt by that of the laſt: For let A. D. E. F. in the laſt Diagram be a Square, whoſe ſide A. D. is equall to the Diameter of another Square, as the ſame line A. D. is the Diameter of another Square, namely, A. B. C. D. Wherefore I ſay, that the Square A. D. E. F. is double in content, or ſuperficiall quantitie, to that other Square. PROB. 102. 106.

THEO-

THEOREME XXXII.

All parallelograms haue their oppoſite ſides, and angles equall one to another; and their Diameters diuide them into equall parts.

*Eu.*1.34.
*Ram.*10.6.
*Ceul.*2.25.

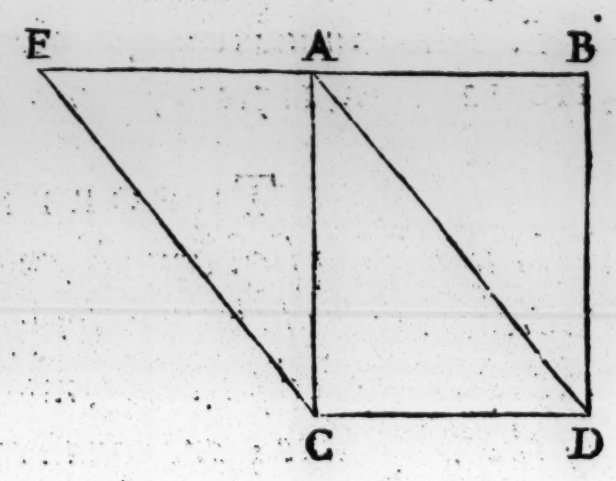


AS theſe two parallelograms A. and B. haue their oppoſite ſides and angles, equall one to another, as in the figure A. the ſides C. D. and E. F. are oppoſite & equall, and likewiſe D. F. and C. E. Alſo the angles thereof at the points C. and F. are oppoſite and equall, and likewiſe thoſe at D. and E. And moreouer, their diameters diuide them into equall parts, as the diameters C. F. and D. E. doe either of them diuide the parallelogram A. into two equall parts: And the like explication and conſtruction is to be made of the figure B. PROB. 115. 116.

THEOREME XXXIII.

Parallelograms which conſiſt on one and the ſame baſe, or on equall baſes, and in the ſame parallel lines are equall the one to the other.

*Eu.*1.35,36.

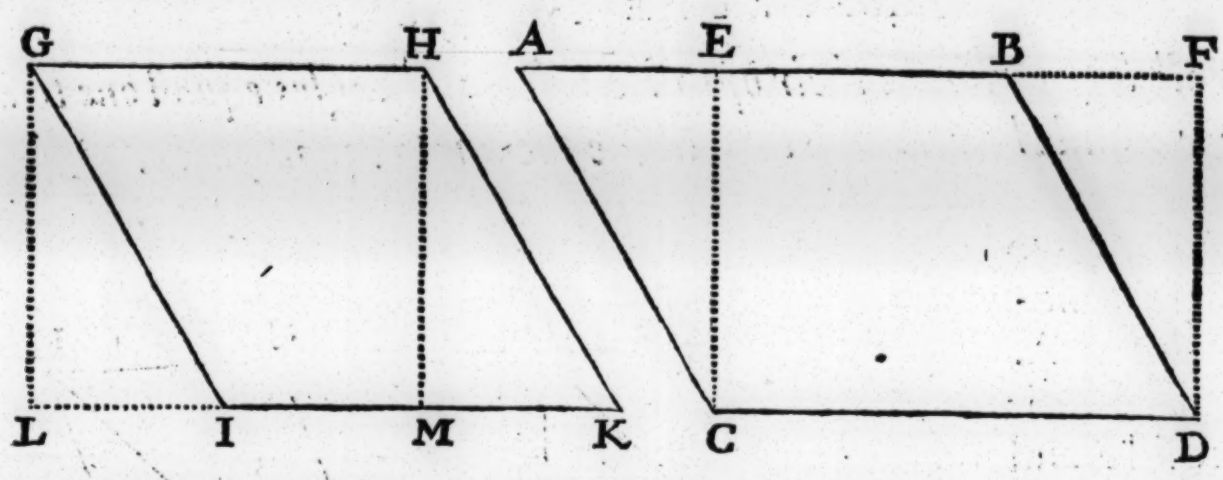


LET A. B. C. D. and E. A. C. D. bee two parallelograms, which conſiſt on one & the ſame baſe, namely C. D. (or on equal baſes which is one and the ſame thing) and in the ſame parallel lines, namely E. B. and C. B. Now, I ſay, that thoſe two parallelograms are both equall the one to the other. PROB. 89. 90.

THEOREME XXXIIII.

Euery Rhombus and Rhomboydes is equall to the long ſquare, whoſe length is one of the ſides, and breadth equall to the parallel diſtance.

LET G. H. I. K. be a Rhombus, and A. B. C. D. a Rhomboydes. I ſay, the Rhombus G. H. I. K. is equall to the long ſquare G. H. L. M. whoſe length is one of the ſides G. H. and breadth the parallel diſtance



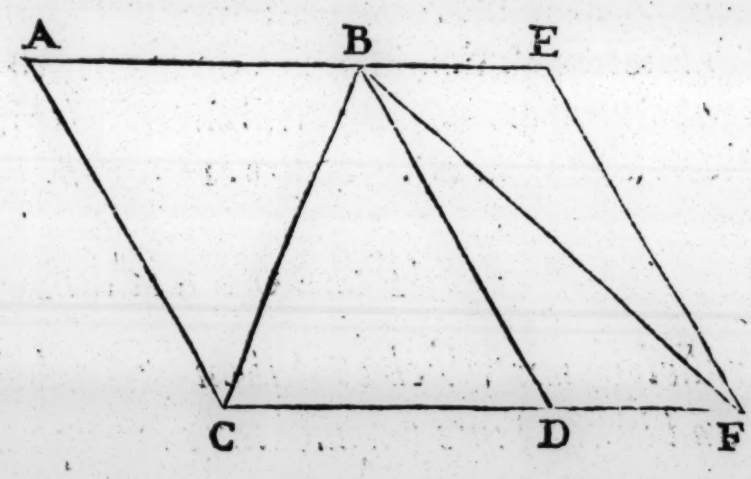
ſtance H. M. And alſo that the Rhomboydes A. B. C. D. is equall to the long ſquare E. F. C. D. whoſe length is one of the ſides C. D. or A. B. and breadth, the parallel diſtance E. C. or F. D. PROB. 58, 59, 60, 61, 89, 90.

THEOREME XXXV.

Parallelograms and Triangles, within the ſame parallels, are in ſuch proportion the one to the other as their baſes are.

Triangula vel parallelogramma æquealta ſunt vt baſis. Eu. 6. 1. Ram. 10. 13. Ceul. 2. 26.

LEt A. B. C. D. and B. E. D. F. be two parallelograms, within the ſame parallel lines A. E. and C. F. and let alſo B. C. D. and B. D. F. bee two triangles within the ſame parallel lines. Then, I ſay, as the baſe C. D. is to the baſe D. F. ſo is the parallelogram A. B. C. D. to the parallelogram B. E. D. F. And ſo alſo is the triangle B. C. D. to the triangle B. D. F. PROB. 29, 110, 123, 124, 125, 126, 127, 128, 129.

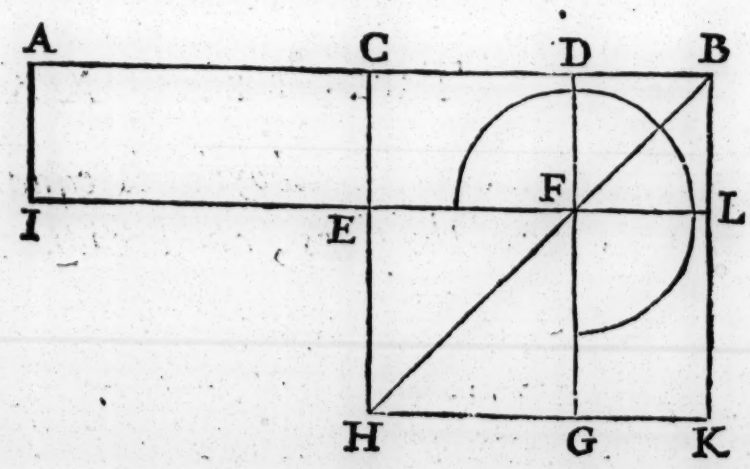


THEOREME XXXVI.

A right line being firſt equally, and then vn-equally diuided; The ſquare which is made of the part lying betweene thoſe ſections, together with the right angled parallelogram, contayned vnder the vn-equall parts of the whole line; are equall to the ſquare of halfe the whole line.

Hereby is demonſtrated that equation, of the greateſt and leaſt karectes or numbers, and their equalitie to the middle. Of great vſe is this Prop. in the rules of Algebar. *Eu.* 2. 5.

LEt A. B. bee a right line diuided, firſt equally in the point C and then vn-equally as in the point D. The ſquare, I ſay, which is made of the part lying betweene thoſe ſections C. and D. namely the ſquare E. F. G. H. together

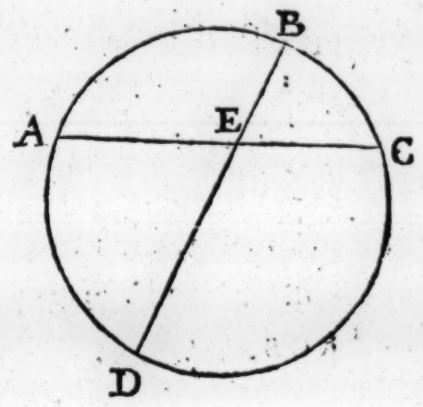


ther with the right angled parallelogram A. D. F. I. contayned vnder the vnequall parts of the whole line as A. D. and D. B. are equall to the ſquare C. B. K. H. being the ſquare of halfe the whole line, as of A. C. or C. B.

THEOREME XXXVII.

Two right lines being drawne in a circle, and the one interſecting the other, either equally or vnequally howſoeuer; The rectangle figure contayned vnder the parts of the one line, ſhall be equall to that, contayned vnder the parts of the other.

LEt A. C. and B. D. be two right lines drawne in the circle A. B. C. D. and let the one interſect the other vnequally at all aduentures in the point E. I ſay, that the rectangle figure contayned vnder the parts of the one line, namely vnder A. E. and E. C. being the parts of the line A. C. ſhall bee equall to that contained vnder the parts of the other line, namely vnder the parts B. E. and E. D. of the line B. D. And the like if thoſe lines had interſected the one the other equally.

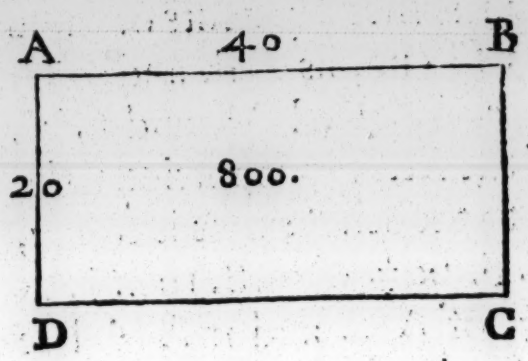


The wonderfull properties of a circle here by appeareth. And many ſtrange concluſions Geometricall from hence may be gathered. *Eu.*3.35. *Ceul.*2.50.

THEOREME XXXVIII.

In all right angled parallelograms, the length thereof being infolded in the breadth, produceth the Area *or ſuperficiall content of the ſame.*

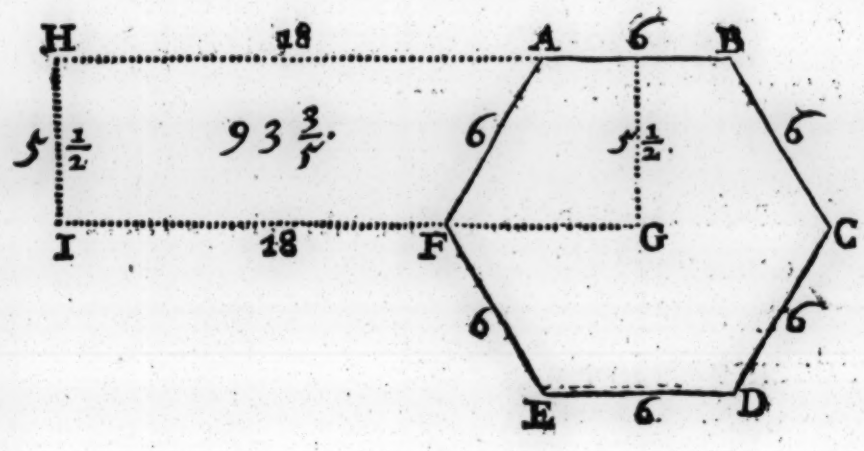
LEt A. B. C. D. be a parallelogram right angled, I ſay, the length there of A. B. 40. being infolded in the breadth A. D. 20. produceth the Area or ſuperficiall content of the ſame 800. PROB. 51. 55.



THEOREME XXXIX.

Euery regular Poligon *is equall to the long ſquare, whoſe length and breadth is equall to halfe the perimeter, and a perpendicular drawne from the center to the middle of any ſide of the ſame.*

LEt the Sexagon A. B. C. D. E. F. be a regular *Polygon*, whoſe three ſides (being halfe the Perimeter) contayne 18. and the perpendicular 6. G. $5\frac{1}{5}$. This *Polygon* is equall to the long ſquare H. 6. G. I. whoſe length H. 6. or I. G. is equall to halfe the Perimeter, and breadth H. I. or 6. G. to the perpendicular 6. G. PROB. 64. 95.

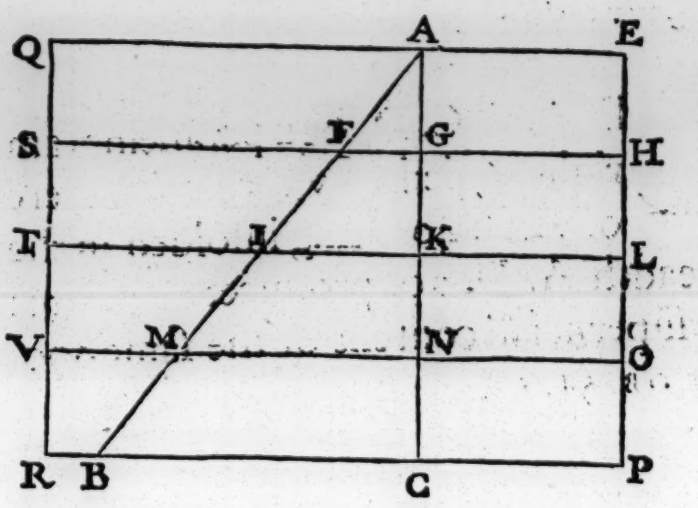


THEOREME XXXX.

If two or more right lines, are cut by diuers parallel right lines; the interſegments of thoſe lines ſo cut ſhall bee proportionall the one to the other.

Pit. 1. 39.

LEt A B. and A C. be two right lines, being cut by diuers parallel right lines as Q E. S H. T L. and the reſt; I ſay, the Interſegments of thoſe lines ſo cut, as A F. and A G. A I. and A K. F M. and G N. and the reſt are proportional the one to the other; that is to ſay, if A F. be $\frac{1}{2}$. of A B. then is A G. $\frac{1}{2}$. of A C. and if A M. bee $\frac{3}{4}$. of A B. then ſhall A N. bee $\frac{3}{4}$. of A C. The reaſon is, becauſe the right line S H. cutteth off $\frac{1}{2}$. of the whole parallelogram Q E P R and the right line V O. $\frac{3}{4}$. thereof; and conſequently the like parts, from all lines drawne ouer-thwart thoſe parallels. And the like conſideration is to bee had of all the other interſegments ſo by thoſe lines cut out.

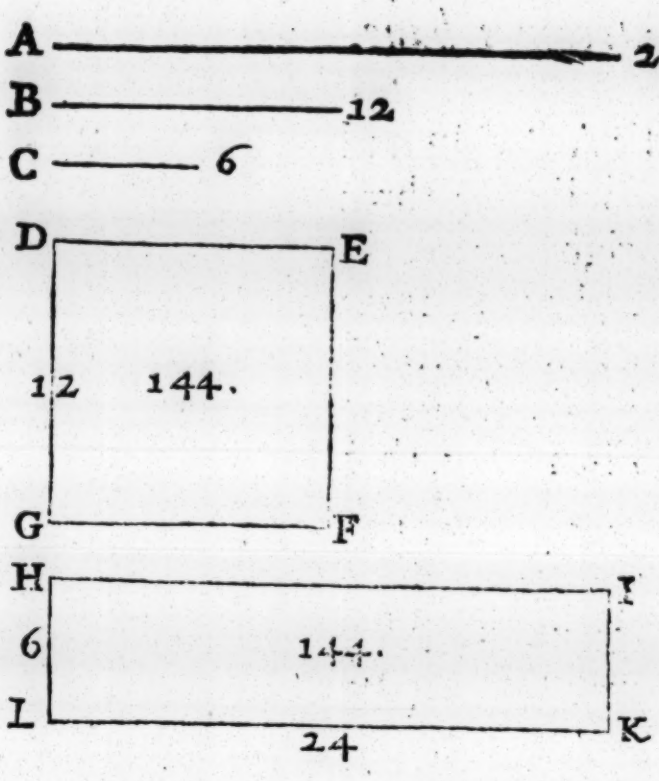


THEO-

THEOREME XLI.

Three right Lines being proportionall, a Square made of the Meane, is equall to the right angled figure, contained vnder the Extreames.

LEt A.B.and C. be three right lines proportionall in continuall proportion, ſo that as A. is to B. ſo let B. be to C. Then I ſay, that the Square, namely, D. E. F. G. made of the Meane B. ſhall be equall to the right angled figure, namely, H. I. K. L. contained vnder the two Extreames A. and C. as appeareth by the Diagram. PROB. 79. 83. 84. 85. 86. 88. 95. 99. 109. 129. 130. 131.

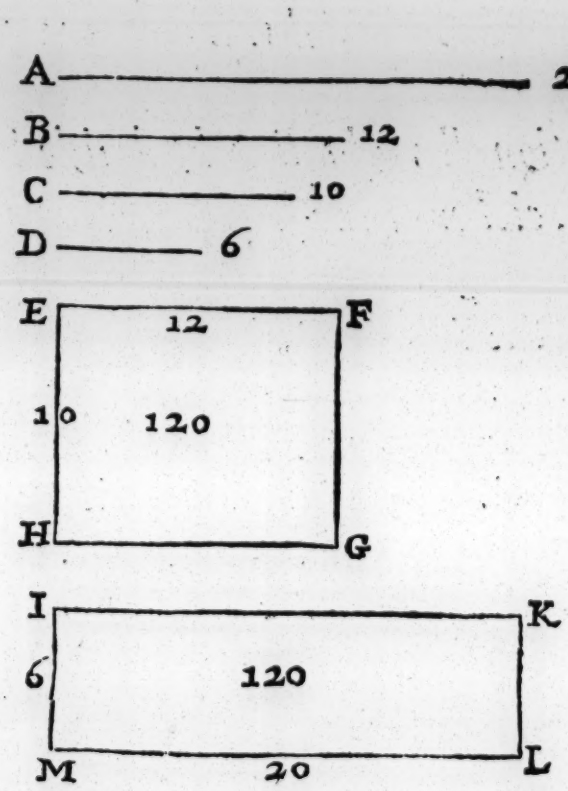


Euc. 6. 17. & l. 7. 20. *Ram.* 12. 4. *Pit.* 1. 43.

THEOREME XLII.

Foure right lines being proportionall, the right angled Parallelogram, contained vnder the two Meanes, is equall to the right angled Parallelogram, contained vnder the two Extreames.

LEt A. B. C. and D. be foure right lines proportionall, ſo that as A. is to B. ſo let C. be to D. Then I ſay, that the right angled Parallelogram, namely, E. F. G. H. contained vnder the two Meanes B. and C. ſhall be equall to the right angled Parallelogram, namely, I. K. L. M. contained vnder the two Extreames A. and D. as appeareth by the Diagram. PROB. 49. 56. 87. 88. 93.



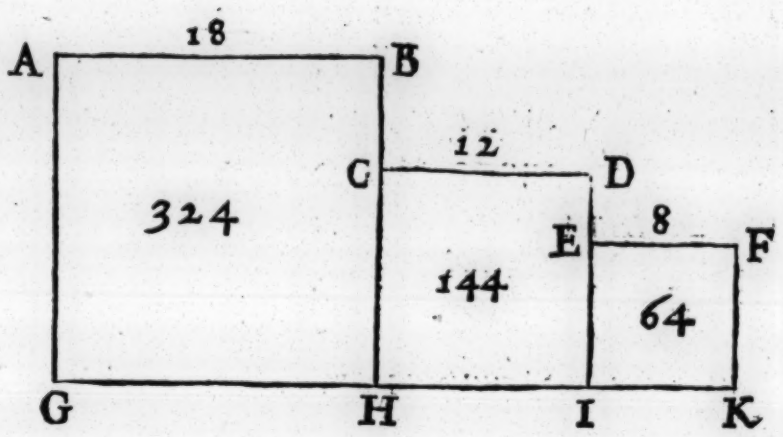
Euc. 6. 16. *Pit.* 1. 42.

THEOREME XLIII.

Of any three proportionall right Lines, the Square which is made of the Meane, and that which is made of either of the Extreames, haue ſuch proportion the one to the other, as the two Extreames haue.

*Eu.*12.2.
*D.P.*1.
*Cam.*2.75.

LEt A.B.C.D. and E.F. bee three right lines, in continuall proportion: ſo that as A.B. is to C.D. ſo let C.D. be to E.F. Then I ſay, that the Square (namely, C.D.I.H.) which is made of the meane C.D. and that which is made of either of the extreames A.B. or E.F. haue ſuch proportion the one to the other (reſpectiuely) as thoſe extreames haue: For the ſame proportion as the greater extreame A.B. hath to the leſſer extreame E.F. the ſame hath the ſquare A.B.G.H. to the ſquare C.D.H.I. and that, to the ſquare E.F.I.K. which in this Diagram is *Dupla ſeſquiquarta*, as thereby appeareth. And the like conſideration is to be had of the proportion of Circles, whoſe Diameters are ſo proportionable. PROB. 79. 81. 82. 94. 129. 130. 131.



THEOREME XLIIII.

If a rationall right line be diuided by an extreame and meane proportion, either of the ſegments is an irrationall reſiduall line.

*Eu.*13.6.

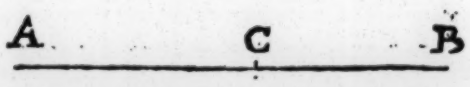
LEt the line A.B. be a rationall right line, and let the ſame be diuided by an extreame and meane proportion in the point C. Then I ſay, that either of the ſegments, namely, A.C. and C.B. is an irrationall reſiduall line. PROB. 20.

THEO-

THEOREME XLV.

If a right line be diuided by extreame and meane proportion, the whole line hath the same proportion to the greater segment, as the same greater segment hath to the lesser.

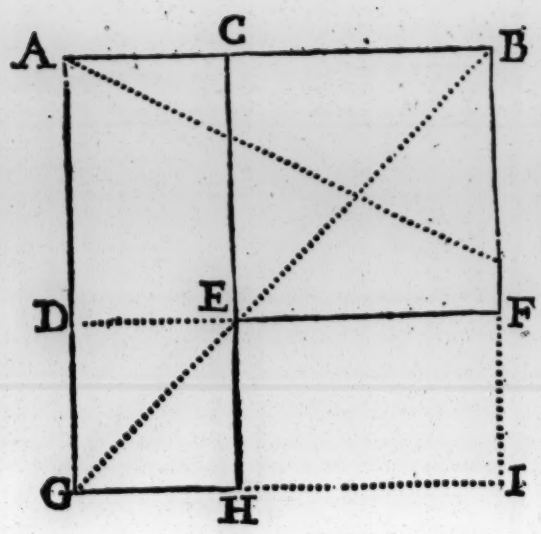
LEt A.B. be a right line, diuided by extreame and meane proportion, as in the point C. I say then, that the whole line A. B. hath the same proportion to the greater segment A. C. as the same greater segment hath to the lesser segment C.B. for as A.B. is to A.C. so is A.C. to C.B. PROB. 20.



THEOREME XLVI.

If a right line be diuided by extreame and meane proportion, the Rectangle figure, comprehended vnder the whole line, and the lesser segment, shall be equall to the Square made of the greater segment.

LEt A. B. be a right line, and let the same be diuided by extreame & meane proportion in the point C. Then, I say, the Rectangle figure, namely, A. C. G. H. comprehended vnder the whole line A. G. (being equall to A. B.) and the lesser segment A. C. shall be equall to the Square, namely, C. B. E. F. made of the greater segment C.B. DEF. 35. PROB. 20.



AB. 10.

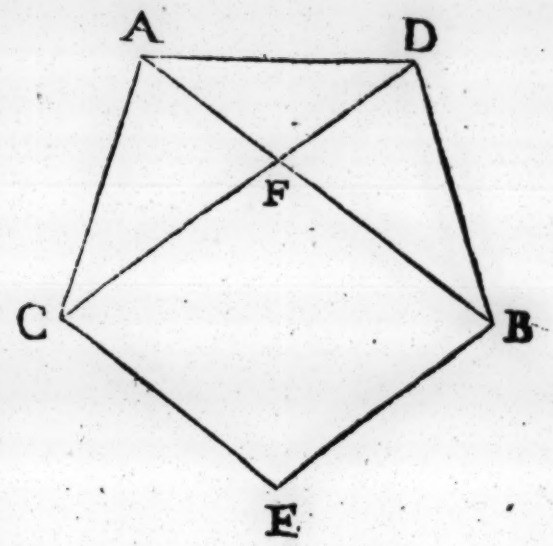
AC. 15—8 125.

CB. 8 125—5.

THEOREME XLVII.

Two right lines being drawne in an equilater Equiangle Pentagon, in ſuch ſort as they ſubtend any two of the next immediate angles, thoſe two lines by their interſections ſhall diuide the one the other by an extreame and meane proportion: and the greater ſegments of either of them ſhall be equall to the ſide of the Pentagon.

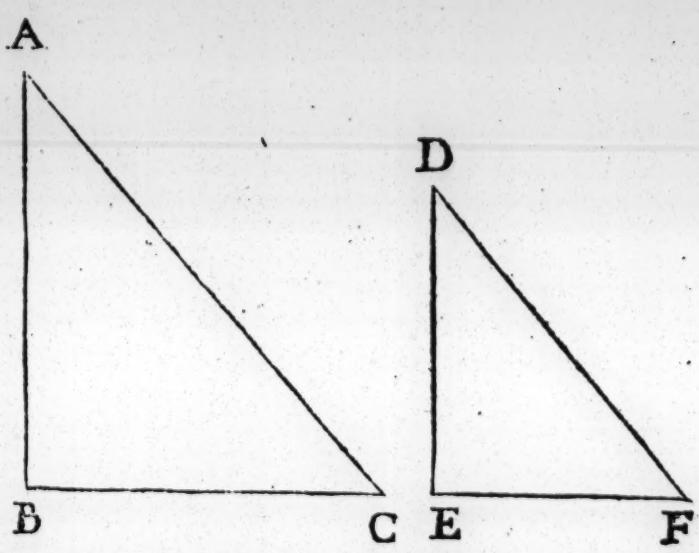
*Eu.*13.8. LEt A.B. and C.D. be two right lines, drawne in the Equilater Equiangles *Pentagon*, A.D.B.E.C. and let the line A.B. ſubtend the angle A.D.B. and the line C.D. the angle C.A.D. being two of the next immediate angles. I ſay then, that thoſe two lines, by their interſection in the point F. ſhall diuide the one the other by an extreame and meane proportion. And the greater ſegments of either of them, as the ſegments F.B. and F.C. ſhall be either of them equall to the ſide of the *Pentagon* A.D.B.E.C. PROB.20.21. 48.62.63.119.120.



THEOREME XLVIII.

Like Triangles are one to the other in double proportion that the ſides of like proportion are.

*Eue.*6.19. LEt A.B.C. and D.E.F. be two like Triangles, and let the angle A. of the one be equall to the angle D. of the other; the angle B. of the one, to the angle E. of the other; and the angle C. of the one to the angle F. of the other; and as the ſide A.B. is to the ſide B.C. ſo let the ſide D.E. be to the ſide E.F. ſo are B.C. and E.F. ſides of like proportion. Now I ſay, that the proportion of the Triangle A.B.C. vnto the Triangle D.E.F. is double the proportion of the ſide B.C. to the ſide E.F. PROB. 45.

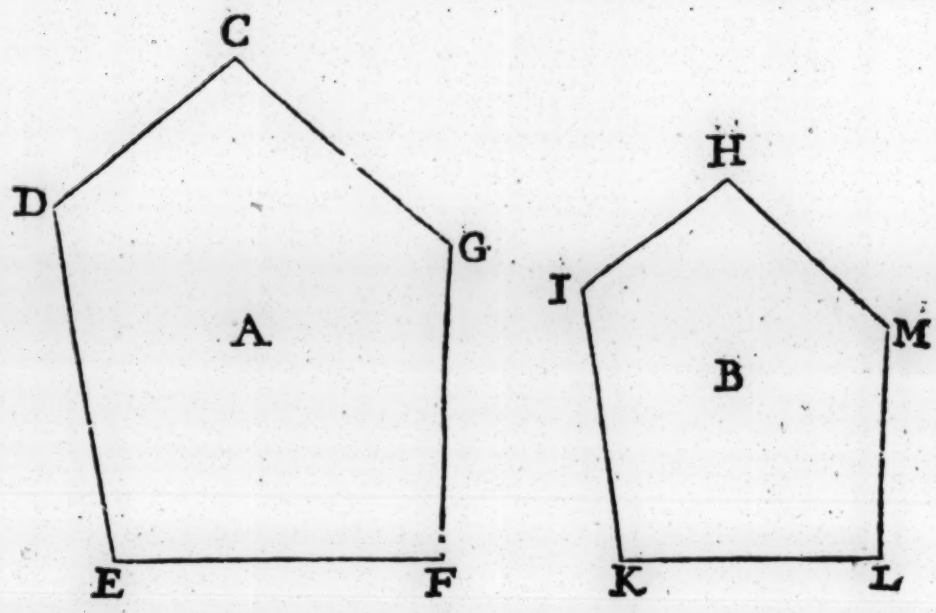


THEO-

THEOREME XLIX.

All like right lined figures whatsoeuer, are the one to the other in double proportion, that the sides of like proportion are.

Euc.6.20.

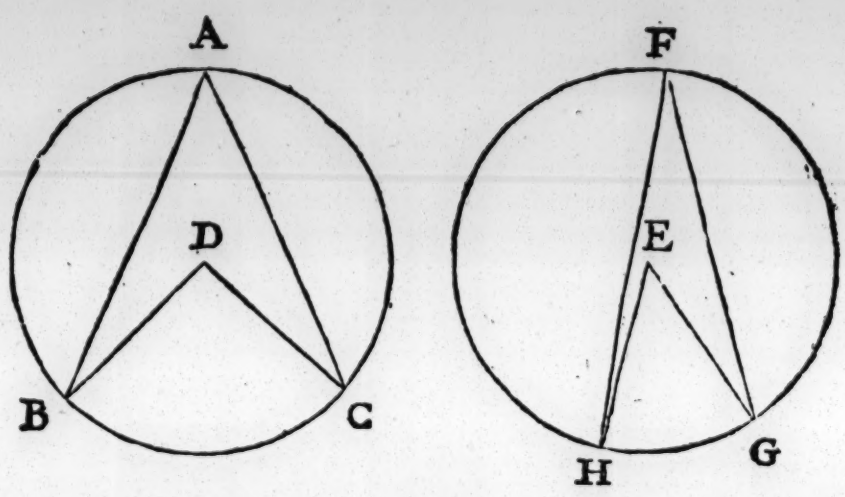


LEt A. and B. bee two right lined figures like, hauing the angle at the point C. equall to the angle at the point H. and the angle at the point D. equal to the angle at the point I. the angle E. to the angle K. and so of all the rest. And also as the side D. E. is to E. F. so let I. K. be to K. L. &c. so are the sides E. F. and K. L. sides of like proportion. Then, I say, that the proportion of the figure A. vnto the figure B. is double, the proportion of the side E. F. to the side K. L. PROB. 45.

THEOREME L.

All angles in equall circles, whether they are in the centers or circumferences, haue the same proportion one to the other as the circumferences haue wherein they consist: And so are the sectors, which are described on the centers.

Euc.6.33.



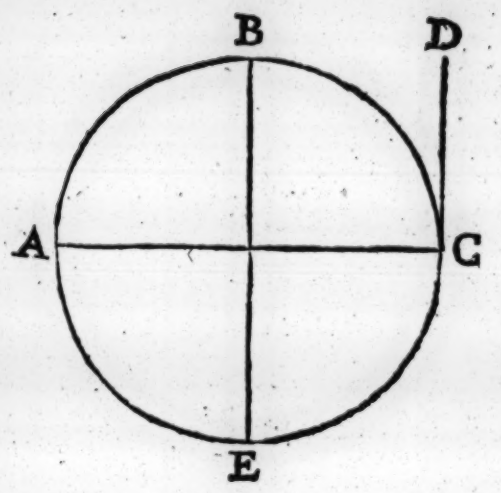
LEt A. B. C. and F. G. H. bee two equal circles whereof let D. and E. be their centers; and let the angles which are in their centers be B. D. C. and H. E. G. and the angles which are in their circumferences B. A. C. and H. F. G. and let the sectors described on their centers be D. B. C. and E. H. G. Then, I say, that the angles B. D. C. and H. E. G. in the centers, and the angles B. A. C. and H. F. G. in the circumferences, haue the same proportion one to the other, as the circumferences haue wherein they consist, that is, as the circumference B. C. hath to the circumference H. G. And the same proportion also hath the sector D. B. C. to the sector E. H. G.

THEO-

THEOREME LI.

If on the end of the Diameter of a circle, a perpendicular bee rayſed, it ſhall fall without the circle, betweene which, and the circumference, another right line cannot be drawne to the Diameter, and the angle within the circle is greater, and that without the circle is leſſer, then any acute angles made of right lines.

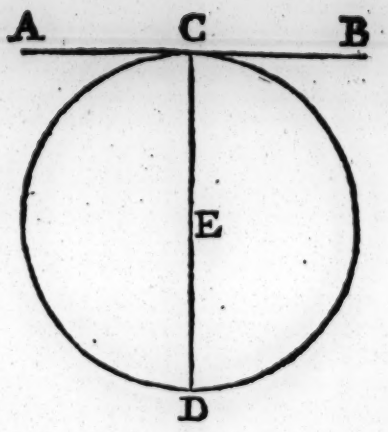
Euc. 3.16. LEt A. B. C. be a circle, whoſe diameter is A: C. and on the ſame diameter let the perpendicular D. C. bee rayſed. Then, I ſay, firſt, that the ſame perpendicular D. C. ſhall fall without the circle; and that betweene the ſame perpendicular and the circumference B. C. another right line cannot be drawne to the diameter A.C. And alſo that the angle within the circle, namely A. C. B. is greater, and that without the circle, namely B.C.D. is leſſer then any acute angles made of right lines.



THEOREME LII.

If a right line bee a tangent or touch line to a circle, and another right line bee drawne by the center to the point of touch, it ſhall bee a perpendicular to the tangent: And if a perpendicular bee let fall from the center to the tangent, it ſhall fall in the point of touch.

Euc. 3.18. LEt the right line A. B. bee a tangent, or touch line to the circle D. E. C. let the point of touch be C. and the center of the circle E. and let another right line, as D.C. bee drawne by the center E. to the point of touch C. Then, I ſay, that the ſame line ſo drawne by the center to the point of touch, ſhall bee a perpendicular to the tangent A. B. And that the perpendicular E. C. being let fall from the center E. to the tangent A. B. ſhall fall in the point of touch C. PROB. 31. 33, 114, 115, 116, 117, 118.



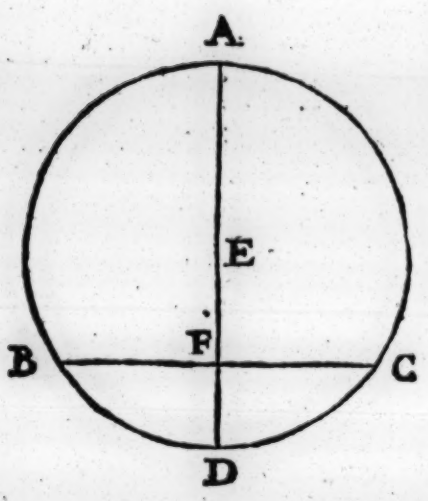
THEO-

THEOREME LIII.

If a right line be drawne in a circle and not by the center thereof, another right line byſecting the ſame by right angles ſhall paſſe by the center of the ſame circle. And if from the center a perpendicular be let fall on a right line drawne in the ſame circle not by the center; the perpendicular ſhall diuide the ſame line into two equall parts.

LEt A. B. D. C. be a circle whoſe center is E. and let B. C. be a right line drawne in the ſame circle, and not by the center thereof, and let another right line as A. D. byſect the ſame by right angles in the point F. Then, I ſay, that the ſame line A. D. ſhall paſſe by the center of the circle. Alſo from the center E. let fall the perpendicular E. D. on the right line B. C. drawne in the ſame circle not by the center; Then, I ſay, further that the perpendicular E. D. ſhall diuide the ſame line B. C. into two equall parts.

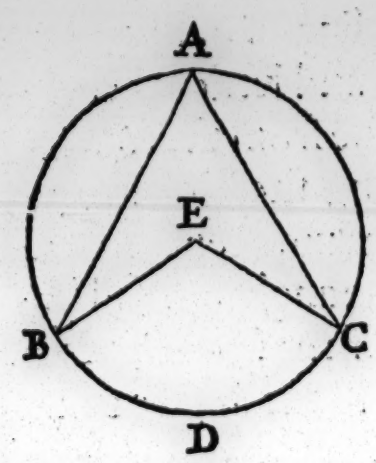
Euc. 3. 3.



THEOREME LIIII.

If one angle be placed in the circumference of a circle, and another in the center thereof, and are both ſubtended by one part of the circumference. That angle in the center ſhall bee double to that in the circumference.

LEt A. B. C. be a circle, and let one angle bee placed in the circumference thereof, as the angle B. A. C. and another in the center thereof, as the angle B. E. C. and let them both bee ſubtended by one part of the circumference as B. D. C. Then, I ſay, that the angle B. E. C. in the center, ſhall be double to the angle B. A. C. in the circumference.



Angulus in centro duplus eſt anguli in peripheria, in eandem peripheriam inſiſtentis.
Euc. 3. 20.

THEOREME LV.

All angles conſiſting in one and the ſame ſegment of a circle are equall the one to the other; If in a ſemicircle, they are right angles; If in a leſſer ſegment, they are greater then a right angle; If in a greater

ſegment

ſegment, they are leſſer. And alſo the angle of a greater ſegment, is greater then a right angle, and the angle of a leſſer ſegment is leſſe then a right angle.

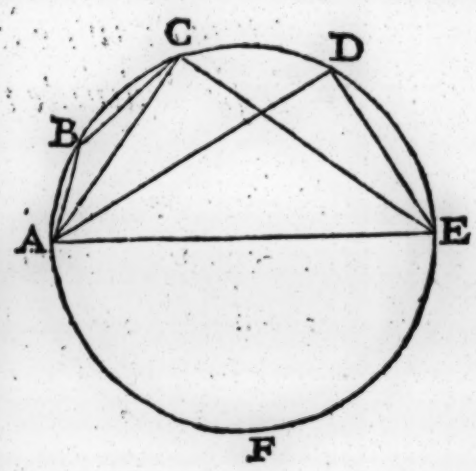
Anguli in eadem ſectione ſunt æquales.

LEt A. B. C. D. E. F. bee a circle, and let A. E. be the diameter thereof, which diuideth the ſame into two ſemicircles or equall ſegments; Then, I ſay, that the angles A. C. E. and A. D. E. conſiſting in one and the ſame ſegment, are equall the one to the other, and being in a ſemicircle they are both right angles: Let alſo the line or cord A. C. diuide the ſame circle into two vnequall ſegments, as A. B. C. the leſſer ſegment, and A. F. E. D. C. the greater, I ſay, the angle A. B. C. in the leſſer ſegment is greater then a right angle, and the angle A. E. C. in the greater ſegment is leſſe then a right angle. And alſo F. A. C. an angle of the greater ſegment is greater then a right angle, and the angle B. A. C. being an angle of the leſſer ſegment, is leſſe then a right angle. PROB. 19. 23. 24. 25. 30. 38. 52. 65.

Eu. 3. 21. 31

Ceul. 2. 46.

Note here the difference betweene an angle in a ſegment, and an angle of a ſegment, See *Def.* 16. 17.



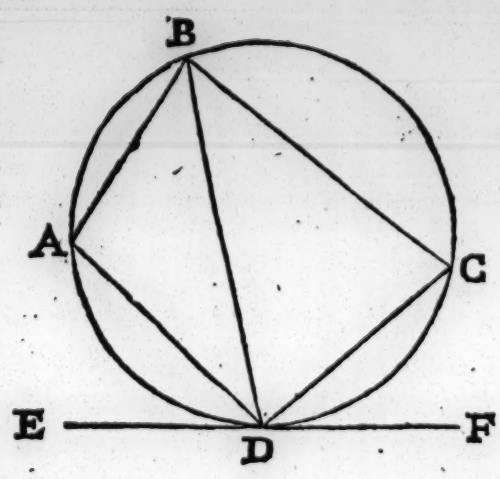
THEOREME LVI.

If a right line be a tangent to a circle, and another right line be drawne from the touch (croſsing the circle) to what point ſoeuer in the circumference; the angles cauſed by interſection or meeting of thoſe two lines, are equall to the angles conſiſting in the alternate ſegments of the circle.

LEt A. B. C. D. be a circle, and the right line E. F. a tangent to the ſame circle; and let another right line as B. D. bee drawne from the touch, namely, the point D. croſſing the circle to what point ſoeuer in the circumference as the point B. Then, I ſay, the angles cauſed by interſection or meeting of thoſe two lines E. F. and B. D. are equall to the angles conſiſting in the alternate ſegments of the circle; that is, the angle B. D. F. ſhall be equall to the angle B. A. D. and the angle B. D. E. to the angle B. C. D. in the alternate ſegments. PROB. 32. 33. 111.

Euc. 3. 32.

Ceul. 2. 47.

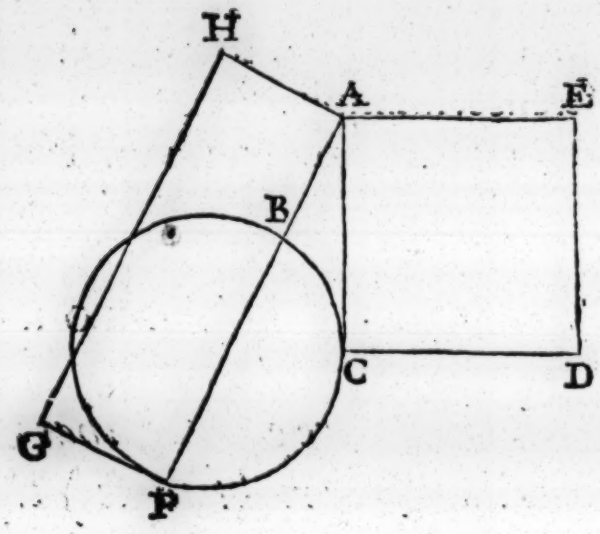


THEO-

THEOREME LVII.

If from a point without a circle, two right lines be so drawne, that the one be a tangent to the circle, and the other deuide the same circle into two equall or vnequall parts: The rectangle figure contayned vnder the whole line which divideth the circle, and that part thereof lying betweene the vtter circumference and the point, is equall to the square made of the tangent line.

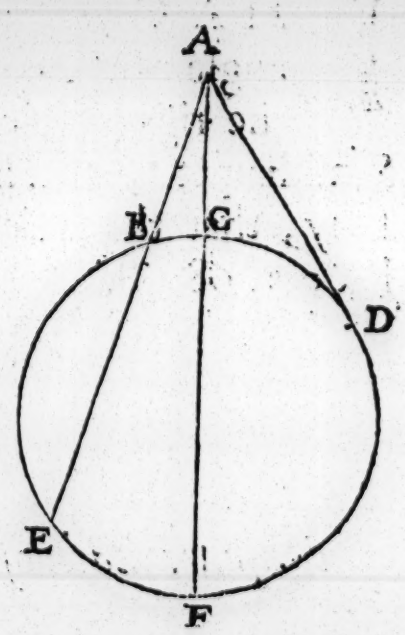
LEt B. C. F. be a circle, and without the same, take a point at all aduentures, as the point A. from whence let two right lines be so drawne, that the one bee a tangent to the circle, as A. C. and the other diuide the same circle, as A. B. F. Then, I say, that the rectangle figure contained vnder the whole line A. F. and that part of the same line, lying betweene the vtter circumference & the point, as B. A. namely, the rectangle figure A. H. G. F. is equall to the square made of the tangent line A. C. namely, to the square A. E. D. C.



THEOREME LVIII.

If from a point without a circle, two right lines be drawne to the concaue circumference of the circle, they shall be reciprocally proportionall with their parts taken without the circle. And another right line drawne from the point as a tangent to the circle, shall bee a meane proportionall betweene either whole line, and the vtter segment thereof.

LEt B. E. F. D. be a circle, and without the same circle take a point at all aduentures, as at A. and from that point to the concaue circumference of the circle, draw the two right lines A. E. and A. F. And let another right line be drawne from the same point as a tangent to the circle, as the line A. D. Then, I say, first, that these two lines A. E. and A. F. are reciprocally proportionall with their parts taken without the circle, that is, as A. E. is to A. F. so is A. C. to A. B. And moreouer, that betweene the lines A. F. and A. C. or betweene the lines A. E. and A. B. the tangent A. D. is a meane proportionall.



Eucl. 6. 33.

THEO-

THEOREME LIX.

Euery circumference of a circle, is more then triple his Diameter, by ſuch a proportion as is more then $\frac{10}{71}$. *and leſſe then* $\frac{1}{7}$ *of the ſame, the neereſt rationall proportion whereof is 22. to 7.*

Corol: Euc. 12. 1.

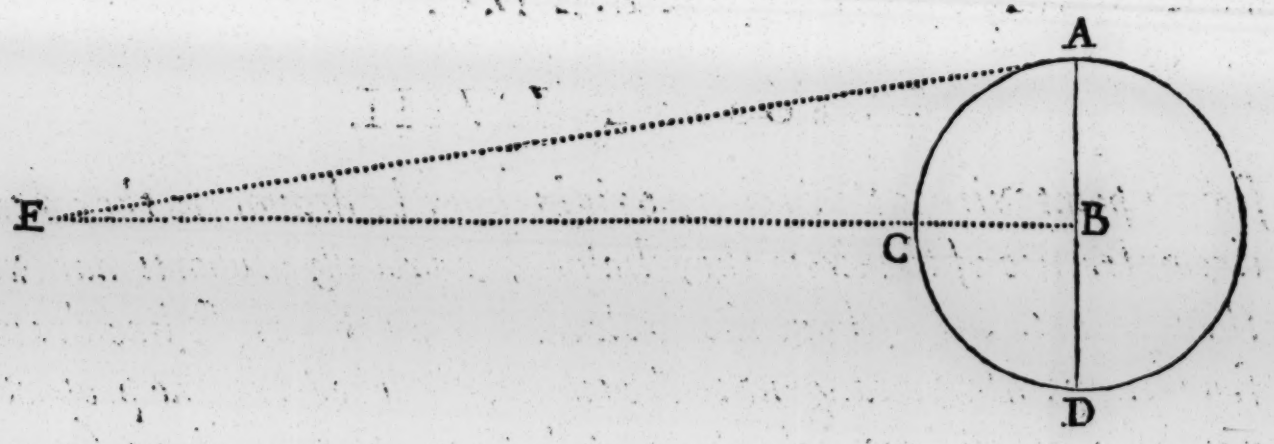
IN the former Diagram, let B. C. D. F. E. be the circumference of a circle, and C. F. the Diameter thereof; I ſay, the ſame circumference is more then triple the Diameter C. F. by ſuch a proportion as is more then $\frac{10}{71}$. and leſſe then $\frac{1}{7}$. of the ſame; and that the neereſt rationall proportion thereof, is 22. to 7. Wherefore to know the quantitie of the circumference, multiplie the Diameter by 22. and diuide the *Factus* by 7. the *Quotient* reſolueth the queſtion. PROB. 34. 67.

THEOREME LX.

Euery circle is neere equall to that right angled Triangle, of whoſe ſides (containing the right angle) the one is equall to the ſemidiameter, and the other to the circumference of the ſame circle.

Corol. Euc. 12. 1.

The preciſe ſquaring of a circle was neuer yet found out; and therfore in this and the 4. next *Theoremes* following, this word *(Neere)* is vſed. But all Concluſions hereby wrought, are without any apparant error.



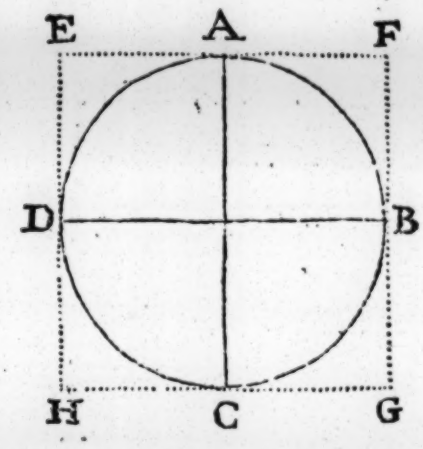
LEt A. C. D. be a circle, and A. E. B. a right angled Triangle, whoſe angle at the centre B. is a right angle, and whoſe ſides containing the right angle, namely, A. B. and E. B. the one is equall to the Semidiameter, as A. B. and the other to the circumference, as E. B. Then I ſay, that the Triangle A. E. B. is neere equall vnto the ſame circle. PROB. 68. 68.

THEOREME LXI.

The Square made of the Diameter of a Circle, is in that proportion to

to the circle (very neere) as 14. *to* 11. *And therefore euery circle is neere* $\frac{11}{14}$. *of the ſquare about him deſcribed.*

LEt A. B. C. D. be a circle, and E.F.G.H. a ſquare made of the diameter A. C. or B. D. Then, I ſay, that the ſquare E. F. G. H. is very neere in the ſame proportion to the circle A. B. C. D. as 14. to 11. And theree fore, the circle very neere $\frac{11}{14}$. of the ſame ſquar- about him deſcribed. PROB. 68.

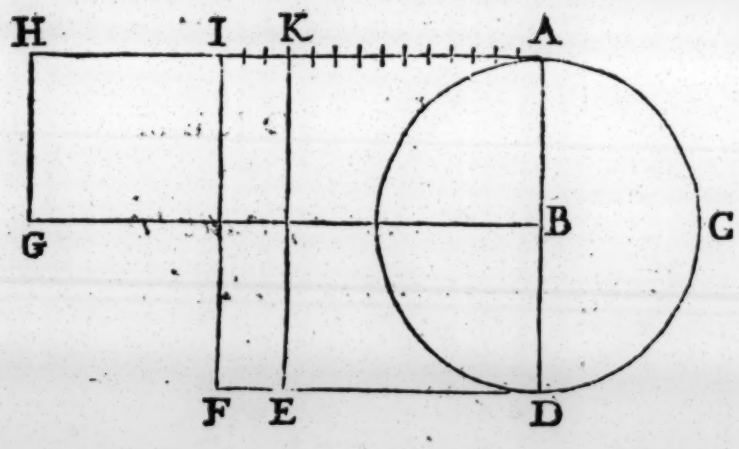


Corol. Euc. 12. 1.

THEOREME LXII.

Euery circle is neere equall to the long ſquare, whoſe length and breadth are equall to halfe the circumference, and halfe the Diameter; or to the whole Diameter, and $\frac{11}{14}$. *thereof.*

LEt A. C. D. be a circle; And let H. A. B. G. be a long ſquare, whoſe length H. A. is equall to halfe the circumfe- rence, and breadth A. B. to halfe the diame- ter; And let alſo K. A. D. E. bee another long ſquare, whoſe length is the whole diameter A. D. and breadth $\frac{11}{14}$. thereof, namely, K. A. Then, I ſay, that either of thoſe two long ſquares are neere equall to the circle A. C. D. PROB. 68.

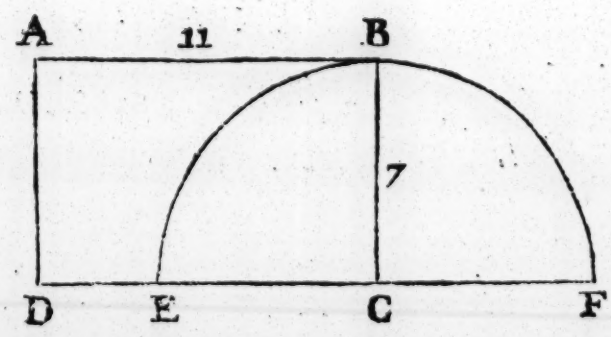


Corol. Euc. 12. 1

THEOREME LXIII.

Euery ſemicircle is neere equall to the long ſquare, whoſe length and breadth is equall to halfe the arch line, and the ſemidiameter.

LEt E. B. F. be a ſemicircle, whoſe ſemidiameter is the line B. C. and the halfe of whoſe arch line is equall to the line A. B. or D. C. Then, I ſay, that the long ſquare, namely, A. B. C. D. (whoſe length A. B. is equall to halfe the arch, and



Corol. Euc. 12. 1.

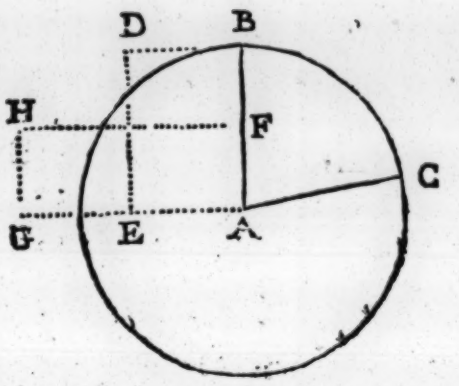
whoſe breadth is B.C. the ſemidiameter) is neere equall to the ſemicircle E.B.F. PROB. 69.

THEOREME LXIIII.

Euery ſector of a circle, is neere equall to that long ſquare, whoſe length and breadth is equall to the ſemidiameter, and halfe the arch-line of the ſame ſector; or the halfe ſemidiameter, and the whole arch line.

Cor. Eu. 12. 1.

LEt B.A.C. bee the ſector of a circle, and let D.B.A.E. bee a long ſquare, whoſe length B.A. or D.E. is the ſemidiameter or equall thereunto, and whoſe breadth D.B. or E.A. is equall to halfe the arch-line B.C. And let alſo H.F.A.G. be another long ſquare, whoſe length H.F. or G.A. is equall to the whole arch-line B.C. and whoſe breadth F.A. or H.G. is halfe the ſemidiameter, or equal thereunto. Then, I ſay, that either of thoſe two long ſquares is neere equall to the ſector B.A.C. PROB. 70.



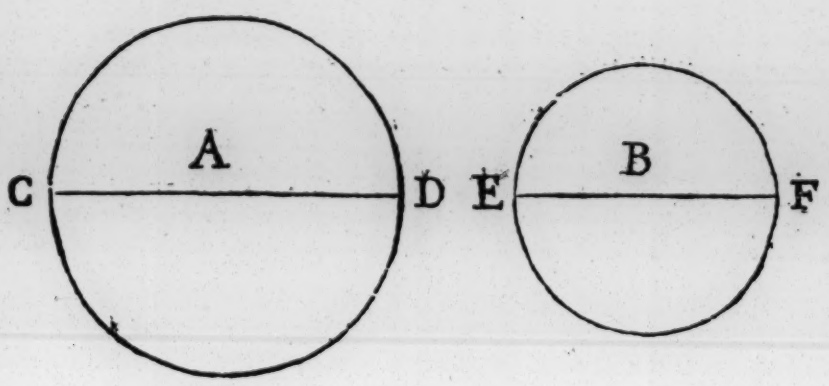
THEOREME LXV.

All circumferences of circles, haue the ſame proportion the one to the other, as their diameters haue.

This *Theor.* is of excellent vſe in the forming of Mill-wheels, clocks, cranes, and other engines for water-workes, &c.

Cor. Eu. 12. 1.

LEt A. and B. be two circles, wher of let C.D. and E.F. bee their ſeuerall diameters, I ſay, that the ſame proportion that the diameter C.D. of the circle A. hath to the diameter E.F. of the circle B. the ſame proportion hath the circumference of A. to the circumference of B. PROB. 65. 66.



THEOREME LXVI.

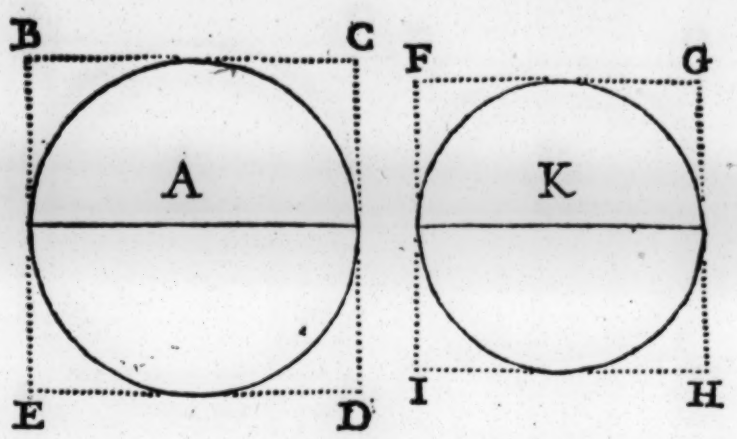
Omnes figuræ ſimiles circulis inſcriptæ ſunt, vt quadrata à diametris circulorum quibus inſcribuntur. Eue. 12. 2.

All circles haue the ſame proportion the one to the other, as the ſquares of their Diameters haue.

LEt A. and K. bee two circles, and let the ſquares circumſcribed about them, bee the ſeuerall ſquares of their Diameters. Then, I ſay, that the circle A. hath the ſame proportion to the circle

K.

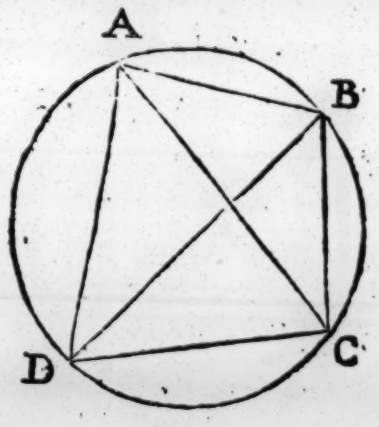
K. as the ſquares of their Diameters haue, namely, as the ſquare B. C. D. E. hath to the ſquare F. G. H. I. PROB. 65, 66, 104. 108.



THEOREME LXVII.

If in a circle be deſcribed a quadrilaterall figure, the oppoſite angles thereof ſhall bee equall to two right angles: and being interſected with two diagonalls, the right angled figure made of thoſe diagonalls, is equall to the two right angled figures, comprehended vnder the oppoſite ſides of the quadrilaterall figure.

LEt A.B.C.D.be a circle,and let therein bee deſcribed the quadrilaterall figure A. B. C. D. let alſo the ſame figure be interſected with the two diagonals A. C. and B. D. Then, I ſay, firſt, that the oppoſite angles at the points A. and C. are equall to two right angles, and likewiſe the oppoſite angles at the points B. and D. And alſo that the right angled figure made of the diagonalls A. C. and B. D. is equall to the two right angled figures (taken together) comprehended vnder the oppoſite ſides A. B. and D. C. and vnder A. D. and B. C.



Exempla illuſtriſſima habebis, Pit. lib. 2. p. 32. 33. 35. 36. 37. 38

Anguli oppoſitis ſectionibus æquantur duobus rectis.

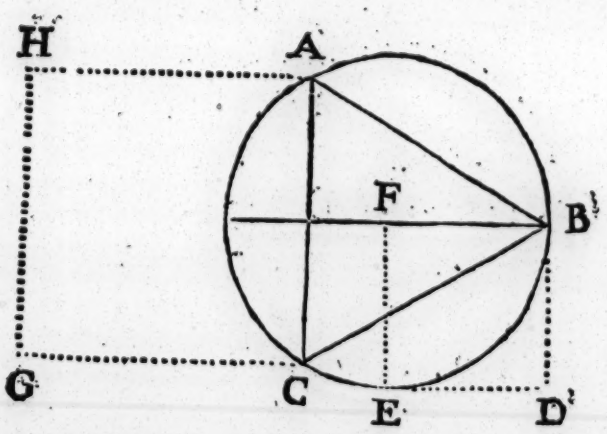
Eu. 3. 22.
Pit. 1. 54.

This Prop. is of very great vſe in trigonometrie.

THEOREME LXVIII.

The power of the ſide of an equilater triangle inſcribed in a circle, hath to the power of the ſemidiameter of the ſame circle triple proportion.

LEt A. B. E. C. be a circle, whereof F. B. is the ſemidiameter, and let A. B. C. be an equilater triangle inſcribed in the ſame circle. Then, I ſay, that the power of the ſide of the equilater triangle A. B. C. namely, the ſquare H. A. C. G. hath to the power of the ſemidiameter F. B. namely, the ſquare F. B. D. E. triple proportion, that

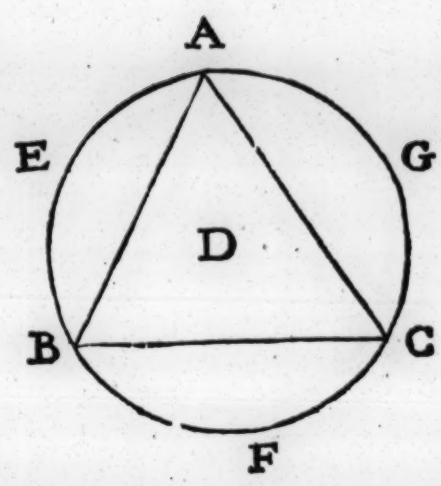


is, as 3. to 1. For the ſquare H. A. C. G. contayneth the ſquare F. B. D. E. three times.

THEOREME LXIX.

A triangle inſcribed in a circle, hath euery of his angles equall to halfe the arch, oppoſite to the ſame angle.

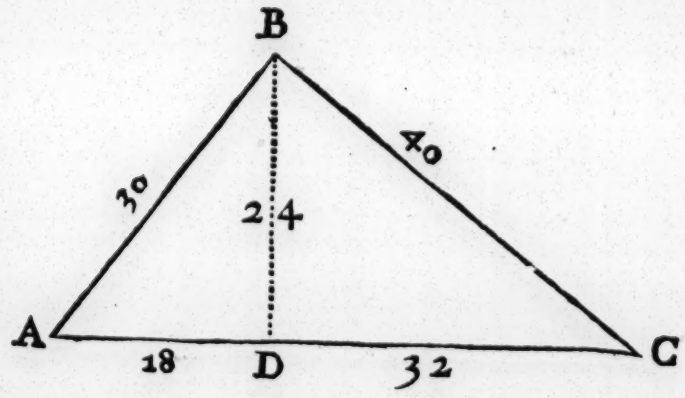
*Pit.*1.53. LEt D. be a circle, and let A. B. C. be a triangle, inſcribed at all aduentures in the ſame circle. Then, I ſay, that the triangle A. B. C. hath euery of his angles equall to halfe the arch, oppoſite to the ſame, as the angle at the point A. is equall to halfe the arch B. F. C. oppoſite thereunto, the angle at the point B. is equall to halfe the arch A. G. C. and the angle at the point C. is equall to halfe the arch A. E. B. For, the whole of euery circle is 360. degrees, whereof the halfe is 180. and the three inward angles of euery right lined triangle, is equall to two right angles, which is 180. degrees. THEOR. 13. PROB. 48. 119. 120.



THEOREME LXX.

If in a rectangle-triangle a perpendicular bee drawne from the right angle to the baſe, the ſame perpendicular is a meane proportionall betweene the ſections of the baſe: And the ſide annext to either ſection, ſhall bee a meane betweene the ſame ſection and the whole baſe.

Eus. Coroll. 6. 8. Ceul. 2. 63. LEt A. B. C. be a rectangle-triangle, right angled at B. from whence let the perpendicular B. D. bee drawne to the baſe A. C. Then, I ſay, that the ſame perpendicular B. D. is a meane proportionall betweene the ſections of the baſe, namely, betweene A. D. and D. C. And alſo that the ſide A. B. annext to the ſection A. D. is a meane proportion betweene A. D. and the whole baſe A. C. and that the ſide B. C. annext vnto the ſection D. C. is a meane proportion betweene the ſame



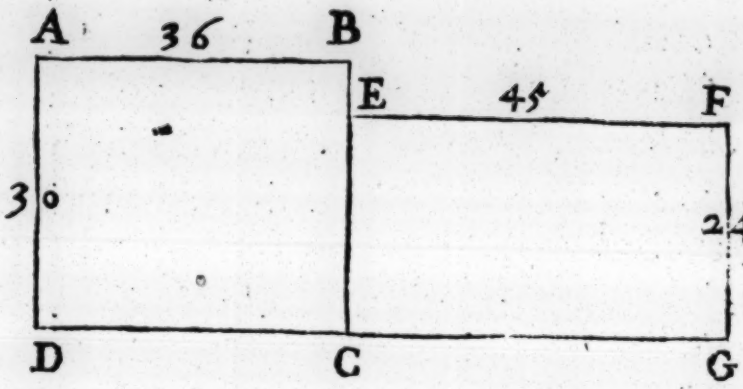
same section D. C. and the whole base A. C. For, as A. D. is to A. B. so is A.B. to A.C. &c. PROB. 17, 19, 23, 24, 25, 30, 38, 43, 65, 66, 109.

THEOREME LXXI.

If in equall parallelograms, one angle of the one, be equall to one angle of the other, the sides which contayne those equall angles, shall bee reciprocall.

Parallelogramma æquiangula æqualia, sunt lateribus reciproca: & contra.

Euc.6. 14.



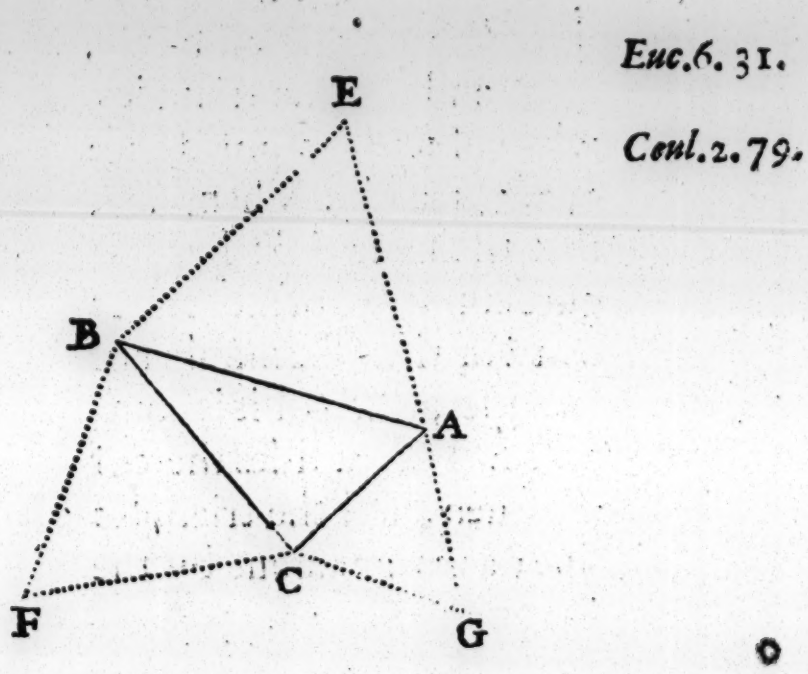
LEt A. B. C. D. and E. F. G. C. bee two parallelograms, equall the one to the other, and let the angle B.C.D. of the one bee equall to the angle E. C. G. of the other. Then, I say, that the sides which contayne those equall angles, are reciprocally proportionall, that is, as D. C. to C.G. so is E. C. to B. C. PROB. 88. 93.

THEOREME LXXII.

In rectangle-triangles, the figure which is made of the subtending side of the right angle, is equall vnto both the figures made of those sides, which contayne the right angle, so as those three figures are like, and in like sort described.

Euc.6. 31.

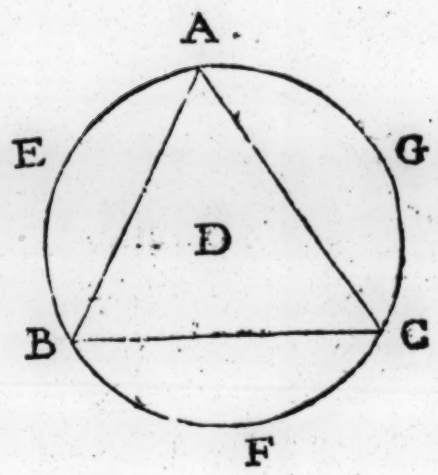
Coul.2. 79.



LEt A. B. C. be a triangle, whose angle at the point C. is a right angle. Then, I say, that the equilater triangle E. B. A. which is made of B. A. the subtending side of the right angle C. is equall vnto both the equilater triangles, made of the containing sides B. C. and C. A. namely, to the triangles B. F. C. and A. C. G. taken together: And the like of squares, and all other like figures, in like sort described.

Theoreme LXXIII.

In all plaine triangles, the ſides are in proportion the one to the other, as the ſubtenſes of the angles oppoſite thereunto; or as the ſines of the angles oppoſite to thoſe ſides.

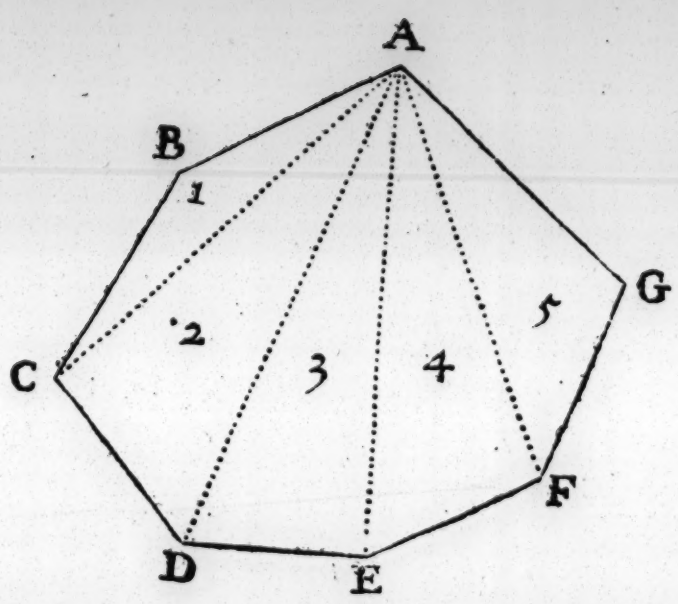


LEt D. be a triangle, and let there be circumſcribed about the ſame triangle the circle A. B. C. by meanes whereof the ſide A. B. is made the ſubtenſe of the angle A. C. B. that is, of the arch A. E. B. which is oppoſite to the ſame angle A. C. B. Alſo the ſide B. C. is made the ſubtenſe of the angle B. A. C. that is, of the arch B. F. C. which is oppoſite to the ſame angle B. A. C. and laſtly, the ſide A. C. is made the ſubtenſe of the angle A. B. C. that is, of the arch A. G. C. which is oppoſite to the ſame angle A. B. C. Then, I ſay, that the ſide A. B. is in proportion to the ſide B. C. as the ſubtenſe of the angle A. C. B. to the ſubtenſe of the angle B. A. C. for the ſides and ſubtenſes, are one and the ſame. And likewiſe of the ſines of thoſe angles; which ſines are the one halfe of their ſubtenſes, and what proportion the whole hath to the whole, the ſame hath the halfe to the halfe. Chap. 14. 3.

Theoreme LXXIIII.

Euery right lined figure, or plat, conſiſteth of more ſides by two; then the number of triangles, whereof the ſame figure is compoſed.

Ram. 10. 1. LEt A. B. C. D. E. F. G. be a right lined figure, and let the triangles, whereof it is compoſed, bee 1, 2, 3, 4, and 5. Then, I ſay, that the ſame right lined figure conſiſteth of more ſides by two, then the number of triangles, whereof it is compoſed. For the ſides thereof are ſeuen, and the triangles fiue; as is apparant by the Diagram. Chap. 37. 3.



The end of the firſt Booke.

THE VSE AND OPERATION OF THE FORMER THEOREMES.

The second Booke.

THE ARGVMENT OF THIS BOOKE.

THIS *Booke consisteth of diuers Conclusions, or Geometricall* PROBLEMES, *here duly placed, by obseruation of naturall course; the cause being formerly, in the first Booke, amply expressed, and here the effect as fully made manifest, hauing either to other due relation. This Booke is diuided into foure Parts, wherein most plainely, briefely, and methodically, is expressed the practicke operation of the precedent* THEOREMES; *as the Distinction, Application, and Diuision of Lines and Angles, and the Description, Mensuration, Reduction, Addition, Inscription,*

ſcription, Tranſmutation, Diuiſion, and Seperation of all ſorts and formes of ſuperficiall Figures, according to their ſeuerall kindes.

THE FIRST PART.

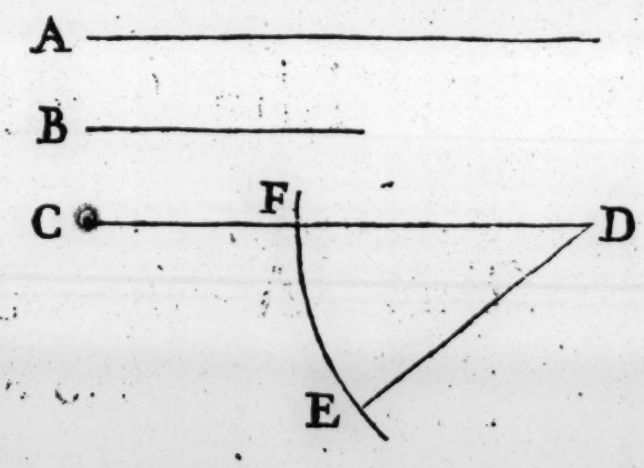
Of the Properties, Paſsions, Diſpoſitions, Applications, and Diuiſions of Lines and Angles.

PROBLEME I.

Two right lines giuen, being vnequall; to take from the greater a line equall to the leſſer.

Euc. 1. 3.

LEt A. and B. be two vnequall right lines giuen, whereof let A. be the greater; from which it is required to take a line equall to the leſſer.



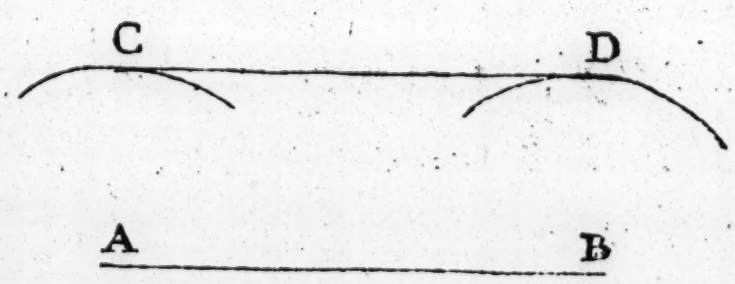
Firſt, ioyne the two giuen lines together in ſuch ſort, as thereby they make any kind of angle, as C. D. E. and making the centre D. and the ſpace D. E. (the length of the line B.) deſcribe the arch line F. E. which ſhall cut off from the greater line, the line F. D. equall to the leſſer line B. which was required to be done. DEF. 10.

PROBLEME II.

To a right line giuen, to draw a parallell line at any diſtance required.

Euc. 1. 31.

SVppoſe the right line giuen, to be A. B. vnto which line it is required to haue a parallel drawn: Open your Compaſſe to the diſtance required, and ſetting one foot in the end A. ſtrike an arch line on that ſide the giuen line, whereon the parallell is to be drawne, and



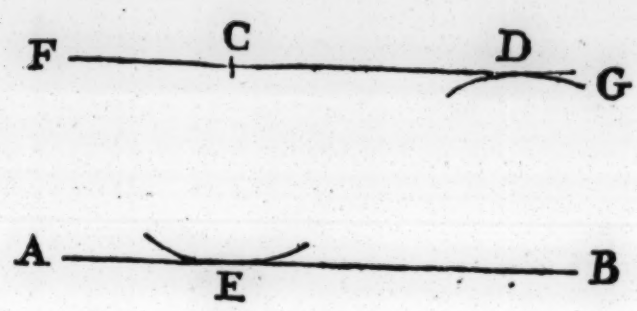
and the like in the end B. as the arch lines C. and D. and by the conuexitie of thoſe arch lines, draw the line C. D. which ſhall be parallell to the giuen line, as was required. DEF. 34.

PROBLEME III.

To performe the former Propoſition at a diſtance required, and by a point limitted.

Euc. I. 31.

LEt A. B. be a right line giuen, whereunto it is required to haue a parallell line drawne at the diſtance, and by the point C. Place one foot in C. from whence take the ſhorteſt extention to the line A. B. as C. E. at which diſtance, place one foot in the end B. and with the other ſtrike the arch line D. by the conuexitie of which arch line, and the limitted point C. draw the line F. G. which ſhall be a parallell to the giuen line A. B. the thing required. DEF. 34.

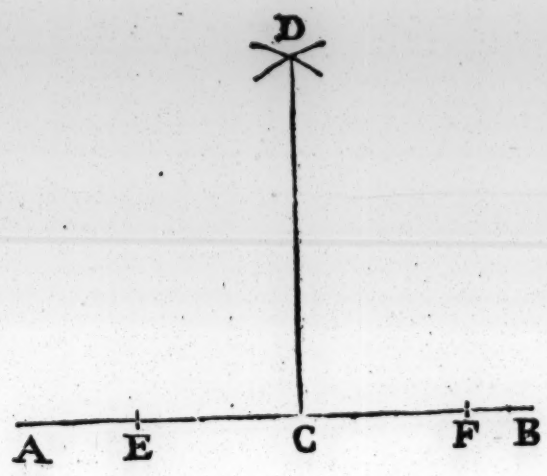


PROBLEME IIII.

To erect a perpendicular on any part of a right line giuen.

Euc. I. 11. *Cenſ.* 2. 7.

LEt A. B. be a right line giuen, and let C. be a point therein, whereon it is required to erect a perpendicular. Open the Compas to any conuenient diſtance, and ſetting one foot in the point C. with the other marke on either ſide thereof, the equall diſtances C. E. and C. F. Then opening the Compas to any conuenient wider diſtance, with one foot placed in the points E. and F. ſtrike two arch lines, croſſing each other, as in D. from whence draw the line D. C. which ſhall be the perpendicular required. DEF. 5.



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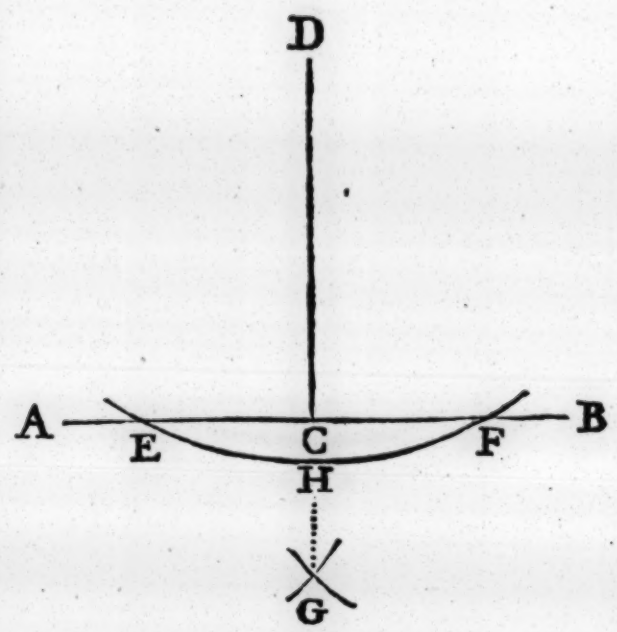
PROBLEME V.

To raiſe or let fall a perpendicular to a line giuen, from a point either aboue or beneath the ſame line.

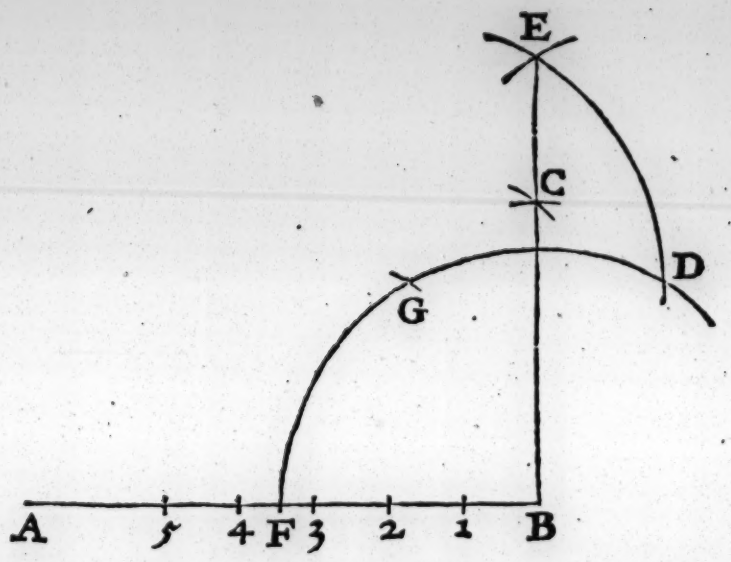
*Euc.*I.11. LEt A. B. be a line giuen, and let D. be a point aboue the ſame line: It is required from the point D. to let fall a perpendicular line to the giuen line A. B. At any indifferent diſtance placing one foot in the point D. deſcribe an arch line, interſecting the giuen line twice, as the arch line H. interſecteth the giuen line, in the points E. and F. Then either with the ſame, or ſome other conuenient diſtance, by placing the one foot in thoſe points E. and F. ſtrike two arch lines, croſſing each other, as in G. By which point, and the giuen point D. draw the line D. C. which ſhall be a perpendicular to the giuen line. And the like conſtruction is to be vſed, if the point were beneath the giuen line. DEF. 5.



PROBLEME VI.

Upon the end of a line giuen to raiſe a perpendicular.

LEt A. B. be a right line giuen, and let B. be the end thereof, whereon it is required to rayſe a perpendicular line. Open your Compas to a conuenient diſtance, and with one foot in B. draw the arch line F. G. D. Then placing one foot in F. at the ſame diſtance marke the arch in G. and on G. draw the arch E. D. Then on D. croſſe the laſt arch in E. ſtill keeping the ſame diſtance; from which interſection draw the line E. B. which ſhall be the perpendicular required. DEF. 5.



reader way thus, vt altering the compas: vz one foote therof in B. draw the arch F g: d: then at the same wideness w one foote in F croſſe the former arch in g: at f: Ion g: plac a ruler w the compas at the ſame distance applied to the ruler setting one foote in the interſection at g extending the other foote towardes E: & there w a point from whence draw a line to B: & this done. To

To performe the ſame another way.

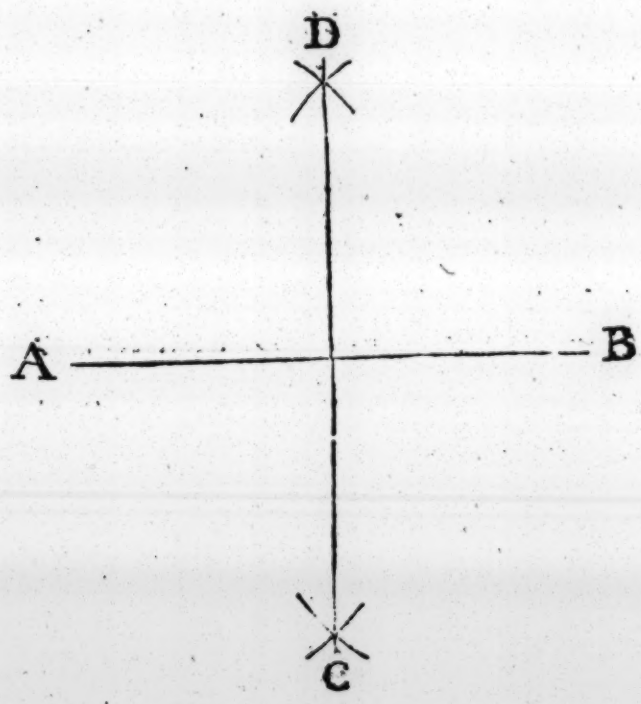
LEt A.B. in the former Diagram be a right line giuen, and B. the end thereof, whereon a perpendicular is to be rayſed. From the end B. pricke out any fiue equall diſtances, and opening the Compas to 4. of them, with one foot in B. ſtrike an arch line towards C. Then opening the Compas to all 5. Diuiſions, with one foot in the third Diuiſion, croſſe the ſame arch line in C. from whence draw the line C. B. which ſhall be the perpendicular required.

PROBLEME VII.

To diuide a right line giuen, into two equall parts.

Euc. 1.10.
Ceul. 2.6.

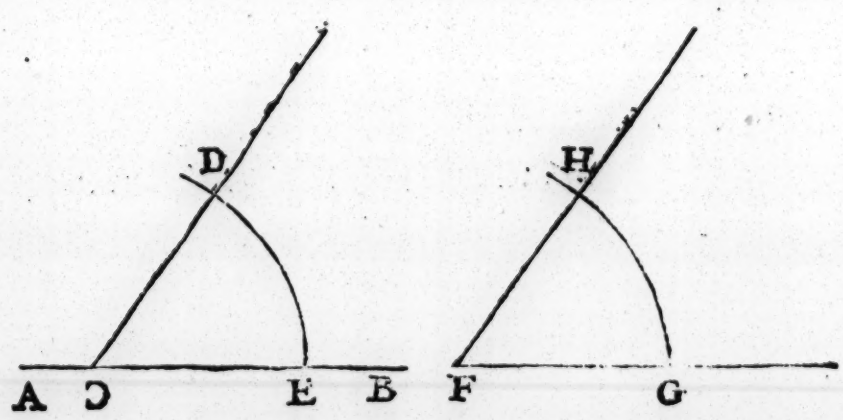
LEt A.B. be a right line giuen, which is to be diuided into two equall parts: Open the Compas to more then halfe of the giuen line, and placing one foot thereof in either of the ends A. or B. with the other ſtrike an arch line towards D. and another towards C. then place one foot in the other end of the giuen line, and with the ſame diſtance croſſe the two former arches in D. and C. by which interſections, draw the line D. C. which ſhall diuide the giuen line A. B. as was required.



PROBLEME VIII.

Vpon a right line giuen, on a point therein limited, to make an angle equall to an angle giuen.

LEt A.B. be a right line giuen, and C. a point therein limited, and let H. F. G. be an angle giuen: It is required on the right line A. B. and on the point therein C. to deſcribe an angle

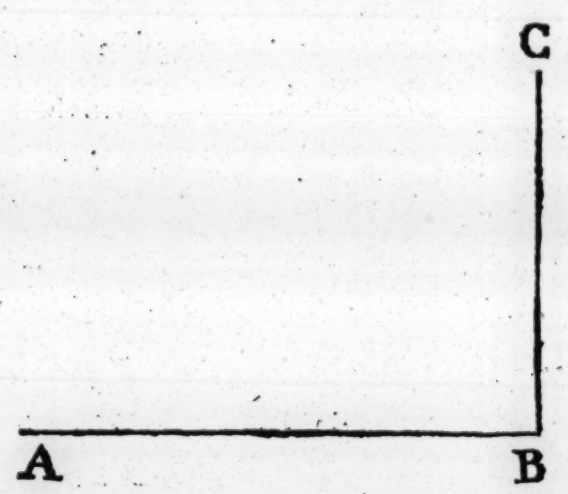


Euc. I. 23. angle equall to the angle giuen H.F.G. At any conuenient diſtance ſetting one foot in F. the giuen angle, ſtrike the arch line H. G. and at the ſame diſtance placing one foot in the limitted point C. make the arch line D. E. Then take the diſtance from G. to H. and place that diſtance on the laſt drawne arch line from E. which endeth in D. by which point draw the line D. C. which ſhall include the angle D. C. E. vpon the giuen line A. B. on the point therein limitted C. being equall to the giuen angle H.F.G. the thing required. DEF. 42.

PROBLEME IX.

To make a right angle vpon a line giuen, and on a point in the ſame line limitted.

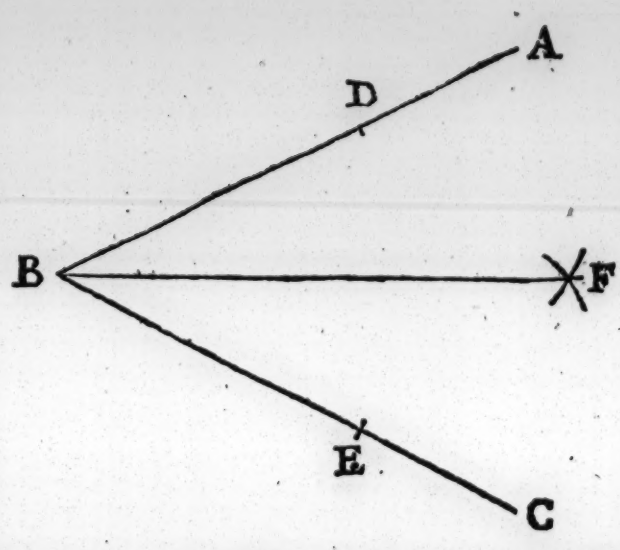
LEt A. B. be a line giuen, and let B. be a point therein limitted. It is required on the line A. B. and to the point in it limitted B. to deſcribe a right angle. By the ſixt PROB. on the point B. raiſe the perpendicular C. B. which with the giuen line ſhall make the right angle A. B. C. on the line A. B. and to the point therein limitted B. which was required. DEF. 5.



PROBLEME X.

To diuide an angle giuen into two equall parts.

Euc. I. 9. *Ceul.* I. 5. LEt A. B. C. be an angle giuen, to be diuided into two equall parts. Hauing opened the Compas to any conuenient diſtance, place one foot in B. and with the other croſſe the two lines B. A. and B. C. in the points D. and E. vpon which two points, ſtrike two arch lines, at any equall diſtance, croſſing one the other, as at F. From whence to the angle B. draw the right line B. F. which ſhall diuide the giuen angle into two equall parts, as was required. THEOR. 18.



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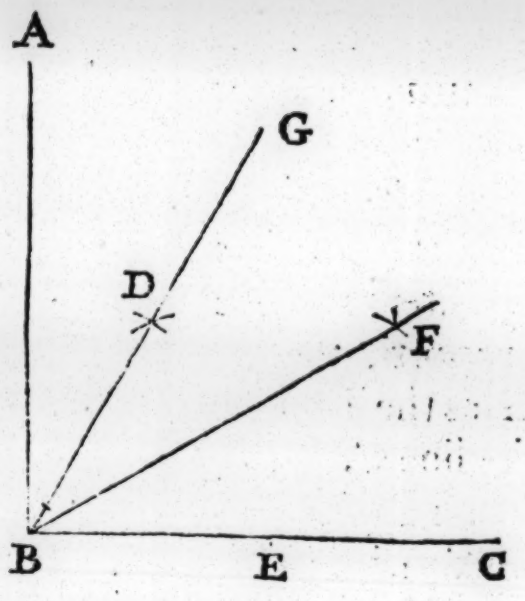
PROBLEME XI.

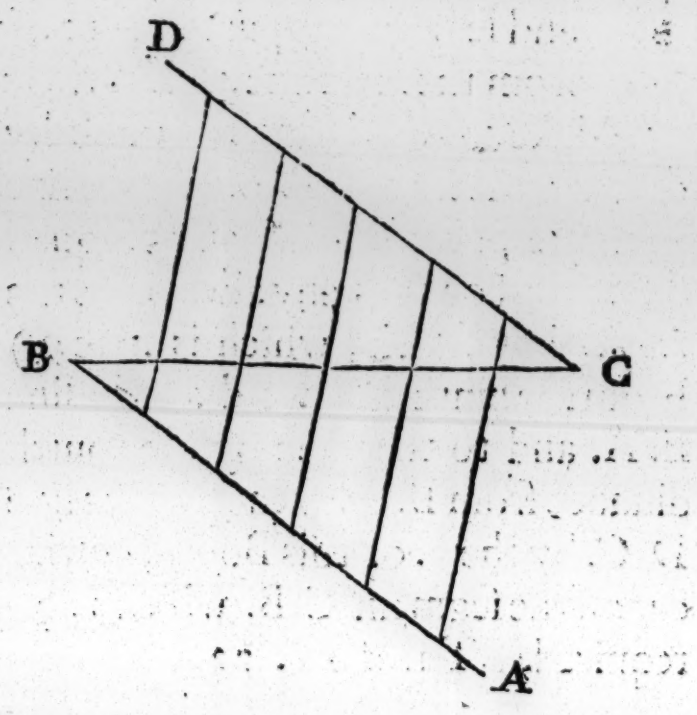
To diuide a right angle giuen into three equall parts.



LEt A. B. C. be a right angle giuen, to be diuided into three equall parts. At any conuenient diſtance, with one foot in B. croſſe the line B.C. as at E. and at the ſame diſtance on the points B. and E. ſtrike two arch lines, croſſing one the other in the point D. by which point, and the angle B. draw the line B. G. Then, by the tenth laſt before going, diuide the angle G.B.C. into two equall parts, with the line F. B. So ſhall thoſe two lines F. B. and G. B. diuide the right angle giuen into three equall parts; which was the thing required. DEFINITION 21. THEOREME 18.

PROBLEME XII.

To diuide a right line giuen into diuers equall parts, as many as ſhall be required.



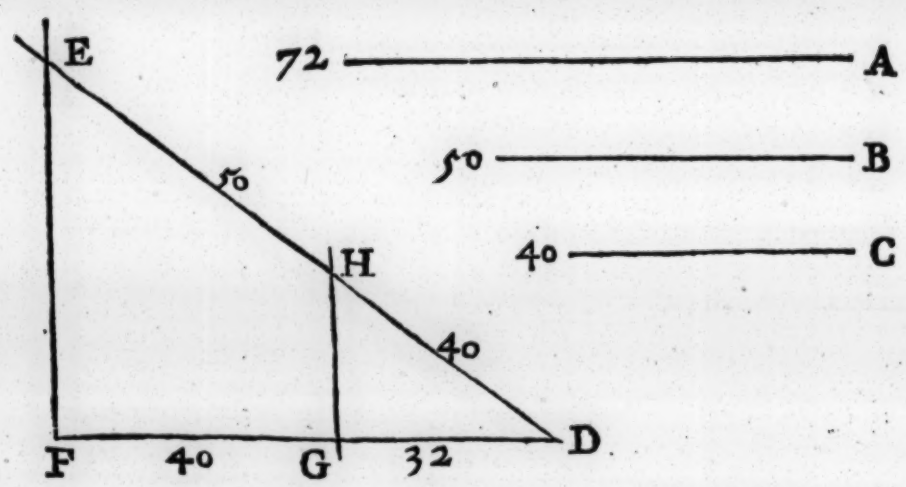
LEt B. C. be a right line giuen, and let it be required to diuide the ſame line into ſixe equall parts. Firſt from B. draw a line at all aduentures, making an angle of any quantitie with the giuen line, as the line B. A. making the angle A. B. C. Then by the 8. of this booke make the angle D. C. B. equall to the angle A. B. C. and from B. towards A. and likewiſe, from C. towards D. at any conuenient diſtance, make 5. equall ſpaces (that is, one alwayes leſſe then the number of parts required) and from point to point reſpectiuely draw lines, interſecting the giuen line: So ſhall you diuide the ſame into ſixe equall parts required. THEOR. 22.

PROBLEME XIII.

To diuide a right line giuen proportionally, according to any proportion required.

Euc.6. 10, 11, 12. Ceul.2. 67.

LEt A. bee a right line giuen, and let it be required to diuide the ſame into two ſuch parts, that the greater may bee in proportion to the leſſer, as the line B. is to the line C. First, make an angle of any quantitie, as E. D. F. whereof make the ſide F. D. equall to the giuen line A. then place on the other ſide the line C. from D. to H. and the line B. from H. to E. from E. to F. draw the line E. F. and laſtly, by the 3. of this Booke, draw a parallel line to E. F. by the point H. as H. G. cutting F. D. in G. So ſhall you diuide F. D. (being equall to the giuen line A.) in the point G. in ſuch ſort that the greater ſegment F. G. hath the ſame proportion to the leſſer G. D. as the line B. hath to the line C. which was required.



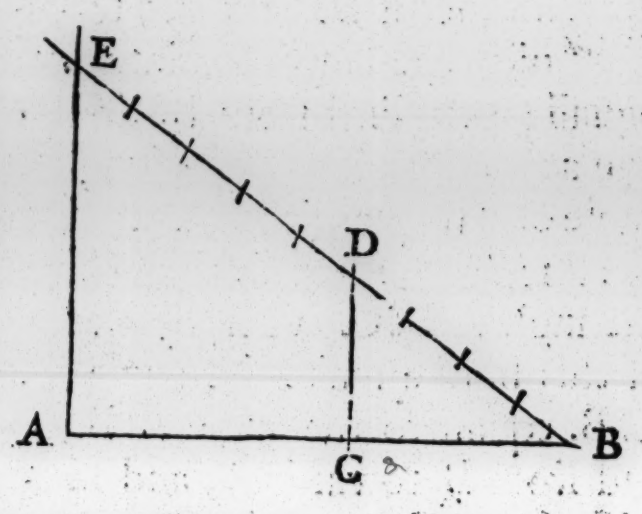
THEOR. 22.

PROBLEME XIIII.

From a right line, giuen to cut off any parts required.

Euc.6.9. Ceul.2.66.

LEt A. B. bee a right line giuen; and let it bee required to cut off from the ſame line $\frac{4}{9}$. parts there of. Make any angle on the end B. by drawing the line E. B. on which line from the point B. make any 9. equall diſtances (or ſo many alwayes as the *Nomen* importeth, which is here 9.) From the ninth diſtance, draw the line E. A. and to that line by the fourth diſtance from B. draw the parallel line D. C. by the 3. of this Booke, cutting the giuen line A. B. in C. So haue you cut off the ſegment C. B. which is $\frac{4}{9}$. parts of the giuen line A. B. the thing required. THEOR. 22.



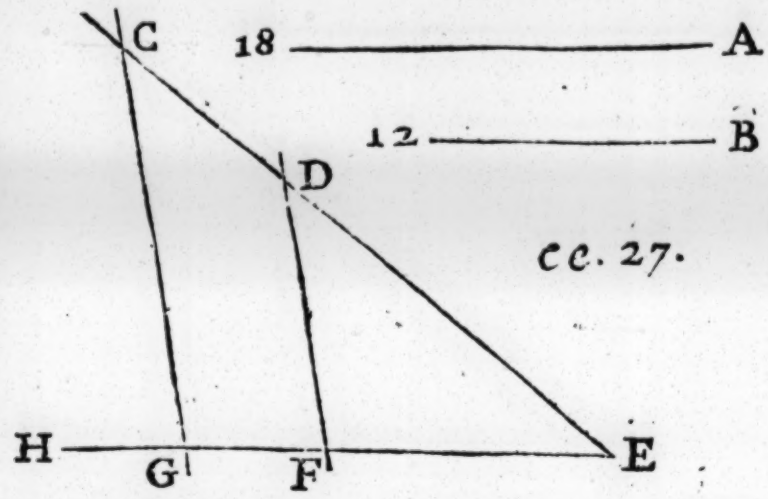
PROBLEME XV.

To finde a third line in continuall proportion vnto two lines giuen.

LEt A. and B. be two lines giuen; and let it be required to finde a third line, to be in ſuch proportion to A. as A. is to B. Make an angle of any quantitie, as H. E. C. then place the line A. from E. to D. and the

line

line B. from E. to F. and draw the line D.F. place alſo the line A. from E. to G. and laſtly, by the 3. of this Booke, by the point G. draw the line G. C. parallel to F. D. So ſhall E. C. bee a third proportionall line to the two giuen lines, the thing required. THEOR. 22.



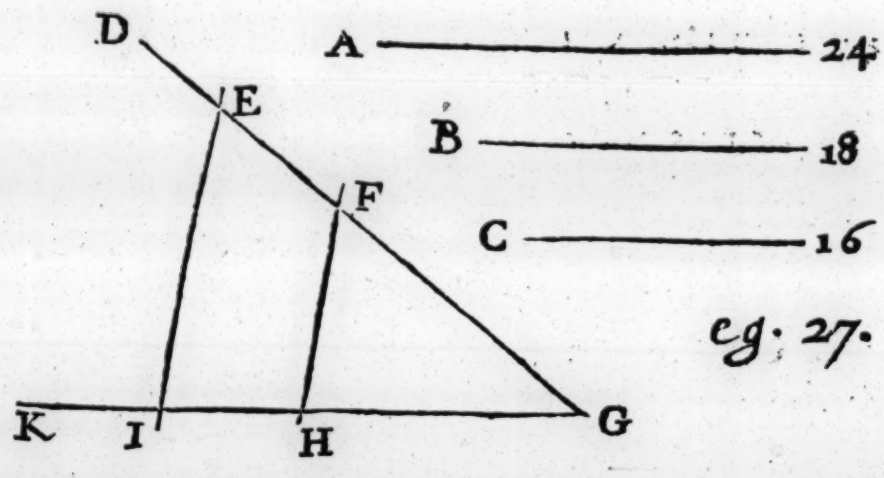
Euc.6.11. Ceul.2.68.

cc.27.

PROBLEME XVI.

To finde a fourth proportionall line to three lines giuen.

LEt A. B. and C. bee three lines giuen; and let it be required to find a fourth line, hauing ſuch proportion to A. as B. hath to C. Make an angle of any quantitie, as D.G. K. And ſeeing it is the greater extreme, which is ſought, place firſt the leſſer extreme C. from G. to H. and the leſſer meane B. from G. to F. then draw the line F. H. and place the greater meane A. from G. to I. by which point I. draw the line E. I. parallel to F. H. which cutteth D. G. in E. So haue you E. G. the fourth proportionall line required. THEOR. 22.



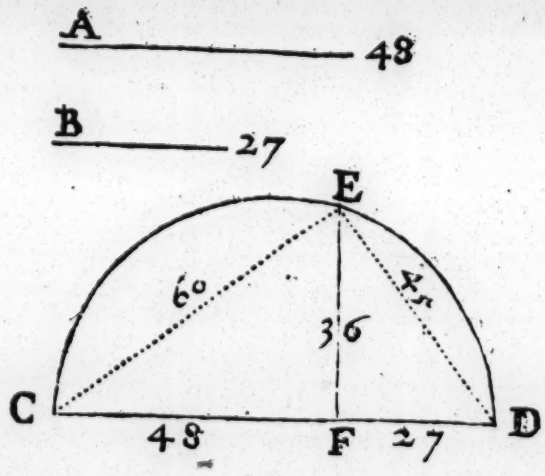
cg. 27.

Euc.6.12. Ceul.2.69.

PROBLEME XVII.

To finde a meane proportionall line betweene any two lines giuen.

LEt A. and B. be two lines giuen, betweene which it is required, to finde a meane proportionall line. Firſt, ioyne the two giuen lines together, ſo as they make both one right line, as C.F.D. meeting in the point F. then deſcribe thereon the ſemicircle C.E.D. and on the point F. by the 5. of this Booke erect a perpendicular to cut the circumference in E. as F.E. which ſhall be the meane proportionall required. DEF. 39. THEOR. 70.



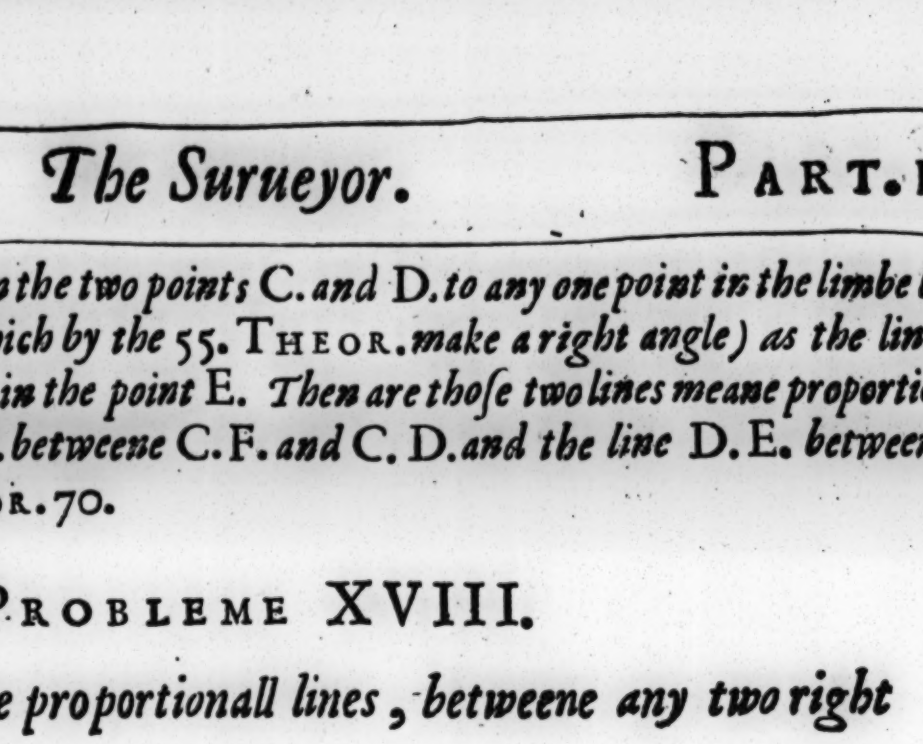
Euc.2.14.& 6.13. Ram.16.19. Ceul.2.64.

Note here, that if from the two points C. *and* D. *to any one point in the limbe be drawne two right lines (which by the* 55. THEOR. *make a right angle) as the lines* C. E. *and* D. E. *meeting in the point* E. *Then are those two lines meane proportionalls, that is, the line* C. E. *betweene* C. F. *and* C. D. *and the line* D. E. *betweene* D. F. *and* D. C. THEOR. 70.

PROBLEME XVIII.

To finde two meane proportionall lines, betweene any two right lines giuen.

ENC. 1. 43. LEt A. and B. be two right lines giuen, betweene which it is required to finde two meane proportionall lines. On the end of the line A. by the *6.* of this book place the line B. perpendicularly, making a right angle, as the angle E. C. D. then drawe the Diagonall line E. D. whereon deſcribe the ſemicircle E. G. D. making F. the center, then inlarge the lines C. E. and C. D. towards L. and M. and taking in your compaſſe the giuen line B. the leſſer extreme, place one foot in D. and with the other ſtrike through the limbe of the ſemicircle in G. and on the point G. lay your ruler, turning it vp and downe on that point till by drawing the line H. K. you may cut the two lines C. L. and C. M. equidiſtantly from the center F. So ſhall E. H. and D. K. bee two proportionall lines, betweene the giuen lines A. and B. as was required. THEOR. 5.

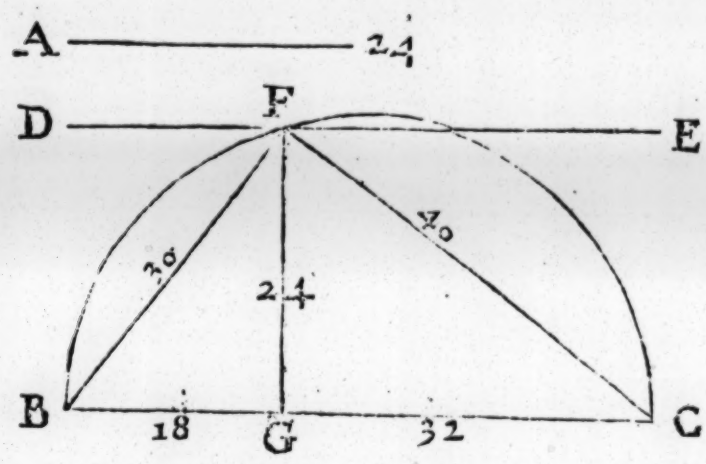


PROBLEME XIX.

To finde out in a line giuen, the two extremes of a meane proportionall giuen: So as the ſame meane bee not greater then halfe the giuen line.

LEt the meane proportionall giuen be A. and the line giuen B. C. It is required in the line B. C. to finde two extremes, betweene which the line A. ſhall bee a meane proportionall. Vpon the giuen line B. C. deſcribe the ſemicircle B. F. C. then at the diſtance of the giuen meane, by the

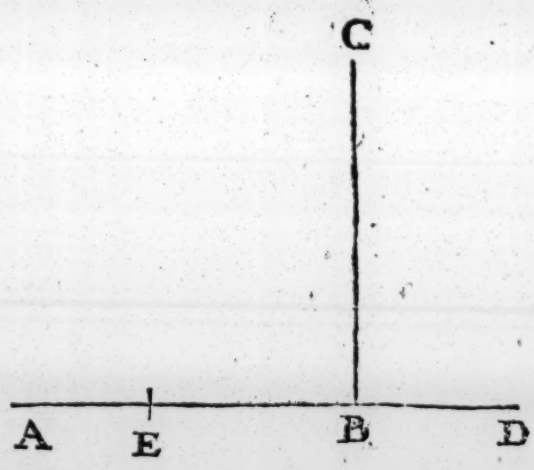
the *ſecond* of this booke draw a line parallel to B.C. (which of neceſſity muſt either touch or cut the ſemicircle) as the line D. E. cutting the ſemicircle in F. From which point F. by the 5. of this booke, let fall the perpendicular F. G. which ſhall ſo diuide the giuen line B. C. in the point G. that the line giuen A. ſhall be a meane proportionall betweene the two ſegments B. G. and G.C. the thing required. THEOR. 9. 55. 70. PROB. 83.



PROBLEME XX.

To diuide a line giuen by an extreme and meane proportion.

LEt A.B. bee a line giuen to be ſo diuided. By the 9. PROB. make of the giuen line A. B. a right angle, as A.B. C. ſetting C. B. equall to A. B. then increaſe the giuen line by halfe the length thereof to D. and opening the compaſſe to the diſtance C. D. with one foot in D. ſtrike through the giuen line at E. which ſhall diuide the ſame line by extreme and meane proportion, whereof the greater ſegment is E. B. and the leſſe A. E. As was required. DEF. 35. 1. THEOR. 44, 45, 46, 47.



AB. 12.

CB. 12.

BD. 6.

EB. √ 180.—6.

AE. 18— √ 180.

Euc. 2. 11. 6. 30. *Ram.* 14. 10. *Ceul.* 2. 36.

To performe the former Prob. *arithmetically.*

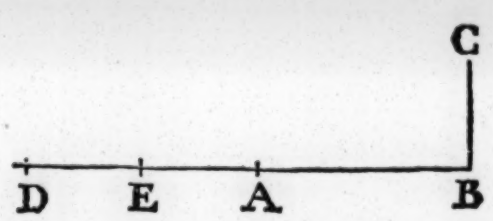
LEt the number giuen to be diuided by extreme and meane proportion be 12. Firſt, infold the ſquare thereof in 5. the factus is 720. diuide that by 4. the quotus is 180. from the ſquare roote whereof deduct halfe the giuen number, the remainder is √ 180—6. and this is the greater portion or ſection, which being deducted from the giuen number, there remayneth 18—√ 180. for the leſſer portion; both which taken together makes 12. the giuen number.

PROBLEME XXI.

To find the leſſer ſegment of a line, diuided by extreame and meane

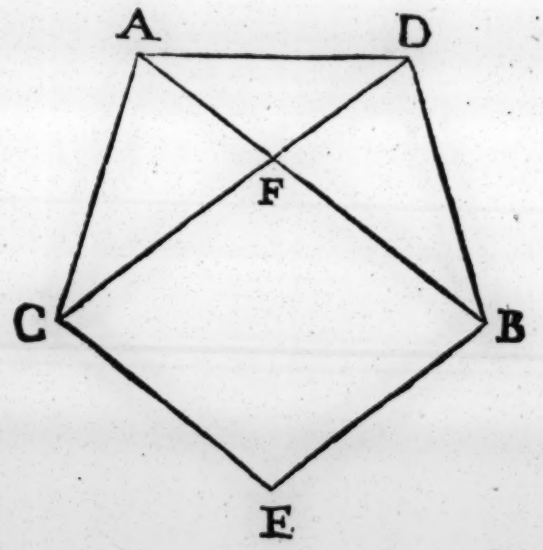
proportion, when onely the greater is giuen; and consequently, to find the whole line.

*Euc.*12. 2. LEt A.B. be the greater segment giuen. It is required to find the lesser segment, and consequently, the whole line. With the giuen line A.B. and halfe the same line, by the 9. PROB. make on the point B. the right angle A.B.C. Then continue the giuen line infinitely towards D. and at the distance A.C. with one foot in A. cut the continued line in D. from whence cut off D.E. equall to halfe the giuen line: So is E.A. the lesser segment, and consequently, E.B. the whole line; as was required.



Or otherwise, thus.

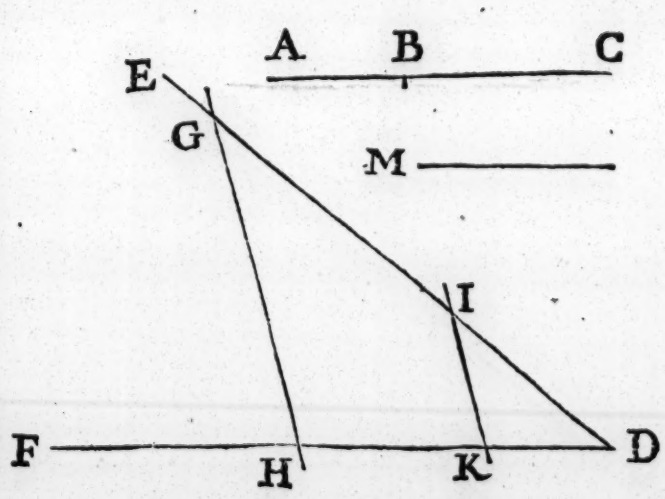
LEt A.D. be the greater segment giuen, it is required to finde out the lesser, and consequently, the whole line. On the giuen line A.D. by the 63. PROB. hereafter expressed, describe the *Pentagon* A.D.B.E.C. and against any two, the next immediate angles, draw subtending lines, which shall intersect the one the other, as the lines A.B. and D.C. in the point F. So shall F.A. or F.D. be the lesser segment; F.B. or F.C. the greater; and consequently, A.B. or D.C. the whole line, which was required. THEOR. 47.



PROBLEME XXII.

Hauing the greater or lesser segment of a line diuided by extreame and meane proportion giuen, to find the other segment, and so the whole line.

*Euc.*6. 2: LEt M. be the lesser segment of such a line giuen; It is required to find the other segment, and so the whole line. First, by the 20. PROB. diuide any line by extreame and meane proportion, as the line A.B.C. Then make an angle of any quantitie, as E.D.F. and place the lesser segment of the diuided line A.B. from D. to K.

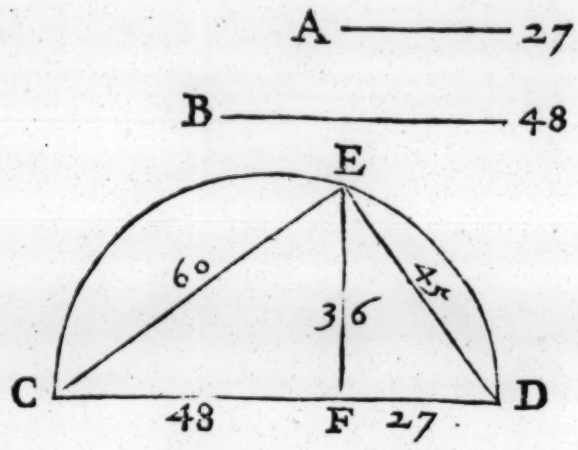


K. and the greater B.C. from D. to I. and draw the line I. K. Then place the leſſer ſegment giuen M. from K. to H. and ſo working forwards, as by the 16. PROB. is taught, you ſhall produce I. G. the greater ſegment, which added to K. H. makes the whole line required. And the like courſe is to be held if the greater ſegment were giuen, and the reſt ſought. THEOR. 22.

PROBLEME XXIII.

To diuide a right line giuen in power, according to any proportion giuen in two right lines.

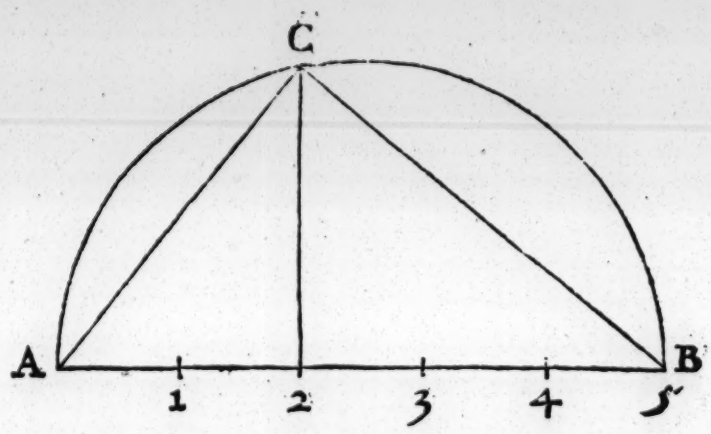
LEt the right line giuen be C. D. and the proportion giuen, that betweene the two right lines A. and B. It is required to diuide C. D. in power, according to the proportion of A. to B. Firſt, by the 13. PROB. diuide the giuen line C. D. in the point F. proportionally, as A. to B. Then on the ſame giuen line deſcribe the ſemicircle C. E. D. and from the point F. rayſe a perpendicular, to cut the lymbe in E. from which point draw the lines E.C. and E.D. which two lines together, ſhall be equall in power to the giuen line, and the power of the line E.D. ſhall be in ſuch proportion to the power of the line E.C. as A. to B. which was required. DEF. 36.37. THEOR. 6.9.55.70.



Eu. 1.47.
3.31.
6.4.
6.8.
6.20. *Cor.* 2.

To performe the ſame another way, according to any proportion required, betweene two numbers giuen.

LEt A. B. bee a right line giuen, and let it be required to diuide the power thereof in proportion, as 2. to 3. Firſt, adde the two giuen termes together, which make 5. wherefore diuide the giuen line A. B. into 5. equall parts, then deſcribe thereon the ſemicircle A. C. B. and on the ſecond part erect the perpendicular 2. C. to cut the limbe in C. from which point draw the line C. A. which ſhall be $\frac{2}{5}$. parts of the power of the giuen line; and the line C. B. which ſhall be $\frac{3}{5}$. parts of the power of the ſame giuen line A. B. which was the thing required. DEFIN. 36, 37. THEOR. 6, 9, 55, 70.



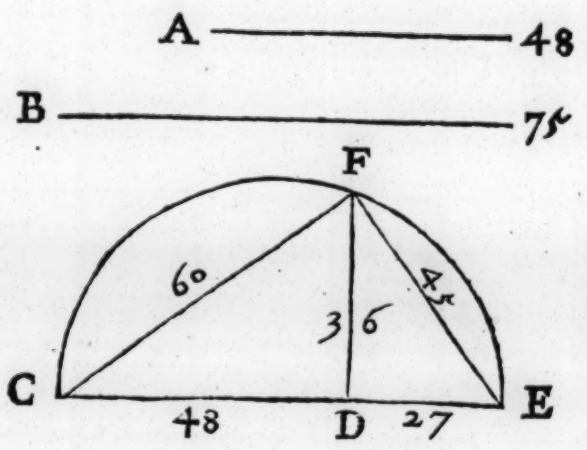
PRO-

PROBLEME XXIIII.

To inlarge a line in power, according to any proportion required.

*Eu.*1.47.
3.31.
6.4.
6.8.
6.20. *Cor.*2.

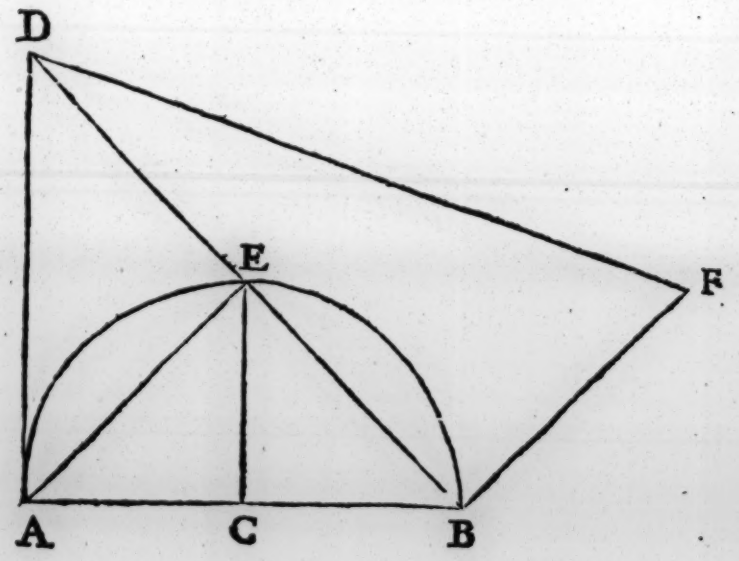
LEt C. D. bee a line giuen, to bee inlarged in power, according to the proportion of A. to B. Seeke firſt by the 16. PROB. a line, which ſhall beare the ſame proportion to the giuen line C. D. as B. to A. which will be found to be C. E. and thereon deſcribe the ſemicircle C. F. E. and, on the point D. erect a perpendicular, to cut the limbe in F. and laſtly, draw the line C. F. which ſhall bee in power to the giuen line C. D. as B. to A. as was required. DEFIN. 38. THEOR. 6, 9, 55, 70.



To performe the ſame another way.

Euc. 10.
*Def.*5.1.47.

LEt A. B. be a line giuen; and let it be required, to encreaſe the ſame line A.B. in power, as 2. to 5. By the ninth PROB. with the giuen line make the right angle D. A. B. placing D. A. equall to A. B. Then draw the line D.B. and by the 23. PROB. before going, take halfe the power of the giuen line, which is A. E. and place it perpendicularly on the line D. B. as B. F. And laſtly, draw the line D. F. which ſhall haue ſuch proportion in power to the giuen line A. B. as 5. hath to 2. as was required. DEFINITION 38. THEOREME 6, 9, 55, 70.

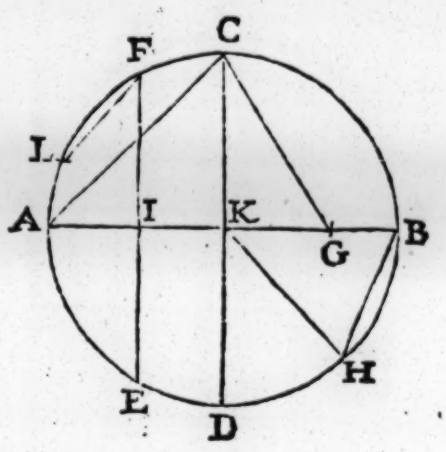


PROBLEME XXV.

To diuide the circumference, or find out all the cord lines of a circle, not exceeding the tenth.

LEt A.B. C.D. be a circle giuen; whereof it is required to find the cord lines. Firſt, the Diameter A. B. being drawne, diuideth the circle into two equall parts: The next is found by opening the Compas to the ſemi-

ſemidiameter, and with one foot in A. ſtrike through the lymbe at F. and E. and draw the line F. E. which will diuide the circumference into three equall parts. Then draw the line C. D. diuiding the Diameter A. B. into two equall parts at right angles, and draw the line C. A. which ſhall be the ſide of an inſcribed ſquare. Then ſetting one foot in I. and at the diſtance I. C. croſſe the Diameter in G. and draw the line C. G. which ſhall be the fift part, or the ſide of a *Pentagon*. The ſixt part, or the cord of a *Sextans*, is the ſemidiameter. The ſeuenth part, or the ſide of an inſcribed *Septagon*, is halfe the third, as F. I. or I. E. The eight being the ſide of an inſcribed *Octogon*, is the line B. H. which is the cord of halfe the Arch or Quadrant B. H. D. The ninth is a cord of the third part of the arch F. L. A. E. as F. L. And the tenth is the line K. G. which is the greater ſegment of the ſemidiameter, diuided by extreame and meane proportion. THEOR. 6, 36, 55. PROB. 118, 119, 121.

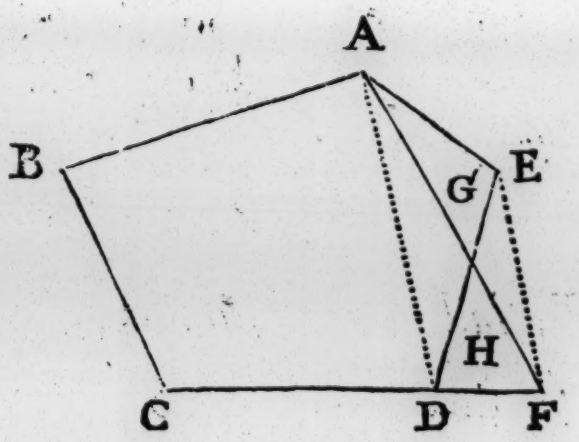


Euc. 1. 47.
2. 6.
4. 6.
4. 15.
13. 9, 10.

PROBLEME XXVI.

To draw a line from an angle in a Plot giuen, which ſhall take in as much as it cuts off.

LEt A. B. C. D. E. be a Plot giuen. It is required by a line drawne from an angle, in the ſame Plot, to take in as much as ſhall be cut off by the ſame line. Increaſe the line C. D. at pleaſure towards F. Then from the point A. draw the line A. D. making the Triangle A. E. D. and by the point E. draw the line E. F. parallell to A. D. cutting the increaſed line in F. from whence draw the line F. A. which ſhall take in the Triangle H. and cut off the Triangle G. each equall to other, as was required. THEOR. 26.



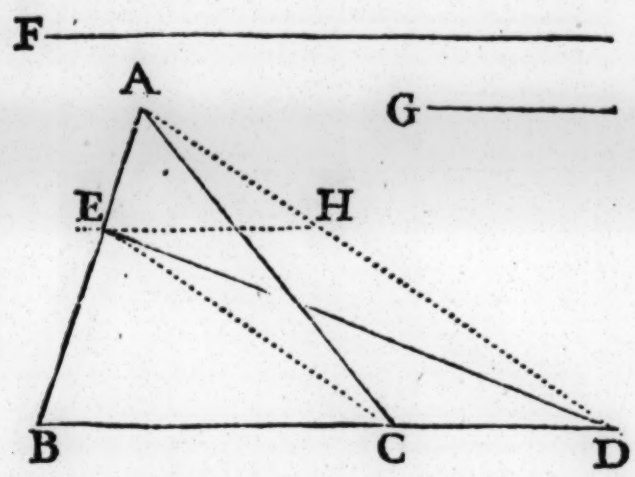
Euc. 1. 37.

PROBLEME XXVII.

To draw a line in ſuch ſort, as to retaine the ſuperficiall quantitie of a giuen Triangle, and yet alter the Baſe to any poſſible length required.

LEt A. B. C. be a Triangle giuen, and let it be required to draw a line in ſuch ſort, as to retaine the ſuperficiall quantitie of the ſame Triangle, and yet alter the Baſe to the length of the line F. Firſt, increaſe the Baſe *Euc.* 1. 37.

Baſe B.C. to D. making B. D. equall to the giuen line F. Then draw the line A. D. and by the 3. PROB. by the point C. draw a parallell line to A. D. as E.C. cutting the ſide A.B. in E. And laſtly, draw the line E.D. including the Triangle E. B. D. which retaineth the ſuperficiall quantitie of the giuen Triangle A. B. C. and yet the Baſe altered, to the length required. THEOR. 26. PROB. 47. 74.



PROBLEME XXVIII.

To draw a line in ſuch ſort, as to retaine the ſuperficiall quantitie of a giuen Triangle, and yet alter the altitude to any poſsible height required.

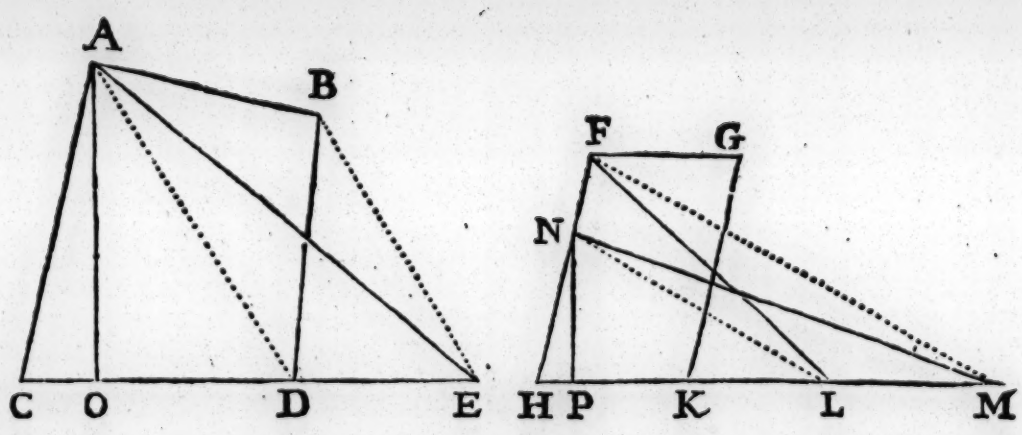
Euc. 1. 37. LEt A.B.C. in the former Diagram be a Triangle giuen; and let it be required to draw a line in ſuch ſort, as to retaine the ſuperficiall quantitie of the ſame Triangle, and yet alter the altitude to the height of the line G. At the diſtance of G. draw the line E. H. parallell to the Baſe B.C. which ſhall cut the ſide A. B. in E. from which point draw the line E.C. Then continue the Baſe at pleaſure towards D. and from the point A. draw a line parallell to the line E.C. as A.D. cutting the line of continuation in D. from which point draw the line E.D. which ſhall make the Triangle E.B.D. retaining the ſuperficiall quantitie of the giuen Triangle A.B.C. and yet the altitude altered to the height required. THEOR. 26.

PROBLEME XXIX.

To find two right lines in ſuch proportion as two figures giuen.

Euc. 1. 17.
6. 1.
1. 38.
1. 41.

LEt the *Trapezium*, A.B.C.D. & the Parallelogram F.G.K.H. be two figures giuẽ. It is required to finde 2. right lines in ſuch proportion the one to the other, as thoſe two giuen figures are. By the 26. PROB. reduce the *Trapezium* A.B.C.D. into the Triangle A. C. E. and let fall the perpendicular A.O. Then increaſe the Baſe of the Parallelogram at pleaſure

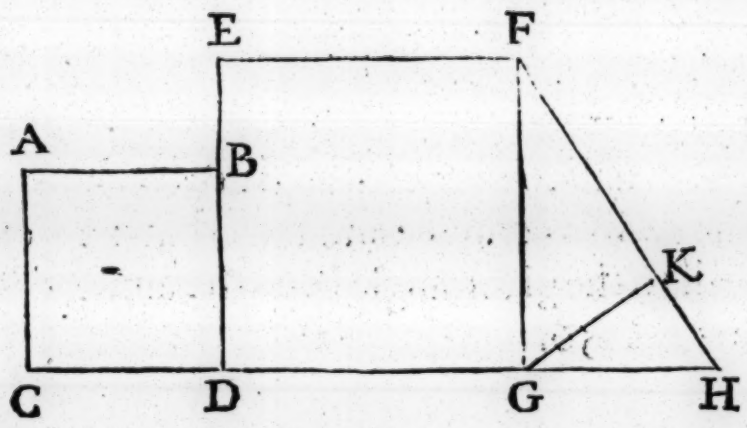


pleaſure towards M. and place on that increaſed line K.L. equall to the Baſe K.H. and draw the line F. L. including the Triangle F. L.H. equall to the Parallelogram F.G. K.H. Then by the 27. PROB. reduce the Triangle F. L.H. to the Triangle N.M.H. making the Baſe thereof M.H. equall to the Baſe C.E. of the firſt reduced Triangle; and laſtly, let fall the perpendicular N.P. So haue you the two perpendiculars N. P. and A. O. in ſuch proportion the one to the other, as the Parallelogram to the *Trapezium* giuen. THEOR. 35,26,28.

PROBLEME XXX.

To find two right lines in ſuch proportion the one to the other, as two giuen Squares.

LEt A.B.D.C. and E.F. G. D. be two Squares giuen. It is required, to find two right lines in ſuch proportion the one to the other, as the two giuē ſquares. Continue the ſide D. G. of the greater ſquare giuen to H. making G. H. equall to the ſide of the leſſer ſquare giuen: Then draw the line F. H. ſubtending the right angle F.G.H. from which right angle let fall a perpendicular, cutting the line F.H. in K. So ſhall H.K. and K.F. be two lines in ſuch proportion, as the two giuen ſquares, as was required. THEOR. 6,9,11,55,70.



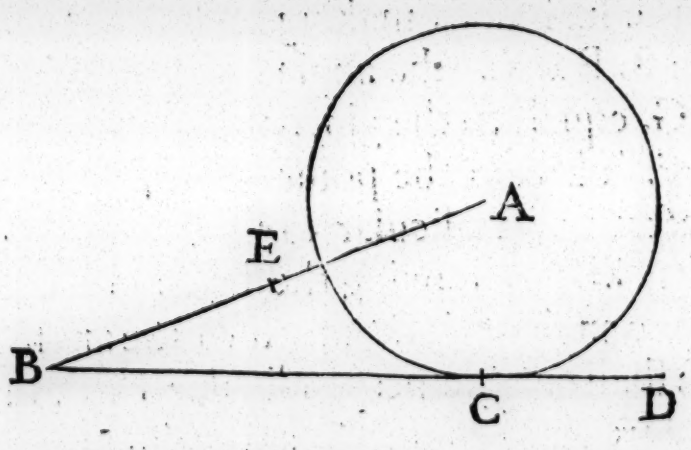
Euc.6.4.8. Ac Cor.19. Euc.12.2. This *Prob.* is generall for all figures reduced into Squares. Which may alſo be thus wrought. Seeke a third proportionall to the ſides C. D. and D.G. which ſhall haue ſuch proportion to either of them, being the other extreame, as the ſquares haue the one to the other. *Euc.* 12.2.

PROBLEME XXXI.

To draw a touch line to a circle giuen, from a point aſsigned.

LEt A. be the circle giuen, and B. the point aſſigned. It is required from the point B. to draw a touch line to the circle A. Draw the line A.B. from the centre to the point aſſigned, and diuide the ſame into two equall parts in the point E. and at the diſtance E.A. or E. B. croſſe the lymbe in C. by which point and the point aſſigned B. draw the line B.D. which ſhall be a touch line to the circle A. as was required. THEOR. 52.

Euc.3.17.



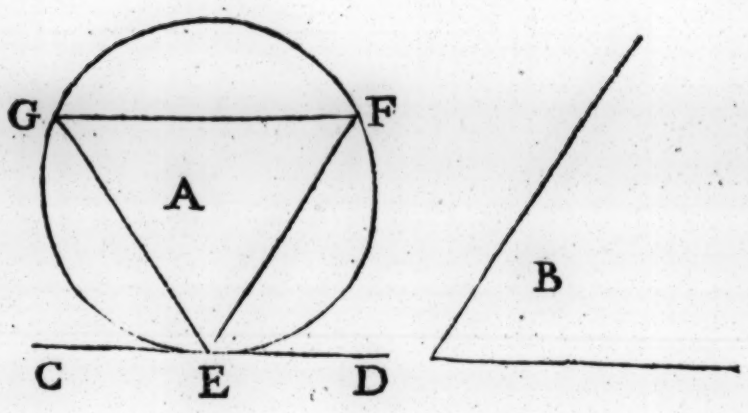
PROB.

PROBLEME XXXII.

To apply a line vnto a circle giuen, in ſuch ſort, as thereby to cut off a ſegment, wherein may be placed an angle, equall to an angle giuen.

Euc. 3. 34. *Ceul.* 2. 48. 49.

LEt A. be a circle giuen. It is required, to apply thereunto a line in ſuch ſort, as to cut off a ſegment from the ſame circle, wherein may be placed an angle equall to the angle B. By the PROB. laſt before going, draw the touch line C. E. D. and on the point E. by the eight PROB. make the angle F. E. D. equall to the giuen angle B. by drawing the line F.E. So ſhall the ſegment F.G.E. containe an angle, drawne from any point in the arch thereof, equall to the giuen angle B. as the angle E. G. F. the thing required. THEOREME 56.

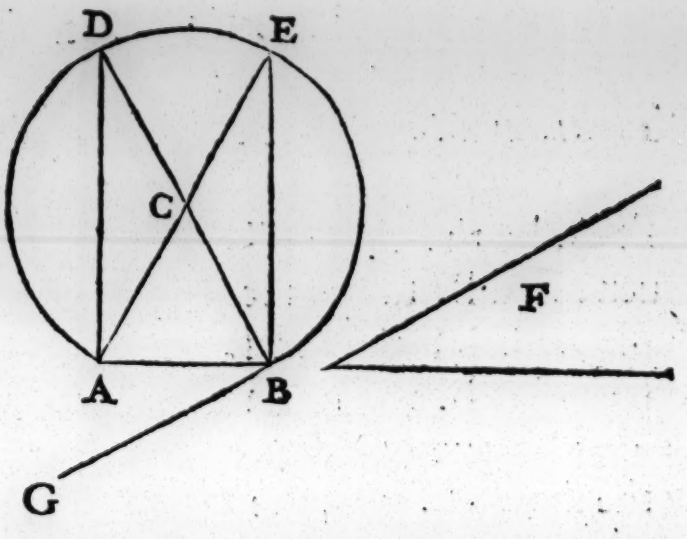


PROBLEME XXXIII.

To deſcribe, vpon a line giuen, ſuch a ſegment of a circle, as ſhall containe an angle, equall to an angle giuen.

Euc. 3. 33.

LEt the line giuen be A. B. and it is required to deſcribe thereon ſuch a ſegment of a circle, as ſhall containe an angle equall to the giuen angle F. To the end B. of the giuen line A.B. draw the line G. B. making by the 8. PROB. the angle A. B. G. equall to the giuen angle F. Then on the point B. erect a perpendicular to the line G.B. by drawing the line D.B. and on the end A. of the giuen line make the angle E.A.B. equall to the angle D. B. A. and draw the line A.E. which ſhall cut the perpendicular D. B. in C. which ſhall be the centre: Therefore on the point C. at the diſtance C.A. or C.B. deſcribe the arch A. D. E. B. which with the giuen line ſhall make the ſegment of a circle, containing an angle equall to the giuen angle F. For if two lines be drawne from any

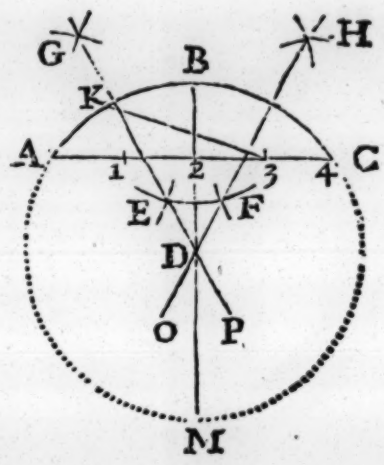


any one part of the circumference to the ends of the giuen line, they ſhal containe ſuch an angle, as thoſe at the points D. and E. which was required. THEOR. 52. 56.

PROBLEME XXXIIII.

A ſegment of a circle being giuen; to find out the centre, and conſequently the diameter and the whole circle.

LEt A. B. C. be the ſegment of a circle giuen. It is required to find out the center thereof. Take a point at pleaſure with moſt conueniencie in the arche A. B. C. as B. and on the point B. at any meete diſtance deſcribe the arch E. F. alſo one towards G. and another towards H. then remoue the compaſſe to A. and with the ſame diſtance croſſe the next two arches in G. and E. and with the ſame diſtance on the point C. croſſe the two next arches in H. and F. And laſtly, by thoſe interſections, draw the lines G. P. and H. O. interſecting in D. which ſhall be the centre required; and conſequently B. M. the Diameter, and A. B. C. M. the whole circle. DEF. 10. 11. 13.



Eu.3.25.

To find out Arithmetically the diameter of the whole circle, (the ſegment A. B. C. being giuen) and conſequently the other parts; worke thus: Suppoſe the corde line A. C. to be 12. and the perpendicular B. 2. 4. Square halfe the corde line, which makes 36. which deuided by the perpendicular 4. quoteth, 9. whereunto adde the ſame perpendicular, which makes 13. the length of the whole diameter, whereby the reſt of the parts are eaſily knowne.

To finde the extention of the arche line A. K. B. C. and to deliuer the ſame in a right line, worke thus; Deuide the corde A. C. into foure equall parts, and place one of thoſe parts on the arche from A. to K. and from K. draw a line to the third part in the corde line, as K. 3. which taken double, ſhall be equall to the arche line, A. K. B. C. THEOR. 59.

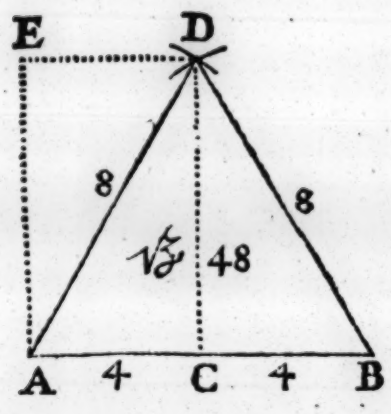
The ſecond Part.

OF the making and deſcription of all ſorts of ſuperficiall figures, with their ſeuerall and particular menſurations.

PROBLEME XXXV.

To make an equilater triangle, the ſide thereof being giuen.

EUC. 1. 1. LEt A B. be a right line giuen, and it is required to make an equilater triangle, whoſe ſide ſhall be equall to the ſame line. At the diſtance A. B. ſetting one foot in A. ſtrike an arche line towards D. and at the ſame diſtance with one foot in B. croſſe the ſame arche line in D. and from the interſection draw the lines D. A. and D. B. which with the giuen line A. B. ſhall make the equilater triangle A.B.D. as was required. DEF. 21. PROB. 36.



PROBLEME XXXVI.

To find the perpendicular of an equilater triangle Arithmetically, the ſide being giuen.

By this Prob. the perpendicular of an Iſoſceles is alſo found, the ſide and baſe being giuen.

IN the former Diagram let the ſide be ginen 8. It is required to find the perpendicular. Square the ſide 8. makes 64. then ſquare the halfe baſe, 4. makes 16. which deduct from 64. reſts 48. whoſe ſquare roote √48. neere rationall 6.$\frac{9}{10}$ is the length of the perpendicular D C. required. THEOR. 6. 19. 29.

PROBLEME XXXVII.

The perpendicular and ſide of an equilater triangle being giuen, to finde the Area or ſuperficiall content.

IN the former Diagram the perpendicular √48. 48. and the ſide 8. is giuen, and the Area is required, Multiplie the whole of eyther by halfe, the other as √48. 48. by 4. the product is √768. neere rationall, 27.$\frac{39}{54}$. the ſuperficiall content required. THEOR. 24.

Or

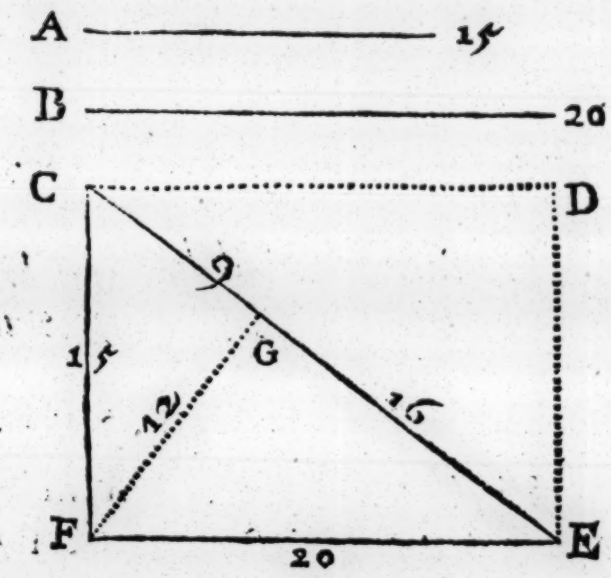
Or thus, without the perpendicular.

MVltiply 4. the halfe of one of the ſides ſquarely, it makes 16. and the ſame product by the former halfe, makes 64. and that by halfe the perimeter which is 12. the product is 768. from whence extract the ſquare roote, which is neere rationall 27. $\frac{12}{14}$. the ſuperficiall content as before. *Ram.* 8. 8.

PROBLEME XXXVIII.

To make a right angled triangle, the two containing ſides being giuen.

LEt A. and B. bee two right lines giuen, for the containing ſides of a right angled triangle required to bee made. By the 9. PROB. of the two giuen ſides A. and B. make a right angle, as the angle C. F. E. then draw the ſubtending line C. E. So haue you included the right angled triangle C. F. E. with containing ſides equall to the giuen lines A. and B. which was required. THEOR. 9. 6. 11. 55. 70.



Eucl. 3. 31.

PROBLEME XXXIX.

The perpendicular and baſe of a right angled triangle giuen to find the ſuperficiall content.

IN the former Diagram the perpendicular F. G. 12. and the baſe C. E. 25. is giuen, and the content required. Multiplle the whole of eyther by halfe the other, as 12. by 12. $\frac{1}{2}$. or 25. by. 6. the product is 150. the ſuperficiall content required. Or multiplie the whole by the whole, the Product is 300. whereof take halfe being 150. as before. THEOREME. 23. 25.

Or thus, without the perpendicular.

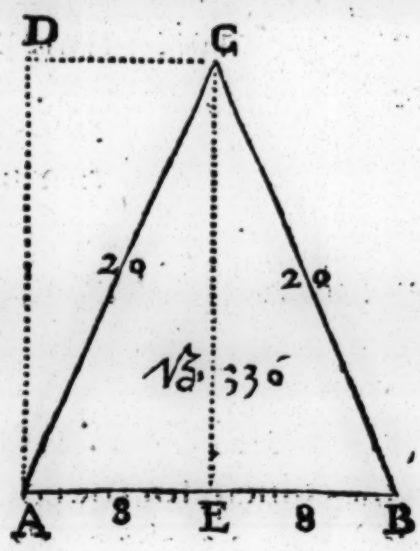
MVltiply the containing ſides 15. by 20. the product is 300. whereof take halfe for your demaund. Or multiply the halfe of the one in the whole of the other, the Product is 150. as before. THEOREME. 23.

PROBLEME XL.

To make an Isosceles triangle on a right line giuen.

LEt the right line giuē be A.B. wheron it is required to describe an Isosceles triangle. Open the compasse at pleasure, and placing one foote in A. with the other strike an arch towards C. and at the same distance placing one foote in B. crosse, the former arch in C. and draw the lines C. A. and C. B. which shall include the Triangle required. THEOR. 10. 18. 19.

Euc. 1. *Def.* 25.



PROBLEME XLI.

The perpendicular and base of an Isosceles Triangle, giuen to find the area, or superficiall content Arithmetically.

IN the former Diagram, let C. E. the perpendicular √8 .336. neere rationall 18. $\frac{1}{3}$. bee giuen (or found by PROB. 36.) and let the base giuen be 16. It is required Arithmetically to finde the content. Multiplie the whole of eyther by halfe, the other as √8 .336. by 8. the production is √8. 21504. neere rationall, 146. $\frac{47}{73}$. the superficiall content required. Or multiplie the whole by the whole, as √8 336. by 16. the Product. is √ 86016. whereof take the halfe which is √ 21504. and neere rationall 146. $\frac{47}{73}$. as before. THEOR. 8. 19. 25.

Or otherwise without the perpendicular, thus.

IN the same former Diagram. Add all three sides together, which makes 56. whereof take halfe which is 28. then take the difference of each side from that halfe, as 8. 8. and 12. And infold those 3. each in other makes, 768. which multiplie by 28. the former halfe, the Product is, 21504. whereof take the square roote, which is neere rationall, 146. $\frac{47}{73}$. for the superficiall content as before, which rule is generall for all right lined Triangles whatsoeuer.

PROBLEME XLII.

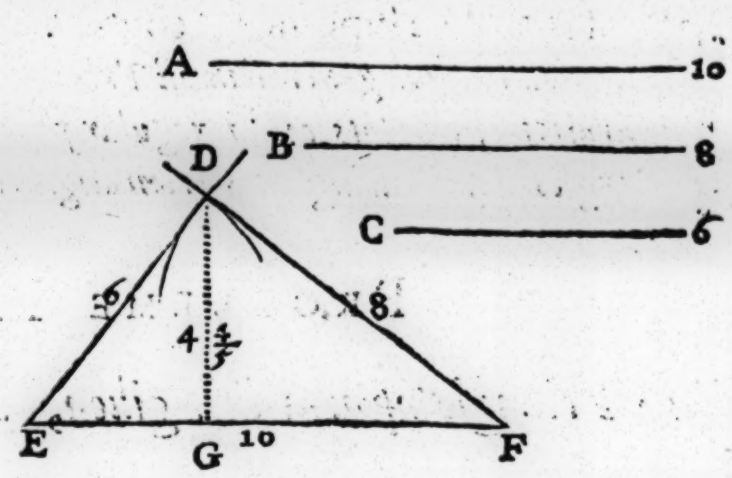
To make a Triangle of three vnequall sides, the lines being giuen, so as the two shortest together bee longer then the third line.

LEt A. B. and C. bee three lines giuen, whereof a Triangle is required to bee made. Place the line A. from E. to F. then taking in the compasse, the line B. with one foot in F. make an arch towardes D. and

Euc. 1. 22.

at

at the diſtance of the third line, and with one foote in E. croſſe the arch line in D. from which interſection to E. and F. draw the lines D. E. and D. F. So ſhall you include the Triangle D. E. F. whoſe ſides are equall to the giuen lines A. B. and C. as was required. THEOR. 15.



PROBLEME XLIII.

To finde the perpendicular of any Triangle Arithmetically, the ſides being giuen.

IN the former Diagram let the ſides giuen be E. D. 6. D. F. 8. and F. E. 10. It is required to finde the perpendicular. Square the three ſides ſeuerally, which make 36. 64. and 100. then adde the ſquare of the baſe, E. F. 100. to the ſquare of one of the ſides, as to 36. the ſquare of the ſide F. D. which makes 136. from whence ſubtract the ſquare of the other ſide D. F. 64. reſts 72. whereof take the halfe 36. which deuide by the baſe 10. *Enc.* 1. 47. producing 3. 3/5. for the leſſer ſegment of the baſe E. G. The ſquare of which ſegment 12 24/25. being deducted from the ſquare of E. D. 36. firſt added, the remainder is 23. 1/25. whoſe radix is 4. 4/5. the length of the perpendicular D. G. required. THEOR. 70.

PROBLEME XLIIII.

The perpendicular and baſe of any Triangle being giuen, to finde the area or ſuperficiall content thereof Arithmetically.

IN the ſame former Diagram let the perpendicular D. G. 4. 4/5. & the baſe E. F. 10. be giuen, and let it be required to find the Area or ſuperficiall content of the Triangle D. E. F. Multiply the whole of eyther by halfe the other, as the whole perpendicular 4. 4/5. by the halfe baſe 5. the Product ſhall bee 24. the ſuperficiall content required. Or multiplie the whole by the whole, as 4. 4/5. by 10. the Product is 48. whereof take halfe, which is 24. as before the ſuperficiall content. THEOR. 25. 7.

Or thus without the perpendicular.

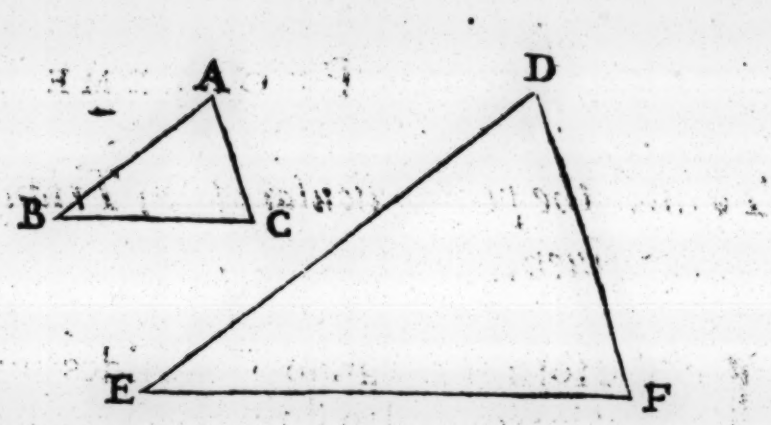
ADde the three ſides together 6. 8. and 10. making 24. whereof take 12. the halfe, and then the difference of each ſide from that halfe, as 6. 4. and 2. and infold thoſe differences each into other, which

bringeth 48. that multiplied by the former halfe 12. produceth 576. The radix whereof is 24. the superficiall content as before.

Regula generalis. *This Propoſition holdeth generall in all Triangles, and is the fitteſt and moſt meete for the menſuration of all plots and irregular figures whatſoeuer, being firſt reduced into Triangles, by drawing lines from angle to angle after the vſuall manner.*

PROBLEME XLV.

To make a Triangle vpon a line giuen, like vnto another Triangle giuen.

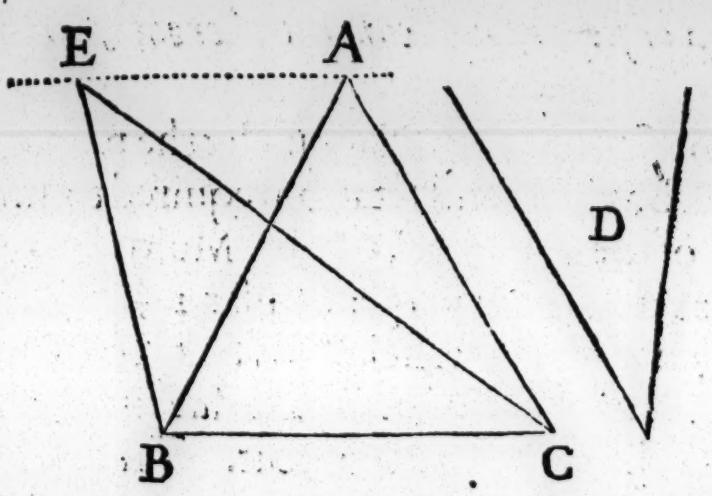


LEt the Triangle A. B C. and the line E. F. be giuen. It is required on the line E. F. to make a Triangle like vnto the Triangle giuen. Vpon the point E. by the 8. PROB. deſcribe an Angle equall to the Angle A. B. C. and on the point F. deſcribe another Angle equall to the Angle A. C. B. in the giuen Triangle, whereby ſhall bee included the Triangle D. E. F. vpon the line giuen E. F. like vnto the Triangle giuen A. B. C. that is with equall angles and lines proportionall, as was required. DEF. 40. THEOR. 11. 49.

Euc. 1. 37. *Ceul.* 2. 78.

PROBLEME XLVI.

To make a Triangle equall to another Triangle giuen, vpon the ſame Baſe, hauing an Angle equall to an Angle giuen.



LEt A. B. C. be a Triangle giuen, whoſe baſe is B. C. and let the Angle giuen be D. It is required vpon the baſe B. C. to deſcribe a Triangle equall to the Triangle giuen A. B. C. hauing an Angle equall to the giuen Angle D. By the point A. by the 3. PROB. draw a parallel line to the baſe B. C. as E. A. and on the point C. deſcribe an Angle equall to the Angle D. whereof the baſe to be one of the containing ſides, and draw the line E. C. till it interſect, the parallel line in E. And laſtly, from the point E. draw the line E. B. which ſhall include the Triangle E. B. C. equall to the giuen Triangle A. B. C. vpon the ſame

Euc. 1. 28. 1. 37.

same base, and hauing the Angle thereof E.C.B. equall to the giuen Angle D: as was required. THEOR. 26.

PROBLEME XLVII.

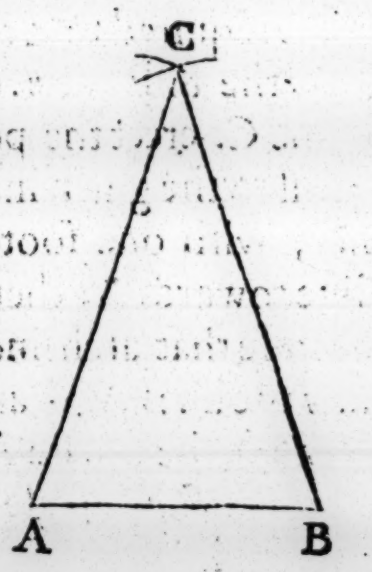
To make a Triangle: equall to another Triangle giuen, with a base or perpendicular limited.

WOrke this Probleme in all respects according to the doctrine taught in PROB. 27. and 28. THEOR. 26. *Euc.* 1.37,

PROBLEME XLVIII.

To make an Isosceles Triangle vpon a line giuen, whose Angles at the base, shall be eyther of them double to the third Angle.

LEt A. B. be a line giuen, whereupon it is required to describe a Triangle, whose Angles at the base shal be eyther of them double, to the third Angle. Suppose the giuen line A. B. to be the greater segment, of a line deuided by extreame and meane proportion, and by the 21. PROB. finde out the whole line, at the distance of which line so found, with one foot in A. strike an arch line towards C. and at the same distance with one foot in B. crosse that arch line in C. from which intersection, draw the lines C.A. & C.B. which shal include the Triangle required. THEOR. 47. 69.

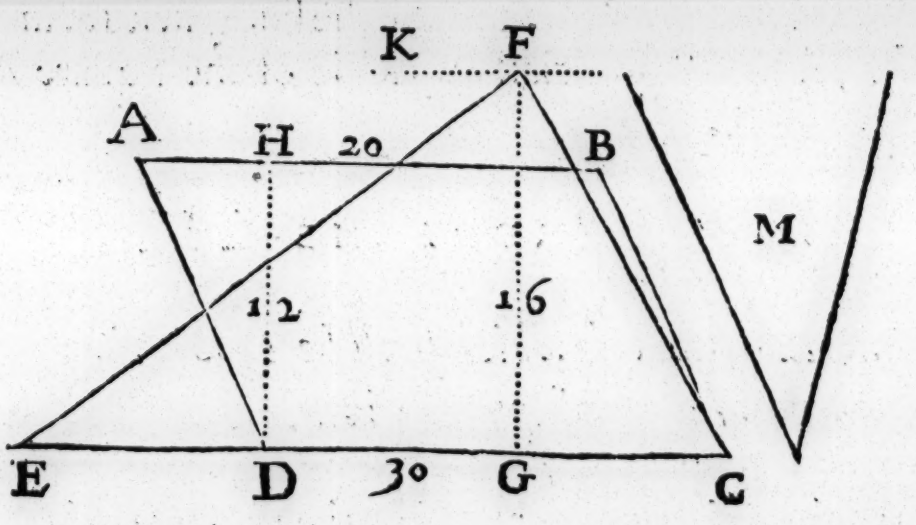


Euc. 4.10.

PROBLEME XLIX.

To make a Triangle equall to a Parallelogram giuen, vpon a line limited, and with an Angle equall to an Angle giuen.

LEt the Paralelogram giuen bee A. B. C. D. the line giuen E.C. & let the Angle giuen be M. It is required vpon the line giuen to make a Triangle equall to the parallelogram giuē, hauing an Angle equall to the giuen Angle M. Take the giuen line E.C. for the



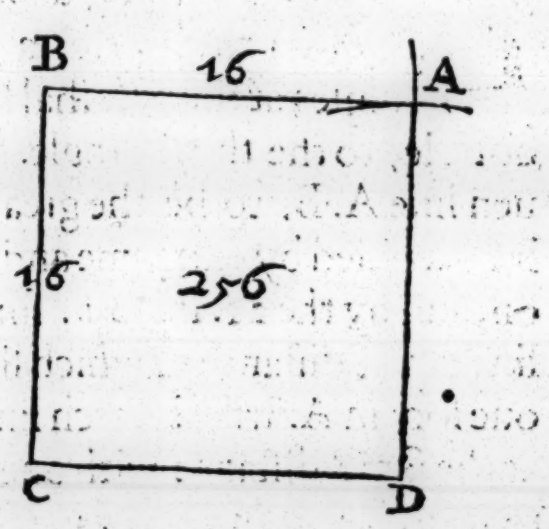
Euc. 1.23.
6. 12.
6. 16.

the baſe, and by the 16. PROB. reaſon thus reciprocally. If halfe the giuen line E.C. yeeld H. D. the bredth of the parallelogram: what giues A.B. the length thereof: The anſwere ſhall bee F. G. the perpendicular of the Triangle to be made; at which diſtance by the 2. PROB. draw the line K. F. parallel to the baſe E. C. and on the end E. by the 8. PROB. deſcribe an Angle equall to the giuen Angle M. and draw the line E. F. which ſhall cut the parallel line in F. from which point of interſection draw the line F. C. which ſhall include the Triangle E. F. C. equall to the parallelogram giuen, vpon the line E. C. giuen and hauing the Angle F. E. C. thereof equall to the giuen Angle M. as was required. THEOR. 42.

PROBLEME L.

To make a Square vpon a line giuen for the ſide thereof.

LEt C. D. be a line giuen, whereon it is required to deſcribe a Square. By the 6. PROB. on the end of the giuen line C. erect the perpendicular B. C. equall to the giuen line C.D. at which diſtance, with one foote in B. ſtrike an *Eu.1.46.* arch line towards A. then with one foote in D. croſſe that arch line in A. And laſtly, draw the lines A.B. and A.D. which ſhall include the ſquare A. B. C. D. vpon the line giuen for the ſide thereof C.D. as was required. THEOR. 2.



PROBLEME LI.

The ſide of a Square being giuen, to finde the Area or ſuperficiall content Arithmetically.

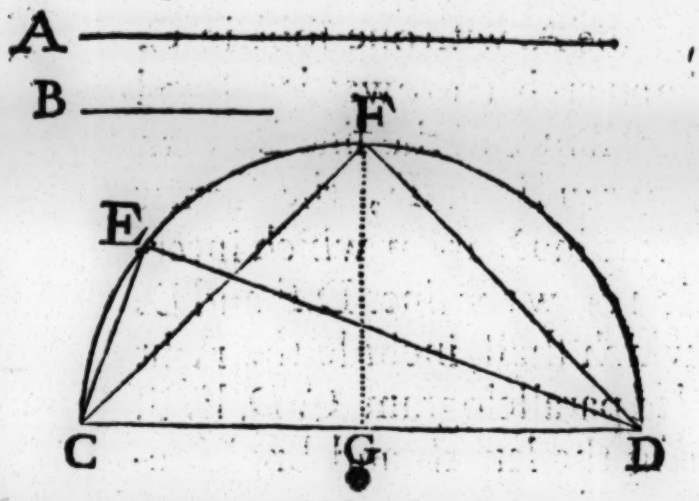
LEt A.B. 16. be the ſide of a ſquare giuen, whereof the ſuperficial content is required. Multiplie the line giuen 16. in it ſelfe, the product is 256. the ſuperficiall capacitie of the Square A. B. C. D required. THEOR. 38.

PROBLEME LII.

To make two Squares which ſhall be equall the one to the other; and alſo to two vnequall Squares giuen.

LEt the right lines A. and B. be the two ſides of two vnequall ſquares giuen; and let it be required to make two other ſquares, which ſhall bee equall vnto them, and alſo the one of them equall to the other. By the

the 9. PROB. of the two giuen lines A. and B. make a right Angle, as the Angle C. E. D. and draw the ſubtending line C. D. on which line deſcribe the ſemicircle C. E. F. D. and on the middle of C. D. erect the perpendicular F. G. to cut the lymbe in F. From which point draw the two lines F. C. and F. D. which ſhall be the two ſides of two ſquares, equall to the two giuen ſquares, and alſo the one of them equall to the other; as was required. THEOR. 6. 23, 55.

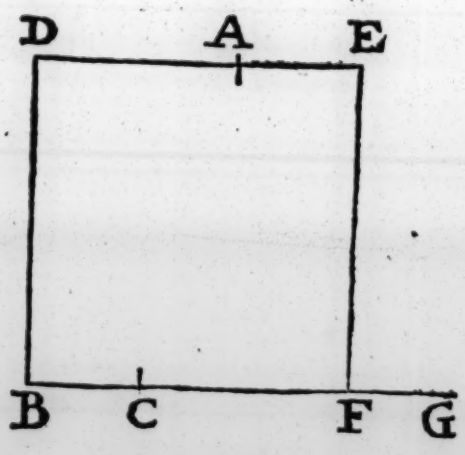


Euc. 1. 5. 1. 32.

PROBLEME LIII.

To deſcribe a Square in ſuch ſort as it ſhall paſſe by any three points giuen.

LEt A. B. and C. be three points giuen, by which it is required to make a ſquare to paſſe. Firſt, by the two neereſt points, as B. and C. draw a line at length as the line B. G. then by the third point A. by the 3. PROB. draw the line D. E. parallel to the line B. G. and on the point B. raiſe the perpendicular B. D. to cut the line D. E. in D. then at the diſtance D B. marke the points E. and F. betweene which draw the line E. F. which ſhall include the ſquare D. E. F. B. paſſing by the three giuen points, as was required. PROB. 3. 6.



Euc. 1. 12. 1. 31.

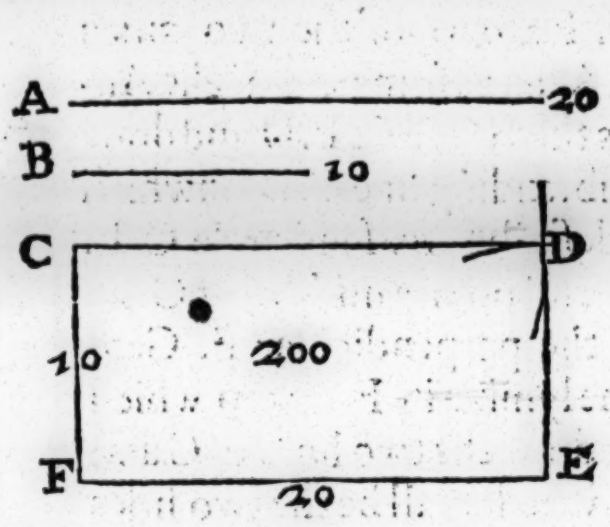
PROBLEME LIIII.

To make a long Square or right angled Parallelogram, the length and bredth being giuen.

LEt A. and B. be two right lines giuen for the length and bredth, it is required to make a right Angled parallelogram, whoſe length ſhall be A. and bredth B. By the 9. PROB. of the two giuen lines A.

and

and B. make a right Angle, as the Angle C. F. E. and at the distance of the line A. with one foot in C. make an arch line towards D. and at the distance of the line B. with one foot in E. crosse, the former arch line in D. from which intersection draw the lines D. E. and D. C. which shall include the right angled parallelogram C. D. E. F. hauing the length and bredth equall to the giuen lines; as was required. THROR. 55.



PROBLEME LV.

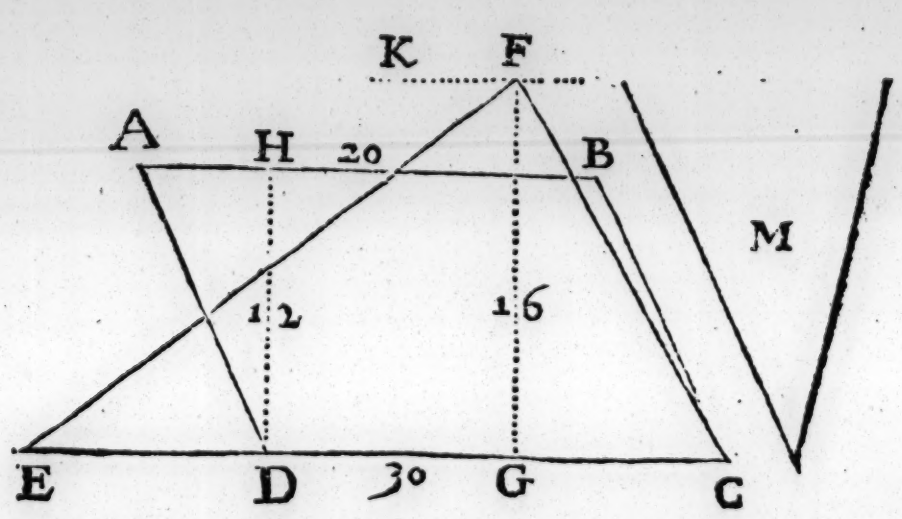
The length and breadth of a right angled Parallelogram or long Square being giuen, to finde the Area or superficiall content thereof Arithmetically.

IN the former Diagram let the length giuen be A. 20. and the bredth B. 10. Multiplie 20. by 10. the Product will bee 200. for the Area or superficiall capacitie of the Parallelogram, C. D. E. F. which is the thing required. THEOR. 38.

PROBLEME LVI.

To make a Parallelogram, whose length is limited, equall to a Triangle giuen, with two opposite Angles each equall to an Angle giuen.

LEt the Triangle giuē be F. E. C let the length limited be D. C. & let the Angle giuen be M. It is required vppon the giuen line to make a Parallelogram equall to the Triangle giuen, hauing two opposite Angles each equall to the Angle giuen. By the 16. PROB. reason thus reciprocally; If the giuen line D. C. yeeld the perpendicular of the Triangle F. G. what giues halfe the base E. C. the answere shall be H. D. the bredth of the Parallelogram to be made; at which distance draw



Euc. 1. 23.
6. 12.
6. 16.
Ceul. 2. 82.

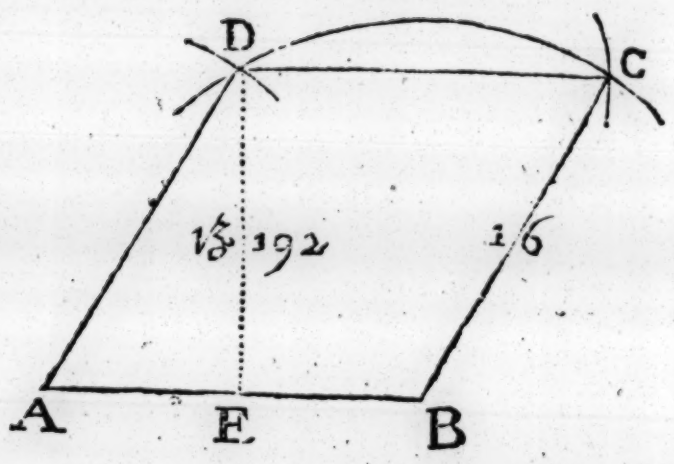
draw the line A. B. Parallel to the giuen line D. C. then on the point C. by the 8. PROB. deſcribe an Angle as the Angle B. C. D. equall to the giuen Angle M. and draw the line B. C. which ſhall cut the Parallell line A. B. in B. from whence marke out the line A. B. equall to the giuen line D. C. and laſtly draw the line A. D. which ſhall include the Parallelogram, A. B. C. D. equall to the Triangle giuen vpon the line D. C. giuen, and hauing two oppoſite Angles, namely, A. and C. each equall to the Angle giuen M. as was required. THEOREME 42.

PROBLEME LVII.

To make a Rhombus, the ſide being giuen.

LEt the line giuen be A. B. wheron it is required to deſcribe a Rhombus. At the diſtance A. B. with one foote in B. deſcribe the arch line D. C. & at the ſame diſtance ſetting one foot in A. Croſſe, the Arch line in D. on which point placing the compaſſe at the former diſtance croſſe the arch line in C. And laſtly, draw the lines D A. D C. and C B. which ſhall include the Rhombus, A. B. C. D. on the giuen line A. B. as was required. DEFINITIOM 26.

Euc. 1. 1.



PROBLEME LVIII.

The ſide of a Rhombus being giuen to finde out the Area or ſuperficiall content thereof Arithmetically.

IN the former Diagram, let the ſide A B. or D C. 16. be giuen, and let it be required to find the Area or ſuperficiall content thereof. By the 36. PROB. finde out the perpendicular D. E. 13. 192. and neere rationall, 13. $\frac{3}{5}$. and multiply the ſame by the giuen ſide, 16. the Product ſhall be 13. 49152. and neere rationall, 221. $\frac{1}{4}\frac{1}{4}$. for the ſuperficiall content required. THEOR. 34.

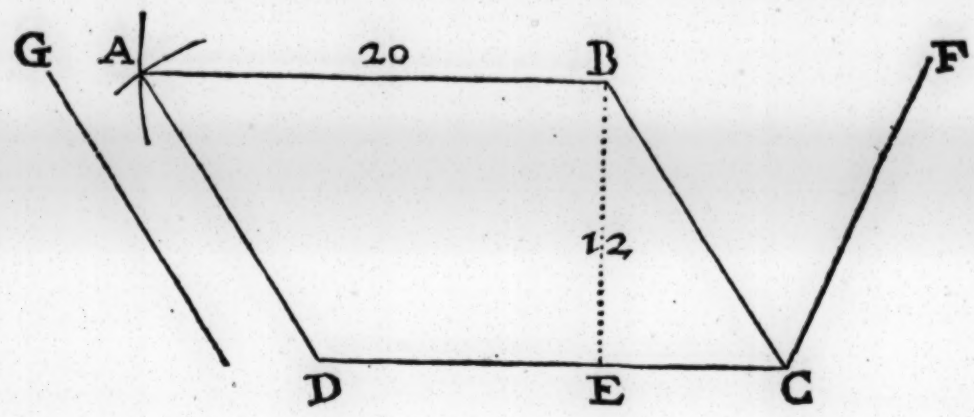
PROBLEME LIX.

To make a Rhomboydes the length and bredth being giuen in two right lines.

LEt the length giuen bee the line D. C. and the bredth, the line G. of which length and bredth it is required to deſcribe a Rhomboydes. At the diſtance of the giuen breadth G. and from one of the ends of the giuen

giuen length, as from C. chooſe a point as B. and at the ſame diſtance with one foot on the other end of the giuen length, as on D. ſtrike an arch towardes A. then at the diſtance of the giuen length with one foot in the point B. croſſe the former arch in A. And laſtly, draw the lines A. D. A. B. and B. C. which ſhall include a Rhomboydes of the length and bredth giuen. PROB. 3.

Euc. 1. 31.



PROBLEME LX.

To make a Rhomboydes with lines limited, hauing two oppoſite Angles, eqnall to an Angle giuen.

LEt the limited lines for the length & bredth be D.C.& G. as in the former Diagram, and let the Angle giuen be F.C. B. and let it be required to make a Rhomboydes of ſuch length and bredth, and with two oppoſite Angles equall to that giuen. On the end C. of the giuen length D.C. by the 8. PROB. protract an Angle equall to the giuen Angle, as D. C. B. making the line B. C. equall to the giuen bredth G. And ſo worke forward in all reſpects as in the former Probleme. PROB. 3.8.

Euc. 1. 23. 1. 31.

PROBLEME LXI.

A Rhomboydes giuen to finde the ſuperficiall content Arithmetically.

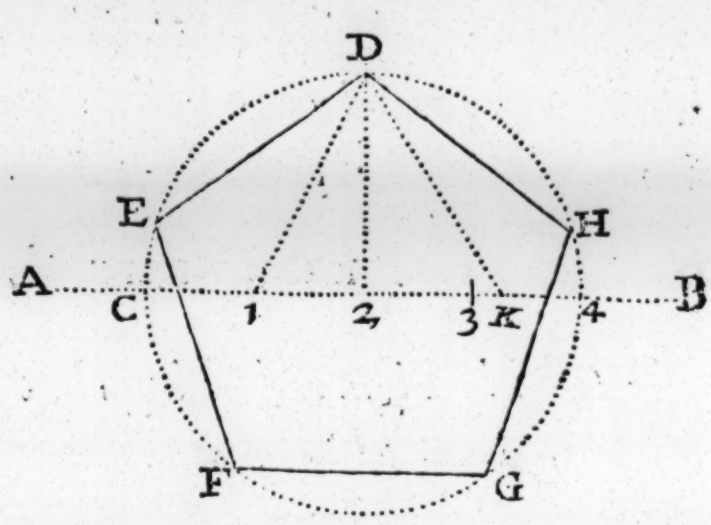
LEt the Rhomboydes giuen bee that in the former Diagram, It is required to find the ſuperficiall content thereof. Take the giuen length D. C. or A. B. 20. and ſeeke out (as hath beene formerly taught) the parallel diſtance or perpendicular B. E. 12. which multiply the one by the other, the Product is 140. the Area of the giuen Rhomboydes required. THEOR. 34.

PROBLEME LXII.

To deſcribe a Pentagon, hauing ſides and Angles equall.

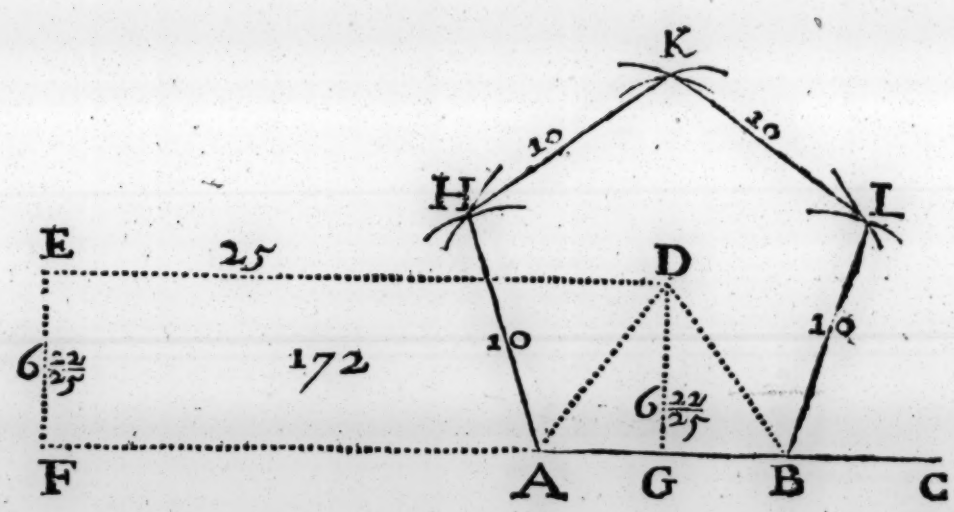
LEt any obſcure line be drawne as the right line A. B. and at any conuenient diſtance place thereon foure equall parts or diuiſions, as from C. to 4. and at the diſtance of two of them, on the ſecond part, as a centre

tre, deſcribe an obſcure circle, on which centre rayſe the perpendicular D. 2. to cut the limbe in D. then at the diſtance D. 1. with one foot in 1. croſſe the line in K. and with the diſtance D. K. marke the limbe of the Circle in the points D. E. F. G. and H. And laſtly, draw lines from point to point, which ſhall include the Pentagon required. DEF. 29. THEOR. 47. PROB. 20. 21. 25.



PROBLEME LXIII.

To deſcribe a Pentagon vpon a line giuen.



Euc. 13. 8.

LEt A. B. be a right line giuē, whereon it is required to deſcribe a Pentagon. Suppoſe the line giuen, to bee the greater ſegment of a right line diuided by extreame and mean proportion. And by the 21. and 22. PROB. finde the whole line; which let be A. C. and at the diſtance of A. C. with one foot in B. ſtrike an arch towards H. and another towards K. and with the foote in A. ſtrike one towards L. then take the diſtance of the giuen line, and on A. and B. croſſe the Arches at H. and L. and on H. or L. croſſe the Arch in K. And laſtly, draw lines from each interſection to other, which ſhall incloſe the Pentagon, as was required. THEOR. 47. PROB. 20. 21, 22, 25.

PROBLEME LXIIII.

The ſide of a Pentagon being giuen, to finde the ſuperficiall content Arithmetically.

IN the former Diagram, let the giuen ſide be 10. It is required to finde the Area of that Pentagon. By the 36. PROB. (ſuppoſing an Iſoſceles deſcribed on any ſide of the Pentagon, whoſe top is the centre, as A. D B.)

seeke the perpendicular D. G. 6. $\frac{22}{25}$. which multiplied in halfe the perimetrie 25. produceth 172. the Area required. This rule is generall in all kind of regular polligons, of how many sides so euer; aswell for their superficiall content, as finding their perpendicular. THEOR. 19. 39. PROB. 36.

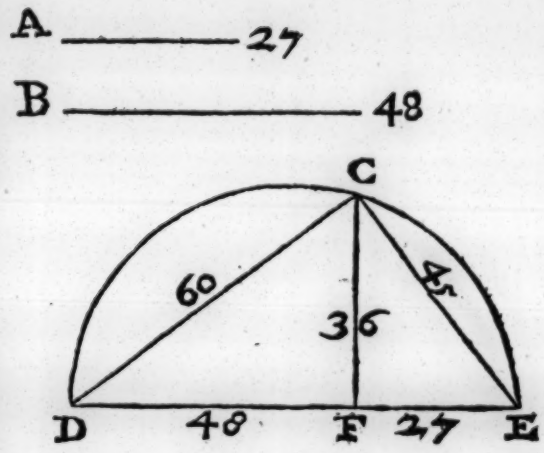
A generall Rule.

PROBLEME LXV.

To make two like Figures, bearing the one to the other, any proportion assigned in two right lines.

LEt A. and B. be two right lines giuen, and let it be required to make two like Triangles, Squares, Circles or other like figures, hauing such proportion the one to the other, as A. to B. make of the two giuen lines one right line, as D. E. and describe thereon the Semicircle C.D.E. and on the point F. where the 2. giuen lines meete, erect the perpendicular C. F. to cut the limbe in C. from whence draw the lines C. D. and C. E. which two lines shall bee the sides of two equilater Triangles, or of two Squares, or other like figures or the diameters of two circles, bearing such proportion the one to the other, as the two giuen lines; which was required. THEOR. 6. 11, 55, 66, 70, 65.

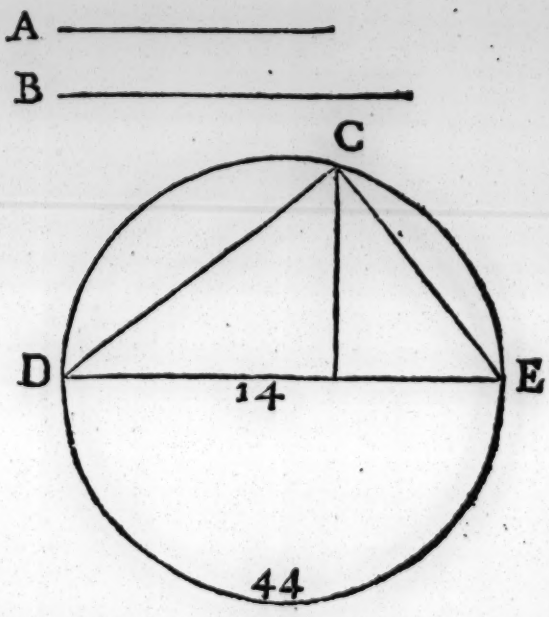
Eue. 6. 4: 6. 8.



PROBLEME LXVI.

Two Circles being giuen, to make one Circle equall to them both.

LEt A. and B. be the Diameters of two Circles giuen. It is required to make one Circle equall to them both. With the lines A. and B. by the 9. PROB. make a right Angle, as D. C. E. then draw the subtending line D. E. And lastly, on the line D. E. describe the Circle D. C. E. which shall bee equall to the two giuen circles as was required. THEOR. 65. 70, 66.



PROB.

PROBLEME LXVII.

The Diameter of a Circle being giuen, to find the circumference thereof Arithmetically.

IN the former Diagram, let the Diameter giuen be 14. It is required to finde the circumference thereof. Multiply the Diameter giuen 14. by 22. the Product is 308. which diuided by 7. bringeth 44. the circumference required. Or multiply 14. the Diameter by $3\frac{1}{7}$. the Product is 44 as before. THEOREME. 59.

If the circumference bee giuen, and the Diameter required, It appeareth by this rule, that the circumference 44. being multiplied by 7. and the product diuided by 22. bringeth 14. the Diameter.

PROBLEME LXVIII.

The Diameter and Circumference of a Circle being giuen, to find the Area, or superficiall content thereof Arithmetically, diuers wayes.

IN the former Diagram let the Diameter of the Circle D. C. E. be 14. and the Circumference thereof 44. It is required to find the superficiall content. Multiply the Semicircumference 22. by the Semidiameter 7. the Product will be 154. the superficiall content required. THEOR. 62.

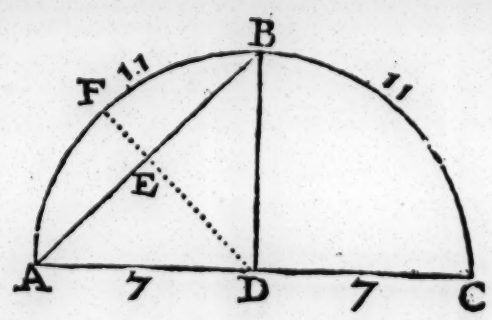
Or multiplie the whole Circumference 44. by the Semidiameter 7. the Product will be 308. whereof take halfe, which is 154. as before. THEOREME. 60.

Or multiply the square of the Diameter 196. by 11. the Product will be 2156. which diuided by 14. bringeth 154. as before. THEOR. 61.

PROBLEME LXIX.

The Diameter and Arch-line of a Semicircle giuen, to find the Area thereof.

LEt A. B. C. be a Semicircle giuen, whose Diameter is A. C. and the Arch line A. B. C. It is required to finde the Area of the Semicircle. Multiplie halfe the arch line 11. by the Semidiameter 7. The Product will be 77. for the Area required. THEOREME. 63.



AB. 13. 98.

A. E. 13. 24. $\frac{1}{2}$.

E. D. 13. 24. $\frac{1}{2}$.

F. E. 7. 13. 24. $\frac{1}{2}$.

PROBLEME LXX.

The Semidiameter and Arch line of a Sector of a Circle giuen to find the Area.

IN the former Diagram, let B. C. D. be the Sector of a Circle, whose Semidiameter is D. C. or D. B. and the arch line B. C. and it is required to finde the Area. Multiply the Semidiameter 7. by halfe the arch line B. C. 5. $\frac{1}{2}$. the Product is 38. $\frac{1}{2}$. for the Area required. THEOR. 64.

PROBLEME LXXI.

Any Segment or part of a Circle being giuen, to finde the superficiall content thereof.

IN the former Diagram, let A. F. B. E. be the Segment of a Circle, the content whereof is required. By the 34. PROB. finde out the Centre, and then draw the lines D. A. and D. B. and cast vp the whole Figure A. F. B. D. as in the last Probleme which will be 38. $\frac{1}{2}$. then finde the superficiall content of the Triangle, A. B. D. by the 41. PROB. which is 24. $\frac{1}{2}$. and deduct the same out of the whole content 38. $\frac{1}{2}$ resteth 14. for the superficiall content of the giuen Segment as was required.

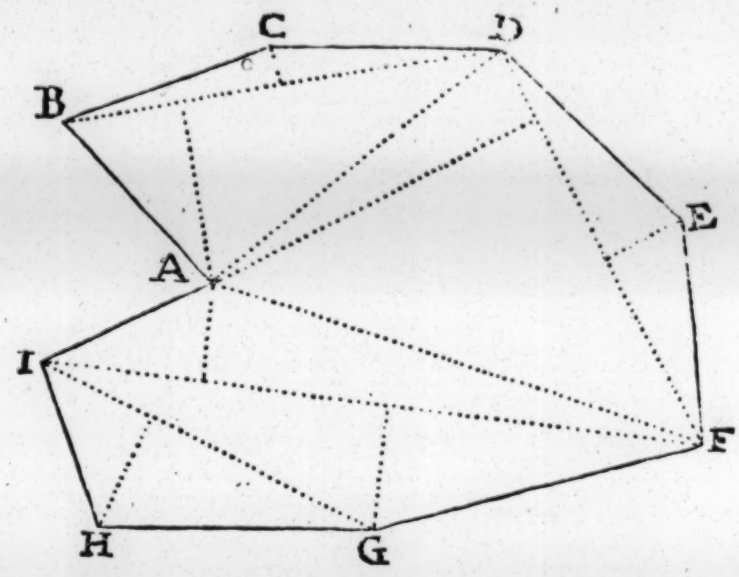
By this rule (obserued with discretion) may all manner of Segments or parts of a Circle, whether greater or lesser then a Semicircle, be easily measured without further instruction.

Note. *But here is to bee noted, that the precedent rules concerning the mensuration of Circles, and their seuerall parts, are not exactly true: for that the proportion betweene the Diameter and circumference is irrationall; and the squaring of a Circle or the meanes thereof (other then mechanically) not yet discouered or found out; yet of such sufficient preciseneſſe as no notable or apparant error can be made or found in the conclusions thereby wrought.*

PROBLEME LXXII.

An irregular plotte or Figure being giuen, to finde the Area or superficiall content thereof.

LEt A. B. C. D. E. F. G. H. I. be an irregular plotte or Figure, whose superficiall capacitie is required. Reduce the same into as many Trapezias as it will containe, as first the Trapezium A. B. C. D. Secondly, A. D E F. Thirdly, A. F. G. I. and there remaineth the Triangle, I. G. H. In which three Trapezias draw the Diagonals B D. D F. and F I.



F. I. which ſhall ſerue as the common baſes to each Triangle on eyther ſide, on which baſes let fall the perpendiculars from the ſeuerall Angles at A.C.E. and G. then in euery Trapezium take the length of the baſe by it ſelfe, and the length of the two perpendiculars theron falling, ioined together in one number by it ſelfe, then multiply the halfe of the one in the whole of the other, the Product is the Area of that Trapezium, which reſerue by it ſelfe, and working in the like ſort with the reſt: And laſtly, the Triangle I. G. H. collect all their Products together, which ſhall ſhew the ſuperficiall content required. PROB. 44. THEOR. 25.

This I hold the beſt manner of Menſuration of plots, aſwell for expedition, as exactnes in auoyding errours, often hapning by multiplicitie of numbers, and many multiplications.

THus in this ſecond part, haue I taught the Deſcription and Menſuration of all manner right lined ſuperficiall Figures, according to the ſtrict and preciſe rules and precepts of abſolute Arte: yet ſeeing that diſpatch and expedition in buſineſſe of import, is much more requiſite then needleſſe niceneſſe; I would not haue my Surueyor ignorant or vnfurniſhed of ſuch other ready and perfect helpes (though more mechanike) as may yeelde him eaſe, and ſaue much labour in furthering his intents. To which end, for the ſpeedie and exact menſuration of all ſuperficiall figures, I would haue prouided a Protractor in Braſſe, whoſe Scale ſhould containe in length about 8. or 10. ynches with equall Diuiſions on the edge of eyther ſide, of 12 in an ynch on the one ſide, and 11. on the other, being numbred by tennes after the vſuall manner of thoſe kind of works (which for mine own part is the Scale, I neuer vſe in all my firſt drawne plots, whether the quantitie be ſmall or great, well knowing the inconuenience of ſmaller Scales) hauing placed thereon a Sextans of a Circle moſt excellent for many vſes, as the ſpeedy laying downe of any Angle required, or the ready finding of any Angle giuen, &c. The order and making whereof is well knowne to Maſter Elias Allen, *who for*

my ſelfe and friends hath made of them. By this Scale with helpe of the middle line thereof, Orthigonally drawne to the edge; you ſhall readily rayſe a perpendicular, and as inſtantly receiue the length thereof, and moſt ſpeedily obtaine the baſe of any Triangle, or the ſide of any Figure giuen by applying the edge of the Scale thereunto: which is much more facile and ſpeedy then the former Precepts; and to bee preferred for exactnes, ſpeede and perfection, before the ordinarie courſe with Scale and Compaſſe. But doubting to exceede the Scale and Compaſſe of my intended purpoſe, I will here conclude the ſecond part of this my ſecond Booke.

The

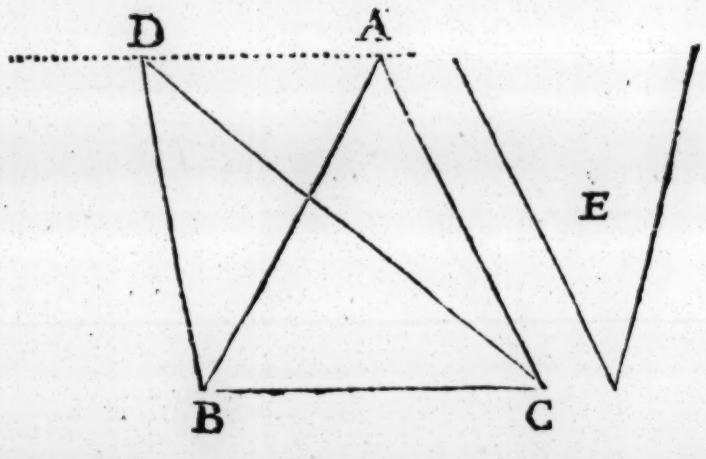
The third Part.

OF the Reduction and Tranſlation of all manner of ſuperficiall figures, from one forme vnto another, retayning ſtill their firſt quantitie.

PROBLEME LXXIII.

To reduce one triangle into another, on the ſame baſe, but hauing an angle equall to an angle giuen.

LEt A. B. C. be a triangle giuen, and let the angle giuen be E. and the baſe of the giuen triangle B. C. on which baſe it is required to reduce the giuen triangle to another, hauing an angle equall to the angle E. From the point A. by the 3. PROB. draw a parallel line to the baſe B. C. as the line A. D. then on the point C. by the 8. PROB. make the angle D. C. B. equall to the giuen angle E. and draw the line D. C. cutting the parallell line in D. and laſtly, draw the line D. B. which ſhall include the triangle D. B. C. equall to the giuen triangle A. B. C. vpon the ſame baſe B. C. and hauing an angle equall to the giuen angle E. as was required. THEOR. 26.

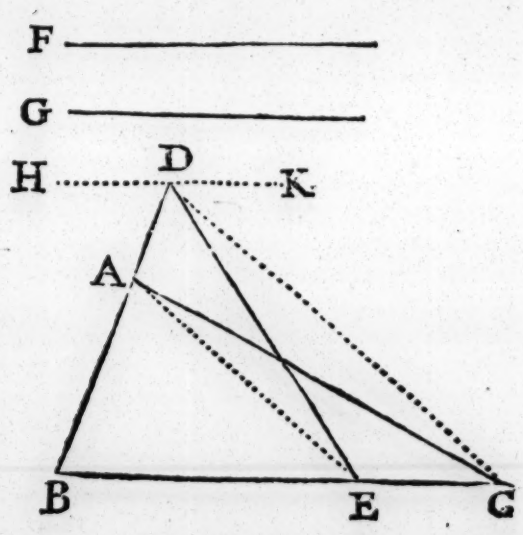


*Eu.*1.37.

PROBLEME LXXIIII.

To reduce one triangle into another, vpon a baſe equall to a baſe giuen.

LEt the triangle giuen be A. B. C. and F. the giuen baſe, whereon it is required to reduce the giuen triangle. Take the giuen baſe F. and place it from B. to E. and from the point E. to the top of the giuen triangle A. draw the line A. E. then increaſe the ſide A. B. of the giuen triangle towards D. and from the point C. by the 3. PROB. draw a parallel line to



*Eu.*1.37.

to A. E. as D. C. cutting the increaſed ſide in D. and laſtly, draw the line D. E. which ſhall include the triangle D. B. E. equall to the giuen triangle A. B. C. vpon the baſe B. E. equall to the giuen baſe, as was required. THEOR. 26. PROB. 27.

If the giuen baſe be greater then the baſe of the giuen triangle, worke in all reſpects, as is taught in PROB. 27.

PROBLEME LXXV.

To reduce one triangle to another, of any poſsible height required.

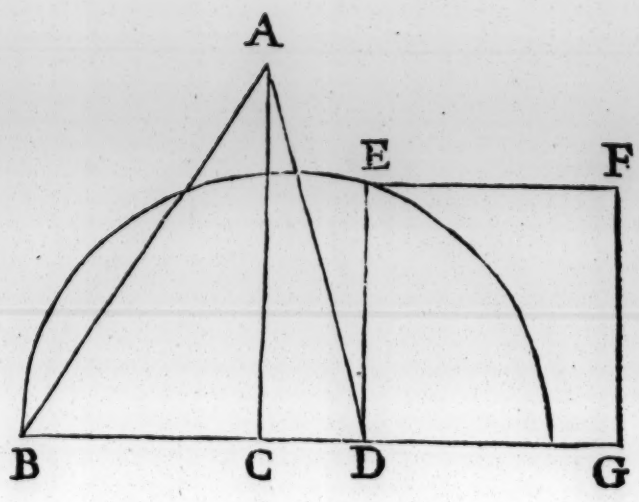
Ceul. 3. 5. LEt the triangle A. B. C. in the former Diagram be giuen, and let the height required be the line G. of which height it is required to reduce the giuen triangle into another of the ſame quantitie. At the diſtance of the giuen height G. by the 2. PROB. draw the line H. K. parallel to the baſe B. C. then increaſe the ſide A. B. till it cut the parallel line H. K. in D. from which point draw the line D. C. then by the point A. draw the line A. E. parallel to D. C. cutting the baſe in E. and laſtly, from the point E. draw the line E. D. which ſhall include the triangle D. B. E. equall to the giuen triangle A. B. C. and of the height G. as was required. THEOR. 26. PROB. 28.

If the giuen height be leſſe then the height of the giuen triangle, worke in all reſpects, as is taught in PROB. 28.

PROBLEME LXXVI.

To reduce a triangle giuen into a ſquare.

Euc. 1. 31. LEt A. B. D. be a triangle giuen. It is required to reduce the ſame into a Geometricall ſquare. By the 17. PROB. finde out a meane proportionall line between the baſe B. D. and halfe the perpendicular A. C. which ſhall be the line E. D. on which line by the 50. PROB. deſcribe the ſquare E. F. G. D. which ſhall be equall to the giuen triangle. THEOR. 25. 27.

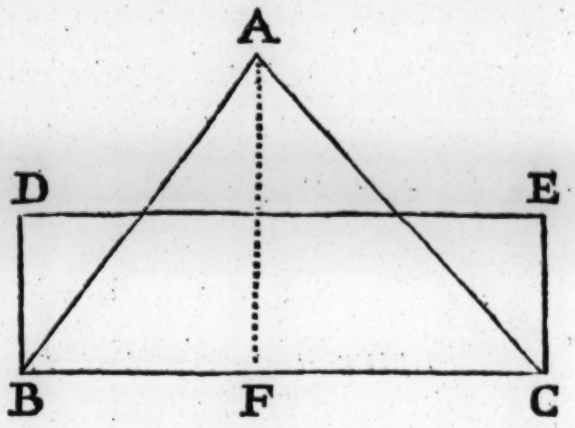


PROBLEME LXXVII.

To reduce a triangle giuen, into a right angled parallelogram.

Euc. 1. 42. *Ceul.* 2. 28. *&* 2. 80. LEt A. B. C. be a triangle giuen. It is required to reduce the ſame into a right angled parallelogram. From the angle A. let fall to the baſe B. C. the perpendicular A. F. then take halfe thereof for the breadth, and the

the whole baſe B. C. for the length, with which breadth and length by the 54. PROB. deſcribe the right angled parallelogram D. E. B. C. which ſhall be equall to the giuen triangle, as was required. THEOR. 25.

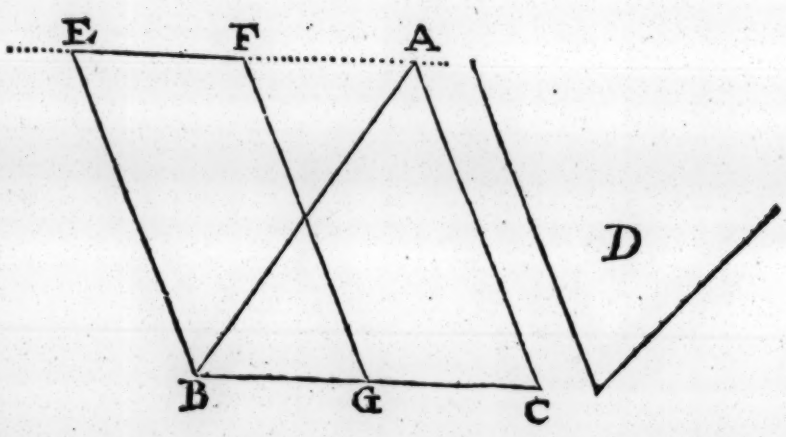


PROBLEME LXXVIII.

To reduce a triangle giuen, into a parallelogram, hauing an angle equall to an angle giuen.

LEt A. B. C. be a triangle giuen, and let the angle giuen be D. It is required to reduce the ſame triangle into a parallelogram, hauing an angle equall to the angle D. By the 3. PROB. from the point A. draw the line E. A. parallel to the baſe B. C. then diuide the baſe B. C. into two equall parts in the point G. on which point by the 8. PROB. deſcribe the angle F. G. B. equall to the giuen angle D. and draw the line F. G. cutting the parallel line in F. from whence draw the line F. E. equall to G. B. and laſtly, draw the line E. B. which ſhall include the parallelogram E. F. G. B. equall to the giuen triangle, hauing an angle F. G. B. equall to the giuen angle D. as was required. THEOR. 28.

Euc. 1. 42.



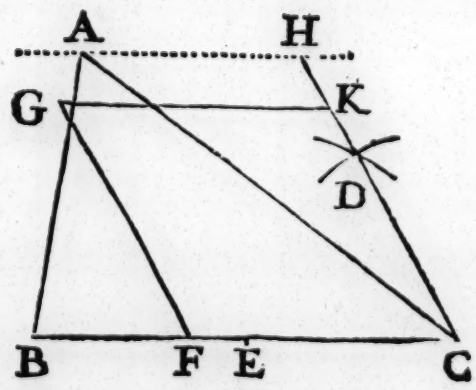
PROBLEME LXXIX.

To reduce a triangle giuen into a Rhombus.

LEt the triangle giuen be A. B. C. and it is required to reduce the ſame into a Rhombus. By the 3. PROB. draw the line A. H. parallel to the baſe B. C. then diuide the baſe B. C. into two equall parts in the point E. on which point at the diſtance E. B. or E. C. ſtrike an arch towards D. and on the point C. at the ſame diſtance croſſe the former arch in D. by which interſection

Euc. 1. 37.

Cor. 6. 19.



and

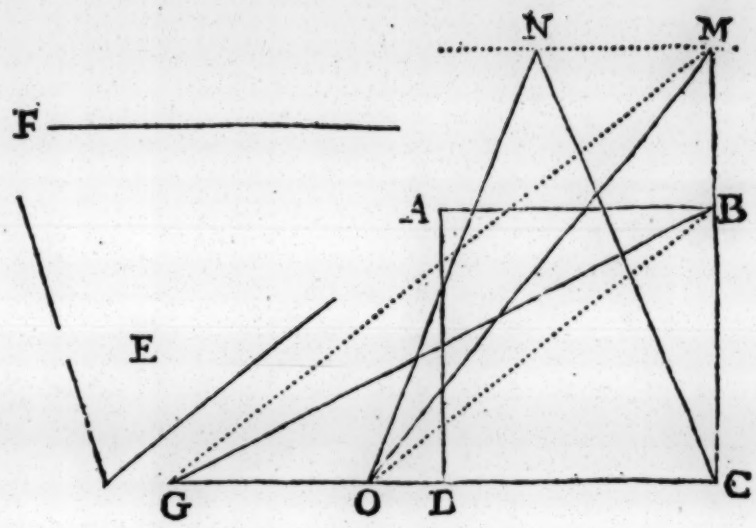
and the point C. draw the line H. C. to cut the parallel line in H. then by the 17. PROB. find out a meane proportionall line betweene C. D. and C. H. which is C. K. vpon which line C. K. by the 57. PROB. describe the Rhombus G. K. C. F. which shall be equall to the triangle giuen, as was required. THEOR. 26. 41. 43.

PROBLEME LXXX.

To reduce a square giuen into a triangle, hauing an angle equall to an angle giuen, and that on a line giuen.

EUC. 1. 37.

LEt the square giuen be A. B. C. D. the angle giuen E. and let the giuen line be F. On which line it is required to reduce the giuen square into a triangle, hauing an angle equall to the angle E. First, continue the side of the giuen square D. C. to G. making G. D. equall to D. C. and draw the line G. B which shall include the triangle B. G. C. equall to the giuen square; then take the giuen line F. and lay it downe from C. to O. and by the 74. PROB. make the triangle M. O. C. then by the third PROB. draw the line M. N. parallel to the base G. C. and on the point C. protract an angle equall to the giuen angle E. as N. C. O. and draw the line C. N. to cut the parallel in N. and lastly, from the point N. draw the line N. O. which shall include the triangle N. O. C. equall to the giuen square, hauing an angle as N. C. O. equall to the giuen angle, and that on the line O. C. equall to the giuen line F. as was required. THEOR. 26.

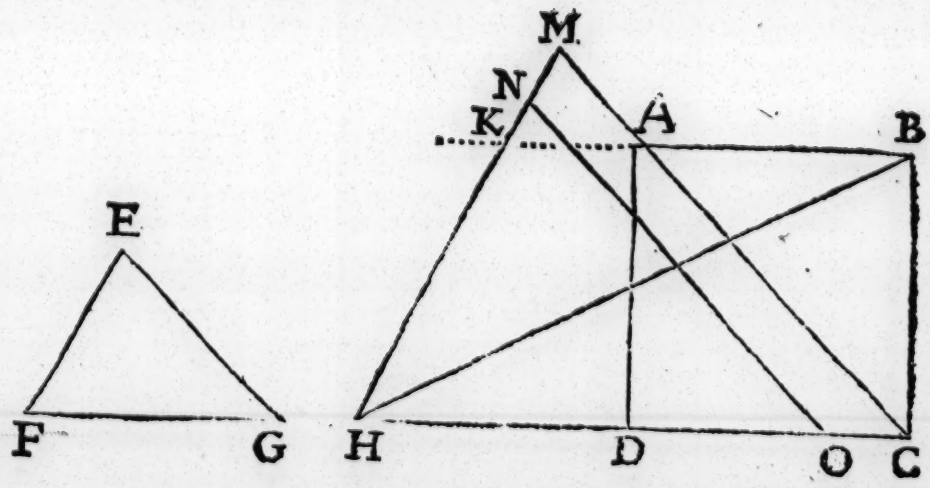


PROBLEME LXXXI.

To reduce a square giuen into a triangle, with angles equall, and lines proportionall to a triangle giuen.

EUC. 1. 37. Cor. 6. 19.

LEt the giuen square be A. B. C. D. and let the giuen triangle be E. F. G. and let it be required to reduce the same square into a triangle, with angles equall, and lines pro-

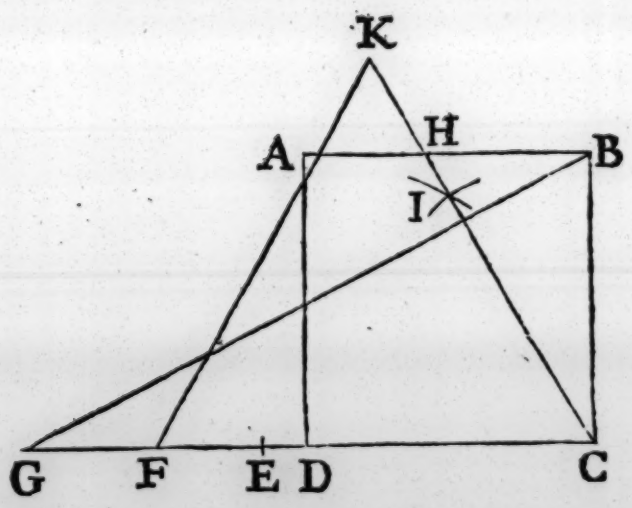


proportionall to the giuen triangle. According to the firſt Part of the laſt PROB. make the triangle B. H. C. equall to the giuen ſquare, then continue the ſide A. B. of the giuen ſquare towards K. and on the point H. protract the angle M. H. D. equall to the angle F. in the giuen triangle, drawing the line M. H. at length to cut the continued ſide A. B. in K. then on the point C. protract the angle M. C. H. equall to the angle G. in the giuen triangle, and draw the line C. M. to cut the line M. H. in M. which ſhall include the triangle M. C. H. with equall angles to the giuen triangle, but of greater content then the giuen ſquare; wherefore by the 17. PROB. finde out a meane proportionall line betweene H. K. and H. M. which is H. N. and from the point N. by the 3. PROB. draw the line N. O. parallel to M. C. which ſhall include the triangle N. O. H. equall to the giuen ſquare, and hauing equall angles and lines proportionall to the giuen triangle, as was required.
THEOR. 26. 43,

PROBLEME LXXXII.

To reduce a ſquare into an equilater triangle.

LEt the ſquare giuen be A. B. C. D. and it is required to reduce the ſame into an equilater triangle. Double the ſide D. C. by increaſing the ſame to G. and draw the line G. B. to include the triangle B. G. C. equall to the giuen ſquare, then at the diſtance of the ſide of the ſquare, with one foot in C. ſtrike an arch towards I. and at the ſame diſtance with one foot in D. croſſe the ſame arch in I. and by the interſection and the point C. draw the line K. C. at length to cut A. B. in H. and take C. H. and place the ſame from C. to E. then by the 17. PROB. finde out the meane proportion betweene C. E. and C. G. which is C. F. at which diſtance deſcribe the equilater triangle K. F. C. which ſhall be equall to the giuen ſquare, as was required. THEOR. 26. 43



Euc. 1. 37. *Cor.* 6. 19.

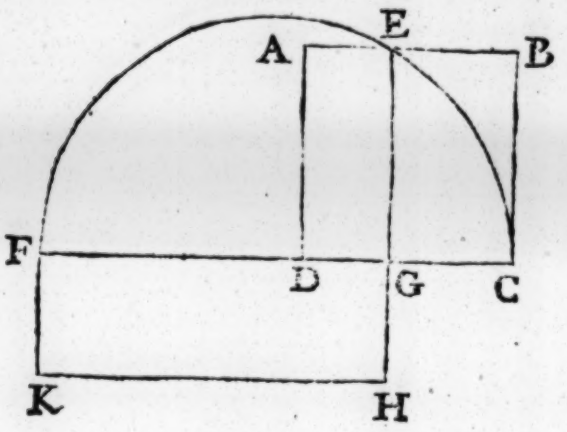
PROBLEME LXXXIII.

To reduce a ſquare giuen, into a right angled parallelogram or long ſquare, the length and bredth being limited in a right line: So as the ſide of the ſquare exceede not halfe the line giuen.

LEt the ſquare giuen be A. B. C. D. and let the right line giuen be F. C. It is required to reduce the ſame ſquare into a long ſquare, whoſe length and bredth together ſhall be equall to F. C. Vpon the giuen line F. C. place the giuen ſquare, as in the Diagram, then deſcribe the ſemicircle

Euc. 1. 47. 6. 13.

circle F.E.C. to cut the ſide A. B. of the giuen ſquare in E. from which point let fall the perpendicular E.G.to cut the giuen line in G. So ſhall F. G. be the length, and G.C. the bredth of the long ſquare to be made, of which length and bredth by the 54. PROB. deſcribe the parallelogram F.G. H. K. which is equall to the giuen ſquare, and of the length and bredth required. THEOR. 41. PROB. 19.

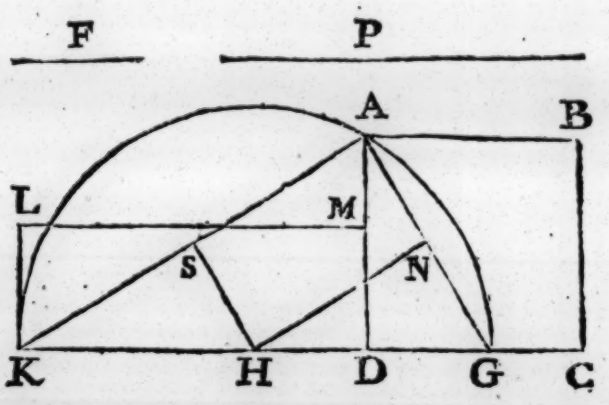


PROBLEME LXXXIIII.

To reduce a ſquare giuen into a long ſquare, whoſe bredth is limited in a right line giuen.

*Euc.*1.47, 6, 13.

LEt the ſquare giuen be A. B. C. D. and let F. be the right line giuen. It is required to reduce the ſame ſquare into a long ſquare, whoſe bredth ſhall be equall to the giuen line F. Continue the ſide C. D. of the giuen ſquare towards K. and place the giuen line F. from D. to G. and from the point G. draw the line G.A. which diuide equally in N. from which point draw the perpendicular N. H. to cut the line of continuance in H. on which point at the diſtance H. G. deſcribe the ſemicircle K. A. G. to cut the line of continuance in K. ſo ſhall K. D. be the length ſought for, with which length, and the giuen bredth F. or D. G. by the 54. PROB. deſcribe the parallelogram L. M. D. K. which ſhall be equall to the giuen ſquare, as was required. THEOR. 41. PROB. 19.



PROBLEME LXXXV.

To reduce a ſquare giuen into a long ſquare, whoſe length is limited in a right line giuen.

*Euc.*1.47, 6, 13.

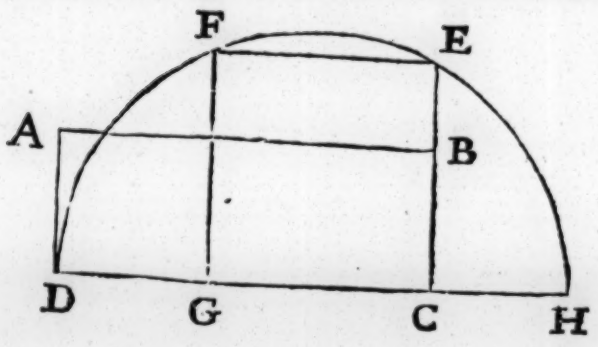
IN the former Diagram, let that ſquare be the ſquare giuen, and the length giuen the right line P. Firſt, continue D. C. as before towards K. and make K. D. equall to the giuen line P. and draw the line K. A. vpon the middle whereof S. raiſe the perpendicular S H. to cut the line of continuance in H. on which point deſcribe the ſemicircle, as before, to cut D. C. in G. ſo ſhall D. G. be the bredth ſought, of which bredth and the giuen length, make the long ſquare, as before. THEOR. 41. PROB. 19.

PRO-

PROBLEME LXXXVI.

To reduce a long ſquare giuen into a geometricall ſquare.

LEt the long ſquare giuen be A. B. C. D. and it is required to reduce the ſame into a geometricall ſquare. Continue the ſide D. C. of the long ſquare giuen towards H. and let the bredth B. C. of the long ſquare be placed on the line of continuation from C. to H. then on D. H. deſcribe the ſemicircle D. F. E. H. and increaſe the bredth of the long ſquare C. B. till it interſect the limbe in E. ſo ſhall E. C. (being the meane proportionall betweene D. C. and C. H.) be the ſide of the ſquare ſought, wherefore, on the line E. C. by the 50. PROB. deſcribe the geometricall ſquare F. E. C. G. which ſhall be equall to the long ſquare giuen, as was required. THEOR. 41. PROB. 19.

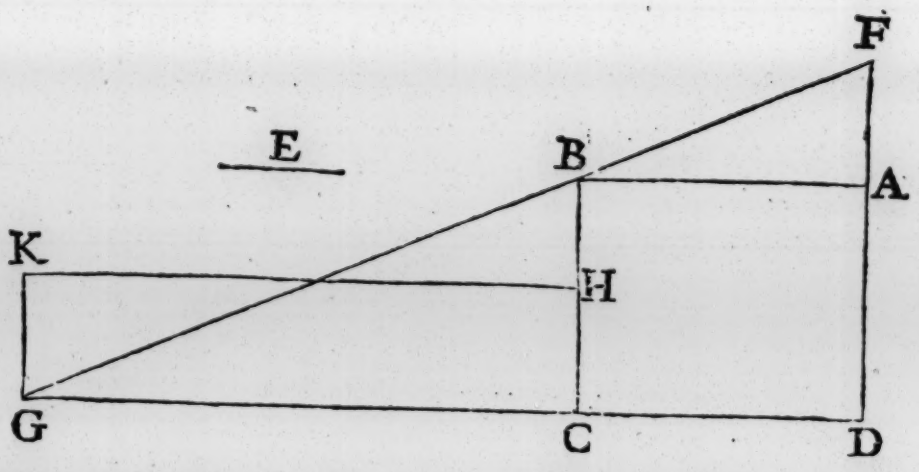


Euc. 2. 14. *Ceul.* 2. 37.

PROBLEME LXXXVII.

To reduce one long ſquare giuen into another, whoſe length or bredth is limited in a right line giuen.

LET the long ſquare giuen be A. B. C. D. and let the bredth giuen be E. and let it bee required to reduce the long ſquare giuen, into another long ſquare of the giuen breadth E.



Euc. 1. 43.

Continue the bredth of the giuen ſquare D. A. towards F. making A. F. equall to the giuen bredth E. alſo continue the length D. C. of the giuen ſquare towards G. and by the point F. and B. draw out the line F. G. to cut the line of continuance laſt drawne in G. ſo ſhall G. C. be the length ſought for, with which length and the giuen bredth E. by the 54. PROB. make the long ſquare K. H. C. G. which ſhall be equall to the giuen long ſquare, and of the limited bredth, as was required. THEOR. 5.

Or otherwiſe, thus.

IN the former Diagram, let it be required to reduce the long ſquare there giuen, into another long ſquare, whoſe bredth ſhall be the giuen line E. *Euc.* 6. 12.

By the 16. PROB. reaſon thus. If E. the giuen bredth giue A. D. the bredth of the giuen ſquare, what giues A. B. the length thereof, the anſwere

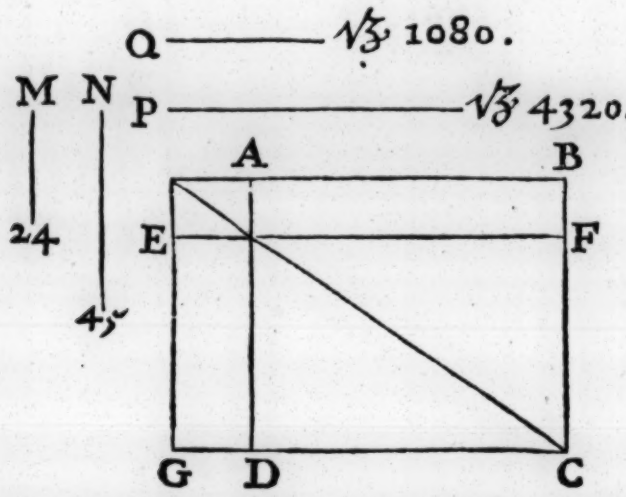
ſhall be G. C. or K. H. the length ſought for, with which length and the giuen bredth, make the long ſquare K. H. C. G. as before. THEOR. 42.

PROBLEME LXXXVIII.

To reduce one long ſquare giuen into another, whoſe length and bredth ſhall haue proportion the one to the other, as two giuen lines.

Euc. 6. 12. 13. 14.

LEt the long ſquare giuen be A. B. C. D. and let the proportion giuen be ſuch, as that betweene the two right lines M. and N. It is required to reduce the ſame long ſquare into another long ſquare, whoſe length and bredth ſhall haue proportion the one to the other, as thoſe two giuen lines. By the 17. PROB. ſeeke out the meane proportion betweene the two giuen lines M. and N. which is Q. ſeeke alſo the meane proportion betweene A. D. and A. B. the bredth and length of the long ſquare giuen, which is the right line P. then by the 16. PRO. reaſon thus. If Q. giues P. what giues N. the anſwere ſhall be E. F. for the length ſought; and againe, if Q. giues P. what giues M. the anſwere ſhall be E. G. for the bredth ſought; of which length and bredth E. F. and E. G. by the 54. PROB. make the long ſquare E. F. C. G. which ſhall be equall to the long ſquare giuen, and the bredth to the length, in ſuch proportion as the line M. to the line N. as was required. THEOR. 5, 41, 42, 71.



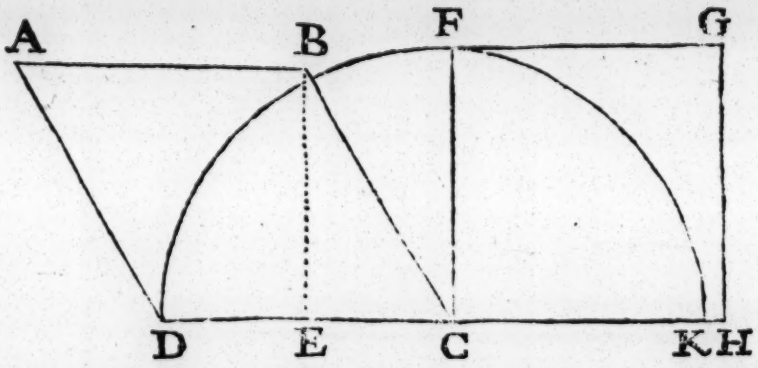
A B. 72.
B C. 60.
E F. 90.
E G. 48.

PROBLEME LXXXIX.

To reduce a Rhombus into a geometricall ſquare.

Euc. 1. 36.

LEt the Rhombus giuen be A. B. C. D. and let it be required to reduce the ſame into a geometricall ſquare. Firſt, by the 17. PROB. finde out a meane proportionall line betweene D. C. the ſide of the giuen Rhombus, and the parallel diſtance, or perpendicular line B. E. which ſhall be the line F. C. vpon which line, by the 50. PROB. deſcribe the geometricall ſquare F. G. H. C. which ſhall be equall to the giuen Rhombus A. B. C. D. as was required. THEOR. 33, 34.



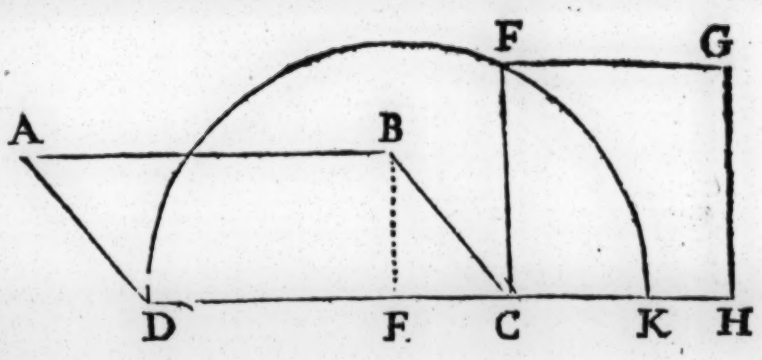
PRO-

PROBLEME XC.

To reduce a Rhomboydes giuen, into a Geometricall Square.

LEt the Rhomboydes giuen be A. B. C. D. and it is required to reduce the ſame into a Geometricall Square. Let fall the perpendicular B. E from the Angle B. to the Baſe D. C. betweene which Baſe and Perpendicular, by the 17. Prob. finde out a meane proportionall line, which ſhall be F. C. vpon which line by the 50. Prob. deſcribe the Square F. G. H. C. which ſhall be equall to the giuen Rhomboydes as was required. Def. 27. Theor. 33. 34.

Euc. 1. 36.

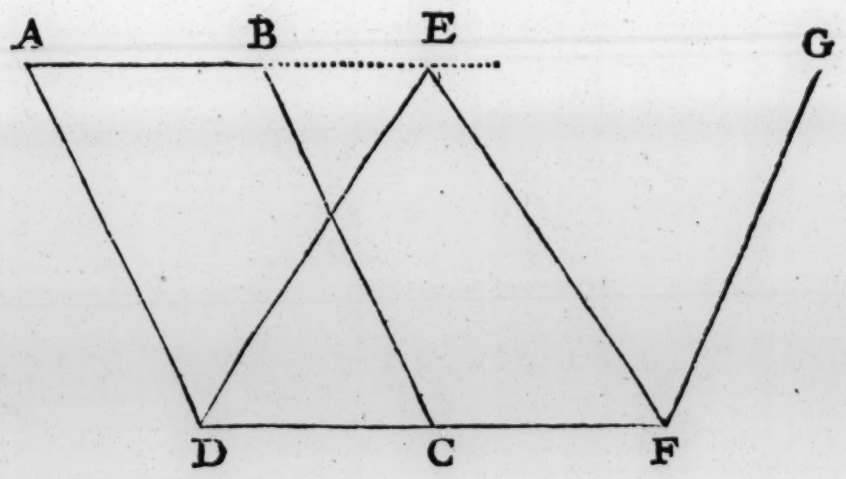


PROBLEME. XCI.

To reduce a Rhomboydes giuen into a Triangle, hauing an Angle equall to an Angle giuen.

LEt the Rhomboydes giuen be A. B. C. D. and let the Angle giuen be E. F. G. It is required to reduce the ſame Rhomboydes into a Triangle, hauing an Angle equall to E. F. G. Firſt, increaſe the line A. B. towards E. and alſo the baſe D. C. towards F. and make C. F. equall to D. C. then on D. protract the Angle E. D. F. by the 8. Prob. equall to the giuen Angle E. F. G. and draw the line D. E. to cut the increaſed line in E. And laſtly, from E. draw the line E. F. which ſhall include the Triangle E. D. F. equall to the Rhomboydes giuen, and hauing an Angle equall to the giuen Angle as was required. Theor. 28.

Eu. 1. 41.



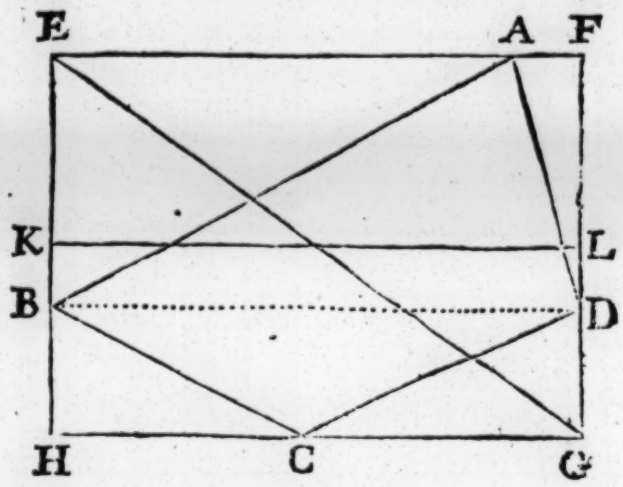
PROBLEME XCII.

To reduce a Trapezium giuen into a right angled Parallelogram, or into a right angled Triangle.

LEt the Trapezium giuen be A. B. C. D. which is to be reduced into a right angled Parallelogram, or into a right angled Triangle. Firſt, draw the Diagonall line B. D. then by the 2. Prob. by the points A. and C. draw

Euc. 1. 41.
Ceul. 2. 29.

draw the lines E. F. and H. G. Parallel to B. D. and by the point B. and D. draw E. H. and F. G. to cut the two laſt lines Orthigonally; ſo ſhal you include the Parallelogram E. F. G. H. which is double to the Trapezium giuen, wherefore diuide the ſame into two equall parts, by drawing the line K. L. So haue you the right angled parallelogram E. F. L. K. or K. L. G. H. equall to the Trapezium giuen. And the Diagonall line E. G. being drawne, ſhall include the right angled Triangle, E. H. G. or E. F. G. likewiſe equall to the ſame Trapezium as was required. THEOREME 23. 27.

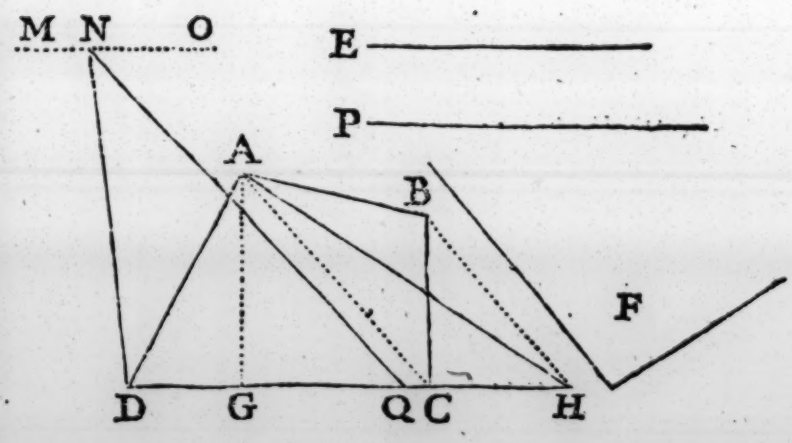


PROBLEME XCIII.

To reduce a Trapezium giuen into a Triangle, vpon a line giuen, and hauing an Angle equall to an Angle giuen.

Eue. I. 23.
I. 13.
I. 37.
I. 44.
6. 12.

LEt the Trapezium giuen be A. B. C. D. the giuen line E. and let the Angle giuen be F. It is required to reduce the ſame Trapezium into a Triangle, on the giuen line E. and hauing an Angle equall to the giuen Angle F. Firſt, increaſe the baſe D. C. at length towards H. then draw the line A. C. and from the point B. by the 3. PROB. draw the line B. H. parallel to the line A. C. to cut the increaſed line in H. and then draw the line A. H. which ſhall include the Triangle A. D. H. equall to the Trapezium giuen, but not hauing the Angle nor Baſe required; wherefore take the giuen Baſe E. and place it from D. to Q. and by the 16. PROB. reaſon thus; if D. Q. the giuen Baſe giues D. H. what giues A. G. the perpendicular; the anſwere will be the line P. for the perpendicular of the Triangle ſought, at which diſtance draw the line M. O. parallel to the line D. H. then vpon the point D. protract an Angle equall to the giuen Angle F. as N. D. Q. and draw the line D. N. to cut the parallel line in N. And laſtly, draw the line N. Q. which ſhall include the Triangle N. D. Q. equall to the Trapezium giuen, vpon a line giuen, and hauing an Angle equall to an Angle giuen, as was required. THEOREME 26. 42. 71. PROBLEME 3. 8. 16. 74.

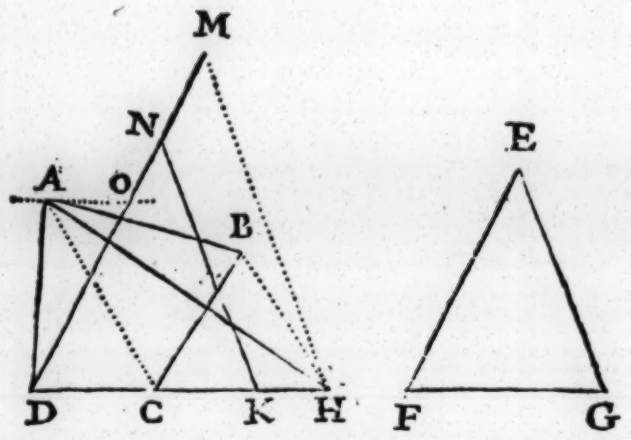


PROB.

PROBLEME. XCIIII.

To reduce a Trapezium into a Triangle, which ſhall be like vnto another Triangle giuen.

LEt the Trapezium giuen be A. B. C. D. and let the Triangle giuen be E. F. G. It is required to reduce the ſame Trapezium into a Triangle, which ſhall be like vnto the giuen triangle E. F. G. Firſt, by the 93. PROB. laſt before going; reduce the Trapezium giuen into the triangle A. D. H. which is equall thereunto, then by the 81. BROB. reduce the ſame triangle A.D.H. into the triangle N.D.K. which ſhall be equall to the Trapezium giuen, and like vnto the giuen triangle E.F. G. as was required. THEOR. 26. 43. PROB. 38. 16. 93.

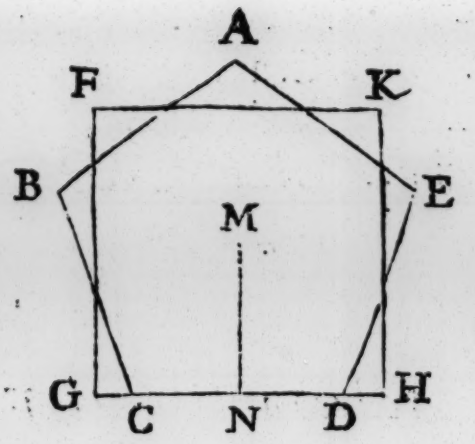


Euc. 1.37. 1, 23. *Cor.*6.19. *Ceul.*3.1.

PROBLEME XCV.

To reduce an equiangled Poligon giuen, into a Geometricall Square.

LEt A. B. C. D. E. be a Pentagonall Poligon giuen, to be reduced into a Geometricall ſquare. By the 17. PROB. find out the meane proportionall line betweene halfe the perimetrie of the giuen Poligon, and the perpendicular thereof M. N. being let fall from the Centre to the middle of any ſide, which meane proportionall is the line G.H. whereon by the 50. PROB. deſcribe the ſquare F.G.H.K. which ſhall be equall to the giuen Poligon, as was required. DEFINITION. 29. THEOREME 39. 41.



Euc. 2.14. *Ceul.*2.30.

This rule is generall for the reducing of all rectangle poligons.

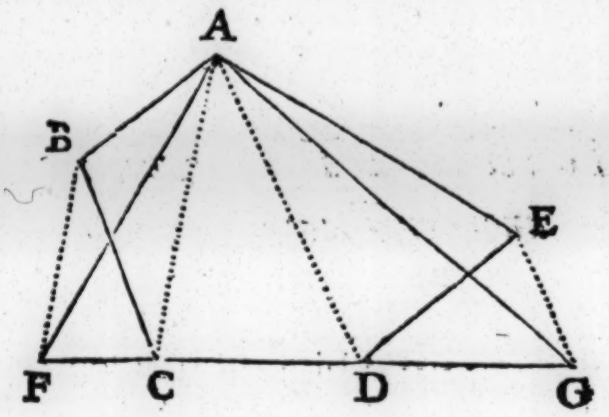
PROBLEME XCVI.

To reduce a plotte giuen into a Triangle, with lines drawne from an Angle aſſigned.

LEt A.B. C. D. E. be a plotte giuen, and let the Angle aſſigned be A. It is required from the Angle A. to reduce the ſame plot into a Triangle. Firſt, increaſe the ſide C. D. of the giuen plot, of conuenient

Cæterorum polygonorum rectangulorum reductio & demonſtratio huic eſt ſimilima: ſemper enim hac reductione ſigillatim vnum latus detrahitur.
Euc. 1.37.
Coul. 3.1.2.

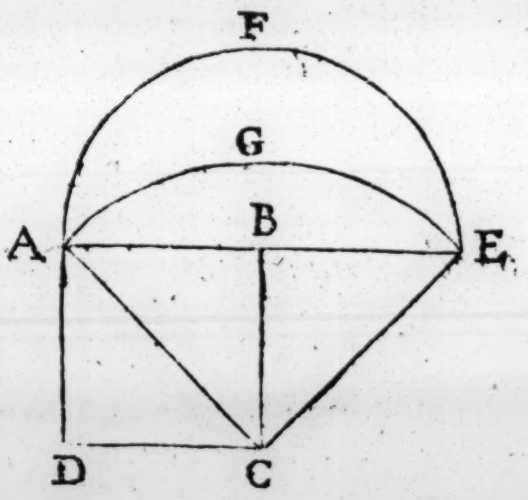
length both wayes towards F. and G. then draw the line A. D. and by the 3. PROB. by the point E. draw the line E. G. parallel to the line A. D. to cut the continued line in G. from which point, draw the line G. A. then draw the line A. C. and by the point B. make B. F. parallel to A. C. to cut the line of continuance in F. from which point, draw the line F. A. which ſhall include the Triangle A. F. G. equall to the giuen plot, with the lines A. F. and A. G. drawn from the Angle A. aſſigned as was required. THEOR. 26.



PROBLEME XCVII.

To reduce a Figure giuen, into a Lunula or Figure of a Lunular forme.

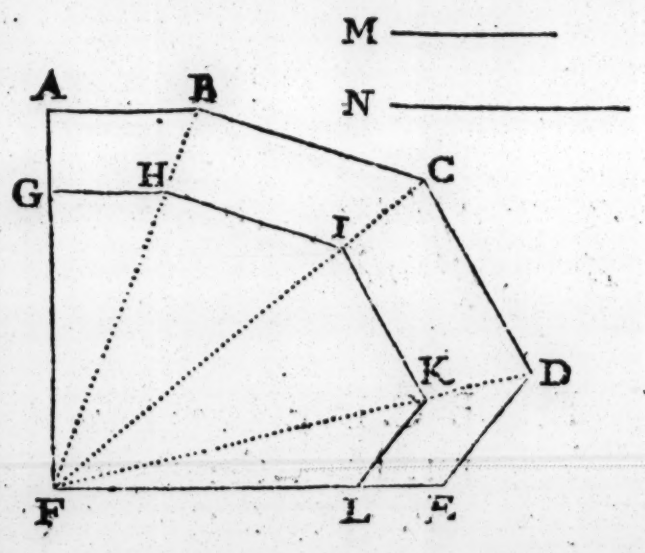
LEt A. B. C. D. be a ſquare giuen, and let it be required to reduce the ſame into a Lunula. Draw the Diagonall A. C. and on C. the end thereof by the 6. PROB. erect the perpendicular E. C. equall to A. C. then continue the ſide A. B. to E. and on the point B. at the diſtance B. A. or B. E. deſcribe the Semicircle A. F. E. And laſtly, on the point C. at the diſtance C. A. or C. E. deſcribe the Arch line A. G. E. which ſhal include the Lunula A. F. E. G. equall to the giuen ſquare, as was required.



PROBLEME XCVIII.

To reduce an irregular Figure giuen, into a greater or leſſer forme, according to any giuen proportion.

LEt A. B. C. D. E. F. be an irregular Figure giuen, and let the proportion giuen bee that betweene M. and N. It is required to reduce the ſame Figure into a leſſer, to be in ſuch proportion to that giuen as M. to N. Firſt reduce the giuen Figure into Triangles, by drawing right lines from any one angle, as from F. to all the oppoſite Angles, as B. C. and D. then by the

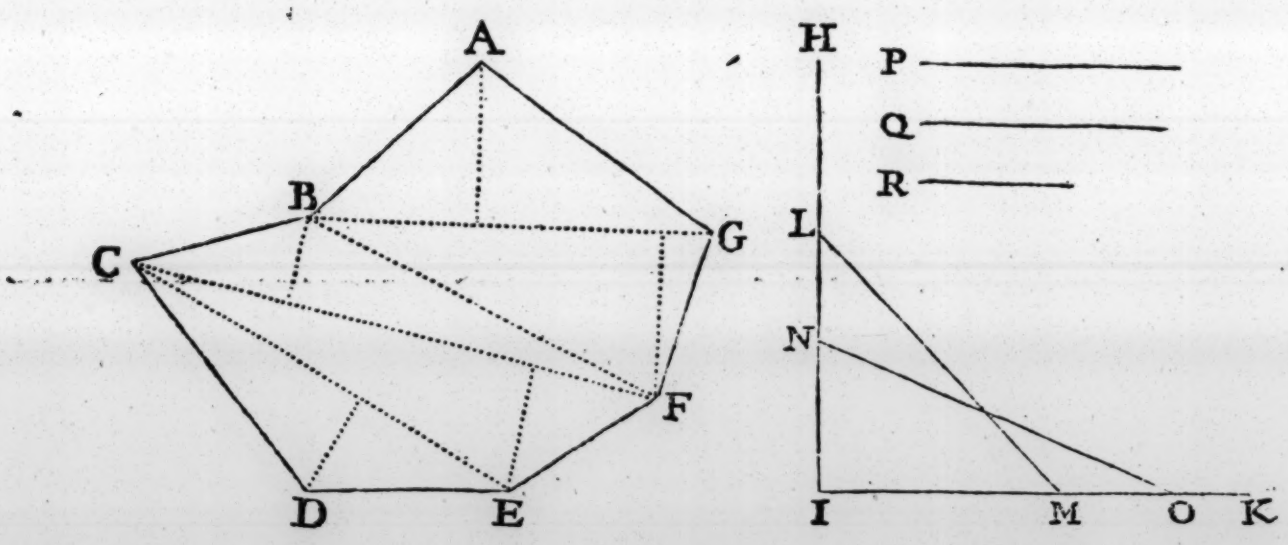


23.

23. PROB. diuide one of the ſides as F.E. in power, as M. to N. ſo that the power of F. L. may be to the power of F. E. as M. to N. Then by the point L. draw the line L. K. parallel to E. D. to cut F. D. in K. and in like ſort proceede with the reſt, as K. I. I. H. and H. G. drawing them parallel to their anſwerable ſides; ſo ſhall you include the Figure F. G. H. I. K. L. being like vnto the Figure giuen, and in proportion to it, as the line M. to N. as was required. But ſuppoſe the leſſer plot were giuen, and let it bee required to reduce the ſame into a greater, according to the proportion of N. to M. then increaſe all the lines from F. towards A. B. C. D. and E. and by the 24. PROBLEME inlarge the line F. L. in power as N. to M. which ſet from F. to E. and by the point E. draw the line E. D. parallel to L. K. to cut F. D. in D. and in like ſort proceede with the reſt. So ſhall you include the irregular Figure A. B. C. D. E. F. like to that giuen, and of the proportion, as was required. THEOREME. 22.

PROBLEME XCIX.

To reduce an irregular Figure giuen, into a Geometricall Square.



LEt A.B.C.D.E.F.G. bee an irregular Figure giuen, to bee reduced into a Geometricall Square. Firſt, draw the lines B. F. and C. E. diuiding the giuen Figure thereby into two Trapezias, and one Triangle, namely, A.B.F.G. B.F.E.C. and C.D.E. then croſſe thoſe Trapezias with the Diagonall lines B. G. and C. F. and let fall perpendiculars thereon from the Angles A. F. E. and B. and likewiſe from D. to the Baſe of the Triangle, then by the 17. PROBLEME finde out the meane proportionall, betweene halfe the Diagonall B. G. and the two perpendiculars thereon falling, which ſhall be the line P. Alſo the meane proportionall betweene halfe the Diagonall C. F. and the two perpendiculars thereon falling, which ſhall be the line Q. and likewiſe the meane proportionall betweene halfe the Baſe C. E. and the perpendicular thereon falling from D. which ſhall be the line R. then by the 9. PROBLEME deſcribe a right Angle at pleaſure, as H. I. K. and take the line P. and place the ſame, from I. to L. and alſo the line

Euc. 2. 14. 6. 13. 1. 47.

line Q. from I. to M. and draw the line L. M. which line place from I. to O. and alſo the line R. from I. to N. And laſtly, draw the line N.O. which ſhall be the ſide of a Square, equall to the giuen Figure as was required.
THEOREME 6. 25. 41. 30.

Here might I fitly inſert the Reduction and manner of tranſlation of large and ſpacious plots, from one Scale to another, diuers wayes, with many other works of this nature, fitte to bee knowne, which for ſome ſpeciall reaſons, I will referre vnto the later end of my next Booke. And in the meane ſpace will here conclude the third part of this Second Booke.

The

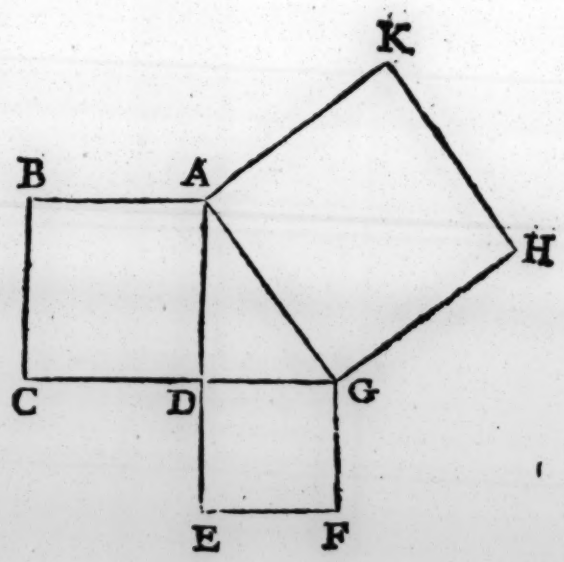
The fourth Part.

HOw diuers ſuperficiall figures, of ſeuerall formes, are brought into one figure, and one forme: Alſo to ſubtract one figure from another, and thereby to know how much the one exceedeth the other in quantitie; and likewiſe, hereby is taught, the inſcription and circumſcription of one figure within and without another; and the diuiſion and ſeparation of figures, into any parts required.

PROBLEME C.

Two Geometricall ſquares being giuen, to adde them together into one ſquare.

LEt the two giuen ſquares be A. B. C. D. and D. E. F. G. and let it be required to adde them together into one ſquare. Firſt take a ſide of either of the giuen ſquares, as A. D. and D. G. and by the 9. PROB. make thereof the right angle A. D. G. (as they are already placed in this Diagram) then draw the diagonal line A. G. and on that line by the 50. PROB. deſcribe the ſquare A. K. H. G. which ſhall be equall to the two giuen ſquares, as was required. THEOR. 6.

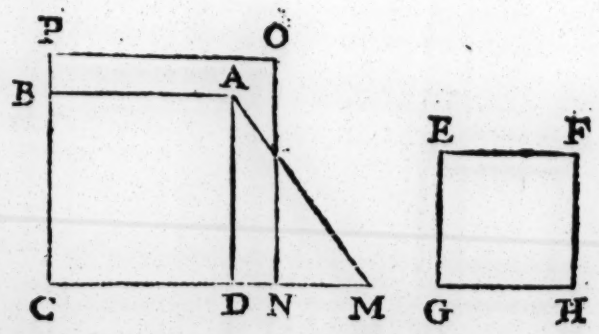


Euc. 1. 47.

PROBLEME CI.

Two Geometricall ſquares being giuen, to adde them together in ſuch ſort, as the one ſhall be a Gnomon vnto the other.

LEt the two giuen ſquares be A. B. C. D. and E. F. G. H. and let it be required to adde them together, in ſuch ſort that the ſquare E. F. G. H. ſhall be a Gnomon vnto the other. Increaſe the ſide C. D. of the greater ſquare to M. making D. M. equall to the ſide of the leſſer ſquare,



Euc. 1. 47.
Cor. 2. 38.

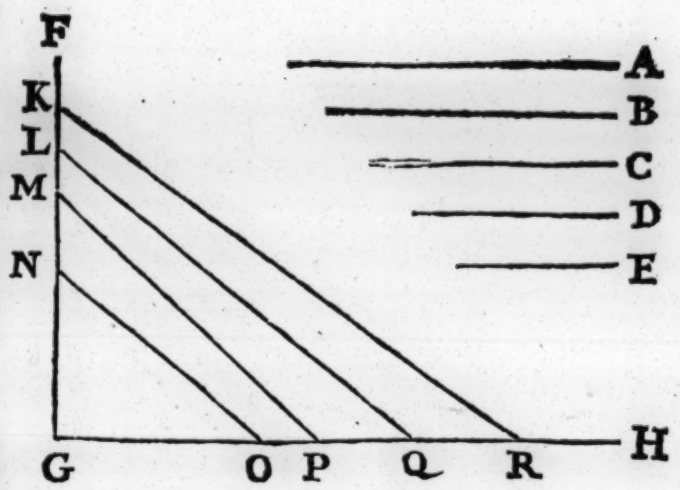
ſquare, then draw the ſubtending line A.M. which take and lay downe from C. to N. and thereon by the 50. PROB. deſcribe the ſquare O.P.C.N. which ſhall performe what was required. DEF. 30. THEOR. 6.

PROBLEME CII.

To adde diuers ſquares together into one geometricall ſquare.

EVC. 1. 47. 6. 31.

Diuers figures of what forme or kinde ſoeuer, being by the former rules reduced into ſquares, may hereby inſtantly be added together into one.

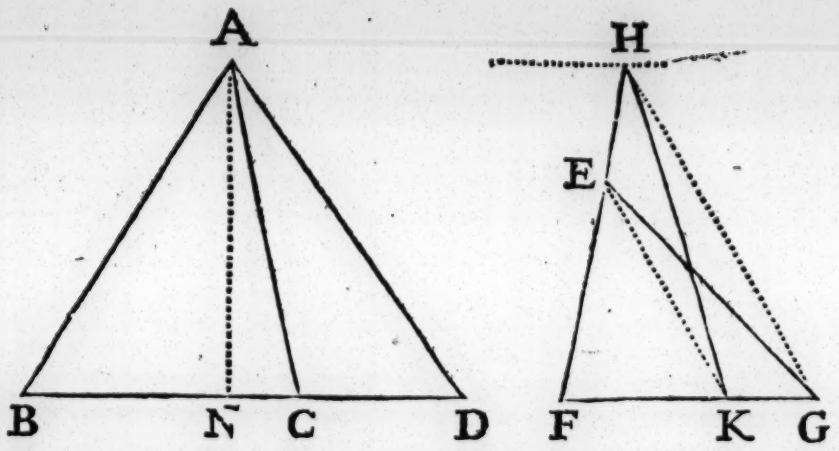


LEt A. B. C. D. and E. be the ſides of fiue ſquares giuen, and let it be required to adde them together into onegeometrical ſquare. Firſt, by the 9. PROB. make a right angle at pleaſure, as F. G. H. then (beginning with the leaſt ſides firſt) take the line E. and place it from G. to N. and the line D. from G. to O. and draw the line N.O. whoſe ſquare ſhall be equall to both the ſquares of E. and D. then take N. O. and place it from G. to P. and the ſide C. from G. to M. and draw the line M. P. which line place from G. to Q. and the ſide B. from G. to L. and draw the line L. Q. which line place from G. to R. and the line A. from G. to K. and laſtly, draw the line K. R. and thereon by the 50. PROB. deſcribe a ſquare which ſhall be equall to all the fiue ſquares, whoſe ſides were giuen, as was required. THEOR. 6. 23. 30. 31.

PROBLEME CIII.

To adde two giuen triangles together into one, which new compoſed triangle, ſhall haue his perpendicular, equall to that of one of the giuen triangles.

EVC. 1. 37. 1. 38.



LEt the two giuen triangles bee A. B, C. and E. F. G. and let it be required to adde thoſe 2. triangles into one, which ſhall haue his perpendicular equall to that of the triangle A. B. C. By the 75. PROB. reduce the triangle E. F. G. to the triangle H. F. K. of equall height to the other giuen triangle, then increaſe the baſe B. C. of the triangle A. B. C. from C. to D. making C. D. equall to F. K. the baſe of the reduced triangle H. F. K. and laſtly, draw the line A. D. which ſhall include the triangle A. B. D. equall to both the giuen triangles, and hauing the ſame perpendicular A. N. as the giuen triangle A. B. C. as was required. THEOR. 26. PROB. 28.

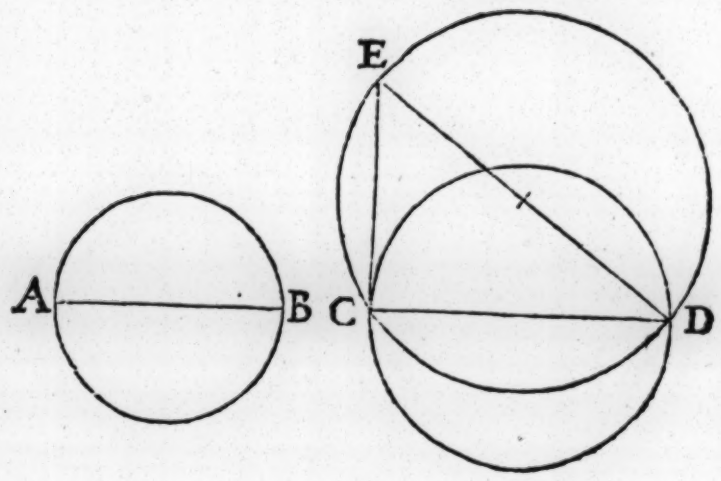
PRO-

PROBLEME CIIII.

Diuers Circles being giuen, to adde them together into one Circle.

LEt A. B. and C. D. bee two Circles giuen, and let it be required to adde them both into one Circle, or to make one Circle, which ſhall be equall to them both. Take the Diameter A.B. and by the 6. PROB. rayſe it perpendicularly on the end of the other Diameter C. D. as E. C. then draw the ſubtending line E. D. on which line as a Diameter deſcribe the Circle E. D. which ſhall bee equall to the two giuen Circles as was required. And in like ſort, by helpe of the 102. PROB. may be added as many Circles as ſhall be required; For Circles are added by their Diameters, as Squares by their ſides. THEOREME 6. 66.

Euc. 1. 47. 12. 2.

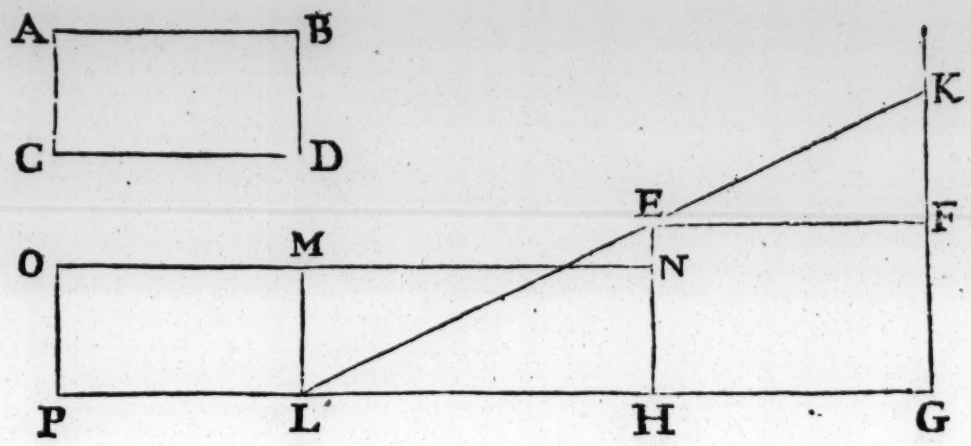


PROBLEME CV.

Two long Squares being giuen, to adde them together into one long Square, whoſe breadth ſhall be equall to that of one of the long Squares giuen.

Euc. 1. 43.

LEt the two giuen long Squares be A.B.C.D. and E. F. G. H. and let it bee required to adde thoſe two Squares together into one Square, whoſe breadth ſhall be equall to the giuen long



Square A.B.C.D. Firſt, increaſe the ſide G. F. of the greater long Square giuen towards K. making F. K. equall to the breadth of the leſſer giuen Square, and ſo working on by the 87. PROB. reduce the giuen long ſquare E. F. G. H. into the long ſquare M. N. H. L. then on the Line M. L. by the 54. PROB. deſcribe the long ſquare O. M. L. P. equall to the giuen long ſquare A. B. C. D. So ſhall the long Square O. N. H. P. bee equall to the two

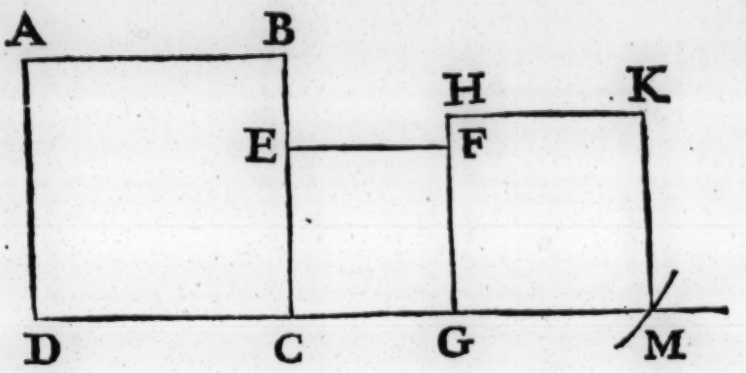
two giuen long Squares, and the breadth thereof O. P. or N. H. equall to the breadth of the leſſer long Square giuen, as was required. THEOREME. 5.

PROBLEME CVI.

Two Geometricall Squares being giuen, to ſubtract the one out of the other, and to produce the remainder in a third Square.

Euc. 1. 47. 6. 31.

LET the two giuen Squares be A.B.C.D. and E.F.G.C. and let it be required to ſubtract the leſſer out of the greater, and to produce the remainder in a third ſquare. Continue out at length the ſide C.G. of the leſſer giuen ſquare towards M. and at the diſtance of the ſide of the greater giuen Square with one foot in the Angle F. of the leſſer ſquare ſtrike an Arch line through the line of continuance in M. And laſtly, by the 50. PROB. on the line G.M. deſcribe the Square H.K.M.G. which ſhall be the remainder of the greater giuen Square, the leſſer being ſubtracted from the ſame, as was required. THEOREME. 6. 23. 30. 31.

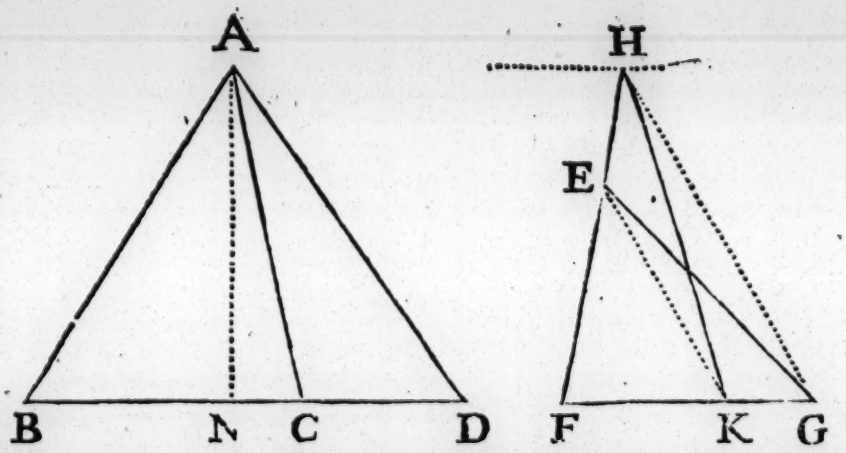


PROBLEME CVII.

Two Triangles being giuen to ſubtract the one out of the other, and to leaue the remainder in a Triangle of equall height to one of the giuen Triangles.

Euc. 1. 37. 1. 38.

LEt the two Triangles giuen be A.B.D. and E.F.G. And let it be required to ſubtract the Triangle E.F.G. out of the Triangle A.B.D. and to leaue the remainder in a Triangle of equall height to the Triangle A.B.D. By the 75. PROB. reduce the Triangle E.F.G. to the triangle H.F.K. of equall height to the other giuen triangle, then take the Baſe F.K. of the ſame reduced triangle, and place it from D. to C. And laſtly, draw the line A.C. which ſhall include and ſubtract the Triangle A.C.D. (equall to the leſſer giuen triangle E.F.G) from the

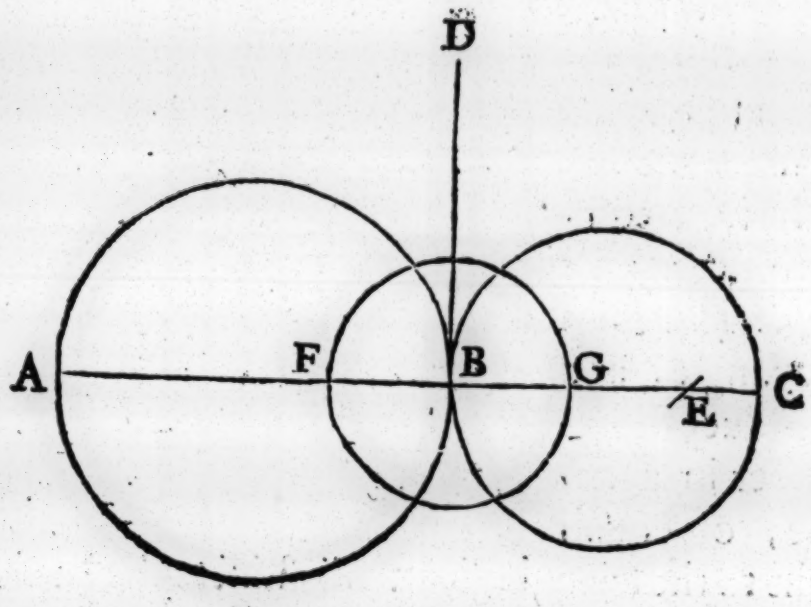


the greater giuen triangle A. B. D. and leaue the remainder in the triangle A. B. C. of the ſame height of the giuen triangle A. B. D. as was required, THEOR. 26. PROB. 28.

PROBLEME CVIII.

Two circles being giuen, to ſubtract the one out of the other, and to produce the remainder in a third circle.

LEt the two circles giuen be A. B. and B. C. and let it be required to ſubtract the circle B. C. out of the circle A. B. and to produce the remainder in a third circle. Take the diameter B. C. and by the 6. PROB. raiſe it perpendicularly on the point B. as D. B. then at the diſtance of the diameter A. B. with one foot in D. ſtrike an arch through the diameter B. C. in E. then at halfe the diſtance of B. E. deſcribe the circle F. G. on the point B. So haue you ſubtracted the circle B. C. out of the circle A. B. and produced the remainder in a third circle F. G. as was required. THEOR. 6, 66.

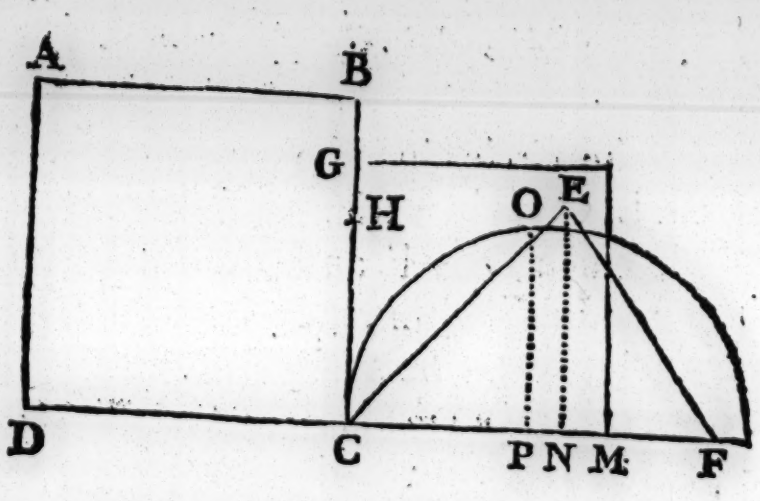


Euc. 1. 47. 12. 2.

PROBLEME CIX.

A geometricall ſquare and a triangle being giuen, to ſubtract the triangle from the ſquare, and to produce the remainder in a ſquare.

LEt the Geometricall ſquare giuen be A. B. C. D. and let the triangle giuen be E. C. F. and let it be required to ſubtract the triangle from the ſquare, and to produce the remainder in a ſquare. By the 17. PROB. find out the meane proportionall, betweene the perpendicular of the giuen triangle E. N. and halfe the baſe C. F. which ſhall be the line O. P. which place from C. to H. on which point H. at the diſtance of the ſide of the giuen ſquare, ſtrike an arch through the baſe C. F. as at M. and laſtly, vpon the line C. M. deſcribe by the 50. PROB. the ſquare G. K. M. C. So haue you ſubtracted the giuen



Eu. 1. 47. 2. 14. 6. 18.

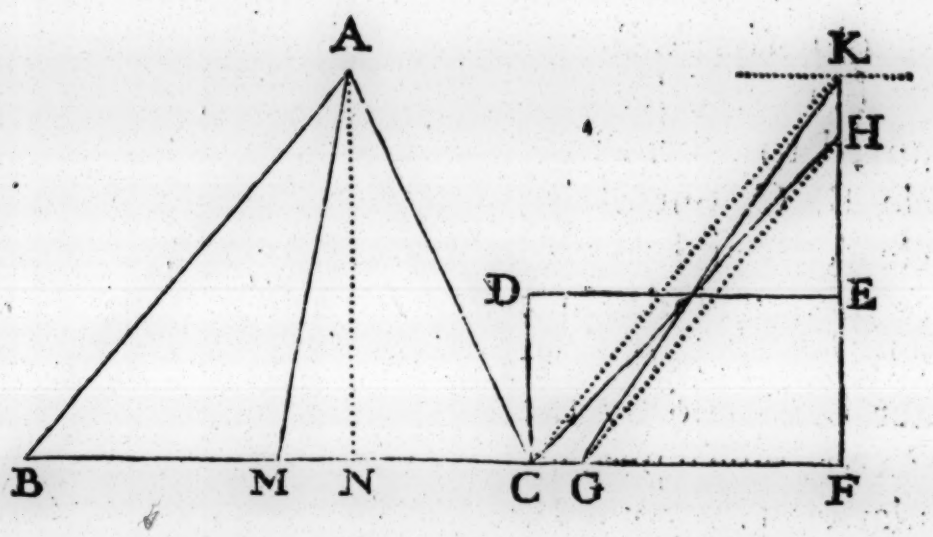
triangle E.C.F. from the giuen ſquare A. B. C. D. and produced the remainder in the ſquare G. K. M. C. as was required. THEOREME 6, 41, 70.

PROBLEME CX.

A triangle and a long ſquare being giuen, to ſubtract the long ſquare from the triangle, and to produce the remainder in a triangle of equall height, to the triangle giuen.

Euc. 1. 38.
1. 41.
6. 1.

LEt the giuen triangle bee A.B.C. and the long ſquare giuen D. E. F. C. and let it berequired to deduct the long ſquare from the triangle, and to produce the remainder in a triangle, of equall



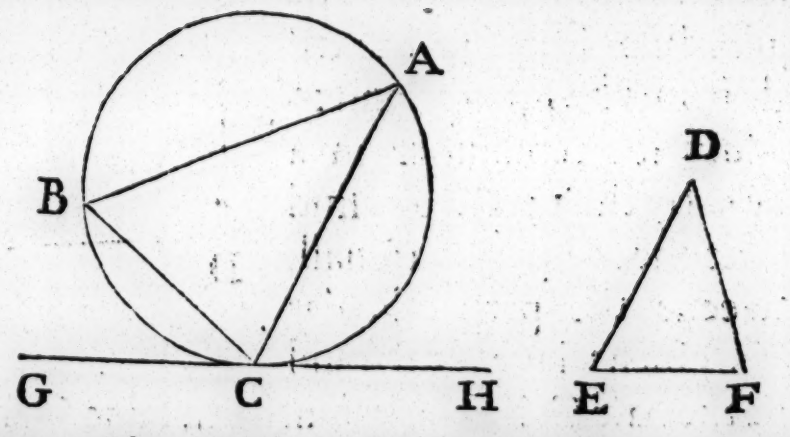
height to the triangle giuen. Firſt, increaſe the ſide F. E. of the long ſquare towards K. and placing E. F. from E. to H. draw the line H. C. including the triangle H. C. F. equall to the long ſquare giuen; which triangle by the 75. PROB. reduce into the triangle K. G. F. of equall height to the giuen triangle A.B.C. then take the baſe G.F. of the laſt found triangle, and place it from C. to M. on the baſe of the giuen triangle; and laſtly, draw the line A. M. to include and ſubtract the triangle A.M.C. (equal to the giuen long ſquare) from the giuen triangle A.B.C. & producing the remainder in the triangle A. B.M of equall height to the triangle giuen, as was required. THEOR. 26, 27, 28, 35.

PROBLEME CXI.

Within a circle giuen to inſcribe a triangle, with angles equall, and lines proportionall, to a triangle giuen.

Euc. 3. 32.
4. 2.
Ceul. 2. 53.

LEt the circle giuen be A. B. C. and the triangle giuen D. E. F. and let it be required within the giuen circle to inſcribe a triangle, with equal angles and lines proportionall to the giuen triangle. By the 31. PRO. draw the giuen circle, the



right line G. H. touching the ſame in the point C. vpon which point vnto the line C. H. deſcribe the angle A. C. H. equall to the angle D. E. F. and likewiſe on the ſame point, to the line G. C. deſcribe the angle B. C. G. equall to the

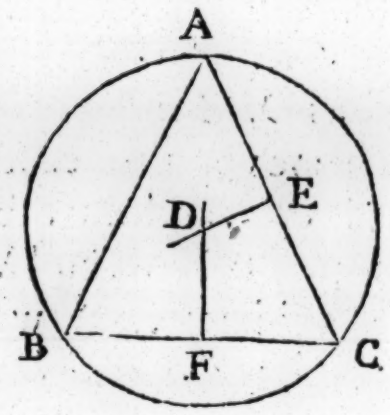
the angle E. D. F. and laſtly, draw the line B. A. which ſhall include the triangle A. B. C. inſcribed within the giuen circle, with equall angles and lines, proportionall to the triangle giuen, as was required. THEOR. 56, 13.

PROBLEME CXII.

To deſcribe a Circle about a Triangle giuen.

LEt the Triangle giuen be A. B. C. about which it is required to deſcribe a Circle, vpon the middle point of any two ſides of the Triangle by the 5. PROB. erect perpendiculars, which being produced will interſect each other, as E. D. and F. D. in the point D. which point of interſection ſhall be the Centre; whereupon at the diſtance from thence to any one of the Angles deſcribe the Circle A. B. C. which ſhall circumſcribe the Triangle as was required. DEF. 32.

Euc. 4. 5.



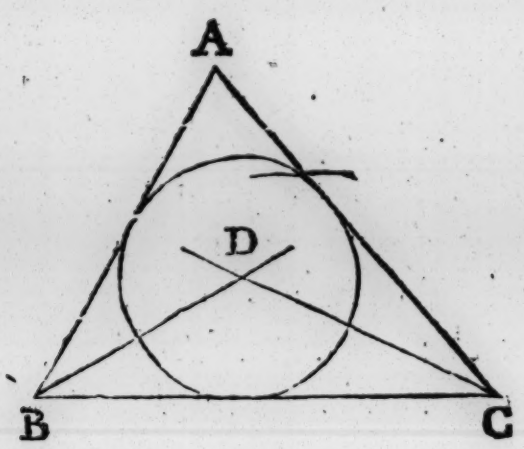
Or otherwiſe the Centre may bee found as is taught in the 34. PROB.

PROBLEME CXIII.

To inſcribe a Circle within a Triangle giuen.

LEt the Triangle giuen be A. B. C. within which Triangle it is required to inſcribe a Circle. By the 10. PROB. diuide any two Angles of the giuen Triangle into two equall parts as A. B. C. and A. C. B. by drawing the lines B. D. and C. D. interſecting in D. which point of interſection ſhall be the Centre, whereon at the neereſt diſtance from thence to any ſide of the Triangle deſcribe the circle D. which ſhall be inſcribed within the Triangle, as was required. DEF. 32. PROB. 10.

Euc. 4. 4:
Ceul. 2. 55.



PROBLEME CXIIII.

To deſcribe a Triangle about a Circle giuen, which ſhall be like vnto a Triangle giuen.

LEt the Circle giuen be H. K. M. and let the Triangle giuen be E. F. G. It is required to deſcribe a triangle about the giuen Circle, like vnto the giuen triangle. Firſt, continue the Baſe F. G. of the giuen

Euc. 4. 3:
Ceul. 2. 54.

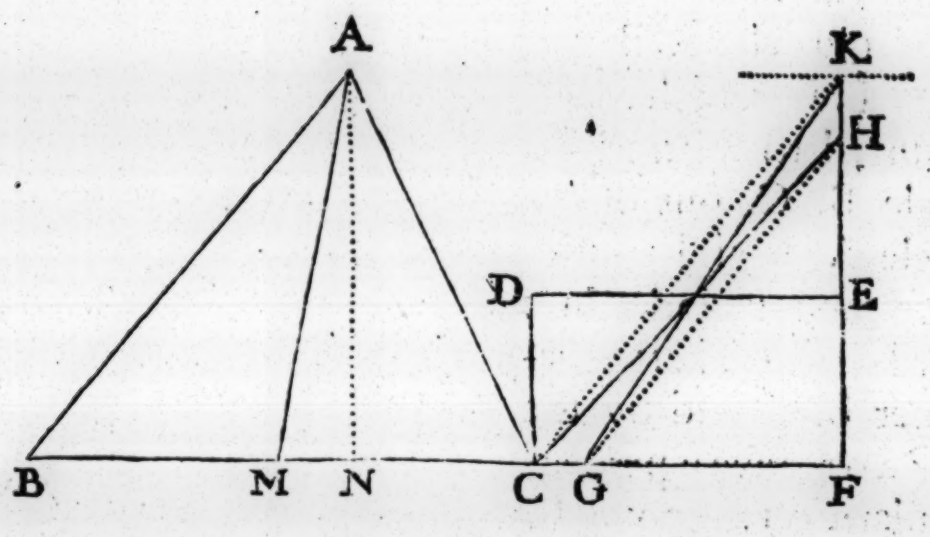
triangle E.C.F. from the giuen ſquare A. B. C. D. and produced the remainder in the ſquare G. K. M. C. as was required. THEOREME 6, 41, 70.

PROBLEME CX.

A triangle and a long ſquare being giuen, to ſubtract the long ſquare from the triangle, and to produce the remainder in a triangle of equall height, to the triangle giuen.

Euc. 1. 38.
1. 41.
6. 1.

LEt the giuen triangle bee A.B.C. and the long ſquare giuen D. E. F. C. and let it berequired to deduct the long ſquare from the triangle, and to produce the remainder in a triangle, of equall height to the triangle giuen. Firſt, increaſe the ſide F. E. of the long ſquare towards K. and placing E. F. from E. to H. draw the line H. C. including the triangle H. C. F. equall to the long ſquare giuen; which triangle by the 75. PROB. reduce into the triangle K. G. F. of equall height to the giuen triangle A.B.C. then take the baſe G.F. of the laſt found triangle, and place it from C. to M. on the baſe of the giuen triangle; and laſtly, draw the line A. M. to include and ſubtract the triangle A.M.C. (equal to the giuen long ſquare) from the giuen triangle A.B.C. & producing the remainder in the triangle A. B.M of equall height to the triangle giuen, as was required. THEOR. 26, 27, 28, 35.

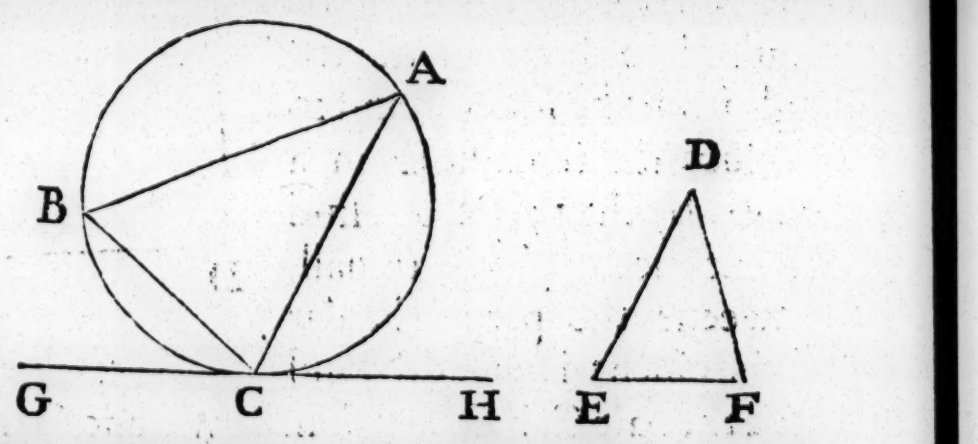


PROBLEME CXI.

Within a circle giuen to inſcribe a triangle, with angles equall, and lines proportionall, to a triangle giuen.

Euc. 3. 32.
4. 2.
Cenl. 2. 53.

LEt the circle giuen be A. B. C. and the triangle giuen D. E. F. and let it be required within the giuen circle to inſcribe a triangle, with equal angles and lines proportionall to the giuen triangle. By the 31. PRO. draw the giuen circle, the right line G. H. touching the ſame in the point C. vpon which point vnto the line C. H. deſcribe the angle A. C. H. equall to the angle D. E. F. and likewiſe on the ſame point, to the line G. C. deſcribe the angle B. C. G. equall to the



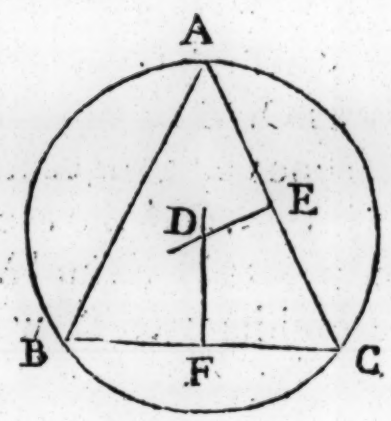
the angle E. D. F. and laſtly, draw the line B. A. which ſhall include the triangle A. B. C. inſcribed within the giuen circle, with equall angles and lines, proportionall to the triangle giuen, as was required. THEOR. 56, 13.

PROBLEME CXII.

To deſcribe a Circle about a Triangle giuen.

LEt the Triangle giuen be A. B. C. about which it is required to deſcribe a Circle, vpon the middle point of any two ſides of the Triangle by the 5. PROB. erect perpendiculars, which being produced will interſect each other, as E. D. and F. D. in the point D. which point of interſection ſhall be the Centre; whereupon at the diſtance from thence to any one of the Angles deſcribe the Circle A. B. C. which ſhall circumſcribe the Triangle as was required. DEF. 32.

Euc.4.5.

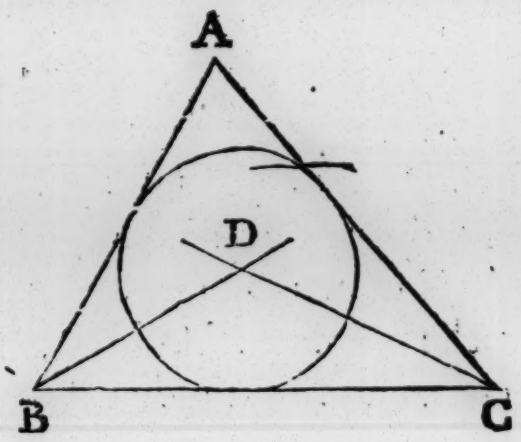


Or otherwiſe the Centre may bee found as is taught in the 34. PROB.

PROBLEME CXIII.

To inſcribe a Circle within a Triangle giuen.

LEt the Triangle giuen be A. B. C. within which Triangle it is required to inſcribe a Circle. By the 10. PROB. diuide any two Angles of the giuen Triangle into two equall parts as A. B. C. and A. C. B. by drawing the lines B. D. and C. D. interſecting in D. which point of interſection ſhall be the Centre, whereon at the neereſt diſtance from thence to any ſide of the Triangle deſcribe the circle D. which ſhall be inſcribed within the Triangle, as was required. DEF. 32. PROB. 10.



Euc.4.4:
Ceul.2.55.

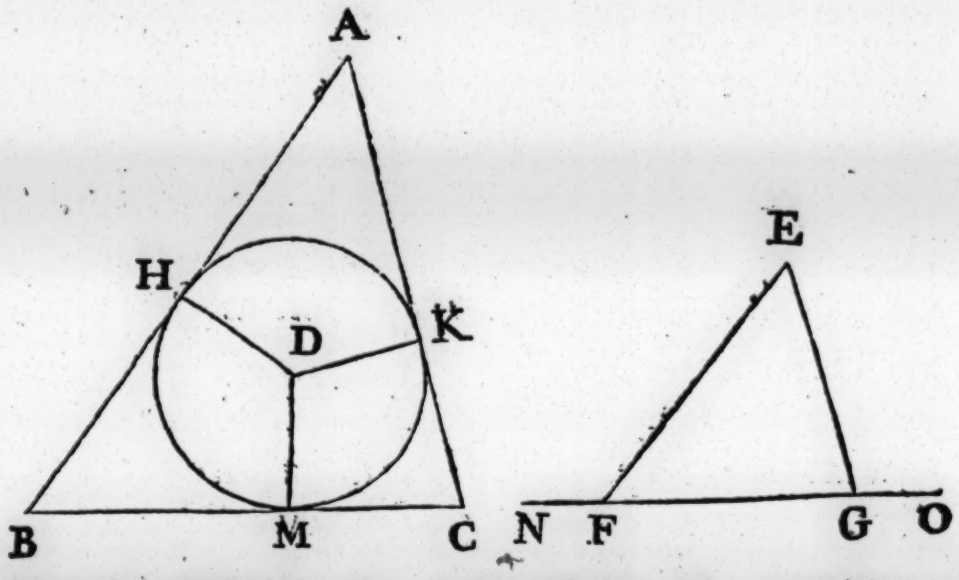
PROBLEME CXIIII.

To deſcribe a Triangle about a Circle giuen, which ſhall be like vnto a Triangle giuen.

LEt the Circle giuen be H. K. M. and let the Triangle giuen be E. F. G. It is required to deſcribe a triangle about the giuen Circle, like vnto the giuen triangle. Firſt, continue the Baſe F. G. of the giuen tri-

Euc.4.3:
Ceul.2.54.

Triangle both wayes towardes N. and O. making the two outward Angles E. F. N. and E. G. O. then from the centre D. of the giuen circle draw to any part of the lymbe a ſemidiameter as D. M. to



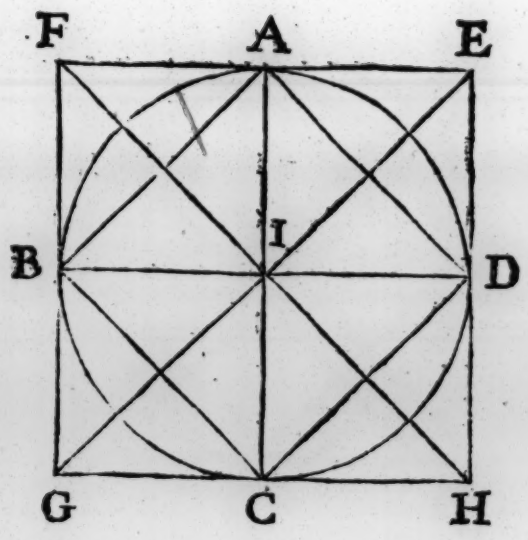
which line vpon the centre D. deſcribe the angle H. D. M. equall to the angle E.F.N. and alſo the Angle K.D.M. equall to the angle E.G. O. then by the points H. K. and M. draw right lines, (making right angles with the three ſemidiameters) which will interſect each other in the points A.B.and C. including the triangle A. B. C. like vnto the giuen triangle, and circumſcribed about the giuen circle, as was required. THEOR. 1. 13. 52.

PROBLEME CXV.

To deſcribe a Square about a Circle giuen.

Euc.4.7. Cent. 2. 57.

LEt the circle giuen be A. B.C. D. and let it be required to deſcribe a ſquare about the ſame circle. Draw the two Diameters A. C. and B. D. cutting each other at right angles in the centre 1. then by the 3. PROB. by the points A. and C. draw parallel lines to the Diameter B. D. And likewiſe by the points B. and D. draw parallel lines to the Diameter A. C. which ſhall interſect each other in the points E.F.G.H. and include the ſquare as was required. THEOR. 32. 52.



PROBLEME CXVI.

Within a Square giuen to inſcribe a Circle:

LEt the ſquare giuen be E.F.G.H. in the former Diagram, within which it is required to inſcribe a circle. Draw the Diagonall lines E. G. and F. H. interſecting each orher in 1. the center, on which point at the diſtance of the ſhorteſt extention to any ſide, deſcribe the circle A B C D. within the ſquare as was required. THEOR. 32. 52.

PROB.

PROBLEME CXVII.

About a Square giuen to circumscribe a Circle.

LEt the Square giuen be A.B.C.D. in the former Diagram, about which it is required to circumscribe a Circle. Draw the Diagonall lines A.C. and B. D. interfecting each other at right Angles in the point I. the centre, on which point, at the distance from thence to any of the points A. B. C. or D. describe the Circle A.B.C.D. which shall circumscribe the Square as was required. THEOREME 1.13.52. *Euc.4.9.*

PROBLEME CXVIII.

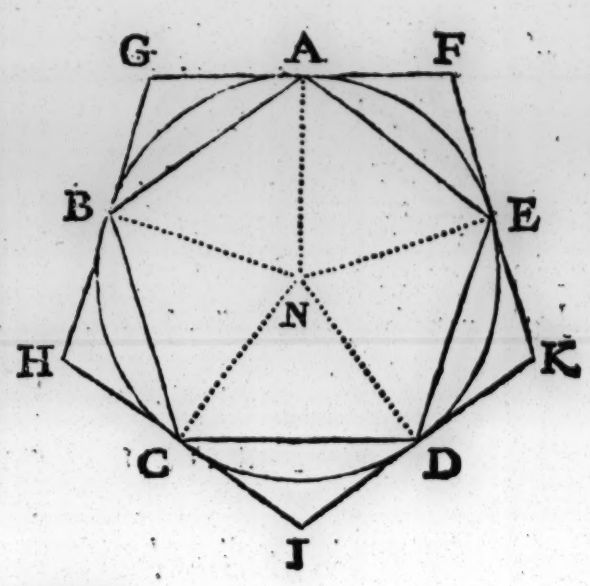
To inscribe a Square within a Circle giuen.

LEt the Circle giuen be A. B. C. D. in the former Diagram, and let it be required to inscribe a square within the same Circle. Draw the Diameters A. C. and B. D. crossing each other at right Angles in the centre I. then betweene the points A.B.C. and D. draw the right lines A B. B C. C D. and D A. including and inscribing the square A. B. C. D. within the giuen Circle as was required. THEOREME. I. 13. 52. PROBLEME 25. *Euc. 4.6.*

PROBLEME CXIX.

To inscribe a Pentagon, within a Circle giuen.

LEt A.B. C.D. E. be a Circle giuen, within which it is required to inscribe a Pentagon. By the 25. PROBLEME Finde out the fift corde line of a Circle, at which distãce passing through the limbe of the circle note fiue marks as at the points A. B. C. D. and E. And lastly, frõ point to point draw fiue right lines, which shall include and inscribe the Pentagon A.B.C.D.E. as was required. THEOR. 47. 69. PROB. 48. *Euc.4.11. Ceul.2.58.*



PROBLEME CXX.

About a Circle giuen to circumscribe a Pentagon.

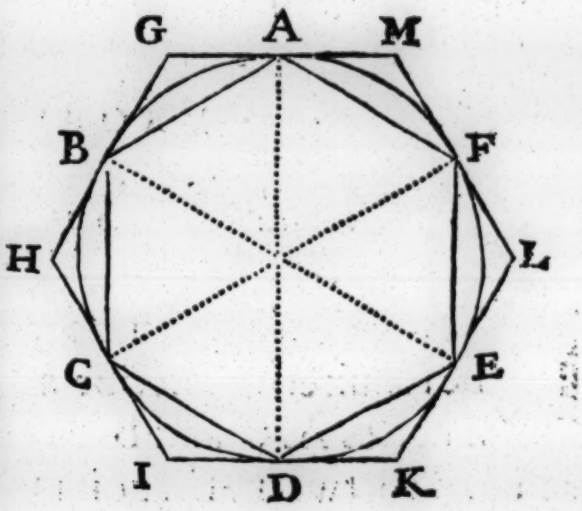
LEt the Circle giuen be A.B.C.D. in the former Diagram, about which, it is required to circumscribe a Pentagon. Let first a Pentagon be inscribed as before, and from the Centre N. draw right lines to euerie Angle

*Eu.*4.12. Angle of the inſcribed Pentagon, as to A.B.C. D. & E. on which fiue points, draw lines Orthigonally to thoſe lines iſſuing from the centre, which will interſect each other in the points F. G. H. I. K. and circumſcribe the Pentagon about the giuen circle as was required. THEOR. 47. 69.

PROBLEME CXXI.

To inſcribe a Sexagon within a circle giuen.

*Eu.*4.15. *Ceul.*2.63. LEt the circle giuen be A. B. C. D. E. F. within which it is required to inſcribe a Sexagon. By the 25. PROB. finde out the ſixt corde of a Circle, which is alwayes the ſemidiameter of the ſame circle; wherefore at the diſtance of the ſemidiameter; Diuide the limbe of the circle into ſixe equall partes, as in the points A.B.C.D.E. and F. and then from point to point, draw right lines, which ſhal include and inſcribe the Sexagon A.B. C. D.E.F. within the giuen Circle, as was required. PROB. 25.



PROBLEME CXXII.

To circumſcribe a Sexagon about a Circle giuen.

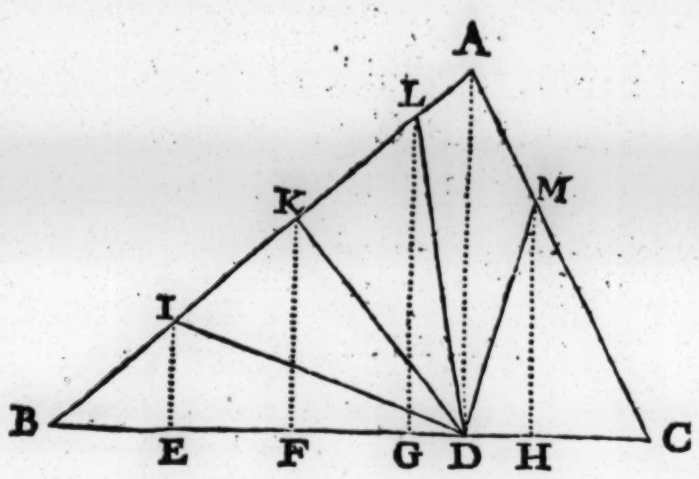
LEt the Circle giuen be A.B.C.D.E.F. in the former Diagram, about which it is required to circumſcribe a Sexagon. Firſt, diuide the limbe of the Circle into ſixe parts, as was taught in the laſt PROB. in the the points A. B. C. D. E. and F. and draw Diameters from each oppoſite point to other, making in all three Diameters, then at euery end of thoſe Diameters draw lines Orthigonally vnto them, which will interſect each other in the points G.H.I. K. L. and M. and include the Sexagon circumſcribed about the giuen circle as was required. PROB. 25.

PROBLEME. CXXIII.

To diuide a right lined Triangle giuen, into any number of equall parts required, from a point limited in any ſide of the ſame Triangle.

LEt A. B. C. be a right lined Triangle giuen; let the limited point be D. in the Baſe B. C. and let it be required from the ſame limited point D. to diuide the giuen triangle into fiue equall parts. Firſt, diuide the Baſe B. C. into fiue equall parts, as in the points E. F. G. and H. (or into as

as many as ſhall bee required) then from the limited point to the oppoſite angle, draw a right line as A. D. vnto which line, by the points E.F.G. and H. draw Parallel lines, as E I. F K. G L. and H M. And laſtly from the points I. K. L. and M. to the limited point D. draw the lines I D. K D. L D. and M D. which ſhall diuide the giuen triangle into

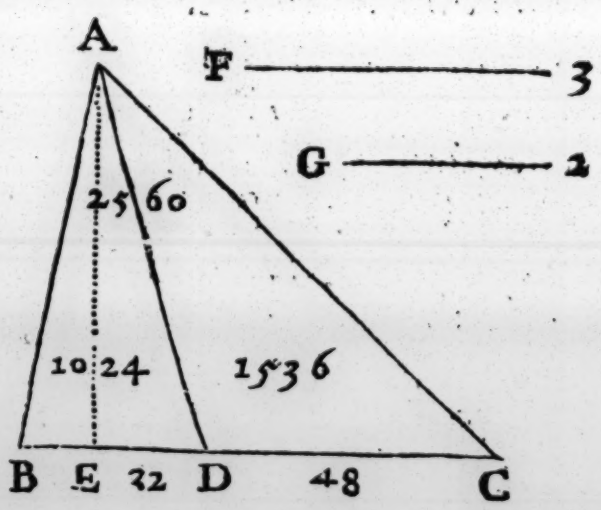


fiue equall parts from the limited point D. as was required. THEOREME. 26. 35.

PROBLEME CXXIIII.

To diuide a giuen Triangle by a line iſſuing from an angle aſſigned, in any proportion required.

LEt the Triangle giuen be A.B.C. the angle aſſigned A. and the proportion that betweene the two right lines F. and G. It is required to diuide the ſame triangle into two parts by a line iſſuing out of A. the one part hauing proportion to the other, as F. to G. By the 13. PROB. diuide the Baſe B C. as F. to G. the point of which diuiſion will fall in D. From which point draw the line D. A. So is the giuen Triangle diuided into two parts, hauing proportion the one to the other, as F. to G. For as the line F. is to the line G. ſo is the triangle A. C. D. to the Triangle A. B. D. as was required. THEOR. 35. PROB. 13.



Euc. 6. 1.
Ram. 10. 13
Ceul. 3. 8.

A. E. 64.
B. C. 80.

PROBLEME. CXXV.

A Triangle being giuen, and the Baſe thereof known, to diuide the ſame into two parts by a line from an angle aſſigned, according to any proportion giuen in two numbers.

LEt the triangle giuen be A.B. C. in the former Diagram, whoſe Baſe is B.C. 80. and the angle aſſigned A. and let it be required to diuide the ſame into two parts, the one in proportion to the other *Seſquialtera*, that

that is, as 3. to 2. Firſt, adde together alwayes the *Termini rationis*, which is $\frac{1}{2}$. makes 5. then multiply the Baſe 80. by 3. the greater terme, the *factus* will be 240. which diuided by the former 5. quoteth 48. for the greater ſegment of the Baſe D C. and that deducted from 80. the whole Baſe reſteth 32. the leſſer ſegment B D. from which point D. draw the line D A. which ſhall diuide the giuen Triangle into two parts, as 3. to 2. that is, the greater A C D. ſhall be $\frac{3}{5}$. parts, and the leſſer A B D. $\frac{2}{5}$. of the whole giuen Triangle, as was required. THEOR. 35.

Eucl.6.1. *Ra.10.13.*

PROBLEME CXXVI.

To diuide a Triangle of knowne quantitie giuen, into any two parts, from an Angle aſsigned, according to any number of Acres, Roodes and Perches required.

LEt the Triangle be A B C. in the former Diagram, whoſe quantitie is 16. Acres; let the Angle aſſigned be A. and let it be required to diuide the ſame into two parts twixt H. and I. viz. to H. 9. Acres. 2. Roodes, 16. Perches: and to I. 6. Acres, 2. Roodes, 24. Perches. Firſt, meaſure the Baſe B C. 80. Perches; then reduce the whole quantitie of the Triangle into Perches, by multiplying 160. (the number of Perches in an Acre) by 16. (the numbet of Acres in the Triangle giuen) the Product will be 2560. Reduce alſo one of the parts into Perches, as the greater part 9. Acres, 2. Roodes, 16. Perches; the reduction will be 1536. Perches. And then by the rule of proportion reaſon thus. If 2560. (the whole quantitie) require 80. (the whole Baſe; what part of the Baſe will 1536. Perches (being the part of H.) require. Multiply 1536. by 80. and diuide the Product by 2560. the *quotus* will be 48. the part of the Baſe ſought, which placed from C. to D. and the line D A. drawne, will diuide the giuen Triangle into the parts required. THEOR. 35.

Eucl.6.1. *Ra.10.13.*

Or to lay out one of the parts giuen (whereby the other is knowne) worke thus.

LEt it be required to lay out the leſſer part, which containeth 1024. perches, diuide that number 1024. by halfe, the perpendicular A E. 32. the *quotus* will be 32. for the part of the Baſe to be cut off for that part; which placed from B. to D. and the line A. D. drawn, performeth the worke. Or double the quantitie giuen 1024. and diuide that double by the whole Baſe 64. the worke will be the ſame.

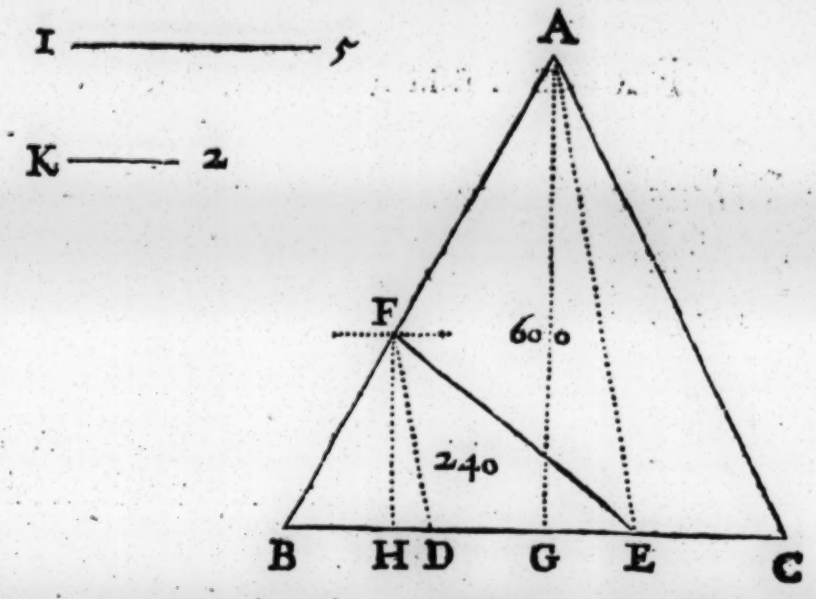
PROBLEME CXXVII.

To diuide a Triangle giuen into two parts by a line drawn, from a point limited in one of the ſides, in any proportion required.

LEt A B C. be the Triangle giuen, the point limited E. in the Baſe B C. and let the proportion be that betweene I. and K. It is required from the point E. to draw a line, which ſhall diuide the ſame Triangle into

two

two parts, hauing proportion the one to the other, as I. to K. Firſt, from the giuen point E. to the oppoſite angle A. drawe a right line, as A. E. then by the 13. PROB. diuide the Baſe B. C. as I. to K. the point of which diuiſion will fall in D. from whence draw the line D F. parallel to A. E. cutting the ſide A B. in F. And laſtly, from thence draw a right line to the limited point E. as F E. which ſhall diuide the giuen Triangle into two parts, and include and ſeparate the Triangle F E B. from the Trapezium A F. E C. hauing ſuch proportion one to the other, as the two giuen lines I. and K. as was required. THEOR. 35.



A G. 40. *Euc.6.1. Ram. 10.13 Ceul.3.9.*

B C. 42.

B D. 12.

D C. 30.

B E. 30.

F H. 16.

PROBLEME CXXVIII.

To diuide a Triangle giuen into two parts, according to any proportion giuen in two numbers from a point limited in any ſide thereof, Arithmetically.

LEt the Triangle giuen bee A B C. in the former Diagram. Let the point limited be E. in the Baſe B C. and let it be required to diuide the ſame into two parts, in proportion one to the other, *dupla ſeſquialtera*, which is, as 5. to 2. Firſt, let the Baſe B. C. be diuided according to the proportion giuen thus; Adde together the two giuen termes of the proportion 5. and 2. makes 7. then multiply the Baſe of the Triangle B. C. 42. by 2. the leſſer terme, the Product is 84. which diuided by 7. the ſumme of the termes quoteth 12. for the leſſer Segment of the Baſe B. D. which deducted from 42. the whole Baſe reſteth 30. the greater ſegment D. C. then (conſidering the leſſer part is to bee layde towards B.) meaſure the diſtance from the giuen point E. to B. which admit 30. and by the rule of Proportion reaſon thus: If the diſtance E. B. 30. giues B. D. 12. the leſſer ſegment what giues A G. 40. the perpendicular of the giuen Triangle, and multiplying 40. by 12. and diuiding the Product by 30. the anſwere will be 16. at which diſtance draw a parallel line to B E. cutting the ſide A B. in F. from which point draw the line F E. which ſhall diuide the giuen Triangle in ſuch ſort that the Triangle F B E. ſhall be $\frac{2}{7}$. parts, and the Trapezium A F. E C. $\frac{5}{7}$. parts thereof, that is the Trapezium, containing the Triangle F B E. twice and a halfe, according to the proportion required. THEOR. 35.

Euc.6.1. Ram.10.13.

PROB.

PROBLEME CXXIX.

To diuide a Triangle of any knowne quantitie giuen, into two parts, from a point limited in any side thereof, according to any number of Acres, Roodes and Perches.

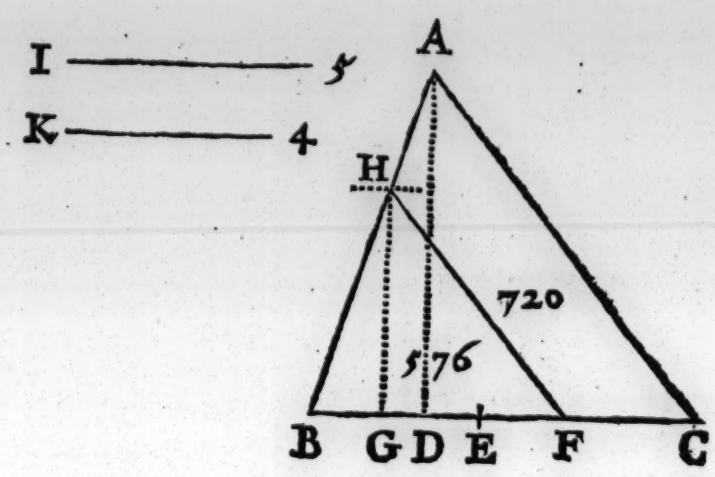
LEt the Triangle giuen be A B C. in the last Diagram, whose quantitie let be 5. Acres, 1. Roode, 0. Perch. Let the limited point bee E. in the Base B C. and let it be required from the same point to diuide the Triangle into two parts betweene M. and N. viz. to M. 3. Acres, 3. Roodes, 0. Perches thereof, and the residue to N. 1. Acre, 2. Roods, 0. Perch. First, reduce the quantitie of N. being the lesser, 1. Acre, 2. Roods, 0. Perch, into Perches, which makes 24. Perches, then (considering on which side of the limited point this part is to bee layde, as towards B.) measure that part of the Base from E. to B. 30. Perches, wherof take halfe, which is 15. and therby diuide 240. the part of N. the *quotus* will be 16. the length of the perpendicular F. H. at which parallel distance from the Base B C. cut the side A B. in F. from whence draw the line F E. which shall include the Triangle, F. B. E. containing 1. Acre, 2. Roods, 0. Perch, the part of N. and so shall the Trapezium A F E C. containe the resideue, namely, 3. Acres, 3. Roodes, 0. Perch, the part of M. as was required. THEOR. 35.

Euc. 6. 1. *Ra.* 10. 13.

PROBLEME CXXX.

To diuide a giuen triangle by a parallel line, to one of the sides, according to any proportion giuen.

LEt A. B. C. be a triangle giuen, and the proportiõ that between I. and K. and let it be required to diuide the same triangle into 2 parts, by a line drawne parallel to the side A. C. the one to be in proportion to the other, as the line I. to the line K. By the 13. PROB. diuide the line B. C. in E. as I. to K. then by the 17. PROB. finde the meane proportion betweene B. E. and B. C. which let be B. F. from which point by the 3. PROB. draw the line F. H. parallel to the side A. C. cutting the side A. B. in H. which line shall diuide the triangle giuen into two parts



A D. 48.
B C. 54.
B E. 24.
B F. 36.
E C. 30.
H G. 32.

Euc. 6. 10. 6. 13. *Cor.* 6. 19. *Ceul.* 3. 10.

parts, the trapezium A. H. F. C. hauing ſuch proportion to the triangle H. B. F. as the line I. to the line K. which was required. THEOREME 43, 41. PROBLEME 13, 17.

PROBLEME CXXXI.

To diuide a giuen triangle by a parallel line, to one of the ſides, according to any proportion giuen in two numbers Arithmetically.

LEt the triangle giuen be A. B. C. in the former Diagram, and let it be required to diuide the ſame by a parallel line, to one of the ſides into two parts, to be in proportion the one to the other, *Seſquiquarta*, that is, as 5. to 4. Firſt, let the baſe B C. 5 4. be diuided, according to the proportion giuen, as is taught in the 127. PROB. ſo ſhall the leſſer ſegment be B E. 2 4. and the greater ſegment E C. 30. then finde out a meane proportionall betweene B E. 24. and the whole baſe B C. 54. by multiplying 54. by 24. whoſe product will be 1296. the ſquare roote whereof is 36. the meane proportionall ſought, which in the former Diagram is B F. then by the rule of proportion, reaſon thus: If B F. 36. giues B E. 24. what A D. 48. the anſwere is H G. 32. at which diſtance draw a parallel line to the baſe, to cut the ſide A B. in H. from whence draw the line H F. parallel to A C. which ſhall diuide the giuen triangle into two parts, in ſuch ſort, as the trapezium A. H. F. C. ſhall be $\frac{5}{9}$. parts, and the triangle H. B. F. $\frac{4}{9}$. thereof, that is the trapezium, contayning the triangle H. B. F. once and a quarter, as 5. doth 4. which was required. THEOR. 43, 41. PROB. 13, 17.

Euc. 6. 10. 6. 13. *Cor.* 6. 19.

PROBLEME CXXXII.

To diuide a triangle of any knowne quantitie, giuen into two parts, by a parallel line, to one of the ſides, according to any number of Acres, Roodes, and Perches required.

LEt the triangle giuen be A. B C. in the laſt Diagram. whoſe quantitie is 8. Acres, 0. Roods, 16. Perches, and let it be required to diuide the ſame (by a parallel line to one of the ſides, as the ſide A. C.) into two vnequall parts, betweene M. and N. *viz.*) to M. 4. Acres, 2. Roods, 0. Perches, thereof, and the reſidue to N. being 3. Acres, 2. Roods, 16. Perches. Firſt, reduce both quantities into perches, which will be 720. and 576. then reduce both thoſe numbers by abbreuiation into their leaſt proportionall termes, which is $\frac{5}{4}$. and according to that proportion diuide the baſe B C. 54. of the giuen triangle, as is taught in the two laſt Problemes, which will be in E. then ſeeke the meane proportionall betweene B E. and B C. as is taught in the laſt Probleme, which is B F. 36. of which 36. take the halfe, and thereby diuide 576. the leſſer quantitie of perches, the quotus will be H G. 32. at which parallel diſtance from the baſe, cut the line A B. in H. from whence draw the line H F. parallel, to the ſide A C. which ſhall diuide the triangle giuen into two parts; the trapezium A. H. F. C. contayning the part of M. and the triangle H. B. F. the part of N. as was required. THEOR. 41, 43.

Euc. 6. 10. 6. 13. *Cor.* 6. 19.

Here

HEre might I now much inlarge and amplifie this latter part, for the diuiding of ſuperficiall figures of all formes and kindes: But ſeeing that all irregular figures and plots are moſt conueniently to be reduced into triangles, before the contents thereof can be had, or any diuiſion thereof made; and generally in matter of Suruey (whereunto my purpoſes chiefly tend) all figures are found irregular; I will content my ſelfe (and intreat my Surueyor to be likewiſe ſatisfied) with theſe few former inſtructions, which being well vnderſtood, with due obſeruation of the precedent rules, will ſerue his turne to whatſoeuer purpoſe can be required. But who ſo deſireth further ſatisfaction, and more varietie in this kinde, I referre him to M. Iohn Speidels *Booke, intituled his Geometricall Extraction, lately by him diuulged; (wherein he hath taken more paynes to excellent purpoſe, then this age, I feare, can afford him recompence) and to himſelfe* M. Henry Brigs, M. Thomas Bretnor, M. Io. Iohnſon, *and others, who are here amongſt vs Profeſſors, and excellent Teachers of theſe Arts. And thus, I conclude, this ſecond Booke.*

The end of the ſecond Booke.

THE EXACT OPERATION OF INSTRVMENTALL DIMENSIONS BY DIVERS MEANES.

The third Booke.

THE ARGVMENT OF THIS BOOKE.

THis Booke tendeth chiefly to matter of Suruey, wherein is first described and declared the seuerall Instruments, fit for that purpose (with their vse in practise) as the Theodelite, the Playne table, and Circumferentor, whereunto I haue added an absolute Instrument, which I call the Peractor, together with the making and vse of the Decimal Chayne, vsed only by my selfe; then is taught the vse of a necessarie Fieldbooke; the taking, protracting, and laying downe of angles diuers wayes; the reducing of customarie measures

into Statute meaſure, and the contrarie, ſhewing their difference; the vſe of the table of Synes, and the diuided ſights on the Circumferentor, withſupply of thoſe defects in the ordinary vſe of the Playne table, by ignorant perſons; the meanes to take Altitudes, Longitudes, Latitudes, and inacceſsible Diſtances, aſwell by ſynicall computation, as diuers other meanes; the Dimenſion and plotting Inſtrumentally of all manner ſuperficiall figures, and irregular plots, by diuers and ſundrie wayes, with their ſeuerall protractions accordingly: how with the Playne table exactly to take a plot of the largeſt Forreſt on one ſheet of paper, without altering thereof: the means of Survaying and plotting of a Lordſhip or Mannor, with the orderly handling of the ſame: the wayes and meanes Inſtrumentally, to reduce Hypothenuſall, to Horizontall lines, and the contrarie: with the beſt and exacteſt courſes to be held in the dimenſion, and plotting of Mountaynous, and vn-euen grounds: the manner of incloſing, diuiding, and laying out of Commons, waſts, or common Fields, into any parts required: the ordering of a plot after the protraction thereof: the reducing of plots from a greater to a leſſer forme, and the contrarie: and laſtly, the ſpeedie reduction of perches into Acres, and thoſe againe into perches: Moſt exactly and artificially wrought, by the beſt and moſt immediate meanes for thoſe purpoſes.

CHAP.

CHAP. I.

Of the ſeuerall Inſtruments in vſe, meete for Suruaie; which of them are moſt fit for vſe, and ſomewhat concerning their abuſe.

BEfore we enter the fieldes to ſuruaie, I hold it neceſſarie wee prouide vs of fitting furniture for the purpoſe, leſt by our neglect therein, thoſe by whom wee haue imployment receyue no leſſe loſſe and preiudice, then our ſelues ſhame and reproch: Wherefore let vs firſt conſider what Inſtruments are moſt vſuall, and then of thoſe, what moſt fit for our preſent purpoſe. The Inſtruments now moſt in vſe are the *Theodelite*, *Plaine-table* and *Circumferentor*, whereunto I will adde one more, which I call the *Peractor*, vſed onely by my ſelfe, and certaine friends by my directions; Of all which I will hereafter make briefe and particular deſcriptions, aſwell concerning their ſeuerall parts and compoſition, as of their vſe in practiſe. Nor will I exclude or wholly neglect the *Familiar Staffe* of *M. Iohn Blagraue*, and the *Geodeticall ſtaffe* and *Topographicall glaſſe* of *M. Arthur Hopton* (though now together dead) or any other Inſtrument which are, or hereafter may be by the art or induſtry of any man, artificially inuented or compoſed for his owne, or others vſe. For, as I tye not my ſelfe to the vſe of any one Inſtrument, at all times, nor on all occaſions, but for a large and ſpacious buſineſſe vſe the *Circumferentor* or *Peractor* (as for many reaſons moſt neceſſary and conuenient) and for a leſſer (where many ſmall Incloſures and Town-ſhips are) the *Plaine Table*; although eyther of the other will well ſerue for performance of eyther kind (and therefore if poſſibly you may, in one and the ſame buſineſſe, vſe euer one and the ſame Inſtrument, for auoyding many inconueniences:) So will I not limit my Surueyor to the vſe of any one Inſtrument, but referre him to all or any at his pleaſure, well knowing they are all compoſed and framed on one and the ſame Theoricall ground: and although in performance and diſpatch, one may be more ſpeedy then another (wherein I finde much difference, with the ſame exactneſſe) yet all or any of them artificially handled, are to excellent vſe, aſwell in ſuruey of lands, as the performance of many other excellent concluſions Geometricall. Wherein I cannot by the way but condemne the folly of diuers, who (deeming themſelues more wiſe and skilfull, then any other ſhall haue iuſt cauſe to conceiue) hauing ſome ſmall ſuperficiall vſe or inſight into ſome one of theſe Inſtruments, are onely wedded to that, and ignorant of the others vſe, will condemne them as vnmeet and inſufficient, the defect conſiſting in their owne vnderſtanding. But I muſt needs acknowledge (which could I as eaſily reforme, I ſhould deſerue well at the hands of many, who pay deerely for it) that by the ignorance of diuers, vſing (or abuſing) theſe inſtruments, infinite groſſe and palpable errors and abuſes are dayly committed, to the great loſſe and preiu-

dice of many, who receiue ſecret and inſenſible blowes (found in their eſtates, though vnfelt in themſelues) who for my part ſhall paſſe vnpittied (ſaue only for their weakeneſſe) the fault being chiefly their owne, when out of a penurious ſparing, to yeeld better ſatisfaction to thoſe which better can; they can be well contented blindly to ſwallow frogges for flyes, and to pay treble damage (nay perhaps ten fold) another way, ſo they neyther feele nor find it in their open view. And thus much concerning the ſeuerall Inſtruments in generall. Next of their Definitions, Parts, and Compoſitions particularly.

CHAP. II.

Of the Theodelite and his ſeuerall parts, with the Deſcription and Compoſition thereof.

THis Inſtrument conſiſteth of foure ſeuerall parts; As firſt, the *Planiſphere* or Circle, whoſe limbe is diuided into 360. equall parts or diuiſions, called degrees, without which it is fitting equidiſtantly to draw and deſcribe ſixe concentricke lines or circles with croſſe Diagonals, by whoſe interſections are had the parts of a degree: vpon which *Planiſphere*, and within the diuiſions before ſpecified, there is deſcribed the *Geometricall ſquare*, being the ſecond part thereof both which together, or eyther of them ſeuerally ſerue to excellent purpoſe, for the dimenſion of lengthes, bredthes and diſtances. The third part is a *Semicircle* or *Quadrant* perpendicularly rayſed, and artificially placed on the former *Planiſphere* (or more properly on the Index thereof) to be moued about circularly at pleaſure on all occaſions; which ſemicircle or quadrant hath the limbe thereof equally diuided into parts or degrees, with like concentricke Circles and Diagonals to thoſe of the *Planiſphere*: And the fourth and laſt part is a *Scale*, deſcribed within the ſame Semicircle or Quadrant, whoſe ſides are diuided into diuers equall parts, the more the better, and to beſt purpoſe; which two later parts ſerue chiefly for the menſuration of altitudes and profundities; All which together compoſed, make an excellent Inſtrument meete for many purpoſes, eſpecially for the deſcription of Regions and Countries, or other ſpacious workes. This Inſtrument would aske a farre more exact and particular deſcription, which for three principall reaſons I will here omit; Firſt, for that I hold it (although a generall and fit Inſtrument for all menſurations, yet in particular) for this our buſineſſe of Suruey, not altogether ſo fitting and commodious as the reſt before named, by reaſon of the multiplicity of Diuiſions therein contained, which will bee ſo much the more troubleſome in vſe and protraction: Secondly, for that M. *Thomas Digges* in his *Pantometria*, hath made a large and particular deſcription thereof: And thirdly and laſtly, for that M. *Allen*, who makes of theſe and all other Geometricall Inſtruments in mettall, is ſo well acquainted with the parts and compoſition thereof, that whoſoeuer purpoſeth to vſe the ſame, needeth only to beſpeake it of him, without further inſtruction or direction to him. The vſe and imployment whereof

ſhall

ſhall be hereafter touched. And thus much for this Inſtrument and the deſcription thereof.

CHAP. III.

The Playne table and his ſeuerall parts, with the deſcription and compoſition thereof.

THis Inſtrument for the playneneſſe and perſpicuitie thereof, and of his eaſie vſe in practiſe, receiueth aptly the name, and appellation of the *Playne Table*. A moſt excellent and abſolute Inſtrument, for this our purpoſe in Suruey; but with all, by reaſon of this his playneneſſe (offering it ſelfe at firſt view, in ſome meaſure, to the weake vnderſtanding of meaneſt capacities, inciting them thereby to the practiſe thereof) is more ſubiect to abuſe, then all, or any of the reſt: For notwithſtanding that theſe, by the common and ordinary vſe and practice of this Inſtrument, may eaſily attayne to a reaſonable truth in Dimenſion and plotting of regular and euen Playnes; yet, if they come to irregular and vn-euen formes, as Hills and Dales, they are ſo farre vnfitting for the true menſuration thereof, that many vſing this Inſtrument, neglect the meanes, and thoſe proper parts of the ſame Inſtrument (being the ſights hereafter ſpecified, whereof they know no vſe) which is their onely helpe and aide in this kinde. But, leſt this relation ſhould be held digreſſion, lets returne to the purpoſe. This Table it ſelfe is diuided into fiue parts, or ſmall bords, whereof three are in the bredth, and the other two imployed as ledges in the length, to keepe the reſt together, whereunto a ioynted frame is artificially applyed, for the faſtning and keeping playne thereon, an ordinarie ſheet of paper for vſe in the fields; of which length and bredth, or rather leſſe, as 14½. inches in length, and 11. in bredth, the whole Table together ſhould conſiſt. But for my purpoſe, I would haue made of theſe three boords in bredth, with helpe of ledges, to be thereon glued, to the back-ſide thereof, and ſtrong ioynts or gemmowes, faſtning them together, an artificiall boxe; which at any inſtant being opened, and the ledges faſtned on, is fit for vſe in the Fields, and afterwards thoſe ledges taken away, may be as inſtantly turned backwards, and incloſed as before, fit for the keeping of looſe papers and ſmall tooles, till further occaſion. Which if any deſire to vſe, *M. Iohn Thomſon*, in *Hoſier*-Lane, will artificially fit him. To this Inſtrument doth alſo belong a Ruler with two ſights thereon, which by diuers men are vſed of diuers kindes, but by the beſt Playne Table men (as occaſion ſerues) thoſe of M. IOHN GOODWYNS *inuention, that excellent and honeſt Artiſt, whoſe liuing Name (though himſelfe be dead) I cannot remember without good reſpect.* This Ruler is to contayne in length about 16. inches, or as long or longer then the table, for drawing parallel lines on the paper faſtned thereon, by the equall and oppoſite diuiſions on the frame; it is likewiſe to contayne in bredth about two inches or vpwards, and in thicknes halfe an inch; the ſights thereof are double in length the one to the other, the longer contayning about 12. inches, and the ſhorter halfe as much; on the head or top

of which ſhorter ſight, muſt be placed a wyer or braſſe pin; and to this ſight there muſt be faſtned a thred and plummet, to place the Inſtrument Horizontally; through the longer ſight muſt be made a ſlit, extending almoſt the whole length of the ſight. Theſe two ſights thus prepared, are to be placed perpendicularly on the Ruler, by ſquare morteſſes, made to that purpoſe through the ſame, in ſuch ſort as the braſſe pin on the ſhorter ſight, and the ſlit through the longer ſight, be preciſely ouer the fiduciall edge of the Ruler; either ſight being equi-diſtantly placed from each end of the Ruler, and the ſpace betweene the ſights to be exactly the iuſt length of the longer ſight, or to ſpeake more properly of the diuided part of the ſame ſight; *(which is alwayes to be vnderſtood when I ſpeake of the length thereof.)* Vpon this longer ſight there is to be placed a vane of braſſe, to be remoued vp or downe at pleaſure; through which a ſmall ſight hole is to be made, anſwerable to the ſlit in the ſame ſight, and the edge of the vane.

By theſe ſights thus placed on the Ruler, there is proiected a Geometrical ſquare, whoſe ſide is the diuided part of the long ſight, or the diſtance betweene the two ſights. In the middle of which long ſight, ouerthwart the ſame, there is drawne a line called the line of leuell, diuiding the ſide of the proiected ſquare into two equall parts, alſo the ſame ſide is on this ſight diuided into 100. equall parts, which are numbred vpwards and downewards, from the line of leuell, by fiues and tens to 50. on either ſide, which diuiſions are called the Scale.

There is alſo on the ſame ſight expreſſed another ſort of diuiſion, repreſenting the Hypothenuſall lines of the ſame ſquare, as they increaſe by vnits, and are likewiſe numbred vpwards and downewards from the line of leuel, from one to 12. as 1, 2, 3, &c. ſignifying 101, 102, 103, &c. which declare how much an Hypothenuſall or ſlope line, drawne ouer the ſame ſquare (that is, from the pinnes head to any ſuch diuiſion) exceedeth the direct horizontall line, being the ſide of the ſame ſquare.

There is moreouer a third ſort of diuiſions on this ſight, repreſenting the degrees of a quadrant (or as many as can well be expreſſed on the ſame ſight being 25.) which are (as thoſe before) numbred, from the line of leuell vpwards and downewards, by fiues and tens to 25. which diuiſions are called the quadrant.

Likewiſe there may be placed on the ſurface, or vpper part of this Ruler, the table of Synes, mentioned in the deſcription of the Circumferentor, next hereafter following; very neceſſary for vſe, as ſhall hereafter partly appeare.

Yet would I further aduiſe concerning this Inſtrument; that in playne and leuell grounds, where is no vſe of reducing Hypothenuſall lines; inſtead of thoſe long ſights formerly expreſſed, to haue vſed ſuch double ſights as are hereafter mentioned in the deſcription of the *Peractor*, which are farre more readie, and leſſe cumberſome for vſe, eſpecially in rough and boyſterous weather. And likewiſe, when occaſion ſhall be offered, by irregular and vn-euen grounds, to reduce thoſe Hypothenuſall lines, to vſe in ſtead of thoſe long ſights, the quadrant hereafter likewiſe ſpecified in the ſame deſcription, which when need requires may be eaſily fixed to the Ruler; and as ſoone laid by, hauing no further vſe, for of all other meanes in taking of heights, and eſpecially for reducing of thoſe Hypothenuſall lines, I hold

that

that quadrant artificially handled to be moſt readie and exact, all concluſions being ſpeedily wrought therewith without Arithmetick, or other collaterall computation whatſoeuer.

To this Inſtrument belongeth diuers other ordinary parts, whereof I ſhall not neede particularly to ſpeake, ſeeing moſt men (if not too many) already know them, being of ordinarie vſe; as the Socket, the Boxe and Needle with the ſtaffe or foote thereof, &c.

CHAP. IIII.

The Circumferentor with his parts, deſcription, and compoſition.

THis Inſtrument for expedition and portabilitie, exceedeth far the reſt, and nothing inferior to any for exactneſſe, if care and arte be vſed; but not ſo vulgarly vſed (though much more generall for vſe) as the playne Table is; the full and perfect vſe thereof, not lying ſo open and apparant to all mens vnderſtanding, as the other is.

It is made and framed of well ſeaſoned boxe, contayning in length about eight inches, in bredth halfe as much, and in thickneſſe about ¼. of an inch; the left ſide whereof is beſyled, and diuided into diuers equall parts, moſt fitly of 12. in an inch, to be vſed as the Scale of a Protractor, the Inſtrument it ſelfe ſeruing well to protract the plat on paper, by helpe of the needle, and the degrees of angles and lengths of lines, taken in the field, and entred into your field Booke, as ſhall be hereafter ſhewne.

About the middle in the ſurface thereof, a round hole is to be turned of the depth of halfe an inch, whereof the Diameter to be about 3½. inches, to place a carde and needle therein, to be couered ouer with cleere glaſſe. The beſt carde for this purpoſe is that, diuided in the limbe into 120. equall parts or degrees, with a Dyall according to the *Azimuths* of the Sunne, wherein the houres are numbred, and the moneths named, ſeruing very aptly to ſhew the time of the day.

Beſides, on the ſurface hereof is placed the table of Synes, calculated from the quadrant of a circle ſo diuided, as the carde before ſpecified; the arch of this quadrant being 30. degrees, and the ſemidiameter or totall ſyne thereof, is diuided into 1000. equall parts, and numbers placed accordingly, anſwering to euery degree and halfe degree, ſeruing to expreſſe the length of euery right ſyne for the Dimenſion and Supputation of Triangles, as ſhall be partly hereafter ſhewne, and in the meane ſpace the Table is here expreſſed.

Tabula

Tabula Sinuum.

½	26		334		609		824		959
1	52	7	358	13	629	19	839	25	969
	78		383		649		853		972
2	104	8	407	14	669	20	866	26	978
	130		430		688		879		983
3	156	9	454	15	707	21	891	27	987
	182		477		725		902		991
4	208	10	500	16	743	22	913	28	994
	233		522		760		923		997
5	259	11	544	17	777	23	933	29	998
	284		566		793		943		999
6	309	12	588	18	809	24	951	30	1000

There is alſo hereunto belonging two ſights, double in length, the one vnto the other, the longer contayning about ſeuen inches or vpwards, being placed, and diuided in all reſpects, as thoſe formerly mentioned in the deſcription of the playne Table; the ſhorter ſight of theſe hauing one propertie and vſe, which that (as needleſſe) hath not, being this, in the edge thereof, towards the vpper part is placed a ſmall wyer, repreſenting the center of a ſuppoſed circle, the ſemidiameter whereof is that part of the ſame ſight, betweene the wyer, and a perpendicular point on the edge of the Inſtrument vnderneath the ſame; which part is imagined to be diuided into 60. equall parts, and according to thoſe diuiſions is the right edge of the Inſtrument, diuided and numbred from the perpendicular point by fiues and tens, as 5. 10. 15. &c. And alſo from the ſame point on the ſurface and vpper edge of the Inſtrument is perfected, the degrees of a Quadrant ſupplying the reſidue of thoſe which could not be expreſſed on the long ſight from 25, to 90. by tens, &c. For the vſe of theſe Diuiſions is there alſo belonging hereunto an Index or ſmall Rule, at the one end whereof is a Centre hole to place on the Wyer in the edge of the ſhorter ſight, and at the other end a ſight is placed, anſwerable to the fiduciall edge of the ſame Index, which edge is likewiſe diuided according to thoſe diuiſions on the edge of the Inſtrument. This ſhort ſight is to haue a plummet of lead hanging in a fine thred, ſeruing to place the Inſtrument horizontally. Where note, that theſe ſights, and their ſeuerall diuiſions ſerue onely for Altitudes, Profundities, and the reducing of Hypothenuſall lines to horizontall, which is to excellent purpoſe, and full of vſe. But when there is no occaſion or vſe of theſe (as ſeldome there is in reſpect of other vſe) I would alwayes haue ready, ſuch double ſights as are hereafter expreſſed in the deſcription of the *Peractor*, which doubtleſſe are of excellent vſe, as I there explaine; with this caution, that you alwayes carefully keepe one and the ſame part of your inſtrument forwards, and reccon euer your degrees in one and the ſame end of your Needle.

The foot of this Inſtrument is that with three ſtaues ioynted in the head, and

and to be taken a ſunder in the middle with braſſe ſockets, according to the vſuall order, moſt fitte for all Inſtruments, except in ſuch caſes, as in the deſcription of the *Peractor* is excepted. And theſe are the ſeuerall parts of this Inſtrumen t,and thus is it compoſed.

CHAP. V.

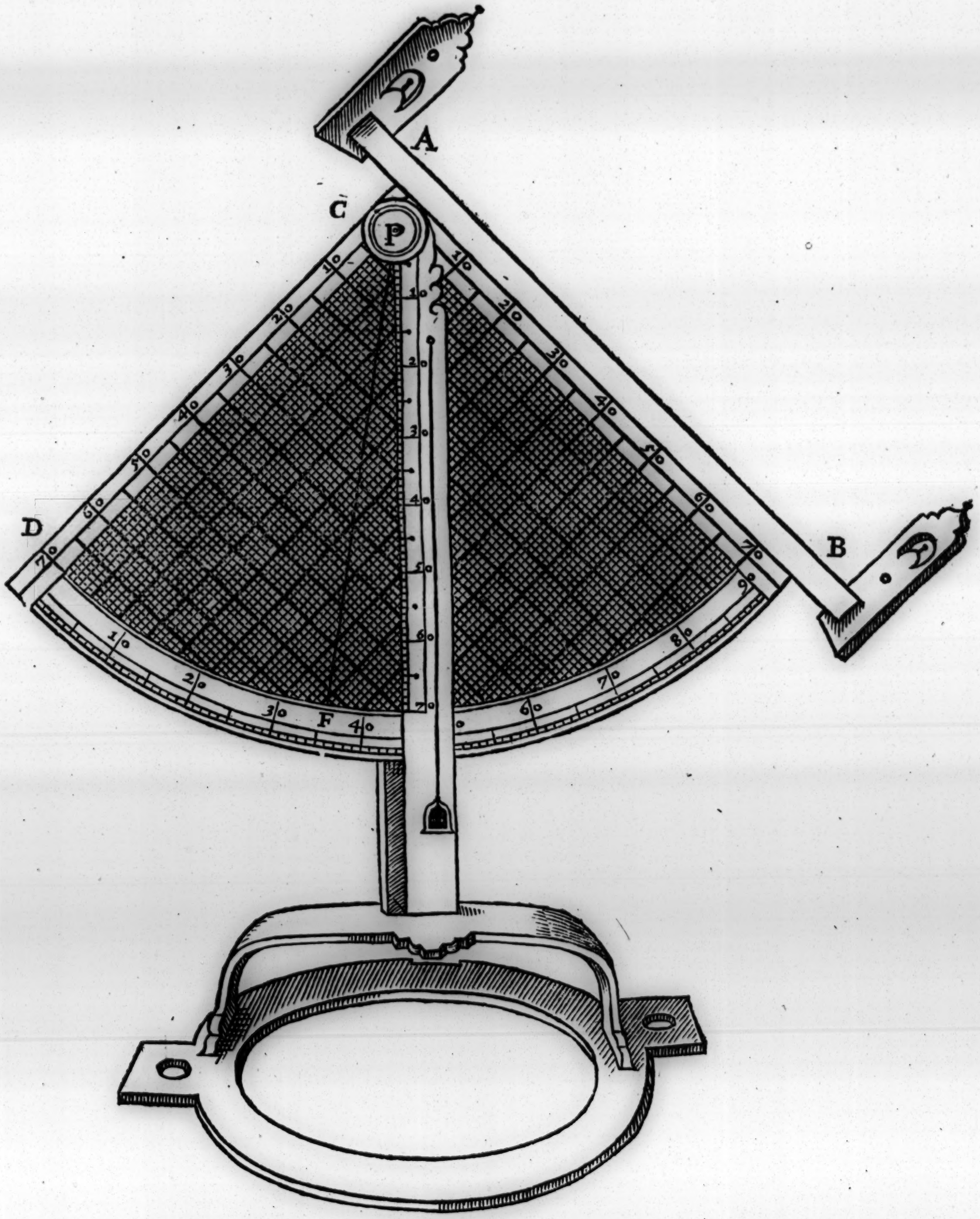
The Peractor, with his ſeuerall parts Deſcription and Compoſition.

AS I will not with the cunning Wine-taſter forbeare commendations, fearing too many partners; So will I refraine wooing any to vſe what I much affect, further then reaſon, and their owne iudgement ſhall rule them therein. But thus much will I boldly ſpeake and maintaine of this Inſtrument, that for generall vſe, perſpicuity, ſpeed and perfection, it may well compare with any hitherto in vſe. It conſiſteth of a *Planiſphere* in braſſe, much like vnto that of the Theodelite, but where the limbe of that is diuided into 360. parts or degrees;this is only into 120. (ſo that each of theſe containeth three of thoſe) and theſe ſubdeuided into halfes and quarters towards the limbe thereof, without which diuiſions, there are drawne and deſcribed three concentricke Circles, being croſſed with Diagonals, by whoſe interſections are exactly expreſſed the third part of euery degree; whereby, and by tripling the former degrees cut (if occaſion require) is had exactly the degrees of the Theodelite conſidering that ten of theſe contain thirty of thoſe, wherewith in matter of Suruey we ſhall little need to trouble our ſelues. *And here haue we large and ſpacious degrees with their exact parts to* $\frac{1}{4}$. *(which in others wee want) by meanes whereof, and with helpe of ſuch a chaine, as I alwayes vſe (which ſhall be next hereafter expreſſed) I will boldly approue and maintaine, to worke with much more facilitie and exactnes, and come neerer the preciſe truth then any other can poſsibly doe, not vſing the ſame or the like, which will appeare moſt manifeſt in practiſe to all mens vnderſtanding.*

There is alſo an Index hereunto belonging, fixed on the Centre as that of the Theodelite, with two ſights thereon placed, with ſliding loopes, eyther of them alike, and of like length, and eyther of them double ſighted, the one hauing a ſlitte beneath, and a thred aboue, the other a thred beneath, and a ſlit aboue, ſeruing to looke backwards and forwards at pleaſure, without turning about or ſtirring the Inſtrument, when the Needle is at quiet; whereby I ſaue neere halfe the labor, and halfe the time that any man ſhall ſpend with other ſights, for that hereby I need plant mine Inſtrument but at each other Angle, which is no ſmall helpe for expedition, and ſuch a meanes for exactneſſe rightly handled, as few will imagine without due proofe; and that without trouble of ſending one before,and leauing another behind, as is vſually accuſtomed. Neyther vſe I theſe ſights with this Inſtrument onely, but with all others as the Theodelite, Plaine-Table, and Circumferentor.

Theſe ſights I alwaies vſe in plain and euen grounds;but if occaſion ſerue for the reducing Hypothenuſall,to Horizontall lines; I then euer vſe this

Qua-



Quadrant here expreſſed moſt fitte for the ſpeedy taking of all maner of altitudes and profundities, and the ſuddaine reduction of thoſe lines without Arithmeticall calculation, or other collaterall account, as is manifeſt in vſe. This Quadrant (the other ſights taken away) is on any occaſiou ſpeedily placed on the Index with ſcrue pinnes, and as ſuddainely taken off, if no further vſe, and the ſights replaced.

Then

Then is there alſo hereunto belonging other vſuall parts, as a boxe of braſſe on the Centre to containe a Carde and Needle therein, ſuch as is formerly expreſſed for the Circumferentor, to be likewiſe couered ouer with cleere glaſſe, and cloſe ſtopped with red waxe about the edge thereof, to defend the Needle from Winde, Wether, and Ayre, the onely enemies thereunto.

Alſo a braſſe Socket to bee ſcrued on with foure ſcrue pinnes on the backe of the Inſtrument, which Socket ought to be preciſely turned, and the head of the ſtaffe therewith, (I meane the braſſe part thereof) which will neuer otherwiſe turne euenly and nimbly about as it ought, the one within the other, without iercking and ſtarting, which much troubleth the Needle in finding his naturall point and place of reſt. And if any doubt the truth of his Needle, let him take backe ſights for his better ſatisfaction therein.

And laſtly, for this as for the reſt the like ſtaffe is to be prouided as before is ſpoken of, which for all purpoſes is the abſolute beſt, except onely for water leuels, and the works thereunto belonging, wherein it is neceſſary at all times, and at each ſeuerall ſtacion to keepe the inſtrument at one and the ſame horizontall diſtance, which otherwiſe may breede much cumber, and no leſſe incertainety in thoſe concluſions; wherefore for thoſe and the like occaſions, a foot with one ſtaffe, hauing three iron pikes therein, after the old order is beſt, and to beſt purpoſe.

The making of this Inſtrument and the reſt in braſſe are well knowne to M. ELIAS ALLEN *in the Strand; and of thoſe in woode to M.* IOHN THOMPSON *in Hoſyer Lane.*

And thus haue I briefly deſcribed theſe ſeuerall Inſtruments with their particular parts, laying them before my Surueyor to take his choyſe as his fancie leades him; But in mine opinion all are better then any; ſo ſhall hee beſt know what is beſt for his purpoſe.

And now let vs conſider, what other neceſſaries are yet to bee prouided, before wee beginne our buſineſſe, for feare wee are to ſeeke when occaſion ſerues for vſe. As the Chaine, Protractor, Field-booke, and the Scale and Compaſſe whereof we will further ſpeake.

CHAP. VI.

The making and diuiſion of a Chaine, called the Decimall Chaine.

THis is the chaine before ſpoken of in the *Peractors* deſcription; which for conueniencie in carryage, and auoyding caſualties often happening to breake it (though made of a full round wyer) I would aduiſe ſhould containe in length but onely two ſtatute Poles or Perches, or three if you pleaſe at the moſt. In the diuiding whereof it is to bee conſidered, that the ſtatute Perch or Pole, (which here we call an vnite, or (*Comencemente*) containeth in length 16 $\frac{1}{2}$. feete. which is, 198. ynches: This quantitie is firſt to be diuided into 10. equall parts called *Primes*, ſo ſhall euery of theſe *Primes* containe in length 19. ynches, and $\frac{4}{10}$. of an ynch: And then theſe *Primes* to bee euery of them

ſub-

ſubdiuided into other 10. equall parts, which wee will call *Seconds*: and ſo euery of theſe *Seconds* ſhall containe in length one ynch, and $\frac{1}{10}$. parts of an ynch. And thus is the whole Perch *vnite* or comencement diuided into 100. equall parts or linkes called *Seconds*.

Which Chaine ſo diuided is thus to bee diſtinguiſhed and marked: Firſt, at the end of euery fifth Prime, or fiftieth *ſecond* or lincke, which is the end of euery halfe Pole, let a large curtaine ring be faſtned, ſo ſhall you haue in the whole Chaine (if but two Poles) three of thoſe rings; the middlemoſt being the diuiſion of the two Poles, which in a Chaine of this length is eaſily and readily diſcerned from thoſe rings of the halfe Poles, though all of one greatnes. Then at the end of euery *Prime*, that is, at the end of euery tenth *ſecond*, or linke, let a ſmall curtaine ring be placed, and not thoſe rings of braſſe wyer, as is vſuall in other chaines, which, with euery buſh and twig are continually broken off, and loſt.

By thoſe diſtinctions this Chaine is now diuided into theſe three termes, *Vnites*, *Primes* and *Seconds*, whoſe Characters are theſe 0.1.2. So that if you would expreſſe 26. *Vnites*, 4. *Primes*, and 5. *Seconds*, they are thus to be written $26^{0}.4^{1}.5^{2}$. or together thus, $26^{0}4^{1}5^{2}$. or more briefly thus, $26.4.5$. making prickes or points onely ouer the Fractions, whereby the reſt may be conceyued to be Vnites, or Intigers, and the firſt point *Primes*, and the next *ſeconds*.

But beſides theſe diuiſions for mine owne vſe, I alwayes at the end of euery $2\frac{1}{2}$. *Primes*, which is the $\frac{1}{4}$. of a Pole, ſowe on a ſmall red cloth, or the like, (thruſt through the ring of the chaine, and at euery $7\frac{1}{2}$. *Primes* being the $\frac{3}{4}$. of a Pole; the like of yellow, or ſome other apparant colour, where with being once acquainted, and thereunto inured, you ſhall moſt ſpeedily at the firſt view reckon the quantitie of euery ring, remembring that if it bee the next ring, ſhort of the red, it is two *Primes*, if the next ouer, three, if the next ſhort of the yellow, it is 7. *Primes*, if the next ouer, 8. if the next ſhort of a great halfe ring, it is 4. the next ouer, 6. And laſtly, if the next ſhort of the middle great ring, it is 9. and if the next ouer, 1. and ſo of the reſt; *Wherein is to bee noted that your Chaine thus marked, is alwayes to be vſed with one and the ſame end forwards.*

This Chaine thus diuided and marked, you haue euery whole Pole equal to 10. *Primes*, or 100. *ſeconds*, euery $\frac{3}{4}$. of a Pole equall to $7\frac{1}{2}$. *Primes*, or 75. *ſeconds*, which is $\frac{3}{4}$. of 100. euery halfe Pole equall to 5. *Primes*, or 50. *ſeconds*, which is $\frac{1}{2}$. of 100. and laſtly euery $\frac{1}{4}$. of a Pole equall to $2\frac{1}{2}$. *Primes*, or 25. *ſeconds*, which is $\frac{1}{4}$. of 100.

And here is to bee noted, that in the ordinarie vſe of this Chaine for meaſuring and platting; I obſerue onely *Vnites* and *Primes* (but on neceſſity) which is much more exact then the ordinary vſe; but hauing occaſion to make diuiſion or ſeparation of lands, or for the dimenſion of common fields in their ſeuerall parts by furlongs or wents and rigges, I vſe my *Seconds*; wherein, what exactnes and moſt excellent vſe I find, I will referre to thoſe who can deſcerne the difference betweene a portion leſſe then two ynches, the length of my *ſecond*, and that of 6. ynches, and $\frac{3}{10}$. of an ynch, the leaſt part of the beſt and exacteſt Chaine now commonly vſed; *but with thoſe of the laſt of long twelues I will not meddle.*

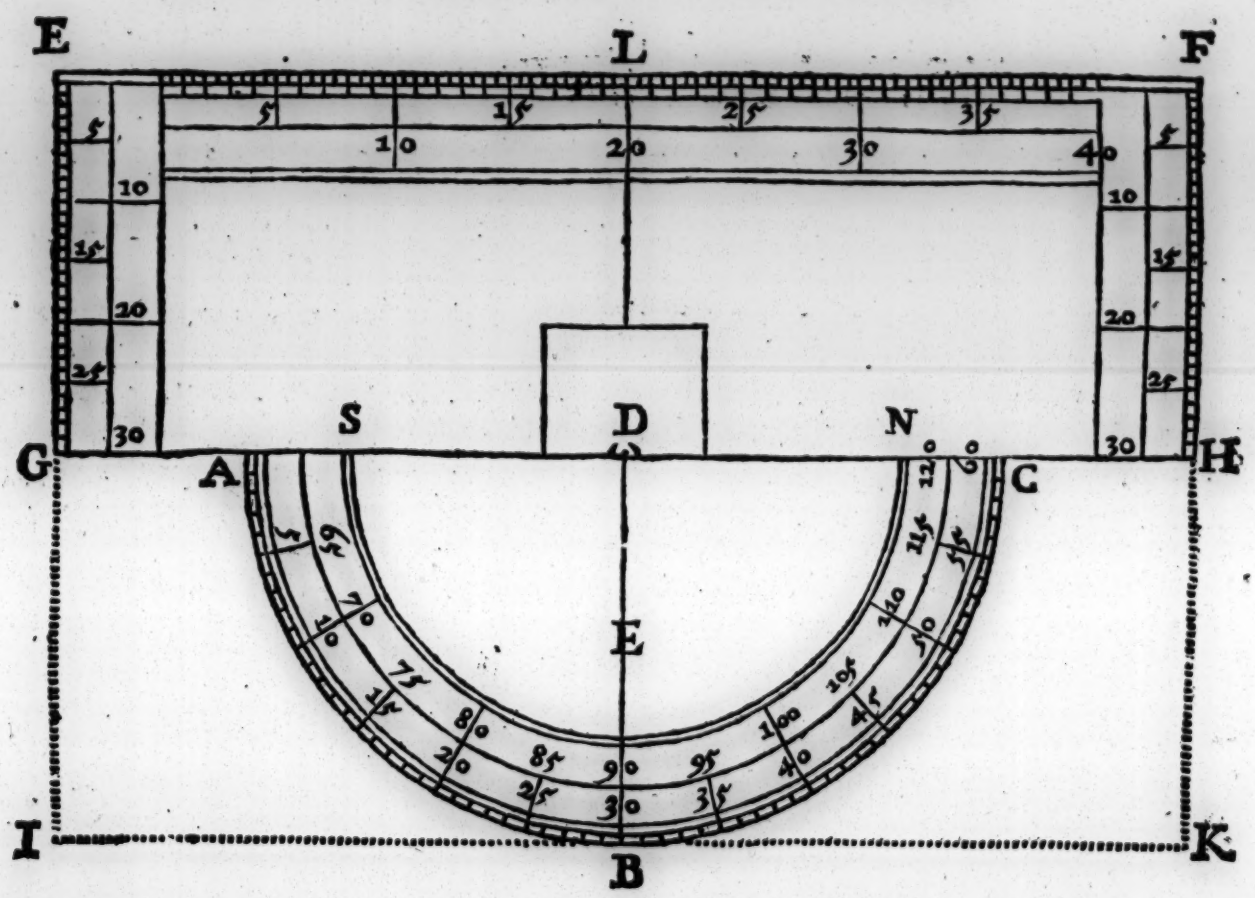
But

But here me thinkes I heare the Aduerſarie queſtion, to what purpoſe ſerues this niceneſſe of ynches in Inſtrumentall obſeruation, when comming to your protraction with a ſmall Scale, you are not able to diſtinguiſh feet? I anſwere (and to purpoſe) thus: If by your ordinary Chaine you take obſeruation in your Field-bookes of $\frac{1}{4}$. $\frac{1}{3}$. and $\frac{3}{4}$. and few or none otherwiſe, (or if they doe, to ſmall purpoſe, as they afterwards handle the matter) then I ſay, I taking my obſeruation of 1.2.3.or 4. *Primes*, or of 6. 7. 8.or 9. *Primes*, can in my Protraction with a ſmall Scale and Protractor (yet mine I muſt confeſſe is none of the leaſt) eaſily diſtinguiſh and expreſſe how much leſſe or more then $\frac{1}{4}$. $\frac{3}{3}$. or $\frac{3}{4}$. thoſe quantities are, as may eaſily appeare, with due obſeruation of my former notes. Yet may it bee further ſayd, What is all this to purpoſe, if there be not as exact a meanes to obtaine or get the true ſuperficiall content in caſting vp the plat, it being thus exactly layde downe? I may anſwere againe, Better one miſchiefe then many; neyther will I ſuffer this; for be well aſſured, I will not be ſo carefull in that, and altogether careleſſe in this; the meanes whereof in due time I may hereafter ſhew, being vnfit for this place; hauing already inlarged my Chaine in length more by a Pole, then firſt I meant; and therefore purpoſe now to be no longer chained therein.

The making of this Chaine is well knowne to M. CHRISTOPHER IACKSON *at the Signe of the Cocke in Crooked-Lane, who by my directions hath made of them for me, and hath the ſcantling thereof.*

CHAP. VII.

Of the Protractor and the Scale thereof.



PRouide for this *Protractor* a fine thinne piece of braſſe well poliſhed, in forme of a long ſquare, as the Figure E F K I. which (for conueniencie in vſe) ought to containe in length from G. to H. about 5 $\frac{1}{4}$. inches, and in bredth from L. to B. ſomewhat better then

3 ½. ynches, whereupon draw two lines as G H. and L B. cutting each other preciſely at right Angles in the point D. diuiding the Square into foure equall parts, then on the point D. as a Centre, at the diſtance of D L. or D B. deſcribe the Semicircle A B C. (for it is not materiall, or of neceſſitie, that the Diameter thereof ſhould agree with the Diameter of the Carde in the Inſtrument, as *M. Hopton* would haue it in the 62. Chap. of his *Top. glaſſe*) then diuide the limbe of the Semicircle A B C. into 60. equall parts or degrees, numbring them by fiues and tens in the outward ſpace to 60. and in the inward ſpace from 60. to 120. as in the figure, the firſt numbers ſeruing for the Eaſt ſide, and the later (being the oppoſite degrees) for the Weſt ſide of the whole Circle, ſo is a labour ſaued of diuiding the other ſide which ſerues to no purpoſe; then let the edges of the Scale as E G. E F. and F H. be ſomwhat beſiled, and made very ſmooth, and preciſely parallel to the firſt drawne lines reſpectiuely, and aboue the reſt, let care be had that the line L B. bee made exactly perpendicular to the edge E F. of the Scale, or otherwiſe great errors may inſue in the vſe; then diuide the Parallel degrees at eyther end of the Scale, betweene E G. and F H. And let the Scale of 12. bee placed on the edge E F. and the ſale of 11. on the edge of the backe ſide, which are moſt neceſſarie and meeteſt for vſe of any other; and laſtly, cut out the ſquare about the Centre D. and likewiſe that between the Semicircle & the pricked lines, hauing care that the line G H. be left perfect, and euen with the Diameter A C. being the meridian line, and the guide of the reſt. And ſo is this worke finiſhed.

Yet would I haue beſides in ſome ſpare place of this Protractor, or on the backe ſide thereof, the *Sextans* deſcribed, which is mentioned in the next Chap.

Here is it to be noted, that this PROTRACTOR *ſerueth without alteration or any difference, aſwell for the* PERACTOR *as the* CIRCVMFERENTOR. *But if you would haue it for the* THEODELITE, *then muſt the limbe of the Semicircle bee diuided into* 180. *equall parts, and numbers placed accordingly, which is all the difference.*

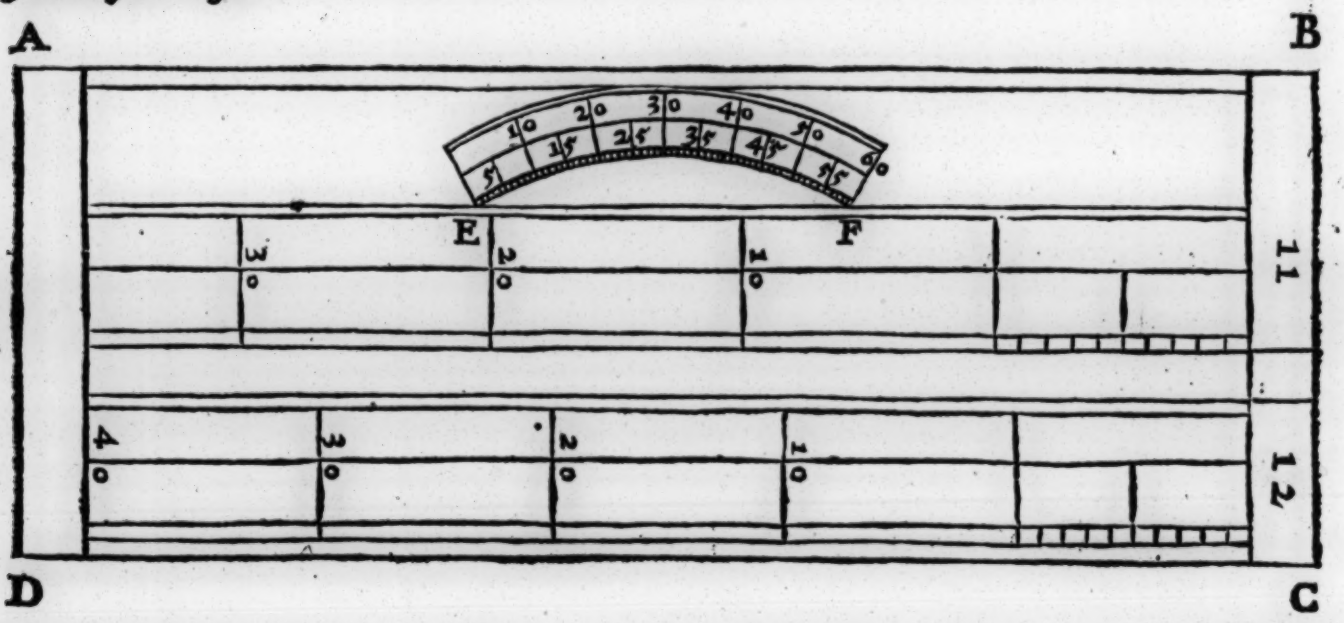
And here alſo would I not haue you forget to prouide that long Protractor formerly mentioned in the concluſion of the ſecond part of my ſecond Booke, which will ſtand you in good ſtead.

Alſo to theſe there belongs a protracting pinne made of a needle (according to the Centre hole of the Protractor) to be placed in the end of a ſmall turned ſticke; or of Iuorie, as beſt likes you. And ſo are you thus farre fitted: wherefore to the reſt.

CHAP.

CHAP. VIII.

Of the ordinarie Scale with the Sextans thereon deſcribed, very neceſſarie for vſe.



FOr this purpoſe let a Ruler of Braſſe or Boxe (but braſſe the better) be prouided, as the Figure A B C D. which let containe in length about 7. or 8. ynches, and in bredth about two ynches, or ſomewhat leſſe, whereupon on the one ſide let bee placed two Scales, the one of 11. the other of 12. in an ynch; and on the ſame ſide, let there be alſo deſcribed a *Sextans* or the ſixt part of a Circle, whoſe chorde E F, (which is alwayes equall to the Semidiameter of the ſame Circle) let containe in length about two ynches or leſſe, and let the limbe be diuided into 60. equall parts or degrees, and numbred by fiues and tens, as in the Figure. On the other ſide thereof there may be placed (after the order of theſe) diuers other Scales, as of 16. 20. 24, &c. as you thinke fitting. So haue you a neceſſary Inſtrument for many purpoſes. And this *Sextans* alſo would I haue deſcribed on ſome ſpare place of your *Protractor*.

To this muſt you haue prouided a neate payre of Compaſſes of braſſe, with fine ſteele points, which muſt alwayes bee ready ſeruing for infinite occaſions.

Beſides theſe ordinary Compaſſes, it is very fitting to bee alſo prouided of a payre of Callem Compaſſes, with ſcrues to alter the one legge at pleaſure, wherein to faſten a penne, blacke leade, a ſteele point, or the like, very fit for many purpoſes.

CHAP. IX.

Of a Ruler, for the reducing of Plats.

ALthough wee are not as yet fitte for the vſe of this Rule, yet ſeeing our buſineſſe now in hand, is to prouide vs of neceſſaries: It is no ill rule to take our buſineſſe before vs: Wherefore repayre to Maſter IOHN THOMPSON in *Hoſier lane*, who without further In-

ſtructions will furniſh you, onely this before you goe: Let the Ruler bee made of drie boxe, if you may, of a yard in length, and let the equall diuiſions thereon bee of 12. in an ynch, to bee numbred with double numbers, as he vſeth for me. So will it ſerue you to good purpoſe, aſwell for caſting vp of large plats. &c.

CHAP. X.

The order of making of a neceſsary and fitting Field-booke, ſeruing aſwell for the Peractor and Circumferentor, as for the Theodelite, with the ordering and vſe thereof in the Fields.

THis Booke may conſiſt of halfe a quire of paper, to bee bound (moſt aptly for vſe) in a long *Octauo*: Let it be ruled towards the left margent of euery ſide, with foure lines, ſo ſhall you deſcribe three Collums, the firſt ſeruing for the degrees; the ſecond (according to my Chaine) for Vnits: and the third and laſt for Primes; or according to the accuſtomed vſe, for degrees, poles, and parts of a Pole.

The order of vſing it is thus: Suppoſe you are to ſuruey the Mannor of *Beauchampe*, and are to beginne with theſe fiue ſeuerall parcels numbred in this plat or figure, with 1. 2. 3. 4. 5. being ſeuerall grounds of ſeuerall Tenants, and of ſeuerall natures, whereof you are to make ſeuerall obſeruation, as appeareth. Firſt, for the title of your Booke begin thus: *Maner. de Beauchampe in Com. Ebor. Pro Rege incip.* 24. *Iunij*, 1616.

Then begin with the firſt field, at A. writing the Tenants name, the name of the field, of what nature, and of what Tenure thus: *Io*: *Iackſons Hall Cloſe*, *Meadow*, *Free*: and expreſſe in the Margent of your Booke *No* (1) ſignifying the firſt Cloſe, then hauing placed your Inſtrument at A. directing your ſight to B. you finde the degree cut 65. which place in the firſt Collum towards the left hand, then meaſuring the diſtance betweene A. and B. you ſhall finde 20. Vnites, and 2. Primes, or 20. Poles, and a little leſſe then a quarter, which place ſeuerally in the two next Collums towards the right hand; then place your Inſtrument at B. and directing your ſight to C. obſerue and expreſſe the degrees and length there found, as $\frac{d.}{65.}$ $\frac{o.\ p.}{20.\ 2.}$ and conſidering that at C. you are to leaue the bounder you formerly went againſt, ſtrike there a ſingle line ouerthwart your booke, and aboue that line againſt the degrees and quantities taken, write what was the bounder you went againſt, whether it be of this or any other Mannor, as here in this example thus, the Mannor of *Barnſey*; and thus proceed with the reſt of this Field, till you come to incloſe at A. where you firſt began; and then ſtrike a double line, and place this marke ⊕ or the like in your booke, at the end of the incloſing line, ſignifying to incloſe. And ſo is that Field finiſhed. Then conſider which Cloſe is next fitting to be taken in hand, which let be (2.) and withall at what Angle thereof it is meeteſt to begin, which let bee at C. and here (for your helpe when you come to protraction) you muſt expreſſe in the Title of this ſecond Cloſe, at what Angle you begin the ſame (vnleſſe you

Maner. de Beauchamp. in Com. Ebor. pro Rege incip. 24. Iunij. 1616.

Iohn Iacksons Hall, Close, Mead. Free.

d	o	1	
			The Mannor of Barn-
(1) 65	20	2	sey.
68	16	6	(1)
			North Breading.
$36\frac{1}{2}$	22	5	(2)
$27\frac{1}{4}$	12	3	
106	14	1	Vpper barne Close.
$21\frac{1}{2}$	18	2	(3)
112	21	--	South-Field.
$32\frac{3}{4}$	20	$2\frac{1}{2}$	⊕ Where began.

(2) *William Bensons* North-breading, Close, Pasture, Cop. from No. (1.)

$55\frac{1}{2}$	14	5	
65	18	9	The Mannor of Barn-
$41\frac{1}{2}$	16	2	sey.
to $5\frac{1}{4}$	35	4	⊕ Vpper Barne Close, at (2.)

(3) *Thomas Brownes* vpper Barne Close, arr. Lease for yeeres.

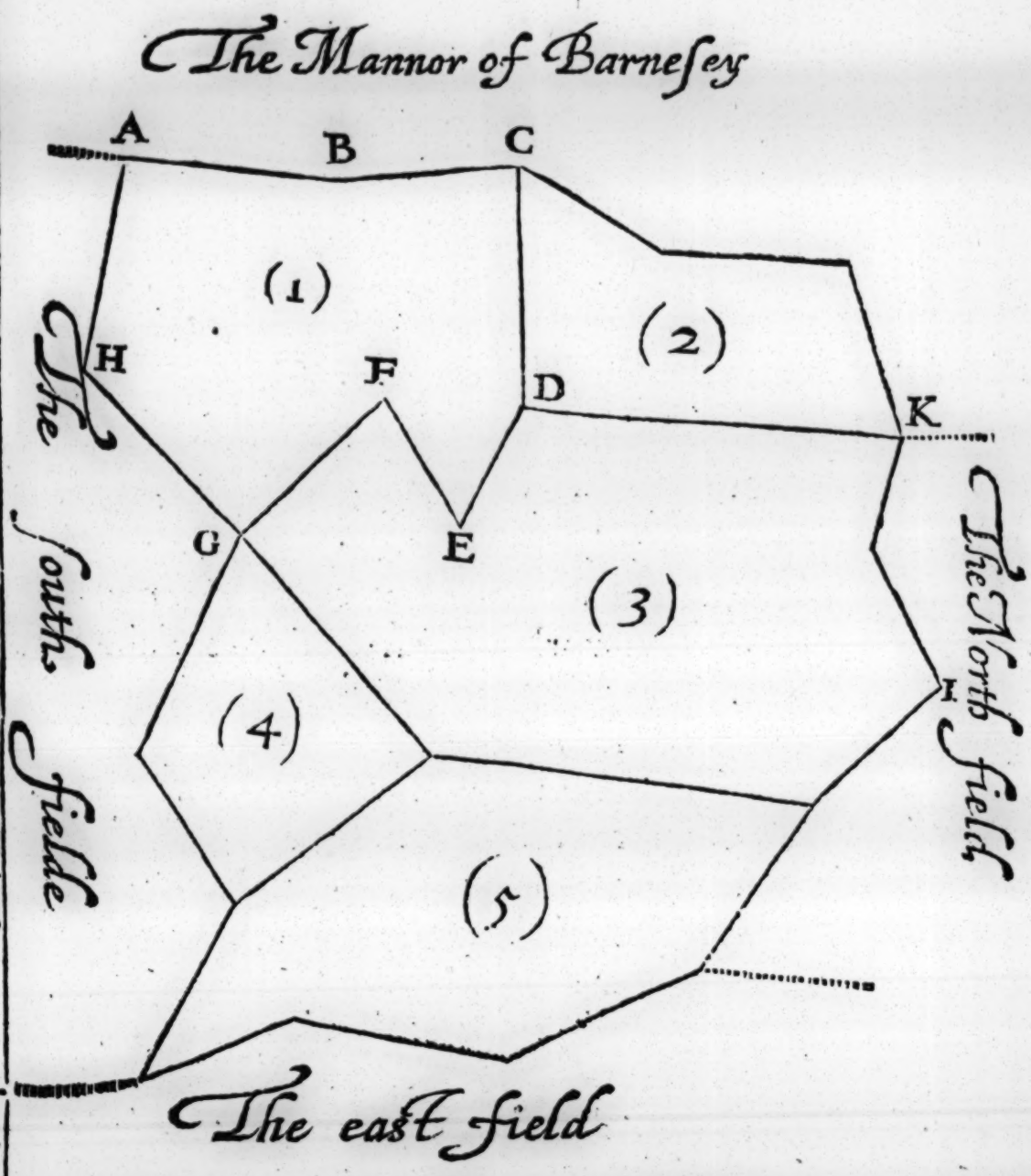
$31\frac{1}{2}$	10	3	
$44\frac{3}{4}$	14	5	North Field.
20	16	2	
$4\frac{1}{2}$	34	6	Oake Close.
$46\frac{3}{4}$	27	3	⊕ At (3.) East garth.

(4) His East garth, Meade, Free.

28	22	2	South Field.
$46\frac{1}{2}$	16	6	
to $78\frac{1}{2}$	23	2	⊕ Oake Close.

(5) His Oake Close-Pasture, Lease for three liues.

$26\frac{3}{4}$	18	5	South field.
74	15	---	
63	20	4	East Field.
75	19	5	
$84\frac{3}{4}$	19	--	North Field. ⊕ On vpper barn close.



you had begunne it where you ended the last, and then it is not materiall) wherefore seeing now you are to begin at C. looke in your booke on the worke of your last Close, at what place the line B C. ended, as at the end of 68. degrees, $\frac{\circ}{16}$. $\frac{1}{6}$. and there expresse No. (1.) on the right side of your down right lines, & then write your title for this Close thus: *W. Bensons north breading close Pasture Copie from No.* (1.) so shall you readily know when you come to Protraction, where to begin with this Close, and in the Margent place N. (2) for the number of this your second Close; and then worke on as before is taught for the last Close, till you come to K. where noting, that you want but one line as K D. to inclose this second Close, and also that this Angle K. is the fittest place to begin your third close at, placing here your Instrument, and directing your sight to D. where you are to inclose, write in the margent of your Booke against the degree here taken, this word (*to*) as thus, *to* $5\frac{1}{4}$. degrees, $\frac{\circ}{35}$. $\frac{1}{4}$. which serues to put you in mind, when you come to Protraction, that you are to seeke no other place, to begin your third Field at, but where now you are at the An-

gle K. where you tooke your laſt ſight. And then hauing incloſed this ſecond Cloſe, proceede to the third, from K. in the title whereof you need not make mention at what place you begin, for the former reaſon, but working in that, and all the reſt, as before is taught, you ſhall with great eaſe, and without confuſion, if there were infinite ſeuerall parcels, diſtinguiſh the worke of the one from the other, and be able by the onely helpe of your Field booke, thus formally ordered (if need were) to protract, and draw a perfect plat thereof many yeeres after: remembring alwayes that thoſe numbers in the Margent of your Booke, are to be placed ſeuerally in your platte, in thoſe Cloſes they ſeuerally repreſent; ſo ſhall you not need to write in the plat the names of the Tenants, or of the Cloſes, nor the quality or tenure thereof, but onely the number, which will alwaies direct you to your booke, where you may find it at large; for much writing in your plat (as is alwayes vſed in working with the Plaine Table) breedeth confuſion, and cauſeth much cumber and trouble in caſting vp the contents.

But ſeeing that practiſe is much more inſtructiue (in works of this nature) then many words; I will referre the reſt to your owne trauell; which by comparing thoſe former notes of the Field-booke with the plat, and often protracting the ſame according to thoſe degrees and lengthes, the whole courſe is very eaſily vnderſtood. The manner and order of which protraction is hereafter taught in the 39. Chapter of this Booke.

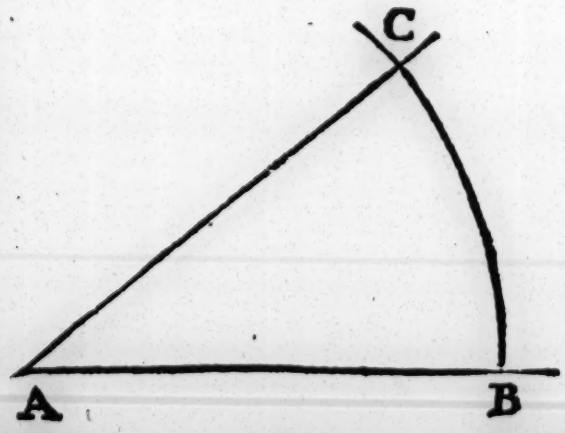
And here note further that in practiſe you ſhall find many helpes, which are too tedious here to expreſſe, as the taking in of diuers ſeuerals together, when they lie in ſuch ſort diuided with regular lines and hedges, that by onely taking true notice of their ſeuerall ends, as you paſſe by them, you ſhall moſt eaſily and ſpeedily ſeuer them on your plat: All which with many others (to auoide prolixitie) I muſt refer to your owne finding out by diligence and practiſe.

And thus are we now reaſonably well furniſhed of neceſſary implements for our purpoſe, and therefore fitting to prepare vs to practiſe: but yet before wee goe into the Fields, we will conſider of ſome neceſſary concluſions and obſeruations, fit to bee knowne and remembred.

CHAP. XI.

To lay downe an Angle of any quantitie required; or to find the quantitie of any Angle giuen, by the Sextans and the Scale.

SVppoſe it be required to lay downe an Angle of 40. degrees. Firſt draw any line at pleaſure, as A B. then opening the Compaſſe to the diſtance E F. the chorde of your *Sextans* (deſcribed in the 8. Chapter) and placing one foot in A. with the other, deſcribe the Arch C B. cutting the line A B. in B. then extend the Compaſſe on the Sextans to 40. degrees, and with



one

one foot in B. crosse the Arch line in C. and draw the line A C. So shall you describe the Angle C A B. containing 40. degrees, as was required.

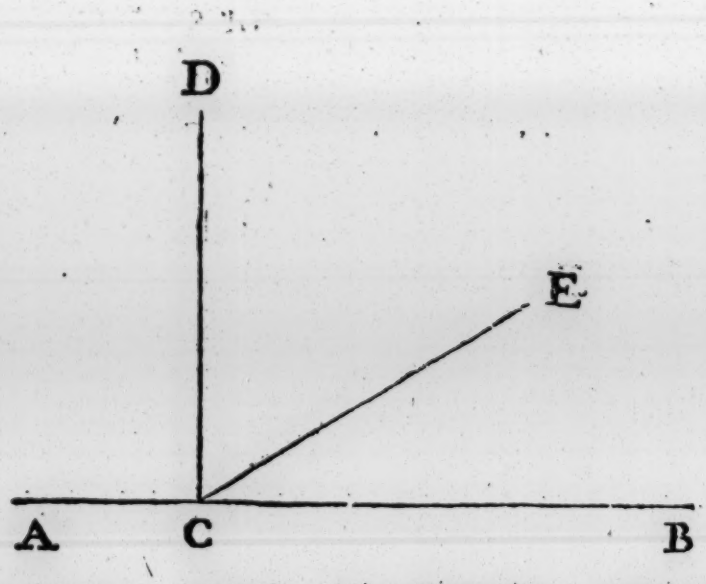
Againe, suppose C A B. be an angle giuen, & let it be required to know the quantitie thereof. Extend the Compasse to the chorde of the Sextans as before, and at that distance with one foot in A. describe the Arch line C B. to cut both sides of the giuen Angle, as in C. and B. then opening the Compasse to C B. and applying them at the same distance, to the degrees in the Sextans. It will appeare that the quantitie of the Angle is 40. degrees, the thing required.

If the Angle giuen or required happen to be more then the whole Sextans, or aboue 60. degrees, yet take still the chorde of the same Sextans, and describe the Arch line as before; and first place the whole Sextans, (which is the chorde thereof) on the Arch line from B. so farre as it will extend beyond C. and thereunto on the same Arch line, adde so many degrees more as the Angle giuen or required, containeth degrees aboue 60. So shall you performe what was required.

CHAP. XII.

To lay downe an Angle of any quantitie required, or to finde the quantitie of any Angle giuen, by the Protractor.

SVppose it be required to lay downe an Angle of 30. degrees with the Protractor. First, draw any right line at length, as A B. then on any point thereof, as on C. place the Centre of the Protractor, on which point moue it about, by helpe of the protracting pinne held in the Centre, till the Meridian line of the Protractor lye precisely on the line A B. (the semicircle of the Protractor being vpwards) and by the edge of the Semicircle at the diuision of 30. degrees, marke with the protracting pinne the point D. and draw the line D C. which shall include the Angle D C B. containing 30. degrees as was required.



Againe, suppose D C B. in the former Diagram bee an Angle giuen, and let it be required to know the quantitie thereof. Place the Centre of the Protractor as before in the Angle C. and the Meridian line thereof on the line C B. and hauing the semicircle vpwards, note what degree on the edge of the Protractor is cut by the line D C. which you shall find to be 30. Shewing that the giuen Angle D C B. containeth 30. degrees, the thing required.

But here is to bee noted, that these degrees thus taken by the Protractor belonging to the *Peractor* or *Circumferentor*, are not the true degrees of a Cir-

Circle; for one degree of a Circle, is but the 360. part thereof, and one of theſe degrees thus taken are the 120. part, ſo that one of theſe containeth three of thoſe; Wherefore if you are to take the quantity of an Angle (according to the degrees of a Circle) by thoſe Protractors, take alwayes a third part vpon the Semicircle, of the number giuen or required; as in the former example, where 30. is giuen, take 10. and ſo ſhall you finde the Angle E C. B. in the laſt Diagram, to be an Angle of 30. degrees, and to be a third part of the Angle D C B. As may be proued, if you apply thereunto a Protractor belonging to the *Theodelite*; yet notwithſtanding theſe Protractors and degrees in all our occaſions in the vſe of the Circumferentor and Peractor are alwayes to bee vſed, which will tend to one and the ſame purpoſe.

CHAP. XIII.

The reducing of ſtatute meaſure into Acres of any cuſtomary meaſure required, and the contrary, ſhewing the difference betweene them.

BY the Statute of 33. *Ed.* 1. It was ordained that an Acre of ground ſhould containe 160. ſquare Perches, to be meaſured by the Pole of 16½. feete, which is the meaſure now receyued, and generally allowed of, and is commonly called Statute-meaſure: yet notwithſtanding in many places of this Kingdome, there are diuers other ſorts retayned and claimed as cuſtomary, whereof ſome are greater, and ſome leſſe then that by Statute. Wherefore I hold it very fitting, and a maine point belonging to a Surueyor, readily to reduce theſe quantities from the one to the other, whereby the difference may appeare; whereof in practiſe hee ſhall find often vſe; which to effect worke thus. Suppoſe there are 5. Acres, 2. Roodes, 20. Perches, of 18. feet to the Pole giuen (called Wood-land meaſure; and let it be required to know the quantitie thereof by ſtatute meaſure, being of 16½. Firſt, finde out the leaſt proportionall termes, betweene 18. and 16½. which by their abbreuiation, by 1½. you ſhall finde to be 12. and 11. then reduce your giuen quantity into the loweſt denomination, which is Perches, ſo ſhall your 5. Acres, 2. Roodes, 20. Perches, be 900. Perches. And conſidering that the ſame proportion which the ſquare of 12. bears to the ſquare of 11. the like proportion beares the Acre of 18. foot Pole to that of 16½. therefore ſquare thoſe two termes 12. and 11. which produceth 144. and 121. then multiply the giuen quantitie 900. Perches by 144. the greater ſquare (becauſe the greater meaſure 18. is to be reduced into the leſſer 16½. the Factus is 129600. which diuided by the leſſer ſquare 121. quoteth 1071 $\frac{9}{121}$. Perches; which reduced into Acres, is 6. Acres, 2. Roodes, 31. Perches, and $\frac{9}{121}$. parts of a Perch; for the quantitie required in ſtatute meaſure, whoſe difference by deducting that from this, appeareth to be 1. Acre, 0. Roode, 11. Perches $\frac{9}{121}$.

But ſuppoſe the giuen quantitie had beene ſtatute meaſure, and the ſame required to be reduced into Wood-land meaſure; then ſhould you haue multiplied the 900. Perches giuen by 121. the leſſer ſquare (becauſe the leſſer meaſure 16½. were to be reduced into the greater 18.) whereof the Product

duct is 108900. which diuided by 144. the greater ſquare quoteth 756 1/4. Perches, which reduced into acres, is 4. Acres, 2. Roodes, 36 1/4. Perches, for the quantitie by Wood-land meaſure; whoſe difference by deducting this from that appeareth to be 0. Acre, 3. Roode, 23 3/4. Perches.

And the like courſe is to be held in all reſpects, with all other quantities of what proportion ſoeuer; as thoſe of 12. 20. 24. 24 1/2. and 28. foot to the Pole, of all which ſeuerall ſorts I haue found in diuers places, whoſe difference in euery Acre, from that of 16 1/2. appeareth by this Breuiat following.

An Acre meaſured by the Pole of theſe feet,	*Containeth of Statute meaſure*
12	0 — 2 — 4 76/121.
18	1 — 0 — 30 50/121.
20	1 — 1 — 35 35/1089.
24	2 — 0 — 18 66/121.
24 1/2	2 — 0 — 32 333/1764.
28	2 — 3 — 20 4/9.

And here is it not amiſſe to note the benefite and vſe of your two Scales of 11. and 12. in an inch formerly deſcribed in the 8. Chapter, which will ſerue you now to purpoſe. For if in your Surueyes (as often hapning) you meete with Wood-land grounds, whoſe quantities are required to be of the Acre of 18. foot Pole, and yet platted with the reſt: In ſuch caſe you may meaſure thoſe Wood-lands with the Pole of 16 1/2. and likewiſe plat the ſame with the Scale of 12. as the reſt, but to caſt vp the contents of thoſe Wood-land grounds by the Scale of 11. which will produce the deſired quantitie; By reaſon that if 11. Perches be meaſured in a right line with the 18. foote pole, the ſame length containeth 12. Perches meaſured with the 16 1/2. foote pole. But if you are conſtrained in the meaſuring of your Wood-lands, to vſe the Pole of 18. foot; then muſt you protract and lay downe the ſame in your plat by the Scale of 11. which otherwiſe will not ioyne with your other works; and the ſame likewiſe to be caſt vp by the ſame Scale of 11. as before. So ſhall you obtaine the true quantitie thereof in Acres, after the meaſure of the 18. foot pole required.

CHAP. XIIII.

Of the Table of Sines expreſſed on the Circumferentor.

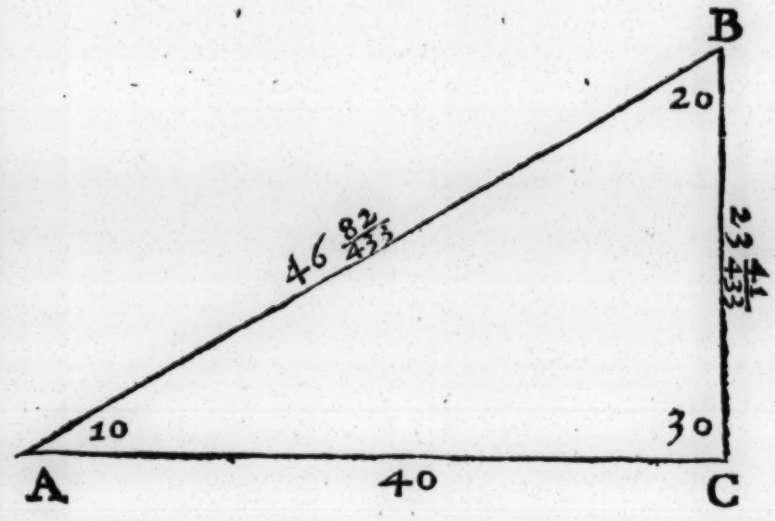
THis Table (as is ſpecifyed in the deſcription of the Circumferentor (Chap. 4.) ſerueth for the calculation, reſolution and dimenſion of Triangles; not in reſpect of the Area or ſuperficiall content thereof; but for the finding out of the vnknown ſides and angles of the ſame; by meanes whereof, all manner of quantities in menſuration of altitudes, profundities, longitudes and latitudes are exactly knowne and diſcouered, conſidering that none of theſe can be had or obtained inſtrumentally, without deſcription of Triangles.

Where-

Wherefore let it firſt bee conſidered, that by the 73. THEOREME of the firſt booke, the ſides in all plaine Triangles are in proportion the one to the other, as the Sines of the Angles oppoſite to thoſe ſides.

And then that in euery Triangle, there are ſixe termes, that is, 3. ſides and 3. angles, whereof any three being knowne (ſo one be aſide) the other three are had, by meanes of this Table, and the rule of proportion thus.

Suppoſe B C. in the Triangle A B C. to be a Tower or perpendicular altitude giuen, and let it be required to finde the height thereof, and the length of the Hypothenuſall line A B. Firſt, meaſure the diſtance from A to C. 40. then by any Inſtrument planted at A. take the quantitie of the Angle B A C. which let be 10. degrees, then conſequently by the 13. THEOREME of the Firſt, the Angle A B C. ſhall be 20. degrees, for that the Angle A C B, is a right Angle: So is there here alreadie had foure parts or termes of the ſixe before mentioned, namely, the three Angles, and the length of the line A C. Wherefore now repayre to the table, and find there the Sines of thoſe Angles which are theſe, of the Angle A B C. 20. degrees, the Sine is 866. of the Angle B A C. 10. degrees, the Sine is 500. and of the Angle A C B. 30. degrees the Sine is 1000. the totall Sine: Then by the rule of proportion reaſon thus. If A C. 866. the Sine of the Angle A B C. giues B C 500. the Sine of the Angle B A C. what giues A C. 40, and multiplying 500. by 40. the Product is 20000. which diuided by 866. quoteth $23\frac{41}{433}$. for the height of A C. required.



Again, if A C. 866. the Sine of the Angle A B C. giues A B. 1000. the Sine of the Angle A C B. what giues A C. 40. and multiplying 1000. by 40. the Product is 40000. which diuided by 866. quoteth $46\frac{82}{433}$. for the length of A B. the Hypothenuſall line, as was required.

And thus much for a ſmall taſte onely of this little Table, which may ſerue to induce and incite a willing minde, not onely to the vſe and exerciſe thereof; but to the further conſideration and practice of the infinite vſe of thoſe moſt excellent Tables and workes de BARTHOLOMAEO PITISCO GRVNBERG. *now partly tranſlated into Engliſh by M.* RAPH HANDSON; *and of thoſe Tables, and more then admirable inuention of* LOGARITHMES, *by that diuine and noble Writer, the Lord* MARCHISTON, *whoſe name and honour will neuer out.*

CHAP.

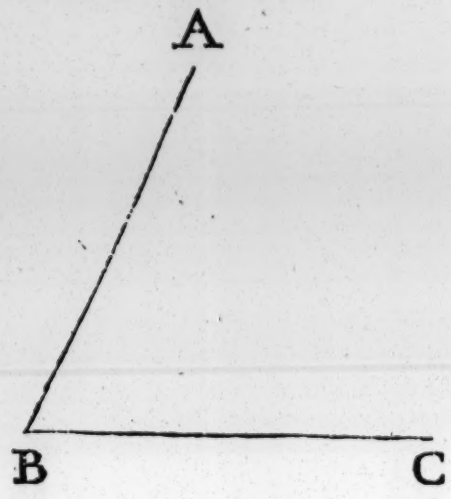
CHAP. XV.

Of the congruitie in vſe betweene the Peractor and Circumferentor; and the meanes to find the quantitie of an Angle by eyther of them.

THeſe two Inſtruments in vſe differ little or nothing, conſidering the degrees of eyther are equally numbered, although thoſe of the Circumferentor are placed on the Carde, and theſe of the *Peractor*, on the limbe of the Planiſphere, whereby they are ſo much the larger, and thereby the fitter for vſe: onely herein they differ, the degrees obſerued and taken by the *Peractor*, are euer cut by the edge of the Index, moued about till through the ſights thereon, the obiect be found, the Needle being alwayes kept on one degree, and that moſt fitly on the Meridian line in the Carde, the North end (being that with the Croſſe) lying euer ouer the Flower deluce, and the ſouth end pointing to the beginning of the degrees; and the degrees obſerued and taken by the *Circumferentor*, are alwayes cut in the Carde by the South end of the Needle, playing about at pleaſure, whileſt the Inſtrument and the ſights thereof are directed to the purpoſed marke. By meanes of which diuerſitie, there is a diuers meanes to be vſed, in taking the true quantitie of an Angle, by theſe two Inſtruments, as followeth.

By the 49. DEF. *of the Firſt. The quantitie or meaſure of an Angle, is the Arch of a Circle, deſcribed from the point of the ſame Angle, and intercepted betweene the two ſides of that Angle, which is found by the Circumferentor thus:*

Suppoſe A B C. be an Angle giuen in the Fields, and let A B. and B C. be two hedges or other lines, containing the ſame Angle; and let it be required, to take the quantitie thereof by the *Circumferentor*. Firſt, place your Inſtrument in the Angle at B. and turning the ſame about on the ſtaffe, direct your ſight towards A. to bee alwayes parallell to the line B A. where obſerue what degree the ſouth end of the Needle cutteth, which let be 40. degrees, then direct your ſight towards C. and there making like obſeruation, let the degree cut bee 18. But yet haue you not the quantitie of this Angle, but onely the degrees cut at each obſeruation. Wherefore now you are to deduct 18. the leſſer from 40. the greater terme, the remainder will bee 22. the true quantitie of the giuen Angle. But had this remainder exceeded 60. (which is two right Angles, or a Semicircle) then muſt you haue deducted that remainder out of 120. the whole circle, and the remainder of that laſt deduction had beene the quantitie required; which circumſtance in the *Peractor* needeth not, wherewithall worke thus.



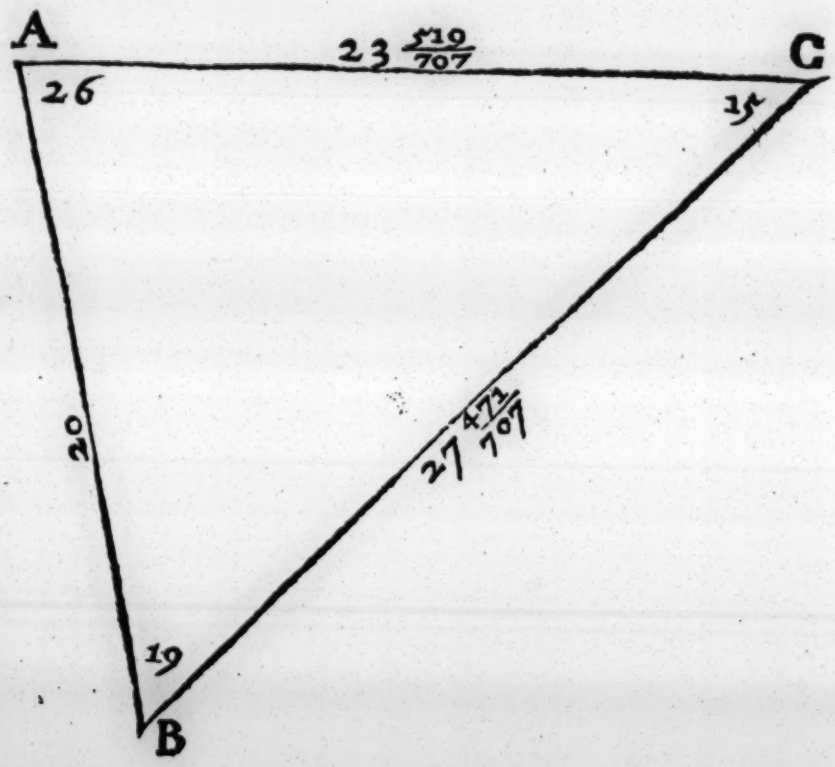
Place your Inſtrument at B. in the former Diagram. (the Index ſtanding on the Diameter where the degrees commence) then turne about the Inſtrument

ment on the ſtaffe (the Index remayning) towards A. till your ſight bee parallel to the line B A. and there your Inſtrument fixed, remoue the Index, directing your ſight towards C. to be likewiſe parallell to B C. where obſerue what degree the edge of the Index cutteth, which will be 22. the quantitie required. And here note an exquiſite diſpatch.

CHAP. XVI.

To take any horizontall diſtance at two ſtacions, by Sinicall computation.

SVppoſe A and B. be two places giuen, & let it be required to know the diſtance from eyther of them to C. Firſt, place your Inſtrument at A. where directing your ſight firſt to C. and then to B. take the quantitie of that Angle, as was taught in the laſt Chap. which ſuppoſe to be 26. degrees, then meaſuring from thence to B. let the length there found be 20. where placing your Inſtrument, take likewiſe the quantitie of that Angle as before, which let be 19. degrees. Now to finde the quantitie of the third Angle at C. (Foraſmuch as by the 13. THEOREME of the firſt, the three inward Angles of euery Triangle is equall to two right Angles) adde together the quantitie of thoſe two Angles at A. and B. already found, 26. and 19. which makes 45. and that deducted from 60. (which is two right Angles or a Semicircle) the remainder is 15. the quantitie of the Angle at C. So haue you gotten the quantitie of euery Angle, and the length of one of the ſides, namely, the Stationary diſtance A B. Now to euery of thoſe Angles in the Table of Sines, finde out their ſeuerall Sines, whereof make collection in your Field booke, or otherwiſe thus.



	d	s
A.	26	978.
B.	19	839.
C.	15	707.
A B.	20. Perches.	

Then by the Rule of Proportion, reaſon thus. If A B. 707. the Sine of 15. degrees yeelde 20. the ſtationary diſtance, what A C. 839. the Sine of B. 19. degrees, and multiplying 839. by 20. and diuiding the Product by 707. the quotus will bee 23. $\frac{519}{707}$. for the diſtance A C. And againe, if A B. 707. the Sine of C. 15. degrees, yeeld 20. the ſtationary diſtance, what B C. 978. the Sine of A. 26. degrees, and multiplying 978. by 20. and diuiding the Product by 707.

707. the quotus will bee 27 $\frac{371}{707}$. the diſtance B C. required.

The Theoricall ground and reaſon of this worke dependeth on the 13. *and* 73. THEOREMES *of the firſt booke.*

Where note in all works of this nature, that if any of the three Angles bee an obtuſe Angle, containing aboue 30. *degrees, then (ſeeing the Table of Sines exceedeth not* 30.*) deduct the exceſſe of the obtuſe Angle aboue* 30. *out of* 30 *(as if it were* 44. *the exceſſe whereof aboue* 30. *is* 14. *which deduct out of* 30. *there remaineth* 16.*) and of that remainder ſeeke the Sine in the Table, which ſerues the turne. The reaſon hereof is, becauſe the right Sine of the Arch in the greater or leſſer Quadrant are all one and the ſame thing.*

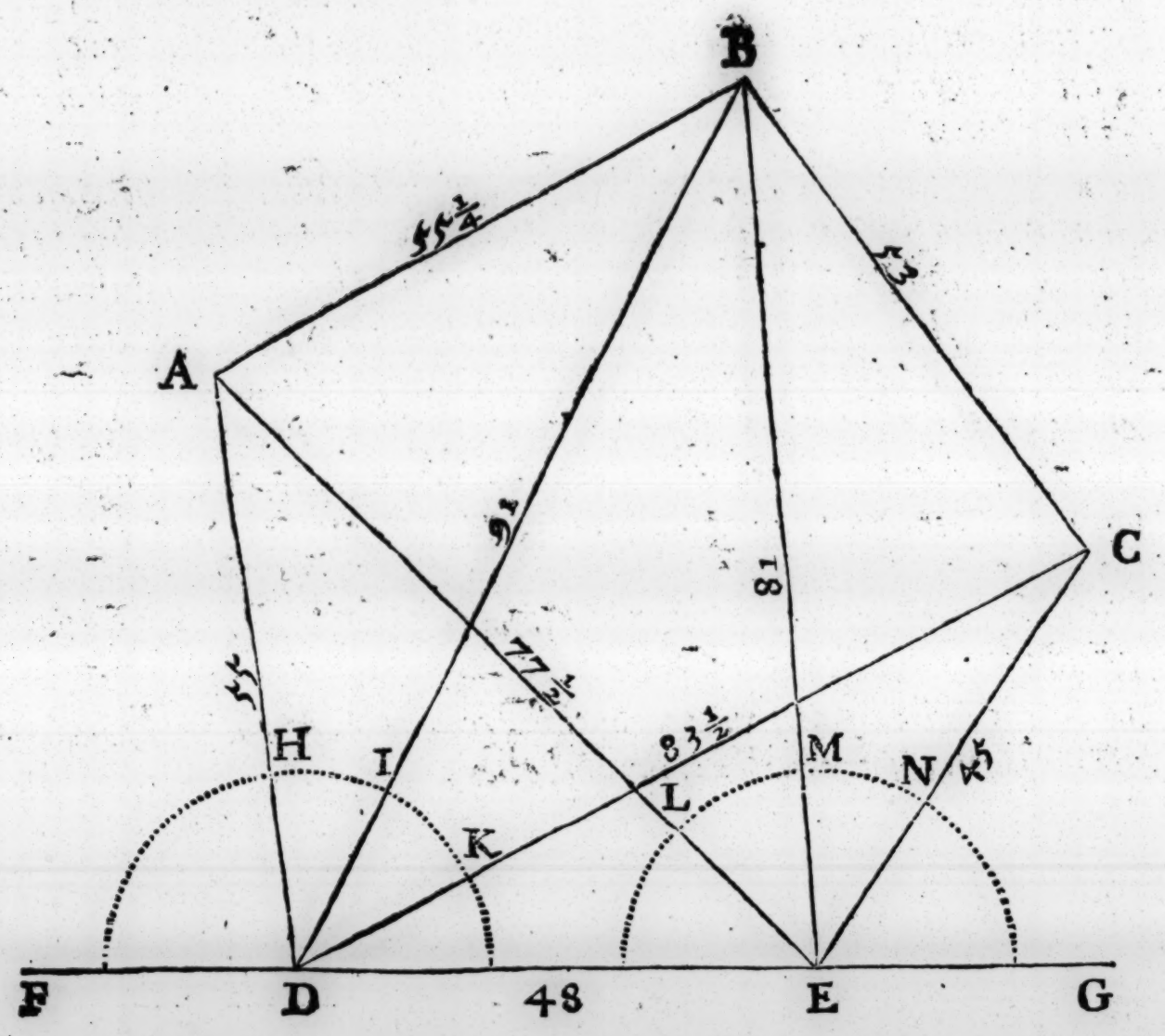
Likewiſe note alwayes in your working by the golden Rule, that the Sine of the Angle oppoſite to the Stacionary line (as in this example 707.*) muſt bee your firſt proportionall number; and moſt fitly (though it may be otherwiſe,* tranſpoſitis terminis medijs*) the diſtance betweene the two ſtacions the ſecond; and the Sine of the Angle, oppoſite to the ſide, whoſe length you ſeeke the third.*

And note alſo, that not onely this, but all other the like Propoſitions are to be performed, aſwell by the Peractor *and* Theodelite, *as the* Circumferentor, *the Table of Sines being had in any voyde paper, or much rather thoſe ſmall Tables of* Logarithmes, *or of* Pitiſcus, *which are imprinted by themſelues in ſmall volumes, being moſt excellent pocket-companions for infinite Concluſions, aſwell Geometricall as Aſtronomicall.*

And if any deſire the performance of this Propoſition, or the like by protraction; let him diligently obſerue the doctrine of the next.

CHAP. XVII.

To take the distance aswell betweene diuers seuerall places remote from your place of being, as betweene your being, and those seuerall places, by the helpe of two stacions.



SVppose A.B. and C. bee three places giuen remote from your place of being, which let be at D. and let it bee required at D. to finde the seuerall distances, aswell betweene A B. and B C. as betweene D A. D B. and D C. First place your Instrument at D. and directing your sight to A. obserue what degree is there cut, eyther by the Needle of the *Circumferentor*, or by the Index of the *Peractor*, which let be 32 ½. degrees, to be noted for your first obseruation, thē turning your sight to B. make the like, where you finde 21. degrees, and the like towards C. obseruing 15. degrees. Then your second Stacion, (not being limited) make choyse thereof with such discretion (if the place will affoord it, as at E.) that your Stationary distance bee no lesse at the least then ⅓. of the other distances from you, how much greater (with reason) makes no great matter; and as neere as you may, let it make a right Angle with the first obseruation of your first Stacion, then for the last worke of that stacion direct your sight to E. obseruing the degrees cut 120. then take vp your Instrument, and leauing a marke at D. measure from D. to E. the stationary distance, which suppose 48. then at E. plant your Instrument precisely as at D. vsing the helpe both of your Needle and backe sights herein, looking backe to your marke at D. whereof speciall care is to bee had, or maine

maine errors may enſue: which done, direct your ſight firſt (as at the firſt ſtacion) to A. obſeruing the degree there cut 45. the like to B. 31 ½. degrees, And laſtly, to C. 15. degrees. So haue you finiſhed; if you omit not the collection of your ſeuerall obſeruations, which in your Field Booke or otherwiſe are thus to be expreſſed.

d

Obſ. { 1. St. { 1 — 32 / 2 — 21. / 3 — 9 ¼. / 4 — 120° } dist. { P 48. } 2. St. { 1 — 45. / 2 — 31 ½. / 3 — 15.

Then prouide a cleane ſheete of paper, and according to theſe collections layde before you, protract the ſeuerall angles or degrees here obſerued, as is taught in the next.

CHAP. XVIII.

To protract any number of Angles or degrees taken by the Peractor, Theodelite or Circumferentor, at ſeuerall obſeruations.

LEt the Angles or degrees taken be thoſe expreſſed in the former Chapter, and let it bee required to protract the ſame, whereby the quantitie of each ſeuerall diſtance there ſought for, may appeare. Firſt, on your paper prouided, draw a right line at pleaſure, as F G. in the former Diagram; then laying your Field-Booke before you, with the former obſeruations, make choyſe of any point in the line F G. to repreſent your firſt Stacion, as at D. then applying the Scale of your Protractor to that line, lay downe your ſtacionary diſtance 48. Perches from D. to E. repreſenting the place of your ſecond ſtacion; and placing your Protractor with the Centre on the point D. (the Semicircle vpwards) turne it about thereon, till that degree on the Protractor which was taken from the firſt to the ſecond ſtacion (which in this example is 120.) lye preciſely on the line F.G. and then looke in your Field-Booke for the degrees obſerued at your firſt ſtacion, which were 32 ½. 21. and 9 ¼. (for the fourth 120. that is ſuppoſed alwayes to fall on the firſt drawn line F G.) and againſt thoſe ſeuerall degrees on the limbe of your Protractor, by the edge thereof with your protracting pinne, make ſeueral pricks or points, as at H. I. and K. then by the point D. and thoſe three ſeuerall pricks with the ſcale of your Protractor, and protracting pinne, draw out at length the lines D H. D I. and D K. ſo haue you finiſhed your firſt obſeruations; then place your Protractor on the point E. in all reſpects as before, at D. and there mark the degrees of your ſecond obſeruations, as 45. 31 ½. and 15. as before, at the points L. M. and N. whereby, and by the point E. draw out at length the

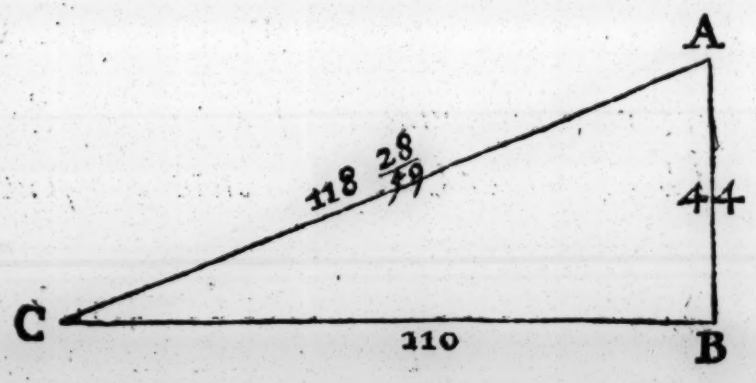
lines E L. E M. and E N. till they interſect the three former lines, drawne from D. as in the points A. B. and C. by which interſections from point to point, draw the lines A B. and B C. So haue you finiſhed your Protraction. And by applying the ſcale of your Protractor, (whereby the ſtacionary diſtance was laide downe) to any line or diſtance, the ſeuerall quantities will appeare to be as they are expreſſed in the Diagram, on the ſeuerall lines thereof, as was required.

But here is to be noted, that if thoſe former obſeruations were made and taken by the Theodelite, *then this Protraction is to be made and layde downe by the Protractor belonging to the* Theodelite, *being dimided into* 360. *degrees, as is before mentioned, which is to be performed in all reſpects, according to the rules and inſtructions before deliuered.*

CHAP. XIX.

To take any acceſsible altitude by the Circumferentor or Plaine Table with the diuided ſights.

SVppoſe A B. to bee a perpendicular height, & let it bee required from C. (whoſe diſtance to B. is acceſſible) to know the altitude thereof. Firſt, place your Inſtrument at C. preciſely horizontall, by helpe of the plummet hanging on the ſhorter ſight; whereunto great care muſt bee had aſwell in this, as all other works of this nature, or maine errors will ariſe; then moue the vane on the longer ſight vpwards and downwards, till through the hole therein, and by the pinnes head on the toppe of the ſhorter ſight, you eſpie the Summitie of the giuen height at A. where note the equall diuiſions cut on the longer ſight by the edge of the vane, which ſuppoſe to be 40. then meaſure the diſtance from C. to B. which let be 110. And the ſame proportion which 40. the part cut hath to 100. the ſide of the ſquare proiected on the Inſtrument, the like hath the altitude A. B. to the meaſured diſtance, 110. Wherefore multiply 110. the diſtance by 40. the parts cut, and diuide the Product by 100. the quotus will be 44. the altitude required.



But it hapneth oftentimes that the altitude required is of that height, that you cannot produce the vane low inough, to ſee the ſummitie of the height, as before. In which caſe you are to vſe the Index to be placed on the wyer-pinne in the edge of the ſhorter ſight, and turning it vp and downe cloſe by the right edge of the Inſtrument, till through the ſight thereof, and by the wyer pinne you eſpie the ſummytie of the giuen height, and then note the parts cut on the ſame edge of the Inſtrument, by the fiduciall edge of the Index. For the ſame proportion which the parts cut, beare to 60. (the imagined parts on the edge of the ſhorter ſight) the like hath the meaſured diſtance

to

to the altitude required. Wherefore, multiply the ſame meaſured diſtance by 60. and diuide the Product by the parts cut, the quotus ſheweth your demand. And if you deſire to know the Viſuall or Hypothenuſall line, multiply the meaſured diſtance by the parts cut on the edge of the Index, and diuide the Product by the parts cut on the edge of the Inſtrument; the quotus ſheweth what you deſire. For what proportion the parts cut on the edge of the Inſtrument, beare to thoſe cut on the edge of the Index, the like doth the meaſured diſtance to the Viſuall line.

And here is to be noted, that of this later worke the plaine Table *hath no vſe; and therefore of all other Inſtruments moſt vnfit for theſe purpoſes of Altitudes and Profundities, without helpe of the quadrant ſpecified in the deſcription thereof,* CHAP. 3.

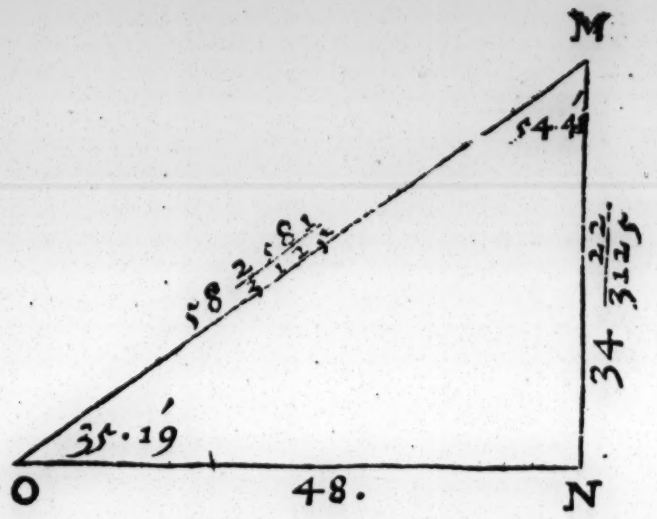
Or with the Circumferentor, by Protraction, thus.

PLace your Inſtrument at C. as before, and there obſerue the quantitie of the Angle of altitude, which being gotten, protract and lay downe the ſame as hath beene taught, and on the baſe line from C. to B. lay downe the meaſured diſtance 110. at the end whereof, as on the point B. either by the 6. PROBLEME of the ſecond Booke, or with helpe of your Protractor erect a perpendicular line as A B. to cut the other ſide of the protracted Angle, as in A. and with applying your Scale thereunto, the altitude appeareth, as was required.

CHAP. XX.

To take any acceſsible altitude diuers wayes, by the Peractor and the Quadrant thereof.

SVppoſe M N. to be a perpendicular height giuen; and let it be required from O. to find the Altitude thereof. Place your Inſtrument preciſely Horizontall at O. as before is taught, then mooue your Quadrant vp and downe, till through the ſmall round hole, in the end of the ſight towards you at B. on the Quadrant (as it is deſcribed in the 5. Chapter) and by the pin in the great round hole of the other end at A. you eſpie the Summytie M. of the giuen height, where letting your quadrant reſt, meaſure the diſtance O N. which ſuppoſe to be 48. then looke on the ſide C D. of the quadrant, for the 48. line, reckoning from the Centre P. and paſſing downe by that line to the edge of the handle or Index (which ſuppoſe to ſtand now on the line P F. drawne from the Centre of the quadrant)



drant) note what line (paſſing from the other ſide A.B. of the quadrant) the former line 48. meeteth and interſecteth on the edge of the Index, ſuppoſed as before the line P. F. and you ſhall find it ſomewhat more then 34. wherefore I conclude, that the altitude A B. is ſo much: and noting what part of the Index is there cut, you ſhall find it ſomewhat more then 58 $\frac{4}{5}$. the length of the viſuall line, or hipothenuſall O. M. Where is to be noted, that in all workes wrought with this quadrant, the ſide thereof A. B. repreſenteth the perpendicular height, the ſide C. D. the horizontall diſtance, and the Index or handle, the hipothenuſall or viſuall line.

Or Sinically thus.

HAuing placed your Inſtrument as before at O. By the degrees on the limbe of the quadrant, obſerue the angle of altitude M O N. 35. degrees 1′9. and meaſuring the diſtance O N. 48. as before (by the helpe of *Pitiſcus*, or any other Canon) Reaſon thus: If O N. the radius 100000. yeeld 48. the meaſured diſtance; what M N. 70848. the tangent of the Angle M O N. 35. degrees, 1′9. and multiplying the tangent 70848. by 48. the meaſured diſtance, you ſhall produce 3400704. which parted by the radius 100000. quoteth 34 $\frac{7\,0\,4}{1\,0\,0\,0\,0\,0}$. or in leſſer termes, 34 $\frac{22}{3\,1\,2\,5}$. the altitude M N required.

Againe, if O N. the radius 100000. giues 48. the meaſured diſtance, what O M. 122554. the ſecant of the Angle M O N. 35. degrees, 1′9. and multiplying the ſame ſecant 122554. by 48. and parting the Product by the radius 100000. you haue 58 $\frac{5\,8\,8\,2\,9\,2}{1\,0\,0\,0\,0\,0}$. or in leſſer termes, 58 $\frac{2\,5\,8\,1}{3\,1\,2\,5}$. being ſomewhat more then $\frac{4}{5}$. the length of the viſuall or hipothenuſall line O. M. as before.

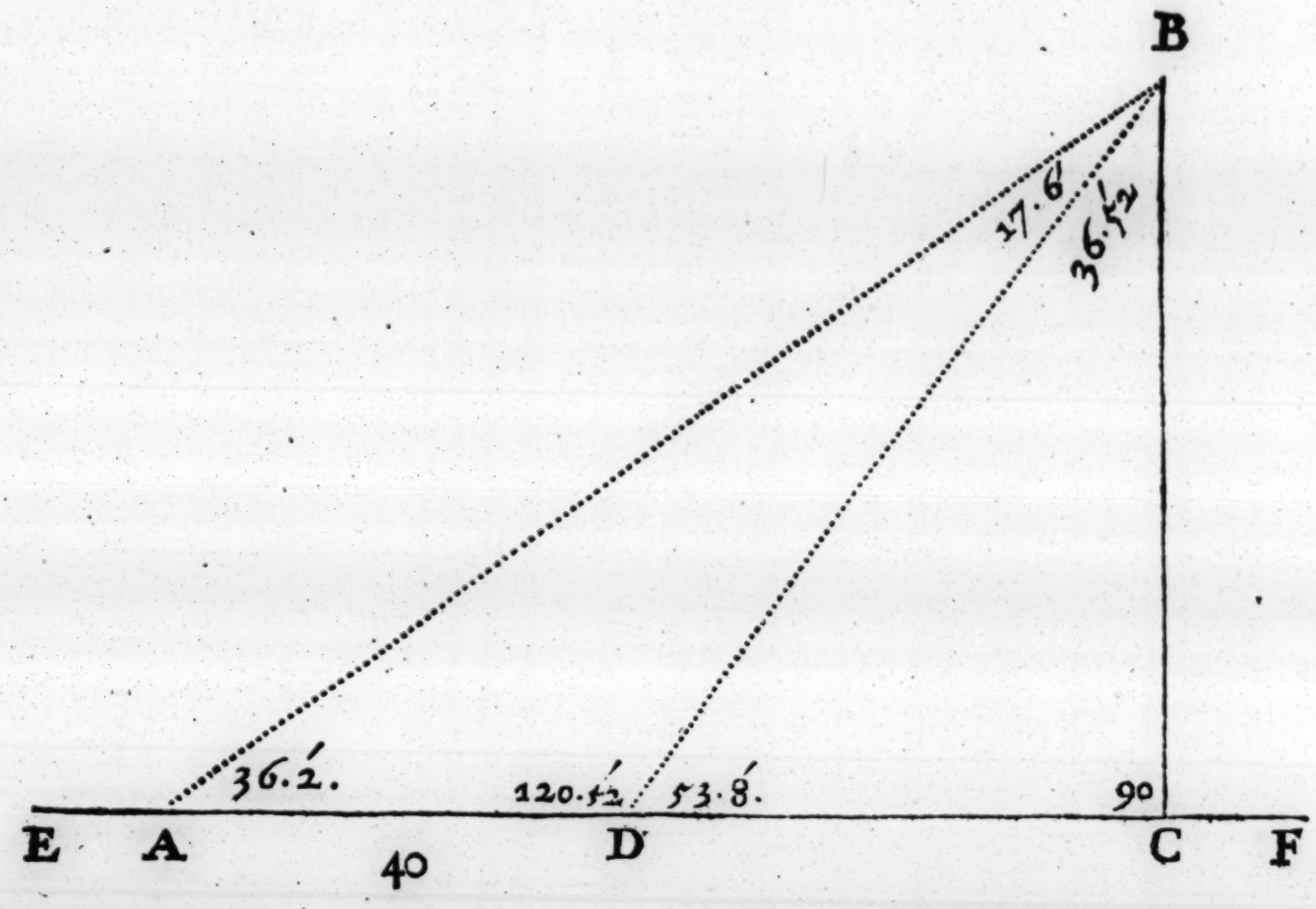
Or by Protraction, thus.

PLacing your Inſtrument at O. as before, obſerue the Angle of altitude M O N. and meaſure the diſtance from O. to N. And then proceed to protracting thereof, as is taught in the later part of the laſt Chapter before going; and the worke is finiſhed.

CHAP.

CHAP. XXI.

To find out any inacceſsible height by the Peractor, Theodelite, or Circumferentor.



IT may oftentimes happen that inacceſſible heights may be required, when by reaſon of waters, trenches, danger of ſhot, or many other impediments, a man cannot approch to the Baſe of the altitude required; yet of neceſſity to bee had and knowne, which to performe, in a moſt abſolute and exact manner, worke thus.

Suppoſe B C. to be a perpendicular height giuen, vnto the Baſe, whereof C. (by ſome impediment) you may not approch neerer then D. yet the altitude is required; wherefore place your Inſtrument at D. preciſely horizontall, and obſerue the Angle of altitude, as is before taught, which let be 53. degrees, 8'. then looking backwards, make choyſe in a right line from C. by D. of a ſecond ſtation, which let be A. and meaſure the diſtance from D. to A. which ſuppoſe 40. then at A. place your Inſtrument as before, and likewiſe obſerue there the Angle of altitude, which ſuppoſe 36. degrees, 2'. ſo is your Inſtrumentall worke already finiſhed. Then repayring to your *Canon of Triangles*, finde the complements of the Tangents of thoſe two Angles taken, which of 53. degrees, 8'. the angle firſt obſerued, is 74991. and of 36. degrees, 2'. the Angle of your laſt obſeruation, is 137470. betweene which two complements take the difference, by deducting the leſſer from the greater, which will bee 62479. and then (for as much as the ſame proportion which the difference of the complements 62479. beareth to the radius 100000. the like hath the meaſured diſtance betweene your two ſtations D A. 40. and the required altitude) multiply the radius 100000. by 40.

the

the measured distance, the Product is 4000000. which parted by 62479. the difference of the complements quoteth 64 $\frac{1144}{1000000}$. for the required altitude.

And if it bee required, to haue the length of a scaling ladder to extend from D. to B. or the length of the visuall line A B. or the inaccessible distance betweene D. and C. by respectiue obseruation of what was taught in the last Chapter, they are easily resolued.

To performe the same by Protraction.

HAuing taken both the Angles of altitude, and measured the stationary distance A D. 40. as before; protract the same thus; Draw a right line out at length, as E F. in the former Diagram, then on any point thereof, as at D. protract the first Angle taken, 53. degrees, 8′. (as before taught) then from D. towards E. lay downe the stationary distance 40. to end in A. on which point A. protract your other Angle 36. degrees, 2′. and continue forth the vpper sides of those two Angles, till they intersect each other as in B. from which point B. by the 6. of the second, or by helpe of your Protractor, let fall a perpendicular, to cut the line E F. as B C. in C. which line B C. shall represent the giuen altitude, the height whereof, and the seuerall lengthes and distances in the whole worke contained, is speedily had by applying thereunto the same scale, whereby your measured distance was laid downe.

And here is to be noted, that in all workes of this kind, it is very requisite to take your stationary distance as large as conueniently you may, for that otherwise by reason of the acuitie of the Angle, as here of A B D. you shall hardly discerne the true point of intersection by the lines B A. and B D. whereby, or from whence you may precisely let fall the perpendicular B C. as before in his due place: by neglect whereof mayne errors may insue.

Wherefore a most excellent, absolute and exact course is that in the former part of this Chapter, for the performance of all manner of conclusions of this kind, and to be preferred before all others.

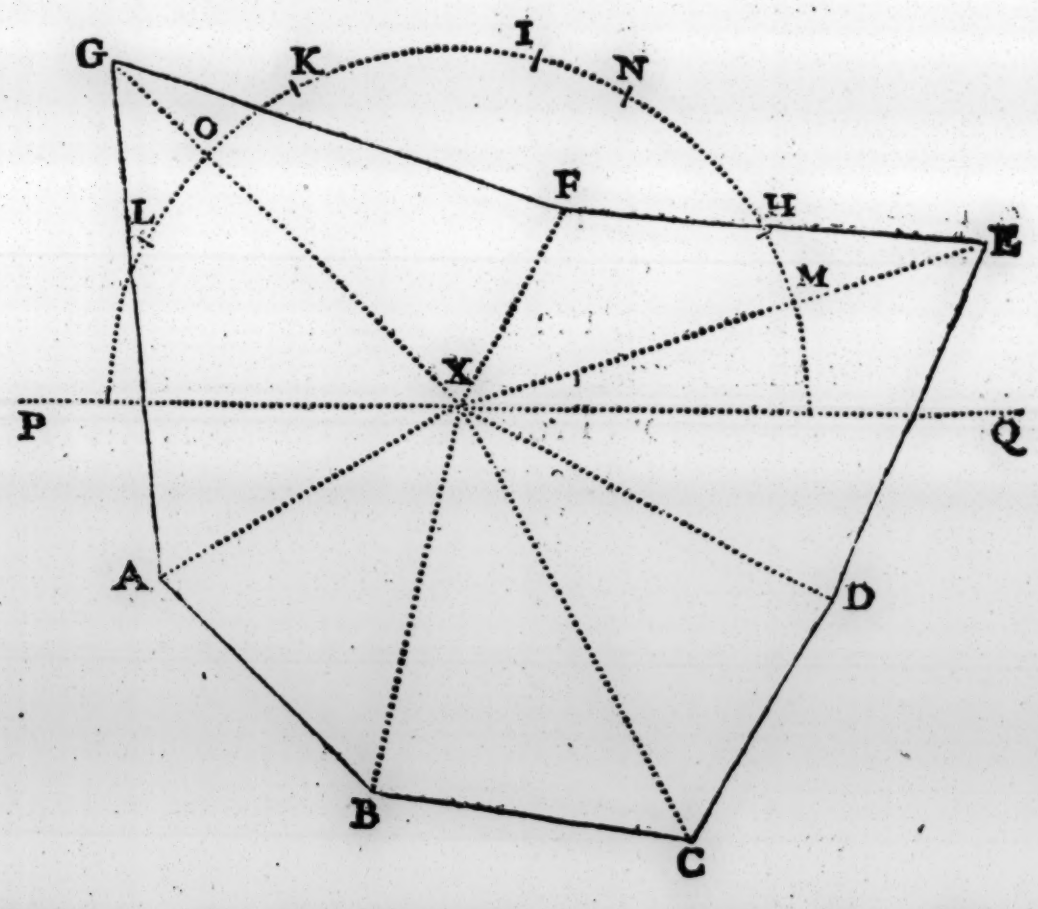
And here now might I much inlarge this worke, by inserting seuerall Propositions for the taking and finding out of distances in heights, with the mensuration of profundities diuers wayes; all which and infinite other conclusions are fully included within the limits of these few former instructions, and with diligent obseruation and practice thereof may bee well vnderstood and performed; for whoso can take one height artificially, may performe another, and by deducting the one from the other, may decerne the difference: and he that can skilfully take an altitude, by the same rule may performe a profunditie, being the one a direct conuersion of the other, without alteration or any difference, either in the Theoricall ground, or practike operation thereof: wherefore to make great shewes, or accumulation of needelesse varieties to one and the

the ſame purpoſe, were but expence of time vnto my ſelfe, and cauſe of confuſion to the learner, ſeeing feweſt Precepts (ſo effectuall) are fitteſt, aſwell for apprehenſion as retention.

But before I paſſe further, let this bee remembred, that in all the former obſeruations in taking of heights: the height of your Inſtrument is alwayes to be added to the altitude taken.

CHAP. XXII.

To take the plot of a Field at one ſtacion; taken in any part thereof, from whence you may view all the Angles, and meaſuring from the ſtacion to euery Angle.



SVppoſe A B C D E F G. to be a Field, whereof it is required to take the Plot. Firſt, cauſe papers or other markes to be placed directly in euery Angle; and then make choiſe of ſome ſuch conuenient place within the ſame, as from whence you may beſt view the ſeuerall Angles thereof, and there as at X. Plant your Inſtrument; if it be the *Peractor*, *Theodelite*, or *Plaine-Table*, faſten the Inſtrument to the ſtaffe with the ſcrue-pinne, that it ſtirre not till your worke be finiſhed, the needle ſtanding on the Meridian line of the Carde, if the *Circumferentor*, that care is already taken; but admit the *Peractor*; then direct your ſight, by turning the Index to any one Angle at your pleaſure, as firſt to A. and obſerue the degree there cut by the edge of your Index, which let be 10. and with your Chaine meaſure from your ſtation to that Angle, which ſuppoſe 30. then direct your ſight to B. and performe the like, and ſo to C. D. E. F. and G. till you haue finiſhed; ſtill entering as you paſſe your ſeuerell obſer-

obſeruations, aſwell of degrees as lines into your Field-booke, as was formerly taught in Chap. 10. which when you haue finiſhed ſhall appeare to bee thus.

Which being layde before you, ſhall moſt ſpeedily and exactly be protracted and layde downe, as is taught in the next.

d	o	I
10	30	—
26	33	2
39 1/2	41	—
51	36	—
66	47	5
81	18	8
105 1/2	41	5

CHAP. XXIII.

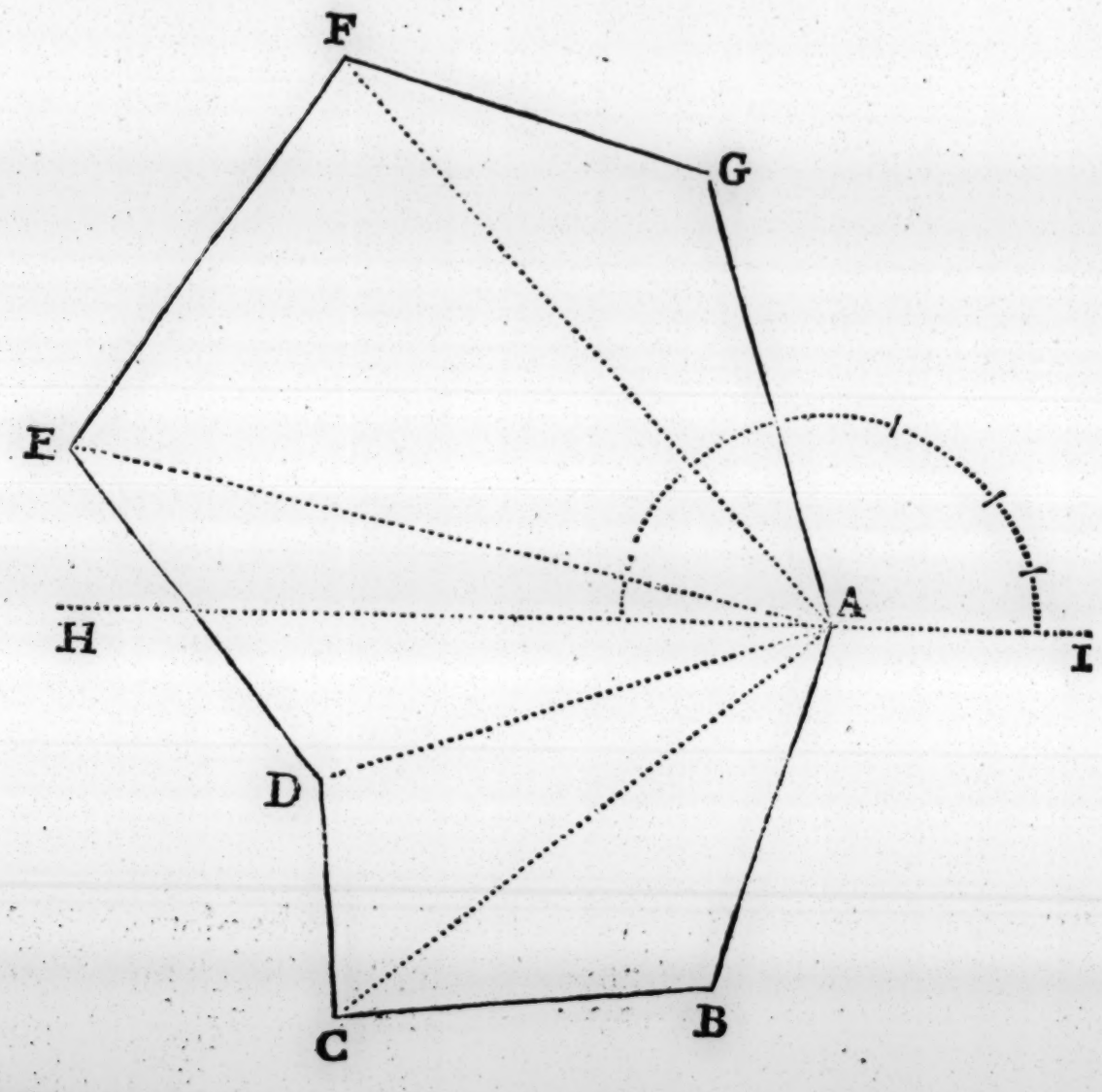
To protract and lay downe the obſeruations made in the laſt Chapter, or any other taken in the like ſort.

TAke a cleane ſheete of paper, and draw a right line thereon at pleaſure as P Q. in the Diagram of the laſt Chapter, repreſenting the meridian Line; and laying your Field-booke before you with the former obſeruations; Firſt, place the Centre of your Protractor (the Semicircle vpwards) on any point of the line as on X. with the Meridian line of the Protractor directly ouer that on the paper; where keeping your Protractor fixed, note all the degrees vnder 60. taken in your Field-booke, as 10. 26. 39 1/2. and 51. againſt which ſeuerall degrees on the limbe of your Protractor by the edge thereof, make ſeuerall prickes or points with your protracting pinne, as at H. I. K. and L. Then conſidering that thoſe degrees are vnder 60. and therefore to lye on the Eaſt ſide of your plot or meridian line P Q. lay the edge of the ſcale of your Protractor on the Centre X. and euery of thoſe points H. I. K. and L. and draw from the Centre X. vnder the meridian line P Q. the ſeuerall lines to A. B. C. and D. laying them downe of the ſeuerall lengthes, belonging to them, according to your Field-booke, as X A. 30° X B. 33° 1/2. X C. 41°. and X D. 36°. making prickes or points at the end of euery ſeuerall length, as at A. B. C. and D. and then from point to point, draw the lines A B. B C. and C D. ſo haue you finiſhed the worke on the Eaſt ſide of your Meridian. Then place againe your Protractor in all reſpects as before, and note all the other degrees being aboue 60. as 66. 81. and 105 1/2. which by the edge of the Semicircle marke out as before, as at M. N. and O. and ſeeing they belong to the Weſt ſide of your worke, draw your lines from the Centre X. vpwards towards thoſe three ſeuerall points, laying them downe with their ſeuerall lengthes obſerued in your booke, as X E. 47° 1/2. X F. 18° 1/2. and X G. 41° 1/2. making points as before at the end of euery length; and laſtly from point to point draw the lines A G. G F. F E. and E D. So ſhall you incloſe the Figure A B C D E F G. with equall Angles and proportional lines to the meaſured field as was required.

CHAP.

CHAP. XXIIII.

To take the plot of any Field at one ſtacion in any one Angle thereof, from whence may bee ſeene all the other Angles of the ſame Field; and meaſuring from the ſtation to euery Angle.



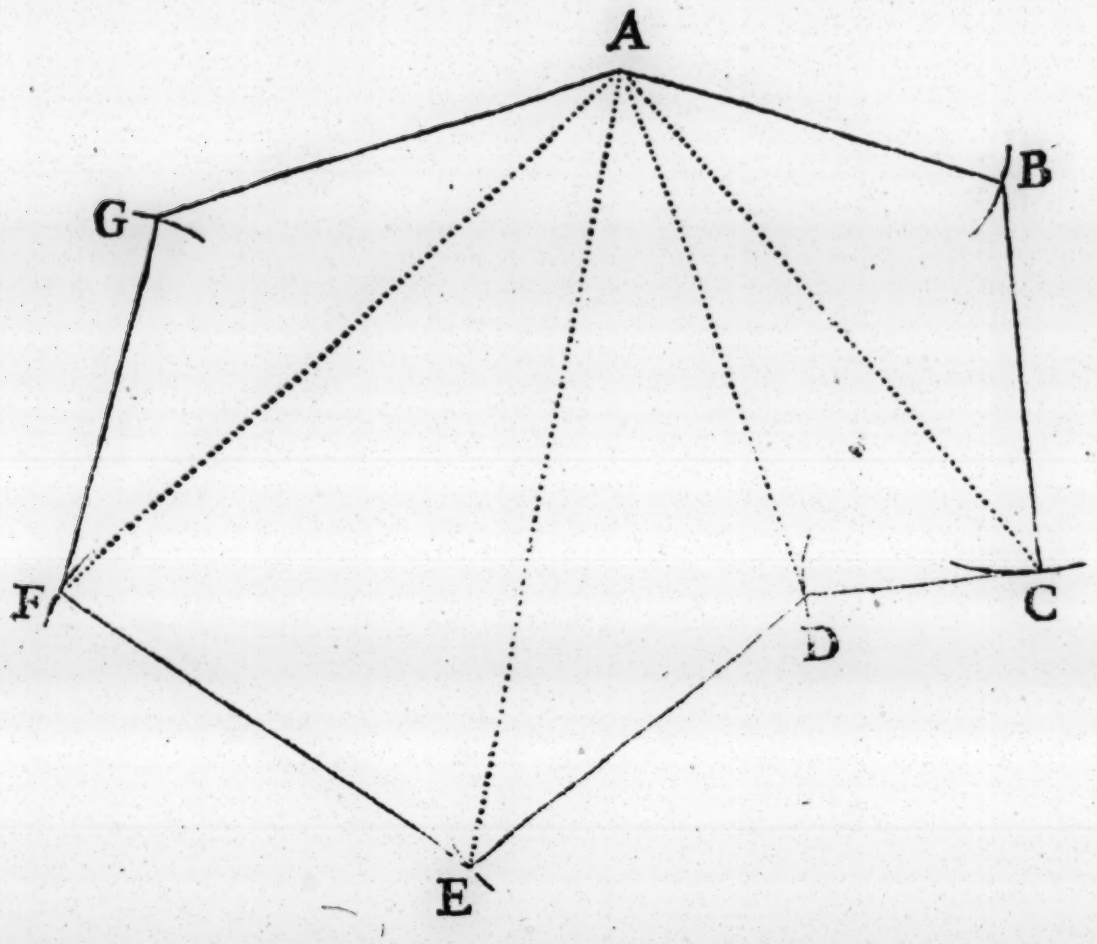
SVppoſe A B C D E F G. to be a field, the plat whereof is required to be taken. Firſt, cauſe whites or markes to be placed directly in euery Angle, then make choyſe of the moſt conuenient Angle, from whence you may beſt view all the reſt, as at A. where place your Inſtrument as before is taught, and directing your ſight to one of the next Angles on eyther hand as to B. obſerue the degrees cut by the edge of the Index, which let bee 24 $\frac{1}{2}$. degrees and meaſure that line A B. $\frac{\circ}{33}\cdot\frac{1}{4}$. then turning your Index to the next, as to C. make the like obſeruation of 13 $\frac{1}{4}$. degrees. and meaſure the length A C. $\frac{\circ}{55}\cdot\frac{1}{2}$. and in like manner proceed to the reſt, as to D. E. F. and G. ſtill expreſſing in your Field-booke your ſeuerall Angles and Lines as before is taught, which hauing finiſhed will thus appeare.

And then protract, and lay downe the ſame in all reſpects, according to the inſtructions of the laſt Chapter. The order whereof appeareth by the Diagram,

d	o	1
24 $\frac{1}{2}$	33	4
13 $\frac{1}{4}$	55	2
6	47	
116	69	2
104	65	6
95 $\frac{1}{2}$	41	6

CHAP. XXV.

To take the Plot of a Field at one ſtation in any Angle, from whence the reſt may be ſeene, and by meaſuring the ſides of the Perimeter.



SVppoſe A B C D E F G. be a Field to be plotted. Firſt; ſet vp markes as before, and chuſe an Angle fitteſt for your purpoſe, from whence you may ſee all the reſt, and there plant your Inſtrument as at A. then direct your ſight to one of the next Angles, on eyther hand, as to B. and note the degree there cut, 24 ½. and meaſure the length of that line A B. $\overset{0}{33}$. $\frac{1}{4}$. then direct your ſight to C. and note the degree there cut 13 ¼. and meaſure the line from the Angle B. to the Angle C. $\overset{0}{33}$. and in like manner worke forwards to D. E. and F. and then note (hauing finiſhed at F.) that you haue yet remayning two lines to meaſure, namely, F G. and G A. and but one degree to bee taken, as from A. to G. (The reaſon whereof dependeth on THEOR. 74. 1.) wherefore meaſure the line F G. $\overset{0}{33}$. $\frac{1}{4}$. and expreſſe the ſame in your Booke without any degree; and laſtly, directing your ſight to G. obſerue the degree cutte 95 ½. and meaſuring the line A G. place the length thereof in your Booke $\overset{0}{41}$. $\frac{1}{6}$. againſt the laſt degree taken. So haue you finiſhed your Field-worke, and your obſeruations ſtand thus.

Which are to be protracted and layde downe as is taught in the next.

d	o	I
24 ½	33	4
13 ¼	33	—
6	21	—
116	36	5
104	41	8
—	33	5
95 ½	41	6

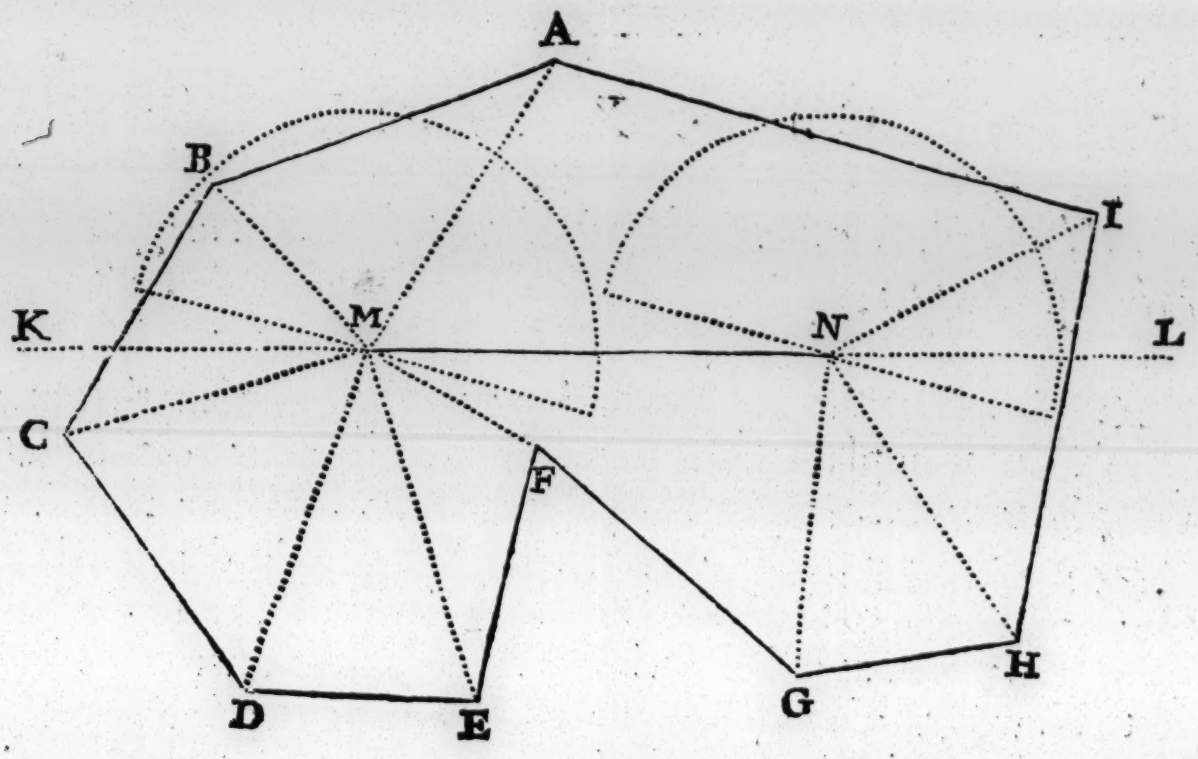
CHAP. XXVI.

To protract and lay downe the obſeruations had, according to the worke in the laſt Chapter, or any other taken by the like meanes.

THe ſeuerall degrees and lengthes ſo had and taken as before, and expreſſed in your Field-booke, being layde before you, worke thus. Firſt, according to your degrees taken, and as is taught in Chap. 23. draw out at length your ſeuerall lines A B. A C. A D. A E. A F. and A G. as in the former Diagram, then opening your Compaſſe on your Scale, take therein the firſt length $\frac{33}{4}$. at which diſtance with one foote in A. ſtrike with the other an Arch through the line A B. cutting the ſame in B. then take your ſecond length $\frac{33}{}$. at which diſtance with one foot in B. croſſe the line A C. in C. and draw the line B C. then take the third length $\frac{21}{}$. and with one foote in C. croſſe the line A D. in D. and draw the line C D. and in this manner worke forwards, laying downe euery length, and drawing each line till you haue incloſed the Figure A B C D E F G. which ſhall be a like Figure to the meaſured Field.

CHAP. XXVII.

To take the plot of a Field at two ſtations, where all the Angles cannot be ſeene at one, and meaſuring as in the 22. Chapter.



SVppoſe this Figure be a Field to bee plotted, which lyeth in ſuch ſort, as from no one place all the Angles thereof can be ſeene. In ſuch caſe make choyſe of a place for your firſt ſtation, where may be viewed as many Angles thereof as poſſibly you may, which let be M. where you may ſee the ſeue-

rall Angles at A. B. C. D. E. and F. then plant your Inſtrument in M. and there obſerue all thoſe Angles, and meaſure the ſeuerall lines, beginning from M. to A. and ending from M. to F. as is taught in the 22. Chapter, ſo haue you finiſhed the worke of your firſt ſtation. Then (before you remoue your Inſtrument) make choice of ſome other conuenient place for your ſecond ſtation, from whence you may ſee all the other Angles not formerly ſeene, as thoſe at I. H. and G. which let be N. vnto which place direct your ſight, and obſerue the degree cut 65. then meaſure the ſtationary diſtance M. N. $\frac{\circ}{40}$. and leauing a marke at M. remoue now your Inſtrument to N. where place it preciſely as it ſtood at M. with helpe of your needle and backe ſight, then obſerue your ſeuerall degrees, and meaſure the ſeuerall lengthes from N. your ſecond ſtation to I. H. and G. as before, and your Field-worke is finiſhed. So as you remember alwayes to expreſſe your obſeruations in your Field-booke, which ſhall thus appeare.

To be protracted and layde downe as is taught in the next.

	d	o	I
I. *Sta.*	83 $\frac{3}{4}$.	29	—
	109 $\frac{1}{2}$.	19	2
	10 $\frac{1}{4}$.	26	8
	28 $\frac{1}{2}$.	30	8
	41 $\frac{1}{4}$.	31	5
	55 $\frac{1}{2}$.	16	4
Sta. diſt.	65	40	—
2. *Sta.*	74 $\frac{1}{4}$.	26	—
	46 $\frac{1}{2}$.	29	—
	33 $\frac{3}{4}$.	27	5

CHAP. XXVIII.

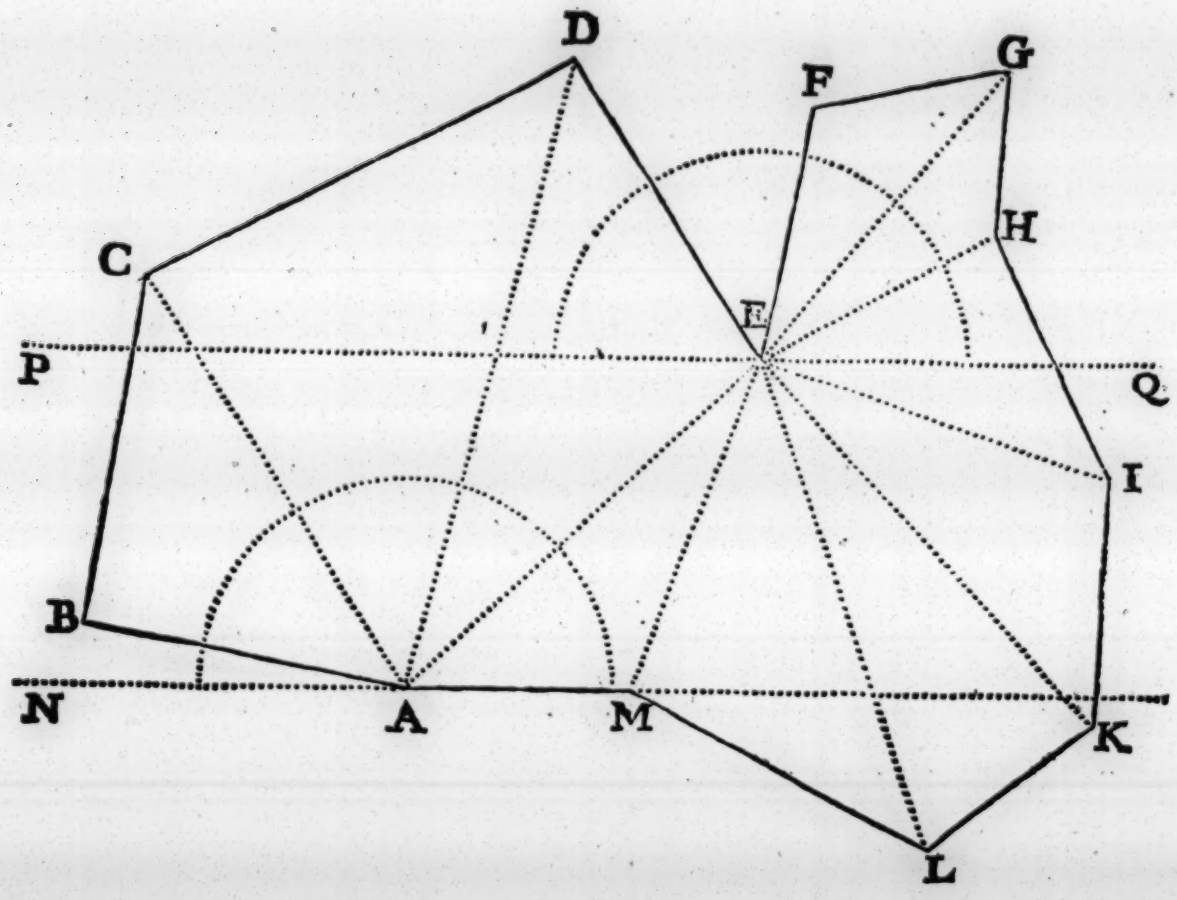
To protract and lay down any obſeruations taken, according to the work of the laſt Chapter.

ON a fayre ſheete of paper draw a right line at length in any conuenient place thereof, as K. L. in the Diagram of the laſt Chapter, which line is not to be taken as before for a meridian line, but ſuppoſed to be drawn according to the degree taken from the firſt to the ſecond ſtation; and therefore call it the ſtationarie line, and thereon lay downe your ſtationary diſtance $\frac{\circ}{40}$. as from M. to N. which two points ſhall repreſent your two ſtations, then on M. the point of your firſt ſtation, place the centre of your Protractor (the Semicircle vpwards) turning it about till the degree 65. on the limbe of your Protractor (being the degree taken from the firſt to the ſecond ſtacion) cut preciſely the ſtacionary line drawne, and there keeping it firme and immoueable, marke out the ſeuerall degrees of your firſt ſtacion; according to your Field-booke; and ſo worke on in all reſpects as is taught in Chap. 23. which effected, remoue your Protractor to N. the point of your ſecond ſtation,

tion, where placed preciſely, as at the firſt, worke forwards with the degrees and lines of your ſecond ſtation, as before; and ſo haue you finiſhed.

CHAP. XXIX.

To take the Plot of a Field at diuers ſtacions in diuers Angles, where all cannot be ſeene from one, and to meaſure as in the 24. Chapter.



SVppoſe this Figure be a Field to be plotted, whoſe angles cannot bee ſeene from any one Angle thereof, wherefore imagine you are now ſtanding in the Angle A. from whence you view and conſider what Angles may there bee conueniently ſeene and taken, which you find to be thoſe at B. C. D. and E. then directly in thoſe Angles cauſe markes to bee placed, and planting your Inſtrument as before is taught in A. direct your ſight firſt to B. then to C. after to D. and laſtly to E. noting the ſeuerall degrees cut towards each ſeuerall Angle, and meaſuring as in the 24. Chapter, from your ſtation to euery of thoſe Angles ſeuerally, and your worke of that ſtation is finiſhed. Now for that you ended your laſt worke at E. remoue your Inſtrument to that Angle, and there plant it preciſely, as at the firſt ſtacion, vſing both your needle and backe-ſight for your helpe therein; And here conſider what Angles from hence may be perfectly ſeene and taken, which on view had, you finde to be all the reſidue not formerly taken, as F. G. H. I. K. L. and M. wherefore hauing your markes placed, take your ſeuerall obſeruations and meaſure your ſeueral lines to euery of theſe Angles, as to thoſe of the firſt ſtation, taking them in order as they lye, and you haue finiſhed. But ſuppoſe at this ſtation, you could haue ſeene onely thoſe Angles at F. G. H. and I.

then here muſt you haue finiſhed thoſe, and remoued your Inſtrument to I. for a third ſtation, and there to haue performed the reſt, or as many as there you might, and if any remaining, to take a fourth, and a fift ſtation, &c. till you haue finiſhed, wherein many words are needleſſe, the matter being apparant. Your obſeruations of this worke are theſe.

A generall Note.

And let it be noted, that where for breuitie ſake in mine inſtructions, aſwell here as elſe where, I omit to expreſſe particularly, the ſeuerall degrees and lengthes obſerued and taken betweene each ſtation and the ſeuerall Angles, (which would be no leſſe tedious, then troubleſome in breeding confuſion) I obſerue a due order in the placing of theſe obſeruations of degrees and angles, according to the order of the letters about the plot, as the firſt degree and length 116½. 28. ¼. *belong to the firſt line* A B. *the ſecond to the ſecond* A C. *and the like of all the reſt, which are to bee protracted and layde downe, as appeareth in the next.*

d	o	ı
116½	28	4
101	41	8
85¼	55	8
74½	41	8
86	22	—
75½	32	8
69⅓	22	6
59	31	5
45¼	42	5
36½	44	4
23	30	4

CHAP. XXX.

To protract and lay downe any obſeruations taken, according to the worke of the laſt Chapter.

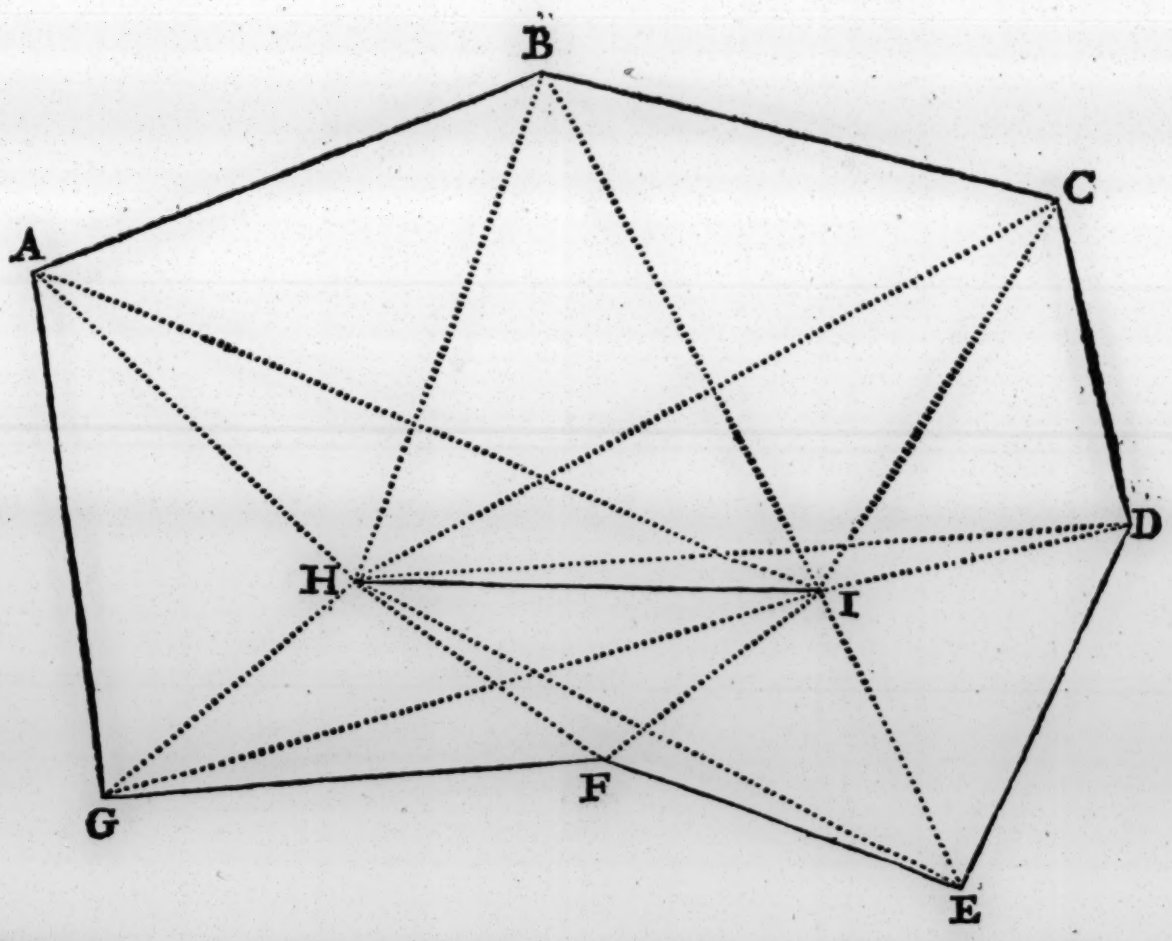
THe ſpeedieſt and exacteſt courſe for protracting works of this nature, conſiſting of diuers ſtations is thus. Firſt, draw a right line at length on your paper, to repreſent the meridian line, as N O. in the laſt Diagram, whereon placing your Protractor, worke in all reſpects as is taught in Chap. 23. for the obſeruations of your firſt ſtation; ſo ſhall you finiſh as much of this worke as is included by the lines A B. B C. C D. and D E. the worke of your firſt ſtation; then by the point E. where you left, draw another meridian line as P Q. which (by PROB. 3. 2.) make Parallel to the firſt line, N O. and then on the point E. place your Protractor in all reſpects, as at the firſt, & worke with the reſt as before, whereby you ſhall finiſh the worke of your ſecond ſtation, and perfect the Figure A B C D E F G H I K L M. with equall Angles, and proportionall lines to the meaſured field. And if there were more ſtations to be vſed in the Field-worke, then at the point where the worke of each ſtation endeth, you are to draw another meridian line parallel to the reſt. Or before you begin your protraction, you may draw diuers parallell lines on your paper, repreſenting ſo many meridians, and by helpe of your parallel diuiſions, placed at eyther end of the Scale of your Protractor, you ſhall on any point falling either vpon or beſides thoſe Meridians place your Protractor parallel as you pleaſe. And this kind of protraction may be vſed in ſtead of that taught in the 28. Chapter, as the better, though eyther will ſtrue, and both tending to one end.

Hither-

Hitherto haue I taught after a perfect and exact manner the mensuration of seuerals (by diuers meanes) where one field or close onely is to bee taken by it selfe: But if many seuerals (as a whole Lordship or Mannor) were to bee measured and plotted together: I hold not these former courses the fittest: but rather those which shall bee hereafter taught. But first I will deliuer some few directions and examples, for the dimension of seuerals after another order, by intersection of lines at seuerall stacions as followeth.

CHAP. XXXI.

To take the plot of any field at two stations, so as all the Angles may be seene from both stacions, by measuring onely the stationarie distance.



SVppose A B C D E F G. be a Field, the plot whereof is required to be taken. Firſt, make choice of two ſuch conuenient places for your Stations, as from whence you may ſee all the Angles about the Field; with theſe further conſiderations, that the diſtance betweene your ſtations bee of conuenient length, the longer the better, that they lye towards the middle of the field; and that neyther of them lye interpoſed in a right line betweene the other, and any Angle of the Field; but to be choſen with ſuch diſcretion, as all lines drawne from eyther ſtation to the ſeuerall Angles, may interſect ech other with as large angles as you may, which let be the two points H. and I. and cauſing marks to be placed in euery Angle, plant your Inſtrument at H. as is before taught, and directing your ſight to A. obſerue the degree there cut; and the like to B. C. D. E. F. and G. and alſo to I. the ſecond ſta-

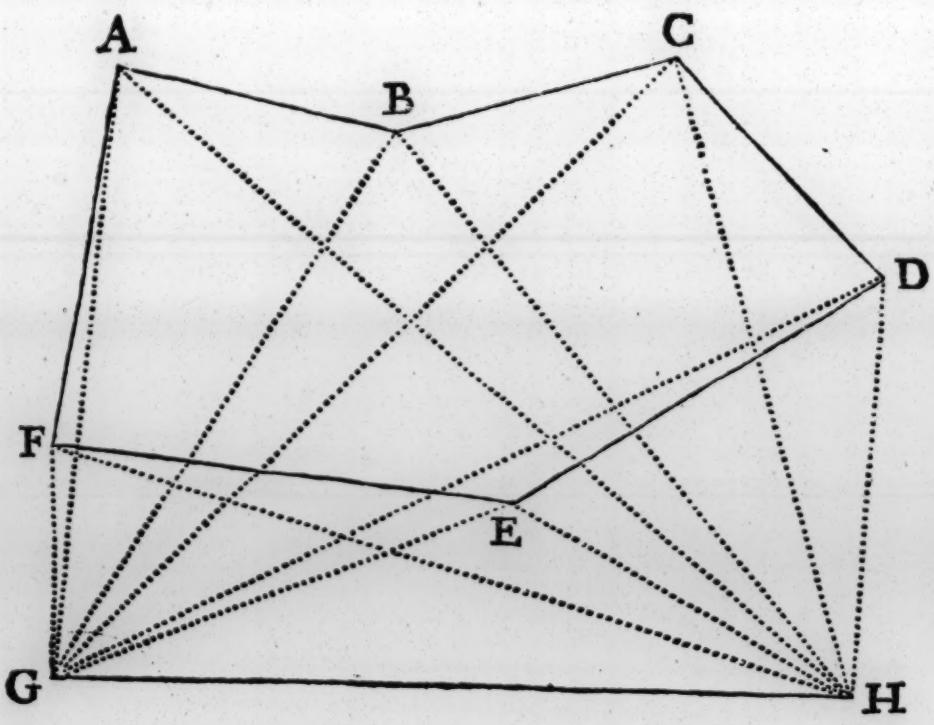
tion, then take vp your Inſtrument, leauing a marke at H. from whence meaſure the ſtationarie diſtance to I. where placing your Inſtrument preciſely as at H. obſerue likewiſe all the degrees cut by your Index, directed to each ſeuerall Angle as before: Of all which ſeuerall obſeruations keepe notice in your Field-booke as hath beene often mentioned; wherewith on a cleane ſheete of paper by the directions of the 18. and 30. Chapters, the plot thereof is ſpeedily protracted, and your buſineſſe fully finiſhed.

And here note the accuitie of diuers of thoſe Angles in the Diagram, cauſed by the interſection of the pricked lines, notwithſtanding all care had therein; and what inconueniences may hereby grow, without good regard; and yet are theſe Angles vſually drawne by many, who make a poore ſhift therewith.

CHAP. XXXII.

To take the plot of any Field remote from you at two ſtations, when eyther by oppoſition you may not, or ſome other impediment you cannot come into the ſame.

SVppoſe A B C D E F. bee a Fielde, whereinto by no meanes you can bee ſuffered to enter, yet of neceſſity muſt the plot thereof be had. In ſuch caſe make choiſe of any two places, eyther neere hand, or further off, all is one; ſo from thence you may well decerne the ſeuerall Angles of the ſame Field; and let your ſtationary diſtance bee the full length of the Field, at the leaſt if poſſibly you may; which two places let be G. and H. Firſt plant your Inſtrument at G. and by directing your ſight in order one after other to A.B.C.D.E. and F. the ſeuerall Angles of the Field, obſerue the ſeuerall degrees there cut, as is before taught; then turne your ſight to H. your ſecond ſtation, and note the degree there cut; which done, take vp your Inſtrument, leauing a marke at G. and meaſure from thence to H. your ſtationary diſtance, and there plant your Inſtrument in all reſpects as before, and make the like obſeruations to all the ſeuerall Angles of the Field, as formerly at G. So haue you finiſhed your Field-worke, which is to be protracted and layde downe according to your Inſtructions of the 18. Chapter.



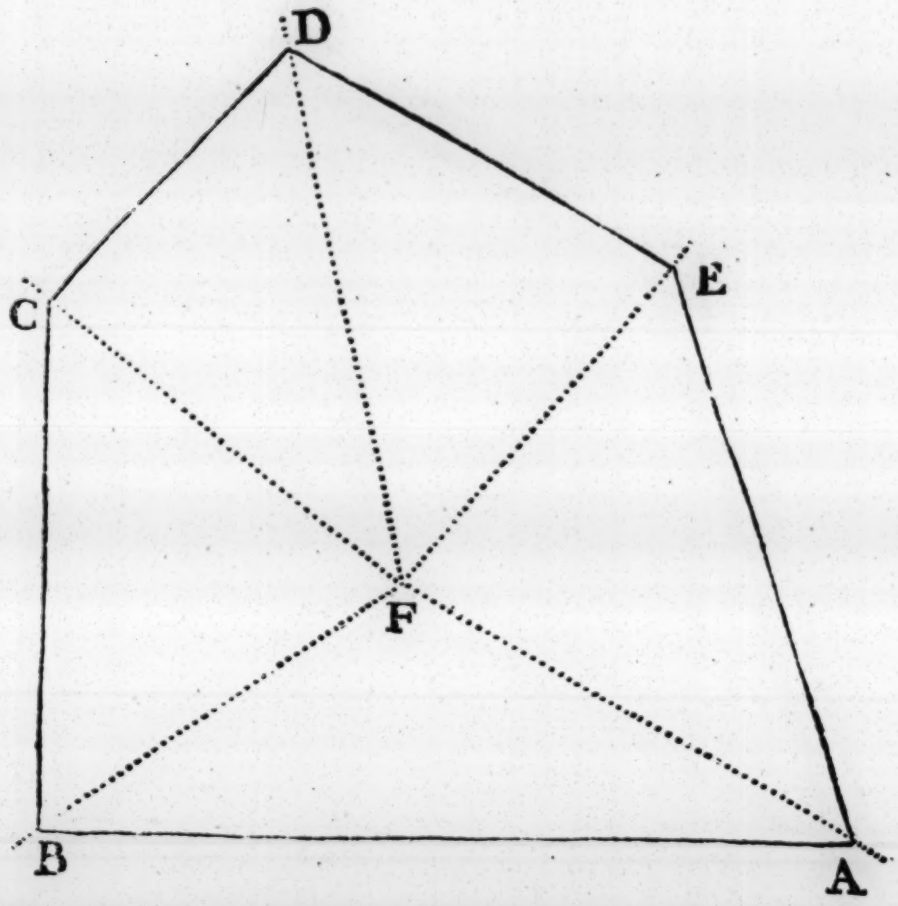
And here note, by reaſon of the length of the ſtationary diſtance, how ex-

excellently the lines iſſuing from thence, interſect each other, which of neceſſitie makes the concluſion abſolute.

CHAP. XXXIII.

To take the Plot of any Field by making obſeruation at euery Angle, and meaſuring onely one line, but no part of the perimeter.

SVppoſe A B C D E. be a Field, & it is required to plot the ſame as before. Firſt, place your Inſtrument at A. and obſerue ſome notable mark towards the middle of the Field; if there be none ſuch, cauſe one to bee ſo placed as it may bee ſeene from euery Angle about the Field, which let be F. then direct your ſight to F. and note the degree there cut 109. and (your Inſtrument remaining) meaſure the diſtance from A to F. 21°. $7\frac{1}{2}$. (which ſhall bee all the menſuration you ſhall need to vſe in this worke) then direct your ſight to B. obſeruing there the degree cut $118\frac{1}{2}$. then remoue your Inſtrument to the Angle at B. and there firſt direct your ſight to F. taking that degree $10\frac{1}{2}$. and then to C. the next Angle $88\frac{1}{2}$. which done, remoue your Inſtrument to the Angle at C. and there as before, firſt direct your ſight to F. taking that degree 106. and then to D. the next angle $74\frac{1}{2}$. and ſo proceed from angle to angle, firſt taking your ſight to F. and then to the next angle, till you haue finiſhed, wherein by the way obſerue this courſe; when you take your degrees to F. reckon thoſe on the North end of your needle, and when you expreſſe them in your Booke, make in the margent againſt them a pricke, or ſome other marke, whereby it may bee apparant, when you come to Protraction which are thoſe degrees, from the reſt, which are to bee reckoned on the South end of the Needle, after the vſuall manner, and belong to the perimeter: ſo haue you very exactly finiſhed your field-worke, and your obſeruations are theſe.



Or take here the right degrees and protract the oppoſite.

	d	o	I
.	109	21	$7\frac{1}{2}$
	$118\frac{1}{2}$		
.	$10\frac{1}{2}$		
	$88\frac{1}{2}$		
.	106		
	$74\frac{1}{2}$		
.	93		
	49		
.	$75\frac{1}{2}$		
	$35\frac{1}{4}$		

Which is to be protracted, as is taught in the next.

CHAP.

CHAP. XXXIIII.

To protract and lay downe any obſeruations taken, according to the worke of the laſt Chapter.

THis kinde of protraction is ſomewhat different from all the reſt formerly taught, wherefore obſerue it thus. Firſt, draw diuers parallel lines ouer all your paper, of conuenient diſtance one from another (not exceeding the bredth of the Scale of your Protractor) which ſhall repreſent the Meridians; then with your obſeruations layde before you, take a point in any conuenient place of your paper, whether vpon or beſides any parallel line, it is not materiall, as at A. in the laſt Diagram, then thereon place the Centre of your Protractor (the Semicircle vpwards) turning it about on your protracting pinne, till you find the match or oppoſite parallel diuiſions on eyther end of the ſcale, to lye eyther preciſely vpon any one line, or equally diſtant ouer or beneath the ſame; then looke in your booke what are the firſt two degrees, which is 109. and 118 ½. againſt which two degrees, by the edge of your Protractors Semicircle, make ſeuerall prickes on your paper, whereby, and by the point A. draw the two right lines A F. and A B. out at length, and from A. to F. lay downe your meaſured length 21°. 17 ½. then on the point F. place the Centre of your Protractor preciſely as before, by helpe of your diuiſions on the end of the Scale, and find in your Field-booke what degrees you haue (more then the firſt already done) marked with points or other markes in the margent (as was taught in the Field-worke) which are 10 ½. 106. 93. & 75 ½. againſt all which degrees on your Protractor make ſeuerall prickes as before, whereby, and by the point F. draw out at length the ſeuerall lines F B. F C. F D. and F E. remembring (as you are taught in Chap. 23.) alwayes to draw thoſe lines proceeding from all the degrees vnder 60. downwards or towards you from the point F. and thoſe aboue 60. vpwards from the point F. Then note, that by drawing out the line F B. you haue interſected the line A B. (formerly drawne) in the point B. on which point now place your Protractor as before, and finde in your Booke the ſecond degree vnmarked in the margent (for the firſt A B. is alreadie done, and likewiſe all thoſe which are marked) which is 88 ½. againſt which, on your Protractor make a pricke, and thereby, and by the point B. draw the line B C. till it interſect the line F C. in C. then place your Protractor on the point C. as before, and finde in your booke the next degree vnmarked, which is 74 ½. and againſt that degree on your Protractor make a pricke, whereby, and by the point C. draw out at length the line C D. to cut the line F D. in D. and in like manner proceed with the reſt; ſo ſhall you include the Figure A B C D E. like vnto the meaſured Field.

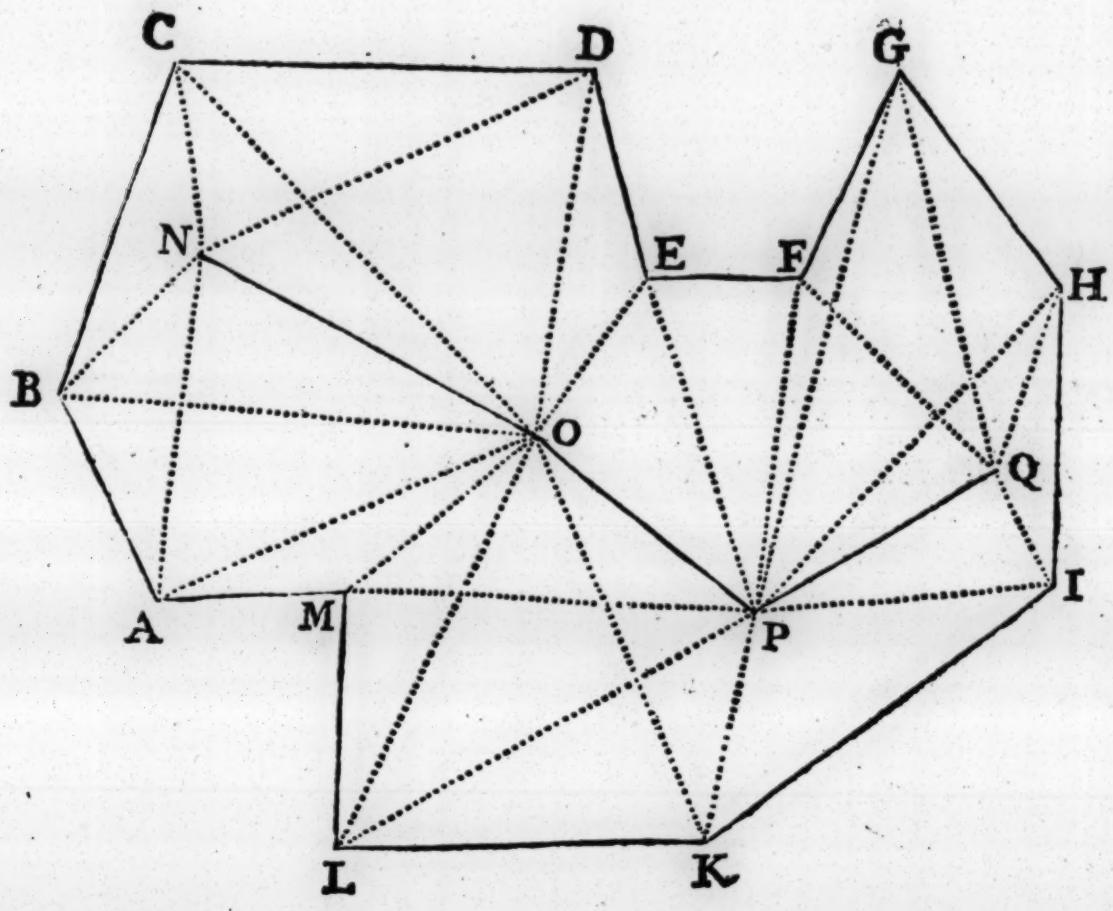
Where note, that now the points in the margent ſerue you to purpoſe, at an inſtant diſtinguiſhing thoſe degrees taken at each angle towards the point F. from the others vnmarked, repreſenting thoſe of the Perimeter.

This kind of worke well handled, is very exact and artificiall.

CHAP.

CHAP. XXXV.

To take the Plot of any Field at diuers ſtations, meaſuring onely the ſtacionary diſtances.

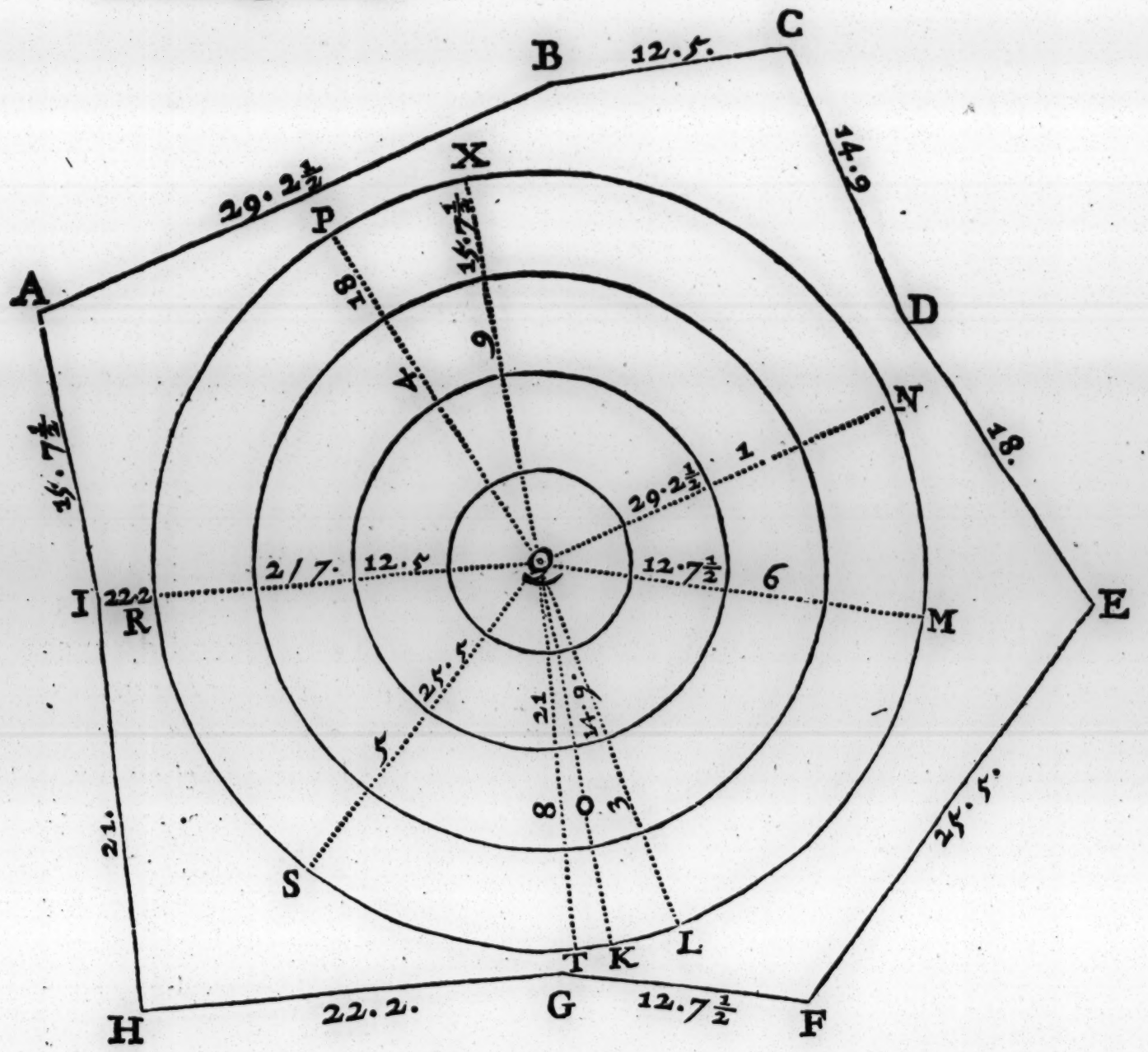


SVppoſe this irregular Figure to repreſent a Field, whoſe plot is to be taken. Firſt, cauſe markes to bee placed in euery Angle thereof, and then (remembring thoſe cautions and conſiderations in this behalfe deliuered in Chap. 31.) make choyce of your firſt ſtation, which let be at N. where planting your Inſtrument, direct your ſight to as many Angles ſeuerally as lyes there within your view, as to A.B.C. and D. and take obſeruation of the ſeueral degrees there cut, and then (before you remoue your Inſtrument) make choyce of your ſecond ſtation, from whence you may not onely ſee all thoſe former angles, but as many more as poſſibly you may, which let be at O. then directing your ſight to O. obſerue the degree there cut, and taking vp your Inſtrument, leaue there a marke, and meaſure the diſtance from thence to O. where againe plant your Inſtrument as before at N. and then directing your ſight to thoſe former Angles A. B. C. and D. note ſeuerally what degrees are cut, and you haue finiſhed for thoſe Angles. Now conſider what other Angles you can here eſpie as thoſe at E. M. L. and K. and thither likewiſe direct your ſight ſeuerally, and make your ſeuerall obſeruations as before, which done (your Inſtrument remayning) make choyce of your third ſtation, (from whence you may not onely ſee thoſe Angles at E. M. L. and K. but as many other vnfiniſhed Angles as you may) which let be at P. then directing your ſight to P. obſerue your degree thither, leaue a marke at O. take vp your Inſtrument, and meaſure from O. to P. where againe plant your In-

Inſtrument, and make your obſeruations of thoſe former Angles E. M. L. and K. ſo haue you thus farre wrought. And now againe conſidering what other Angles you can here eſpie, you ſhall finde within your view F. G. H. and I. being all the reſidue vnfiniſhed; wherefore here alſo make your ſeuerall obſeruations of theſe laſt Angles, and choyſe of your laſt ſtation, as at Q. whither directing your ſight, and making obſeruation, take vp your Inſtrument, and with a marke at P. meaſure the diſtance to Q. where laſtly plant your Inſtrument, and making your obſeruations of thoſe laſt ſeuerall Angles at F. G. H. and I. as before; your Field-worke is finiſhed. And by the directions of the 18. and 30. Chapters, you may protract the ſame accordingly.

CHAP. XXXVI.

To take the Plot of a Forreſt, or any ſpacious Common or Waſte, of whatſoeuer quantitie, by the Plaine Table, on one ſheete of paper without altering thereof.



SVppoſe A. B. C. D. E. F. G. H. I. to bee ſome large irregular ground, and it is required to plot the ſame. Firſt, a ſheete of paper being placed on your Plaine Table after the vſuall manner, Take a point at all aduenture about the middle thereof as at Q. in the Diagram, and thereon deſcribe foure concentricke Cir-

Circles of conuenient diſtance, the vttermoſt extending neere the bredth of the Table, whereby you ſhall produce three ſeuerall circular ſpaces, which are thus to bee employed; In the vttermoſt, expreſſe the ſeuerall lengthes of all ſuch lines, which in paſſing forward in your worke ſhall bee found declining or extending outwards, as the lines D E. G H. and I A. which lines alwayes iſſue from an Angle greater then a Semicircle, the inclination whereof cauſeth the declination of the line: the ſecond which is the middlemoſt ſpace, let ſerue for the number of each line; and in the third and innermoſt ſpace, expreſſe the ſeuerall lengthes of all ſuch lines, as in paſſing forwardes ſhall be found inclining or extending inwards, as all the other lines in the figure not formerly mentioned.

And theſe thus vnderſtood, proceede forwards in this manner. Plant your Inſtrument after the vſuall order in the Angle, where you purpoſe to begin, which let be A. and let the Centre Q. on your Table, alwaies in your whole worke repreſent your place of ſtanding, then lay your Ruler on the Centre Q. and turning it about, thereon direct your ſight to I. and by the edge of the Ruler, draw the line Q. K. (which ſhall be your incloſing line) extending it to the vttermoſt Circle, and in the middle ſpace, on that line (which is to beare no number) place the Cipher o. then keeping ſtill the Centre point Q. turne about your Ruler, and direct your ſight to B. and by the edge of your Ruler draw the line Q N. and in the middle ſpace on that line expreſſe the Figure 1. the number of the firſt line you goe on, ſo haue you on the Table the Angle K Q N. equall to the Angle I A B. in the field, then take vp your Inſtrument, and meaſure the line A B. $\frac{4}{29}$. $\frac{1}{2}$ $\frac{1}{3}$. which length expreſſe on your firſt line Q N. and for as much as the line is inclining, place the ſame in the innermoſt ſpace, as in the Diagram; then place your Inſtrument preciſely at B. and turne it about on the ſtaffe (your Ruler lying on your firſt line Q N. till you finde the ſame line to lye parallel with the line A B. of the Field, hauing Q. towards you, and N. towardes the Angle A. (for the Centre Q. in euery Angle muſt repreſent the point of the ſame Angle) then your Table there fixed, direct your ſight to C. and draw your line which ſhall bee the line Q R. and expreſſe thereon in the middle ſpace the figure 2. for the ſecond line, ſo haue you alſo the Angle N Q R. on the Table, equall to the ſecond Angle A B C. in the Field: then take vp your Inſtrument, and meaſure the line B C. $\frac{8}{12}$. $\frac{1}{5}$. which place on the ſecond line Q R. in the innermoſt ſpace, and planting your Inſtrument at C. direct your laſt line Q R. towards B. as you did, the firſt towards A. and then take the quantitie of this Angle, as before, and you ſhall haue the Angle R Q L. on your Table, equall to B C D. in the Field. And thus proceede from angle to angle, making alwayes the laſt ſide of the laſt Angle to bee the firſt of the next, with all their points concurring in the Centre Q. and meaſuring and expreſſing the ſeuerall lengthes of each line, as before; you ſhall obtain the quantitie of euery Angle, and the length of euery line throughout the whole worke: whereby you may ſpeedily protract and lay downe the ſame, as is taught in the next.

But in the meane ſpace it is to be noted, that if in your former worke it happen at any time, that one line fall directly on any other, formerly drawn on your Table (as in the former Diagram, the ſeuenth line Q R. falleth on the

the ſecond line formerly drawne) then in ſuch caſe expreſſe the number of the ſame laſt line in the middle ſpace, where the firſt is numbred, with a ſtroke betweene them; but place the numbers, expreſſing the length of the ſame laſt line, without the vttermoſt Circle, if it be declining (as in the former example $_{22}^{\circ}.\ _{2}^{1}.$) and within the innermoſt, if it be inclining.

This kind of menſuration of ſpacious works with the Plaine Table (which likewiſe may be wrought with any other Inſtrument according to Chapter 15.) is wonderfull neceſſary both for ſpeed and exactneſſe, if artificially handled: but if it be required to haue notice taken of the ſeuerall lands and grounds abutting and confining hereon, you muſt then haue prouided a Field-booke for that purpoſe.

CHAP. XXXVII.

To protract and lay downe any obſeruations taken, according to the worke of the laſt Chapter.

HAuing before you the paper of your Field-worke as you wrought on the Table: If you imagine that one ſheete of paper will not ſerue the turne, you may with mouth-glue, lay as many together as you pleaſe; and then (conſidering which way your worke will extend) draw a right line accordingly on your paper, whereon, with your Scale and Compaſſe, lay down the length of your firſt line Q N. $_{29}^{\circ}.\ _{2}^{1}\ _{3}^{1}.$ as the line A B. in the Diagram of the former Chapter, and then on the end B. of that line, by PROB. 8. 2. protract an Angle, equall to the Angle N Q R. as the Angle A B C. and on the ſide B C. of that Angle, place by your Scale the length of your ſecond line R Q. $_{12}^{\circ}.\ _{5}^{1}.$ from B. to C. and on the point C. where your laſt length ended, protract another Angle, equall to the Angle R Q L. your third Angle taken, as the Angle B C D. and on the ſide C D. extended, place the length of your third line L Q. $_{14}^{\circ}.\ _{9}^{1}.$ from C. to D. And ſo proceede from Angle to Angle, protracting your Angles equal (and in order by the number of lines) to thoſe anſwerable in your concentricke Circles, and laying downe duly by your Scale on each line, the length thereunto belonging, you ſhall produce the Figure A B C D E F G H I. like vnto the meaſured quantitie.

And hauing thus finiſhed; if you doubt or make queſtion, whether you haue wrought exactly or not, and deſire to be reſolued therein, make approbation thus. Collect the quantitie of all your Angles in your whole worke, and adding them together, note the totall thereof; which in this former worke is 1260. then multiply 180. (the number of degrees in a Semicircle) by a number leſſe by 2. then the number of Angles in your worke, which here is 7. (for the number of Angles is 9. as appeareth by the line O Q. which ſheweth the number of the laſt line, and conſequently of the Angles) and if the product of that multiplication agree with the former totall; then by THEOR. 74. 1. you haue done rightly, otherwiſe not; as the Product of 180. by 7. is 1260. agreeing with your firſt number; and therefore may you confidently affirme to haue wrought exactly, which rule is generall for all other plottes, and ſuperficiall figures whatſoeuer.

If your degrees be thoſe of 120. as obſerued by the Peractor, or Circũferentor, then in ſtead of 180. you muſt take 60.

Thus

Thus hitherto haue wee dealt in the plotting and dimenſion of ſeueralls; and that by ſuch ſeuerall, exact and artificiall wayes and meanes, as may moſt ſufficiently ſerue aſwell for the abſolute performance thereof, as (if well vnderſtood and practiſed) of many other excellent concluſions. But were you to ſuruey and plot great quantities, and many ſeuerals together as a whole Lordſhip or Mannor, or to deale in menſuration of impaſſible wood-grounds, wherein you are debarred from croſſing ouer or working within the ſame; I cannot aduiſe you therein to the vſe of theſe former Precepts, (though otherwiſe to excellent purpoſe) but rather to vſe and obſerue the meanes and courſes preſcribed and taught in the next following Chapter, which of all other is the moſt generall, abſolute and exact, for the menſuration of all manner ſuperficiall Figures, of what forme, quantitie, or number ſoeuer; and therefore to bee obſerued with good reſpect.

CHAP. XXXVIII.

To take the plot of a Lordſhip or Mannor, conſiſting of diuers ſeuerals, of what nature or kind ſoeuer, whether Wood-groundes or other.

THe precepts and inſtructions taught, and deliuered in the 10. Chapter of this booke, concerning the deſcription and vſe of a neceſſarie Field-booke, might well ſerue with diligent obſeruation for the performance of this worke: But ſeeing that there (according to the propoſed matter) my chiefe indeuours tended rather to the explanation thereof, then to the forme and order of menſuration, wee will here make vſe of the Figure there expreſſed; and by inſerting certaine neceſſarie obſeruations there omitted (as needleſſe to that purpoſe) referre you for a full ſatisfaction, to the conſideration and due obſeruation of thoſe, and theſe conioyned. Wherefore ſuppoſe that the figure or Diagram there expreſſed, were a Mannor, or part of a Mannor to bee meaſured and plotted. Firſt, write your Title, as there is mentioned, then planting your Inſtrument in A. direct your ſight to B. and hauing obſerued the degree there cut, take vp your Inſtrument, and meaſure to B. entring your degree and length into your booke, then plant not your Inſtrument at B. but onely meaſure from thence to C. and there place your Inſtrument, & direct ypur ſight backwards to B. obſeruing your degree; but with this ſpeciall regard, that in taking your backe ſights, you alwayes reckon the degree cut by one and the ſame end of the Index, as you reckon on, when you direct your ſight forwards; or otherwiſe you take the oppoſite degree to that you ſhould, which will much trouble you in protraction: then here conſider, that you are to leaue the bounder, which you went againſt from A. where you began, to this place, & therfore draw a ſingle line, and as you are

there taught, write the bounder paſt, as in the example of your booke; then direct your ſight to D. and obſeruing your degree, meaſure thither, which hauing entred, make there another ſingle line, for that here you leaue that bounder alſo, which let be expreſſed; then meaſure from D. to E. and there plant your Inſtrument, and as before direct your ſight backewards to D. and obſerue the degree with the former caution, which done, turne your ſight to F. and hauing your degree, meaſure from E. to F. and likewiſe from F. to G. expreſſing in your Booke thoſe lengthes ſeuerally, then plant your Inſtrument at G. and taking your degree from thence backwards to F. as before, here ſtrike a ſingle line, and write your third bounder, then take your degree to H. and meaſure thither, where alſo (that being your laſt Angle) you muſt plant your Inſtrument, and worke as before to A. where you began, and then ſtrike a double line, ſignifying you haue finiſhed that Field. And in like manner proceed with all the reſt from one Cloſe to another, till you haue finiſhed the whole worke, as you are moſt plainely directed in the 10. Chapter, remembring alwayes to meaſure euery line; and to place your Inſtrument at each other Angle, taking your backe-ſights to that very point or marke, whereunto you directed your ſight laſt before. So ſhall you moſt exactly, and with great expedition performe your deſire. *And your worke is to bee protracted as is taught in the next.*

CHAP. XXXIX.

To protract and laye downe a Plot of many ſeuerals, of what quantitie or number ſoeuer.

ACcording to the quantitie of your plot, or the largeneſſe you ſuppoſe it will be of, glue papers together; but if very large, lay firſt together but 4. or 8. ſheetes onely, and rule them all ouer with parallel lines, repreſenting Meridians of ſuch conuenient diſtance, as they exceede not the bredth of your Protractors Scale. Then laying your Field-booke before you, ſuppoſe you are to protract the obſeruations mentioned in the tenth Chapter, and conſidering towards what point of the Compaſſe your worke will moſt incline or extend, begin your protraction accordingly; as in that example; it inclineth towards the Eaſt, and North-eaſt from the place of beginning; wherefore beginne your protraction towards the Southweſt part of your plot; and there make a point, whereon place the Centre of your Protractor (with the Semicircle eyther vpwards or downewards, as you beſt fancie) and holding your protracting pinne in that point, moue about your Protractor thereon, till you finde one and the ſame parallel diuiſion on eyther end of your Protractors Scale, to lye eyther directly vpon any one parallel line, or equidiſtantly aboue or beneath the ſame; and there, with your left hand keepe firm your Protractor, whileſt you finde in your Field-booke the firſt degree 65. againſt which, on the limbe of your Protractor place the point of your protracting pinne, and there keepe it, bringing the edge of your Protractors Scale thereunto,

unto with the firſt diuiſion of the Scale on your choſen point, and then draw a line by the edge of your Scale of your firſt length in the booke $_{20}^{\circ}$. $\frac{1}{2}$. as the line A B. But with this reſpect, that (as before is taught) the lines belonging to euery degree vnder 60. bee drawne from the Centre point downewards, or inclining towards you, and the lines belonging to euery degree aboue 60. (as the laſt line A B.) bee drawne from the ſame point vpwards, or reclining from you: Then place your Protractor on the point B. (being the end of your laſt line) in all reſpects as before at A. and finding your next degree and length in your booke to bee 68. $_{16}^{\circ}$. $\frac{1}{6}$. againſt 68 degrees on your Protractor, place the point of your protracting pinne, and applying the Scale of your Protractor thereunto, with the beginning of the diuiſions thereof on the point B. draw your line of the length $_{16}^{\circ}$. $\frac{1}{6}$. as B C. and here (conſidering you are to leaue the bounder you went againſt) make a ſmall ſtroke or marke at C. with your penne, and finding in your Booke, at the end of your laſt length the figure (1) place that at C. with blacke lead (the vſe whereof ſhall partly appeare) then place your Protractor on the point C. (the end of your laſt line) as before, and find your next degree and length 36 $\frac{1}{2}$. $_{22}^{\circ}$. $\frac{1}{5}$. and againſt that degree on your Protractor, place the point of your protracting pinne, and bringing the edge of your Protractor thereunto as before, from the point C. draw the line C D. of the length $_{22}^{\circ}$. $\frac{1}{5}$. which ſeruing to a degree vnder 60. is to bee drawne from the point C. downwards as before. And ſo proceede with your ſeuerall degrees and lines, in order as you finde them in your booke, till you come to the point A. where you firſt began, and hauing wrought truly, you ſhall there iuſtly incloſe your firſt ſeuerall. Then looke in your Booke where you are to beginne your next incloſure; and you ſhall be thereby directed to beginne from N°. (1.) Wherefore ſeeke in your laſt protraction where you placed that number, which you ſhall finde at C. and there you are to beginne your ſecond parcell; wherewith, and with the reſt proceed in all reſpects as with the firſt. Small practice with good obſeruation (whereunto I will leaue you) is much more auaileable then many words. And therefore will I ceaſe to ſpend further time herein.

CHAP. XL.

The order and meanes of meaſuring and taking the ſeuerall and particular quantities in common fields, with a briefe inſtruction concerning the vſe of my Chaine.

THE whole plot and quantitie of common fields are to be taken and plotted as they lye among other the adiacent grounds, according to the directions of the 38. Chapter, without regard of the ſeuerall and particular quantities therein contained; which afterwards are to be had and obtained after this maner.

Let a Booke bee purpoſely prouided, which call your Common-Field-Booke to bee ruled as in the example, containing eight Collums. The firſt

The West field arr.

Broad furlong.

Tenants *names.*	Bredth. 0	1	2	Length. 0	1	2	Quantity a. r. p.
Io. Woods from the Church lane eastwards free.	1	3	2	16	2	—	0-0-21
wil. Browne by Copie.	6	8	2	16	2	—	0-2-30
Fra. Jacksons for 3. liues.	8	4	1	14	2	3	0-3-0
Tho. Coakes for yeeres.	7	5	2	15	4	2	0-2-36
wil. Iones at will.	5	6	3	15	4	2	0 2-7

towardes the left hand, ſeruing for the Tenãts names and the tenure whereby they hold the ſame (1) lands; the next three for the bredth of eue- (2) ry parcell intituled with theſe Signes, (3) or Characters, 0. 1. 2. ſignifiing Vnites, (4) Primes and Seconds, as is taught in the (5) 6. Chapter of this Booke; the next three for the length with the like Title; and the eighth or laſt towards the right hand, for the content of each ſeuerall parcell, the length and bredth being multiplied together. In this worke there is no vſe of any other Inſtrument, then your chaine onely: And beginning with any one furlong or went, expreſſe firſt in your booke the name of the field, & then of the ſame your firſt furlong, and ſo the reſt of the Title, as in the example, then in the firſt Collum write the Tenants name, whoſe land you firſt meaſure, & withall from what place you begin, & on what point of the compaſſe you paſſe from thence, wherby you ſhalbe able afterwards (obſeruing the ſame courſe in the beginning of euery furlong) to abut and bound euery parcell if need require, & likewiſe in the ſame firſt collum, expreſſe by what tenure it is held, thẽ cõſider how the whole furlong lyeth, if al of one length, then need you take the length but once for all, although there were twentie Tenants lands in the ſame; but if irregularly, as in ſome places ſhorter, and others longer; then at euery ſecond or third bredth, (or oftener if occaſion require) take the length thereof, and expreſſe the ſame vnder your title of length, as for the ſeuerall bredthes, you may onely croſſe ouer the whole furlong about the middle therof, taking euery mans bredth, and entring the ſame as you paſſe, vnleſſe you finde extraordinary difference betweene the bredthes at eyther end; if ſo, then meaſure the bredth of both ends, adding thoſe two bredthes together, whereof take halfe for your bredth, & enter it in your booke, or you may enter both bredthes, and take halfe thereof when you caſt vp the contents. And thus proceede from one furlong to another, till you haue finiſhed the whole field. And when you haue done (or at any time after at your pleaſure) by multiplying thoſe lengthes and bredthes together (which is moſt ſpeedily and exactly performed, as hereafter followeth according to the order of decimall multiplication) you haue your ſeueral contents to bee expreſſed in the laſt Collum. And laſtly, number all the ſe-

uerall

uerall parcels in the whole booke by Figures in the margent, from 1. forwards as in the example, which will ſerue you to good purpoſe, in the collecting of euery mans parcels together, as ſhall be hereafter declared.

And here ſhal you find (and in all other workes of this nature) moſt excellent vſe of my decimall chain deſcribed in the ſixt chapter of this booke.

But leſt you ſhould bee abſolutely ignorant of the manner and order of caſting vp of the ſeuerall contents, according to the lengthes and bredths ſo taken and obſerued as before, and conſequently the chaine with the ſeuerall parts and fractions thereof may ſtand you in little ſtead, I will here briefly touch the order thereof, in two examples thus.

Suppoſe your length taken to bee $\overset{\bullet}{16}\cdot\frac{1}{2}\cdot$ and your bredth $\overset{0}{1}\cdot\frac{1}{3}\cdot\frac{2}{2}\cdot$ being thoſe firſt ſpecified in the former example. Firſt, let your numbers bee placed in all reſpects, as if they were whole numbers to bee multiplied the one by the other thus; but ouer euery Fraction of your multiplicand, place a pricke or point, being in this example onely one, as that ouer the figure 2. and at the end of your multiplier, place as many prickes or points as there are Fractions in that number, being in this example two, then multiplie the one number by the other after the vſuall manner, as if they were whole numbers, and your worke will ſtand thus; The Product of your multiplication being 21384. and conſidering there are belonging to your two firſt numbers three prickes, repreſenting ſo many fractions, you are therefore to cut off from the Product of your multiplication, with a ſtroke, three Figures towards your right hand, which ſhall bee the Numerator of a Fraction, whoſe denominator is ſo many Ciphars, with an Vnite placed before them, and the other two figures towards your left hand are the Intigers of your multiplication: So ſhall the Product thereof (being the content ſought for) be 21. Perches, and $\frac{384}{1000}$. parts of a Perch, which is ſomewhat more then $\frac{1}{3}$. and your worke ſtandeth thus. But to expreſſe the quantitie of theſe fractions, which you find in the Product of your multiplication, were needleſſe niceneſſe, in buſines of this nature; yet hauing wrought your multiplication, if you find your fraction to bee neere an vnite, increaſe your intire number by an Vnite, and let paſſe the Fraction as not to bee regarded.

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    .
  162
  132..
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  324
 486
162
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21|384
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 1000
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Againe for your better practice, let your meaſured length be $\overset{0}{15}\cdot\frac{1}{4}\cdot\frac{2}{2}\cdot$ and your bredth $\overset{0}{7}\cdot\frac{1}{5}\cdot\frac{2}{2}\cdot$ being the fourth ſort expreſſed in your former example in the beginning of this Chapter. Of which length & bredth you deſire the content by multiplication: firſt, as before, place your numbers, as if they were intigers to bee multiplied, the one by the other thus, as in this exãple: The product whereof is 1159584. and conſidering there are foure points or prickes belonging to the two firſt numbers, that is, to eyther of them two, ſignifying

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    . .
  1542
   752..
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  3084
 7710
10794
-----------
115|9584.
  10000.
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ſo many fractions; you are therefore to cut off foure Figures from the Product towards the right hand, which is the numerator of your fraction as before, and thoſe three Figures towards your left hand are ſo many Perches: ſo ſhall the Product of your multiplication (being the content ſought for) bee 115. Perches, and $\frac{5534}{10000}$. parts of a Perch, which being very neere an Vnite, I therefore adde an Vnite to the former number of Perches, which makes 116. Perches; that is, by reduction 2. roodes. and 36. Perches.

And after this manner (with due obſeruation) may you moſt eaſily and aptly apply this chaine, and the ſeuerall parts and fractions thereof to all the ordinarie rules of Arithmetike, as Addition, Subtraction, Multiplication, and Diuiſion, working any dimenſion thereby, as if they were Intigers, or whole numbers.

And thus much for a taſte onely of the neceſſarie and infinite vſe of this Chaine thus diuided.

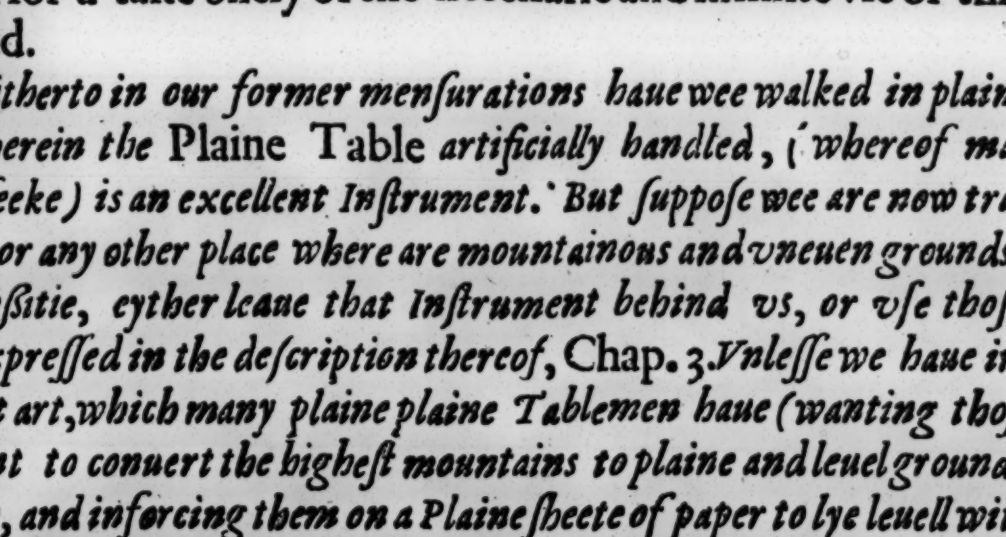
All this time hitherto in our former menſurations haue wee walked in plaine and euen leuelles, wherein the Plaine Table *artificially handled, (whereof many vſing it, are to ſeeke) is an excellent Inſtrument. But ſuppoſe wee are now trauelling into* Wales, *or any other place where are mountainous and vneuen grounds; then muſt wee of neceſsitie, eyther leaue that Inſtrument behind vs, or vſe thoſe meanes, or the like expreſſed in the deſcription thereof,* Chap. 3. *Vnleſſe we haue inſight in that excellent art, which many plaine plaine Tablemen haue (wanting thoſe meanes) at an inſtant to conuert the higheſt mountains to plaine and leuel grounds, preſſing them downe, and inforcing them on a Plaine ſheete of paper to lye leuell with the reſt; which they eaſily performe by onely thruſting out their bordering grounds from their due and naturall place, where,* ab initio *they haue remained.*

Yet let vs conſider of ſome better meanes for the performance thereof, which ſhall be hereafter taught.

CHAP. XLI.

To reduce Hipothenuſall to Horizontall lines by the Peractor.

SVppoſe A B C D. be a hill, or mountaine to be protracted and layde downe in your plotte amongſt your other grounds: It is apparant by the Figure, that the hipothenuſall lines A B. and B C. cannot be layd down exactly in a right line betweene the other grounds which bounder on this hill at the points A. and C. Wherefore we are to find the true leuell and horizontall diſtance betweene A. and C. which is a right line, extending ouerthwart



thwart the ground whereon the hill ſtandeth. Which to performe, worke thus. Plant your Inſtrument at A. the foot of the hill preciſely horizontall, by help of your plummet (wherin great care muſt be had) then cauſe a marke to be placed on the top of the hil at B. to be of equal height from the ground, with the Centre of your Quadrant, whereunto direct your ſight, mouing the Quadrant vp and downe, till you perfectly decerne the ſame, where letting it ſtand, meaſure the Hipothenuſall line A B. which ſuppoſe to bee $\overset{6}{30}$. then ſeeke 30. on the Index of your quadrant, and note what line iſſuing from the left ſide C D. of the Quadrant is cut by the ſame diuiſion or number of 30. on the edge of the Index, which you ſhall finde to be 18. and that number is to be obſerued, and kept for the horizontall line A D. to be protracted and layde downe in ſtead of the hipothenuſall A B. And if the ſame hill from B. continue not plaine and horizontall, but deſcendeth againe on the other ſide, as this from B to C. then muſt that hipothenuſall line B C. bee likewiſe taken, by planting your Inſtrument at B. and cauſing a marke to be placed at C. as before, and then direct your ſight to the marke, and meaſure the hipothenuſall line B C. which ſuppoſe to be $\overset{\circ}{40}$. then note as before what line cutteth 40. on the Index, as 32. and take that for your horizontall line C D. which added to the former 18. maketh $\overset{\circ}{50}$. for the whole line A C. which is to be protracted and layde downe for the two lines A B. and B C.

And if at any time it hapneth (as often it may) that the meaſured diſtance of the hipothenuſall line exceedeth the greateſt number on the Index: In ſuch caſe take halfe, or a third part of the meaſured diſtance, and finding that number on the Index, note what line from the left ſide of the quadrant interſecteth therewith on the edge of the Index, aud the double or triple of the number of that line, is your horizontall line ſought for.

CHAP. XLII.

To reduce hipothenuſall to horizontall lines by the Circumferentor, or by the Plaine Table, with vſe of thoſe meanes expreſſed in the deſcription thereof.

PLant your Inſtrument as before, at the foote of the hill, and let a marke be placed in the toppe thereof, in manner as is directed in the laſt, then directing your ſight to that marke, moue the vane vp and downe on the longer ſight, till through the ſmall hole thereof, and by the pinnes head in the ſhorter ſight, you eſpie the marke, then note among the hipothenuſall diuiſions, what is then cut by the edge of the Vane, which ſuppoſe to be 7. ſignifying (as is expreſſed in the deſcription, Chap. 3) 107. Then meaſuring the hipothenuſall line, which ſuppoſe to be 40. by the rule of proportion, reaſon thus. If 107. the hipothenuſall in the Inſtrument, yeeld 100. the ſide of the ſquare thereby proiected, what 40. the hipothenuſall meaſured and by increaſing 100. by 40. and parting the Product by 107. your anſwere will be 37. and very neere $\frac{2}{7}$. the length of your horizontall line ſought for. And

the

the ſame courſe in all reſpects which you haue here helde in this Angle of aſcention, the like is to bee obſerued in all works whatſoeuer for Angles of deſcention.

Or otherwiſe it may be performed thus.

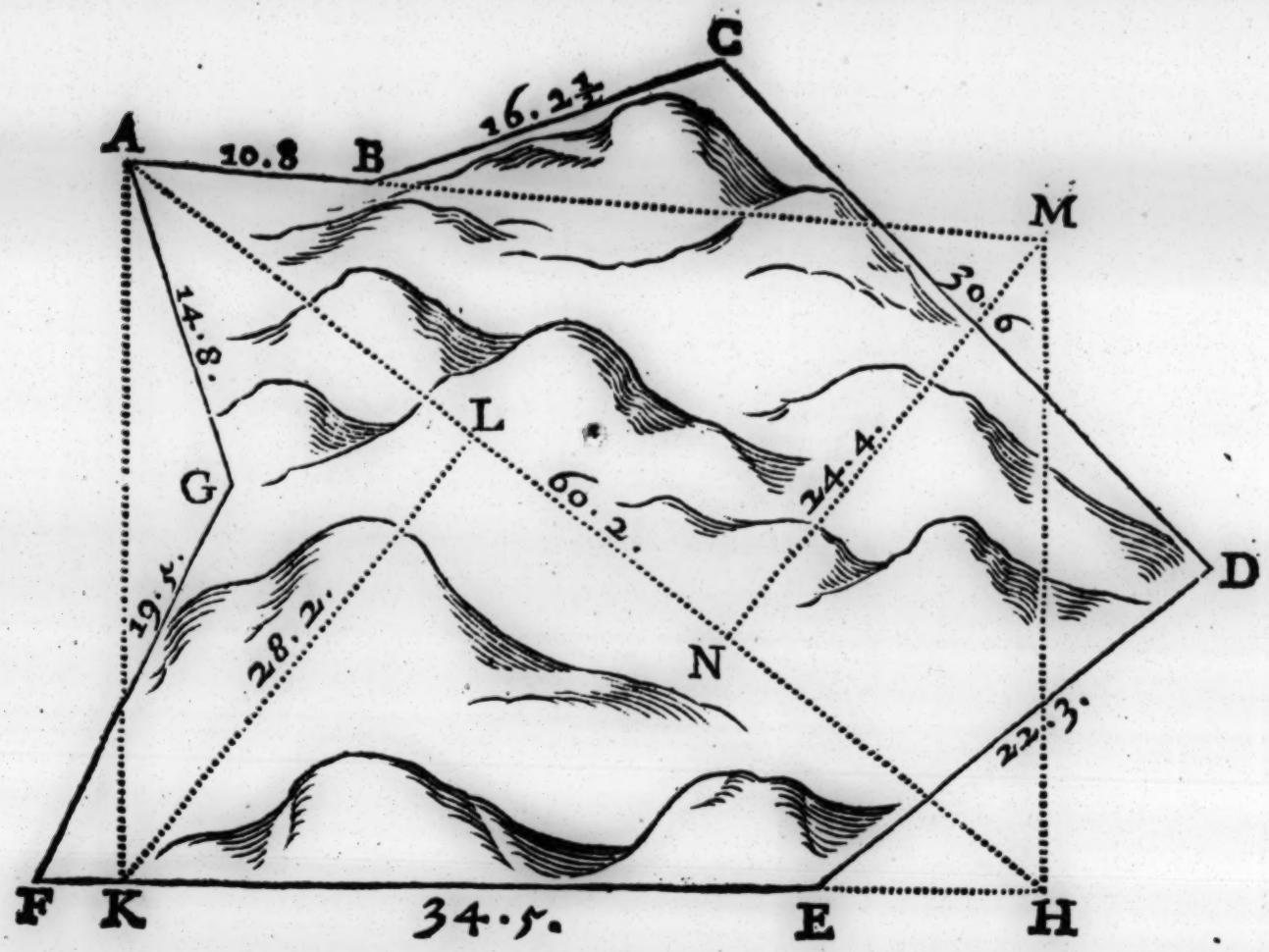
BY any Inſtrument take the quantitie of the Angle, eyther aſcending or diſcending as before taught, and then by the 11. or 12. Chapter protract the ſame Angle, and on the one ſide, from the point thereof, lay downe the meaſured length of your hipothenuſall line, and from the point where thoſe meaſures end, by PROB. 5. 2. let fall a perpendicular on the other ſide of the Angle; and the Segment of that other ſide, betweene the interſection of the perpendicular, and the point of the Angle ſhall bee your horizontall line required.

And thus hauing ſhewed the meanes for the reducing of theſe lines; let vs now conſider of the application and vſe thereof; which for our preſent purpoſe is to finde and expreſſe the true content of irregular and vneuen grounds, and withall notwithſtanding their irregularitie, in protraction to plot and lay them downe in ſuch ſort, as neyther in themſelues they exceede the bounds of their owne Perimeter (but may truly incloſe though expreſſed in Plano) *nor diſplace or thruſt out of order the grounds adiacent. And ſeeing it is impoſſible, and againſt the rules of art and nature, preciſely to expreſſe and limit a ſolid, or bodie within the bounds or termes of one viſuall ſuperficies, which is compriſed and compoſed of many: It is therefore not to be expected, wee ſhould truely expreſſe the irregular capacitie of mountainous and vneuen grounds in a plaine ſheete of paper: for if the plot bee exact and anſwerable to the reſt; the ſuperficiall content muſt needes be wanting, or if the true content of lines and angles bee expreſſed, the plot of neceſſitie muſt bee erronious: yet notwithſtanding we are now to reſolue of ſome direct and immediate courſe, aſwell for the obtaining of the true ſuperficiall content, as for the orderly expreſſing and laying downe of ſuch diſordered Figures, which ſhall be amply and plainely taught and deliuered in the next following Chapter.*

CHAP. XLIII.

The beſt and exacteſt means for the dimenſion & protraction of mountainous and vneuen grounds, and the obtaining of their true Contents by the Plaine Table.

SVppoſe A B C D E F G. bee a mountainous and vnleuelled ground to bee meaſured and plotted: Firſt, plant your Inſtrument at A. and directing your ſight to B. and meaſuring the line A B. draw the ſame by the edge of your Ruler, and after the vſuall manner finding the line Horizontall, place thereon your meaſured length found, from A. to B. $10^{\circ}\ 8'$. then place your Inſtrument at B. where finding an aſcent to C. direct your ſight to a marke there placed as is taught in Chapter 40. and hauing drawne your line at length towards



wards C. as if it were leuell ground, measure the Hypothenusal distance frō B. to C. $16^{\cdot}\frac{1}{2}\cdot\frac{1}{2}$. but withal finde out the horizontall distance as before is taught $15^{\cdot}\frac{1}{6}\cdot$ of which length lay down your line B C. but expresse theron in figures your measured Hypothenusall length $16^{\cdot}\frac{11}{22}\cdot$ so haue you the horizontall truely as it is laid down & your hypothenusal length, for the obtaining of the true content expressed thereon; then plant your Instrument at C. and finding from thence a descent to the next angle D. direct your sight thither to a marke there placed as before, and drawing your line at length, measure the Hypothenusall distance, which found to be $30^{\cdot}\frac{1}{6}\cdot$ expresse it on the line drawne; but searching out the horizontal distance $21^{\cdot}\frac{1}{4}\cdot$ make your line C D. of that length, and in like sort should you proceede if there were diuers other ascending and descending lines: but seeing all the rest in this figure are horizontall and leuell lines, proceede with your worke in all respects as is taught in Chap. 38. So shall you exactly inclose with your other grounds this irregular figure.

But yet are we further to cōsider, that notwithstanding we haue obserued the difference betweene the Hypothenusall & horizontall lines happening in the Perimeter of this figure, wherby we are able to place the same in his due scituation, yet are there within the compasse of this Perimeter many hils & dales wherof we haue hitherto taken no notice, sauing only in cōpassing thē about: And if we should, with these lines alreadie had, cast vp the superficial content after the vsual maner, we should come far short of the true quantitie thereof; which to redresse, worke thus. Before you take your work from the table, reduce the same into the largest *Trapezium* you may, by drawing the lines A M. M H. H K. & K A. as in the former Diagram, then crosse the *Trapezium* with the diagonall line A H. and theron let fall the Perpendiculars M N. and K L. then by direction of your Instrument or otherwise let those lines be exactly measured with the chaine ouer Hils & Dales in a right extention; which by reason of the vneuennesse of the ground you shal find to cōtaine much more in length then your lines alreadie laide downe; as the line A H. in the plot con-

containing but $\frac{51}{4}$. by measure is found to bee $\frac{60}{2}$. also M N in the plot but $\frac{22}{3}$. by measure $\frac{24}{4}$. and L K in the plot but $\frac{24}{8}$. by measure $\frac{28}{2}$. which seuerall lengths so found by measure expresse in figures seuerally on the lines in the plot whereunto they properly belong; and by those lengths so expressed cast vp (after the vsuall manner) the superficiall content of the Trapezium, whose true quantitie you shall finde to be 9. Acres, 3. Roodes and 23. Perches. But let the same be cast vp according to those lines protracted it will containe thereby but 7. Acres, 2. Roodes, and 10. Perches. Whereby is manifest the vsuall errors committed by omission of these meanes. Then cast vp the seuerall contents of those small triangles about the Perimeter, adding those left out, and abating those taken in by the Trapezium, and your worke is finished.

Yet were it necessary to distinguish these frō the rest in your plot, shaddowing them off with Hils, &c. and likewise to expresse therein the true content, with some note of instruction concerning the same, for that otherwise a stranger not herewith acquainted, applying your Scale to the plot, may causelessely taxe you of errors committed.

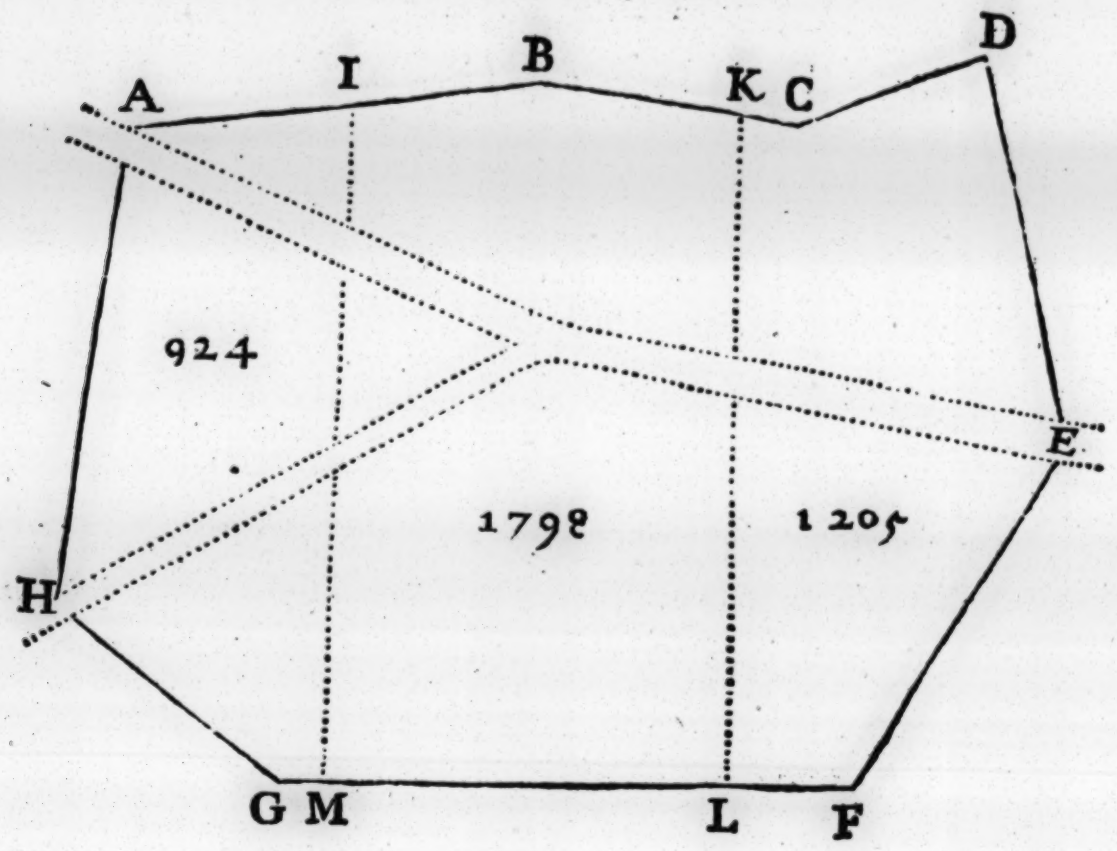
And here note that notwithstanding I haue directed this worke to be wrought with the plaine Table; (by reason that these former courses and obseruations are more vsually omitted in the vse of that Instrument then any other) yet I doubt not but hee who can performe the same by that, (with due obseruation of what hath beene formerly deliuered) will be able to effect the same by any other Instrument. For your obseruations being had and taken in the field, and then laide downe accordingly; you shal thereby afterwards measure your Diagonall and perpendicular lines in the field; as before is directed.

CHAP. XLIIII.

To diuide a Common of pasture, or a common field into any parts required.

SVppose ABCDEFGH bee a stinted pasture, or a common field in the vse and occupation of three men, as A. B and C. and let it be required to diuide the same betweene them proportionally, either according to their seuerall stints (beeing a stinted pasture) or according to their seuerall quantities, being an arrable field in common: First measure and plot the whole quantitie, and as is taught in the second part of the second Booke, finde the superficiall content thereof which suppose to bee 4268. Perches, or by reduction 26. acres 2. roodes 28. perches, then seeke the content of the high wayes A E and H E passing through the same, which let be 341. perches to be deducted out of the whole quantitie: and there resteth 3927. Perches, which to diuide according to their seuerall portions, reason thus with the rule of proportion. If all the whole number of beast-gates of A. B and C. together yeeld the whole quantitie 3927. what the number of those belonging to A. and the answere will be his part: & working thus seuerally for their three seuerall parts suppose them thus, to A. 924. Perches, to B. 1798. and to C. 1205. Then as

is



is taught by the ten laſt Problemes of the ſecond Booke, diuide your plot accordingly, by the lines I M. and K L. laying euery mans part in ſuch place as they ſhall mutually agree: which beeing performed on your plot, you are then to effect the like on the ground, wherein you are onely to lay out the lines I M. and K L. in their due places, which is thus to be performed. Finde out by your Scale the diſtance on the plot betweene the next angle, and the beginning of your firſt line as G M $\frac{\cdot}{3}\cdot\frac{1}{6}$. then find alſo the angle G. in the field, and with your chaine meaſure out that length from G. on the line G F. to end in M. where place your inſtrument, and finding by your Protractor on what degree the line M I. paſſeth on the plot, on the ſame degree of your Inſtrument place the Index: and looking through your ſights, cauſe dooles and markes to be made and placed in a rig t line from M. to I. and for the more exactneſſe, and your better ſatisfactio , note in your plot alſo what angle is next vnto the ende I. of the line M I. as A. from whence take your diſtance by the Scale on your plot to I $\frac{\cdot}{18}\cdot\frac{1}{8}$. and finding the Angle A. in the field, from thence meaſure by your chaine that length $\frac{\cdot}{18}\cdot\frac{1}{8}$. on the line A B. which if you finde to ende in I. is an infailable aſſurance you haue truely wrought; if not, reforme it by reducing the ende I. of your line M I. to that place. And by the like meanes, you are to lay out the line K L. and more, if more required. And after this ſort ſhall you lay out and diuide any common or waſte whatſoeuer; into what parts ſoeuer the ſame is required, to bee deuided, and layde out.

CHAP.

CHAP. XLV.

To know the houre of the day by the Peractor or Circumferentor with the Sunne.

PLace the Index of your Peractor on the Meridian line of the Planiſphere, and direct the North part of the Inſtrument towards the Sunne, turning it about on the Staffe til the ſhaddow of the thrid, or ſhining of the Sunne fall directly through the ſight, on the fiduciall edge of the Index or Meridian line: then obſerue in the Carde the next concentrick circle aboue the preſent moneth, and note what houre-line (which are thoſe ſphericall lines iſſuing from the centre) interſecteth that concentrick circle, vnder the North ende of the needle; which point of interſection, lying directly vnder that end of the needle, ſheweth directly the houre of the day, expreſſed at the vpper ende of that houre-line, or being ſhort or ouer, it ſheweth how much the time of the day is ſhort or ouer that houre.

And the like is performed by the Circumferentor with turning the North ende of the Inſtrument towards the Sunne, till through the ſight, the Sunne ſhineth on the Meridian line; and making your obſeruations as before.

CHAP. XLVI.

The ordering of a plot after the protraction thereof.

THe order and courſe of protraction and laying downe of plots according to the diuerſitie of obſeruations taken, is alreadie in this Booke ſufficiently declared; and likewiſe the meanes of obtaining the true ſuperficiall content of all ſorts of Figures by the Second part of the Second booke; (the concluſion of which part, I would aduiſe you to make vſe of for your further eaſe therein.) Now it reſteth (the plot being thus protracted and caſt vp) next to conſider whether it were fitting, to draw it fayre of the ſame Scale and ſcantling wherein it is, or firſt to haue it reduced into a leſſer forme. If the buſineſſe be not ſo large, but that with conueniencie it may bee drawne in the firſt Scale wherewith it was plotted and caſt vp, then prouide you a piece of ordinarie new linnen cloth of a reaſonable fineneſſe, and thereon paſte you cleane paper, according to the order of Maps (but thoſe for the moſt part are too courſe) of the largeneſſe of your plot; which beeing well waſhed with Allome water, and dryed, faſten your rough plot, ſlenderly thereon with mouth glue at each corner thereof; and with a bodkin or pin of braſſe (which I hold the beſt for theſe purpoſes) trace out all your lines with a reaſonable hand; for a light hand with diſcretion wil make ſufficient impreſſion on your new plot, whereby you may well diſcerne to draw your lines either with Pen or penſill as you pleaſe; and then expreſſe your houſes, buildings, woods, riuers, waters, wayes, and all other remarkeable things in their due proportion perſpectiuely; not placing your houſes and trees euery way, whereby

here

here the tops and there the bottomes ſhall ſeeme ſtanding vpwards, as is vſually accuſtomed; and then garniſh your plot about with ſome neate border, and within with Copartments for your Scale and Title; and in ſome conuenient place thereof deſcribe a Carde, ſhewing the ſcituation according to the points of the Compaſſe; and then let it be neately coloured about the ſeuerall lines, lightly waſhed off, and not dawbed all ouer, as ſome painting Surueyors vſe. But aboue the reſt, forbeare much writing in your plots either of names or quantities (beeing abſurd and groſſe) but onely numbers of reference to your ingroſſed booke concerning the ſame, if you make any; if not, then it may ſerue, as Chalke on a trencher.

But if it bee required to haue your firſt plot reduced into a leſſer or greater forme; vſe the helpe of the next Chapter.

CHAP. XLVII.

To reduce any plot from a greater to a leſſer quantitie, and the contrarie.

IN the third part of my Second booke I haue at large declared the manner and meanes as well of reducing and tranſlating of all ſuperficiall Figures from one forme into another, retaining the ſame quantitie; as one quantitie into another, retaining the ſame forme. But thoſe being of particular figures, are not ſo meet or neceſſarie for the Reduction or tranſlation of a plot conſiſting of many (being often in vſe.) Wherefore I will here deliuer you a very ſpeedie and exact meanes for the performance thereof. According to the proportion whereunto you would reduce your firſt plot, lay papers together with mouth-glue; as if you would reduce it into a fourth part, make your new plot to be in quantitie a fourth part thereof. Then place your new made plot on your rough plot in ſuch ſort, and with ſuch diſcretion, as the middle of the one may be about the middle of the other, whereby all the worke of the one may be reduced the better into the other, and there faſten the one to the other ſlenderly at the corners with mouth-glue, in ſuch ſort, as when occaſion ſerueth, any one of thoſe glued places may be eaſily taken aſunder, and the paper folded in; then take your reducing ruler mentioned in Chap. 9. and faſten the ſame with a needle or blue pin ſtriken into the table about the middle of the plots through the centre hole of the ſame rule, in ſuch maner as both the plots together may bee turned about at pleaſure vpon the table; wherein you muſt take great care of renting or tearing out your centre point in the plots; which to preuent, would require to bee ſtrengthened with a ſmall piece of a Carde or paſt-boord, to bee glued thereon, vnderneath the firſt plot. And being thus prepared, you ſhall finde good part of the worke in your firſt plot, to lye without the vtter edges of your cleane paper, which let be firſt reduced; wherein (hauing reſolued into what proportion you will make your Reduction, as into $\frac{1}{4}$. of the firſt) worke thus; bring the edge of your Ruler to any Angle in your firſt plot, and note what number of diuiſion on the edge of your Ruler is there cut, which admit 40. then take halfe

therof which is 20. and againſt that diuiſion by the edge of your Ruler make a pricke on your cleane paper, then remoue your Ruler to the next Angle in your firſt plot, and note the diuiſion there cut, whereof take likewiſe the halfe, and againſt that halfe by the edge of the Ruler make another prick; and betweene theſe two prickes draw a line, which ſhall repreſent the line betweene thoſe two Angles of your firſt plot; and ſo proceede from Angle to Angle, and from Cloſe to Cloſe, till you haue reduced all the worke on your olde plot, lying without the edges of the new, or ſo much as lies without the ſame towards any one ſide or corner thereof; and then vngluing one of your corners or ſides, fold in backwards towards your olde plot, that part of your new plot as is wrought; ſo ſhall you come to worke that which formerly lay vnder the ſame; and thus by folding in, and working one ſide after another to the centre; whilſt the other ſides are faſtned together with the olde plot, you ſhall ſpeedily reduce the whole plot into your deſired proportion: For in taking the halfe of euery line in your firſt plot your new by THEOR. 49. 1. ſhall be $\frac{1}{4}$ of the olde.

And this kinde of Reduction hold I the ſpeedieſt and exacteſt of all other: Which by ſmall practice, you ſhall much better finde and vnderſtand, then with many words of Relation.

And after this manner may you make your Reduction, into any other proportion, or reduce your plot from a leſſer to a greater forme in any proportion required; by increaſing your ſecond number, proportionally, as in this worke you decreaſed the ſame.

CHAP. XLVIII.

To reduce any number of Perches giuen into Acres, and the contrarie.

SVppoſe 5496. Perches were giuen to bee reduced into Acres. Firſt after the vſuall manner (conſidering that a Statute Acre containeth 160. ſquare Perches) diuide the giuen number by 160. the quotus will be 34. and the remainder 56. which remainder diuided by 40. (the number of Perches in a Roode) quoteth 1. and the remainder 16. So is the whole Reduction 34. Acres, 1. Roode, 16. Perches.

Or more briefely thus.

THis rule is much briefer in operation, though not in demonſtration then the former; which is thus. Frõ your giuen number cut off with a ſtroke, the firſt Figure towards the right hand thus, 549|6. then diuide the other three Figures by 4. and your worke will ſtand thus. the 137. being ſo many Roodes, and 16. Perches remaining, then diuide likewiſe that 137. by 4. and your worke is finiſhed and will ſtand thus. Where note that your firſt remainder with the Figure cut off, are alwaies the odde Perches; and the laſt remainder (if any be) are Roodes.

```
   1
5 4 9 | 6
1 3 7

   1
5 4 9 | 6
1 3 7
-------
3 4 -- 1 -- 16.
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And

And if you would reduce theſe Acres, Roodes, and Perches, into their leaſt Denomination, as into Perches. Firſt multiply your number of Acres 34. by 160. the product is 5440. then multiply the number of Roodes 1. by 40. produceth 40. which together with the number of Perches 16. added to 5440. makes 5496. the firſt number. And the like of all others.

Here might I now much inlarge this worke by ſhewing many other neceſſarie concluſions fit for a Surueyor to know, as the menſuration of Timber, Boorde, Glaſſe, Pauements, and the like; alſo the ſeuerall waies and meanes of plotting of countries and large continents; of carrying of mynes and trenches vnder ground; of Water-workes, and the conueying of the water from any Fountaine to appointed places of whatſoeuer poſſible diſtance; of the taking and making of the formes and models of Plot-formes, Forts, Caſtles, Houſes, and the like. But ſeeing that not onely theſe, but infinite other Concluſions Geometricall, may be eaſily performed and wrought by the former rules and inſtructions well vnderſtood and practiſed; and for that they are without the ſcope and limits of Suruey, whereunto I chiefly bend the ſubiect of this Booke; I will leaue them to your owne indeuours, and diligent practice.

And hauing thus performed at large the Mathematicall part of Suruey in generall; we will next conſider of the Legall; and in the meane ſpace here conclude this Third Booke.

The end of the third Booke.

THE LEGALL PART OF SVRVEY.

The fourth Booke.

The Argvment Thereof.

I Would not bee miſtaken, or haue it vnderſtood; that I here vndertake (as a Lawyer) to inſtruct or teach the rules or Inſtitutions of the Law (being out of mine element) but as a Surueyor, briefely and truely to expreſſe and deliuer herein what I hold fit and meete for a Surueyor to know and vnderſtand. As firſt what a Mannor is, and the ſeuerall parts and members thereof, with the appendants therevnto: Next, the perquiſites, caſualties and profits of Court, and their ſeuerall natures: Then the diuerſitie of eſtates, whereby any Lands or Tenements may bee holden, occupied or enioyed; and the ſeuerall tenures depending on thoſe eſtates; with the Rents and ſeruices incident and belonging to thoſe tenures: Alſo what repriſes, paiments, and deductions may bee iſſuing out of a Mannor, and the

conſiderations thereof to bee had: Likewiſe what courſes are to bee obſerued and taken, before the beginning of a Suruey: The order and manner of keeping thoſe Courts: The entrie of the Tenants euidence and eſtates; and the orderly manner of ingroſſing the ſame: with other briefe and neceſſarie Rules, and Obſeruations tending to thoſe purpoſes.

CHAP. I.

Of a Mannor with his ſeuerall parts, and of the name and nature therof: how made and maintained, and how diſcontinued and deſtroyed.

AS in my workes concerning the Mathematicall part of Suruey, compriſed in the three former Bookes, I premiſe the definitions, principles and grounds thereof: ſo in this *Legal part* I hold it anſwerable to order, and a good decorum, (before we abruptly enter on the Suruey of a Mannor) firſt to conſider what a Mannor is, and the ſeueral parts thereof (leſt being queſtioned of our preſent imployment, wee diſcouer our owne weakeneſſe in vndertaking we know not what) and then to informe our ſelues of the ſeuerall natures, qualities and conditions, of the eſtates, tenures, and ſeruices of land; and of the ſeuerall profits, rents, and commodities therevnto incident and appertaining; with ſuch other meete and neceſſarie obſeruations, as are moſt fitting for a Surueyor to know and vnderſtand, before he aſſume and take vpon him the name, or at leaſt the office or function to a Surueyor belonging. Of all which in order; and firſt of a Mannor what it is, and of the parts thereunto belonging.

Manner whence deriued.

Concerning the Deriuation or Etymologie of the word, I will not ſtand, whether it be of *Maneo manere* to remaine in a ſettled place; or of *Mano manare* to proceede or ſpread abroad out of the bountie of thoſe Princes liberalitie who in the beginning beſtowed them; or of *Manuarius* gotten by labor of the hand; which I hold the beſt; becauſe there is more skil in getting then keeping; and with *Manerium* I will not meddle, ſeeing (as I take it) Mannors were created before the word was made: But from whenceſoeuer deriued, A Mannor is now that which hath therunto belõging, meſſuages, Lands, Tenements, Rents, Seruices & Hereditaments; wherof part are Demeanes, being thoſe which anciently and time out of minde, the Lord himſelfe euer vſed, occupied and manured with the Mannor houſe; the reſidue are Free-holds,

What a Manner is.
Parts of a Manner.

Farmes,

Farmes, and cuſtomarie or coppihold tenements; and theſe haue commonly diuers ſeruices beſides their rents properly belonging thereunto, whereof I will hereafter ſpeake.

Parts of a Mannor.

There is moreouer belonging to a Mannor a Court Baron, and to diuers a Court Leete; which is of more worth and efficacie, and is alwaies granted from the King, or held by preſcription. To theſe Courts, and conſequently to the Mannor is there vſually belonging; Fines, iſſues, amerciaments, heriots, waiues, ſtraies, excheates, reliefes and other perquiſites and profits of Court; whereof likewiſe I will further ſpeake.

Appendants to a Mannor.

Beſides, thoſe there are often appendant and belonging to a Mannor (which are not of neceſſitie to be taken as the proper parts thereof) Wards, Marriages, aduowſons, patronages, free-gifts or preſentations of parſonages, Vicarages, Chappels, Prebends, &c. alſo Commons of Paſture, Moores, Mariſhes, free Warrens, Cuſtomes, Liberties, Franchiſes, and Priuiledges; likewiſe yeerely Rents, ſuites of Court, tenths and ſeruices iſſuing and repriſed out of other Mannors. And of theſe, a Mannor is neither made by them; nor deſtroyed or marred for want of them; wherefore they are termed rather appendants then parts of a Mannor.

No preſent meanes to make a Mannor.

Neither doe thoſe parts formerly named, properly of themſelues make a Mannor: For ſhould any man at this day alott and appoint out any competent quantitie of Land, and diuide the ſame into demeanes and tenement Lands; in feoffing Tenants in Fee of ſome part, and granting others by copie of Court-Roll, and perfecting the reſt which before is ſaid to belong vnto a Mannor; yet all this will not make a Mannor; for that it is the office of time by long continuance to make and create the ſame.

Continuance of time may perfect a Mannor.

How a Mannor may be deſtroyed and diſmembred

But a Mannor at this day may bee diſmembred, and vtterly deſtroyed both in name and nature, by eſcheating the Free-holds, and Copie-holds; for if of Free-holds or Copie-holds there are not two at the leaſt, then are there no Sutors, and if no Sutors, no Court, and conſequently no Mannor; and then may it bee termed a Seigniorie, which can keepe no Court Baron at all.

How one Mannor may be diuided, and made diuers Mannors.

Alſo it is to be vnderſtood, that one Mannor may bee diuided into diuers Mannors; whereof wee haue diuers examples at this day; as where a Mannor deſcendeth to co-heires, and they make diuiſion and partition thereof; allotting to euery of them demeanes and ſeruices; whereby euery of them hath a ſeuerall Mannor, and may keepe ſeuerall Courts Baron thereon, as if anciently entire.

How diuers Mannors may be reduced into one Mannor.

And in like manner two diſtinct and ſeuerall Mannors may bee conioyned and made one entire Mannor, if formerly the one held of the other; and that Mannor ſo holding of the other doe eſcheate; but otherwiſe not. And thus much concerning the name, nature, and parts of a Mannor.

CHAP.

CHAP. II.

Of Perquiſites Caſualties and profits of Court, and their ſeuerall natures.

IN the former Chapter I declare, that (among other things) there is belonging to a Mannor a Court Baron at the leaſt, and to ſome a Leete or Law-day, commonly called the *view* of *Franke pledge*: Now herein will I ſhew what perquiſites, caſualties, and profits are incident and belonging to thoſe Courts; wherein I would haue it vnderſtood, that it is not of neceſſitie, that all theſe hereafter mentioned, muſt be in euery Mannor, but that they may be in any. And firſt of Fines.

Fines of Land.

Fines of poſt mortem.

FInes of lands are of diuers kindes; As firſt, if a man holding to him and his heyres, or otherwiſe certaine Lands and Tenements, by Copie of Court Roll, according to the cuſtome of the Mannor dye, his heyre vpon his admittance by the Lord, ſhall pay a Fine for ſuch his admittance: And theſe Fines are of two ſorts, either certaine, or arbitrable; if certaine, as one or two yeeres rent, or the like; there is then no other queſtion to bee made, but the Lord by his Steward is to admit him, and hee to pay ſuch certaine Fine accordingly; if vncertaine or arbitrable, then is the Tenant to vndergoe what Fine the Lord ſhall in reaſon impoſe or require; and theſe are called *Fines of poſt mortem.*

Fines of alienation.

Alſo a Tenant by Copie of Court Roll, hath not power to Alien or ſell his eſtate or intereſt vnto any other man, without he ſurrender the ſame into the Lords hands to the vſe of him vnto whom he ſhall ſo ſell the ſame; for which Alienation the Lord is alſo to receiue a Fine, which in ſome Mannors are likewiſe certaine, and in others arbitrable, but being arbitrable, they are vſually rated at a lower and more reaſonable value then thoſe after death, and theſe are commonly called *Fines of Alienation.*

Likewiſe if a cuſtomarie Tenant let or ſet his lands vnto another for any terme of yeeres, not warranted by the cuſtome, hee is firſt to obtaine licence of his Lord in this behalfe; and is to pay a Fine in reſpect thereof.

And moreouer if the Lord of a Mannor grant a Leaſe of any lands vnto a Tenant for any number of yeeres or for life or liues; and beſides his annuall Rent, make compoſition for a Fine to be in hand paide; this is alſo a *Fine of Lands.*

Alſo in ſome places, the Cuſtome is, that if a cuſtomarie Tenant alien and make ſurrender of his whole eſtate hee ſhall pay and yeeld vnto the
A farewell paid. Lord the beſt beaſt hee hath, or a certaine piece of money, in name of a *Farewell.* And in ſome places as well Free-holders, as cuſtomary Tenants on euery alienation ſhall pay a certaine ſumme of money for a Fine in
Offare & onfare name of *Offare*, and *onfare*, and all theſe and the like are *Fines of Land.*

Amerciaments.

Amerciaments.

AMerciaments are alſo perquiſites of Court, whereof there are diuers ſorts; which in generall are ſuch Fines, penalties, and amerciaments as by the homage or afferers of the Court Leete, or otherwiſe are impoſed on ſuch Tenants as are found offenders within the Mannor; As if the Free ſutors, Copie-holders, or other Tenants, make default or bee abſent from the Lords Court, they are therefore amerced. Wherein is to be noted, that many Free-ſutors make compoſition, and are at their Fine certaine in reſpect of their ſeruice of ſuite of Court; and theſe are called common Fines.

Common Fines quid.

Heriots.

AN Heriot is properly the beſt beaſt which any heriotable Tenant is poſſeſſed of at the time of his death, whether it be Horſe, Oxe, Cow or the like; for which in many places a ſumme of money is paide by ancient compoſition, and in ſome places for default of liue Cattell (or the beſt beaſt not being to the Lords minde) it is in his choiſe to take the beſt of any other goods, implement, or commoditie the Tenant hath at the time of his deceaſe.

Of theſe Heriots there are two ſorts, *Seruice* and *Cuſtome*: Heriot ſeruice is commonly mentioned and expreſſed in the Tenants grant: and therefore the Land anſwerable for ſatisfaction thereof; and Heriot cuſtome is that which time out of minde hath beene euer paide vpon and after the death of any Tenant dying ſeiſed of any ſuch heriotable Lands: And theſe Heriots of either kinde, are by the homage of euery Court to bee preſented as they fall due; and ſeiſed by the Lords Bayliffe accordingly.

Heriot Seruice.

Heriot Cuſtome.

And it is to bee vnderſtood, that if a Tenant dieth ſeiſed of diuers tenements or lands, which haue beene anciently charged with diuers Heriots; the Lord at the time of the death of ſuch Tenant ſhall receiue ſo many ſeuerall Heriots, as thoſe lands at any time then-to-fore were anciently charged or chargeable to yeeld.

One Tenant may be chargeable with diuers Heriots.

And moreouer, if any heriotable Tenement ſhal be ſeuered & diuided into diuers parts, amongſt ſeuerall Tenants; the Lord ſhal haue of euery ſuch Tenant particularly a ſeuerall Heriot, for and in reſpect of thoſe ſeuerall heriotable parcels; which the Lord may ſeiſe and take, whereſoeuer hee ſhall finde the beſt for his beſt aduantage.

Heriotable lands diuided, are ſeuerally chargeable.
Lib. Aſſ. 27. 24.

Reliefes.

RElIefes are likewiſe accompted amongſt perquiſites of Court: but ſeeing it is a ſpeciall ſeruice tyde to the tenure of Lands; I will here omit to ſpeake thereof; referring you for your ſatisfaction therein to the Title of *Warde*, *Marriage* and *Reliefe*, in the 4. Chap. following.

Eſcheates.

Eſcheates.

Eſcheates what they are.

THeſe are likewiſe perquiſites of Court; and are ſuch as if a Free-holder or Copie-holder of inheritance, commit any manner of Felonie, and be thereof attainted; his Lands are eſcheated and forfeited to the Lord of the Mannor of whom they are holden; but the Lord ſhall not immediately enter thereinto; for the King is firſt to haue *annum diem & vaſtum*; after which time expired, it then remaineth to the Lord and his heyres for euer.

Alſo if any ſuch Tenant dye without heyre generall or ſpeciall, all his Lands and Tenements ſhall fall vnto the Lord by eſcheate, to remaine vnto him and to his heyres for euer.

Forfeitures.

Forfeitures of diuers kindes.

FOrfeitures are of diuers kindes; As if a Copie-holder or cuſtomarie Tenant deny, or will fully refuſe the paiment, doing or performance of his Rents, Seruices and Cuſtomes; or if hee fell or cut Timber on his Copie-hold Lands contrarie to the cuſtome; or doe or commit waſte in the houſes or otherwiſe; or if he grant or ſell his Copie-hold eſtate by deede; or alien or let the ſame without licence of the Lord, beyond limitation of the Cuſtome; In all or any of theſe, the cuſtomarie Tenant ſhall forfeite his Copie-hold eſtate into the Lords hands: Which offences are to bee found and preſented by the homage at the next Court; and thereupon ſeiſure made accordingly.

Alſo Tenant for terme of yeeres, life, or liues, may forfeit his eſtate for making a larger eſtate of Free-hold then he hath, or for not performance of ſuch prouiſoes and conditions, as are expreſſed in his Leaſe or Deede, if any be.

Waiues.

IF any man felonioufly ſteale or take any goods or chattels of what nature or kinde ſo euer; and by earneſt proſecution he is inforced in flying to leaue the ſame behinde him; theſe goods are called *Waiues* or waiued goods; and in what place ſoeuer they are ſo left and waiued, they ſhall be taken and ſeiſed for the vſe of the Lord of that Mannor, if by his grant, charter or preſcription, they belong vnto him (or otherwiſe they are the Kings) and being ſo ſeiſed by the Bailiffe or other Officer, they are to bee preſented and found at the next Court by the Homage there. But if the right owner make freſh ſuite after the thiefe, and attaint him at his ſuite for ſtealing thereof, hee ſhall haue his goods againe, although they be waiued. And the like in all reſpects is if any goods be taken by an officer, from any whom hee ſuſpecteth to haue ſtolne the ſame, though there be no purſuite made or proſecuted.

Freſh ſuite. Waiued goods reſtored.

Eſtraies.

Eſtraies.

ESTraies are when a Horſe, an Oxe, Sheepe, or any other Cattell of what kinde ſo euer come into a Lordſhip or Mannor, no man knowing from whence, nor the owner thereof; ſuch are to bee ſeiſed to the Kings vſe, or to the vſe of the Lord of the Mannor who hath the ſame by grant or preſcription; and if the owner come and make claime within a yeere and a day, then hee ſhall haue the ſame againe, paying for the charge thereof; or elſe after ſuch time expired, the propertie thereof ſhall be to the King; or the Lord of the Mannor hauing the ſame by graunt or preſcription; So that Proclamation bee thereof made in the next markets and the Pariſh Church, according to the Lawes in that behalfe.

Pleas and Proces of Court.

THeſe are where the Lord of a Mannor in the Court Leete, or Court Baron, holdeth plea of his Tenants for actions of debt, of treſpaſſe or other cauſes, not exceeding the value of xl*s*. debt and damage.

And vnder this title of perquiſites is compriſed all other caſualties whatſoeuer, which may happen to grow or ariſe within any Mannor; as profits ariſing by mines of Copper, Tynne, Leade, Cole, and quarries of Stone; alſo by ſale of Woods, Turbarie, and Pannage; likewiſe profits of Fayres and Markets, Fiſhing, Fowling and the like. All or any of which may become certaine; by beeing letten and diſpoſed of for yeerely rents. *Caſualties may become certaine.*

CHAP. III.

Of the diuerſitie of eſtates, and their ſeuerall natures.

HAuing alreadie ſhewed in the two former Chapters what a Mannor is, with the ſeuerall parts thereof, and the appendants thereunto: I hold it fitting here now to conſider of *Eſtates*; as how and by what meanes a man may bee eſtated either in a Mannor or any other Lands or Tenements: wherein it is to bee vnderſtood that all eſtates in generall conſiſt of two principall kindes as *Free-holds* and *Chattels*: which more particularly are ſub-diuided into diuers other parts or branches; as firſt *Feeſimple* and *Feetaile*, which are termed Free-holds of inheritance: alſo eſtates *After poſsibilitie of iſſue extinct*; *By Curteſie*; *In dower*; and *for terme of life*: Which foure laſt mentioned, are called Free-holds, but not of Inheritance: Likewiſe eſtates by *Copie of Court Roll*, being claimed & held by cuſtome, and are diuided into the like parts, as Free-holds at the common Law: and laſtly eſtates *For terme of yeeres*, and *at will*, which two laſt are Chattels. Of all which briefely in order as followeth.

1. *Fee*

1. *Fee-ſimple.*

Defined. FEe ſimple of all other eſtates is the moſt large, ample, and abſolute that wee haue in this Kingdome, or that can by our Lawes bee inueſted or made; and is that which is granted to any man and his heyres for euer, without any further or other limitation of vſe or vſes; and therefore if ſuch Tenant hath iſſue of his bodie, the land deſcendeth to him, if not, to the next of kinne within the degrees of limitation hereafter ſpecified.

But if a man purchaſe in Fee-ſimple to him and his aſſignes for euer, omitting this word heyres; here hath he but an eſtate for terme of life; for heires is the word which carrieth the inheritance.

Yet it is otherwiſe if lands be ſo deuiſed by Will; for the Law intendeth that learned counſell cannot alwaies be preſent in ſuch caſes; and therefore is ſuch deuiſe conſtrued for the beſt, according to the Teſtators meaning and intention, and not to the ſtrict letter of the Will.

Alſo if Lands be granted to any man with a woman in Franke-marriage, this word implyeth an eſtate of inheritance without mention or addition of the word Heyres; Or to a man and to his bloud the like.

And here is to bee conſidered, who are thoſe which are ſaid to bee within the degrees of limitation before ſpoken of; that is, who are vnderſtood to be a mans heires by the common Law. Suppoſe A. B. dieth ſeiſed of a ſtate of inheritance without iſſue of his bodie: Neither his brother, or ſiſter of the halfe bloud, nor their iſſue ſhall be his heyre; nor his baſtard; nor his father, mother, grandfather, or grandmother; for inheritance may lineally or collaterally deſcend, but by no meanes lineally aſcend by our Lawes; but the brother or ſiſter of the father of A. B. (which is called a collateral deſcent) ſhal be his heyre; and then they dying ſeiſed without iſſue, the father of A. B. ſhall haue the land as heyre to his vncle or aunt, but not as heyre vnto him.

The halfe bloud. A baſtard no heire. Lineall and collaterall deſcent.

Likewiſe it is to be vnderſtood, that by the lawes of this Realme the eldeſt ſonne is wholly to inherite; and he dying without iſſue, the ſecond ſon, and ſo of the reſt; and if no ſonnes but daughters, they ſhall ioyntly inherite as coparceners; but if no iſſue neither ſonne nor daughter, then ſhall the eldeſt brother be heyre, for want of ſuch all the ſiſters; and in default of them, the vncle by the fathers ſide, if the Land came by the father, or bee of the purchaſe of him ſo deceaſed: But if there be no heyre of the fathers ſide, the purchaſed lands goe to thoſe of the mothers ſide; But if none ſuch, then all thoſe lands ſhall Eſcheate to the Lord of whom they are holden.

Coparceners.

Eſcheate.

2. *Fee Tayle.*

Fee taile of two kindes. Generall tayle. THis eſtate of Fee-taile is diuided into two kindes or ſorts *Generall* and *Speciall.* The firſt, being Fee-tayle generall, is when Lands or Tenements are granted vnto any man, and the heyres of his body begotten without limitation, or expreſſe mention made by what woman; wherefore if ſuch Tenant marrieth diuers wiues, and hath iſſue by them ſeuerally; they ſhall all be capable of the Inheritance of thoſe Lands. But if it be mentioned and expreſſed in the graunt by what woman, his Heyres ſhal proceed

or

or be begotten, as if the gift be made to A. B, and to the heyres of his body lawfully begotten on C. his wife, this is an especial In-taile, for any of his issue begotten by another woman, shall not inherite by force or meanes of this graunt or tayle. And the like in all respects if Lands bee granted to a woman in the like kinde. *Speciall taile.*

Also if Lands be granted vnto A B. and C. his wife, and to the heyres of their two bodies lawfully begotten; here are the man and his wife Ioynt-purchasers, and this is also a speciall tayle both in him and her.

Likewise if any man grant Lands or Tenements to another man with his Daughter in Franke-marriage, this is also a speciall tayle; and both the man and woman shall be here Tenants in the speciall tayle, for the word Franke-marriage implyeth as much. *Franke mariage.*

Also if Lands be granted to a man and the heyres Males of his bodie; this is an estate tayle, and here the Female shall not inherite. *Descent by heires males.*

3. *After possibilitie of issue extinct.*

IF Lands or Tenements be granted to a man and to his wife, and to the Heyres of their two bodies lawfully begotten, and either of them dye without such issue betweene them; then is he or she suruiuing Tenant in tayle of those Lands, but are without all hope and past possibilitie of hauing such Heyre to inherite those Lands as was limitted in the graunt; & therefore is he or she so suruiuing and ouer-liuing the other, called Tenant in tayle after possibilitie of issue extinct: and from and after the death of him or her so suruiuing; the estate taile so made and granted vnto them, shal be vtterly voide, extinct and dead, as if the same were neuer granted; and the estate of inheritance of and to those lands, shal reuert and turne vnto the first Donor thereof and his heires. *Free-holds.*

4. *By courtesie.*

IF a man marrie a wife, being an Inheritrix, and hath issue by her, and she die; by our lawes he shall hold, occupie, and enioy such lands as his wife died seised of either in Fee-simple, or Fee taile, during his naturall life; and he is called Tenant by courtesie of England, because no other Nation admitteth the like estate. Wherin the Law requireth that such issue be vital, & brought forth into the world aliue, although it immediately die, and also it is requisite that the husband bee in actuall and reall possession of those lands, and seised of them in the right of his wife, at the time of her death, or otherwise he shall not be admitted Tenant by courtesie thereunto.

But if any such Tenant by courtesie commit or suffer any stripe or waste, he is punishable in that behalfe, by action of waste.

Also it is to be vnderstood, that no man can bee Tenant by courtesie of a reuersion: for if a woman solye seised in Fee, granteth a Lease to A. B. for terme of his life, and afterwards marry and hath issue, and then dye; the Tenant or Leasee for life suruiuing; her husband in this case shall not be Tenant by courtesie.

5. *In Dower.*

BY the common Law of this Kingdome, if a man marry a wife, and at any time during the time of couerture he be directly and lawfully seiſed, either by purchaſe or deſcent of any lands or tenements, either in fee-ſimple or fee-tayle, and being ſo ſeiſed die, his wife ſhall be indowed of a full third part of all thoſe lands and tenements, during her life; and being thus indowed, ſhe is called *Tenant in Dower*, and this is by the common Law.

Dower at the common Law.

Dower by cuſtome.

Beſides this, there is Dower by cuſtome, for in ſome places the woman ſhall haue a moytie, and in ſome places more, and in others leſſe during her life, of all the lands her huſband was ſeiſed of at any time during the couerture, according to the cuſtome of the place.

But if the wife be not aboue the age of nine yeeres, at the time of her huſbands death; the common Law will not permit her indowment.

A woman ſhall haue no dower.

And for diuers cauſes a woman may be defeated of her dower; as if ſhee or her huſband, commit treaſon, murder, or felony, and be thereof attaynted (yea, though they haue their pardon:) alſo if ſhe forſake her huſband and liue incontinently; and be not againe reconciled without conſtraint of law; or if ſhee detayne and with-hold the deedes and euidences from the heire, of thoſe lands, whereof ſhee claymeth dower, and the like.

And ſome things there are, whereof a woman is not capable of indowment; as of Commons, Annuities, Eſtouers ſans number, Homages, Seruices, and the like.

There are beſides theſe other kindes of dowers, as one called dowment, *Ex aſſenſu patris*: another termed dowment *ad oſtium Eccleſiæ*: and a third, *de la plus bel part*: As appeareth at large by our common Lawes; whereunto I referre you.

6. *For terme of Life.*

A Tenant for terme of life, is he, who holdeth lands or tenements, either for terme of his owne life, or for terme of another mans life; but for diſtinctions ſake, he who holdeth for his owne life, is termed barely *Tenant for life*, and hee that holdeth for anothers life, is called tenant for terme *d'auter vie*, that is, of anothers life.

Waſte.

And if either of theſe kinde of tenants commit or ſuffer waſte, the leaſor or he in reuerſion, ſhall bring and maintayne againſt him an action of waſte, and thereby recouer treble damages.

7. *By Copie of Court roll.*

THeſe tenants are ſuch, as in diuers Mannors hold lands and tenements to them and to their heires, ſome in the nature of fee-ſimple, others in fee-tayle, or for terme of life or liues, at the will of the Lord, according to the cuſtome of the Mannor; and in ſome Mannors they hold by copie for terme of yeeres: And all theſe haue no other euidence to ſhew concerning the tenure of their lands, ſaue only the copies of the rolls of their Lords Court; and therefore are they called tenants by *Copie of Court roll.*

Alienation of eſtate.

And if any of theſe tenants alien or ſell his lands or eſtate by deede, he ſhal abſolutely forfeit the ſame into the Lords hands: wherefore if he will alien his

his copy-hold eſtate, he muſt come into the Lords Court and there ſurrender the ſame into the Lords hands, to the vſe of him vnto whom he alienateth the ſame.

But in diuers Mannors the ſurrender may be made out of Court, vnto any copy-hold tenant, in preſence of two of the homage (to the vſe as aforeſaid) who are to preſent the ſame vnto the Steward at the next Court, and admittance made accordingly. *Surrender out of Court.*

And theſe tenants can neither ſue or be ſued, in any of the Kings Courts, by writ or otherwiſe, for theſe lands ſo holden. But they muſt implead and ſue for the ſame, by way of plaint, in the Lords Court.

And ſome are of opinion that theſe tenants are but in the nature of tenants at will of the Lord; who at his pleaſure may diſplace them, and they without remedie, but by the Lords fauour. Yet others are of a contrarie minde, who maintayne, that if any ſuch cuſtomarie tenant (but thoſe for yeeres, their terme being expired; paying and doing their ſeruices) ſhall without iuſt cauſe be eiected and diſplaced by the Lord; he may bring and maintayne his action of treſpaſſe againſt him, at the common Law. *Tenants at will.* *Remedie for copy-holder put off.*

And if any of theſe cut timber, growing on his lands, without licence of the Lord (but only for repaire of his tenement) it is a waſte, and an abſolute forfeiture. *Waſte.*

And in moſt Mannors if any ſuch tenant ſhall farme or let out his land, for any longer time then a yeere, without the Lords licence, it is likewiſe a forfeiture vnto the Lord. But of theſe and many other the like, we are to be guided according to the cuſtome of the Mannor, where ſuch tenants are. *Forfeiture.*

And generally theſe tenants, for that they haue no free-hold at the common Law, but by cuſtome, are termed tenants of baſe tenure. *Baſe tenure.*

And thus much concerning free-holds and eſtates of inheritance; and next of Chattels.

8. *Terme of yeeres.*

A Tenant for terme of yeeres, is he, vnto whom an eſtate is granted of lands, for any number of yeeres agreed vpon, betweene the Lord and Tenant; which terme is alwayes expreſſed in the leaſe ſo granted.

On which leaſe there is vſually reſerued ſome annuall rent, payable either halfe yeerely or quarterly, according to their contract. For the recouerie and obtayning of which rent, if it happen to be ariere and vn-paid, the leaſor is at his choyce; whether he will enter and diſtrayne, or bring his action at the common Law for the ſame. *Rent reſerued.* *Diſtreſſe or action of debt.*

And in theſe leaſes for terme of yeeres, whether by writing or otherwiſe, there neede no liuery of ſeiſon, but the tenant may immediately enter by vertue of his leaſe without further ceremonie. But in leaſes for terme of life or liues, it is otherwiſe. *No liuery of ſeiſin.*

Alſo if this tenant commit or ſuffer waſte; the leaſor may bring his action of waſte againſt him; wherein he ſhall recouer *locum vaſtatum*, and his treble damages. *Waſte.*

And if this tenant ſhall grant vnto any other man a greater or larger eſtate

in thoſe lands he holdeth, then he hath therein himſelfe; whereby hee conueyeth the fee-ſimple to himſelfe, hee ſhall forfeit his leaſe and the ſtate, and terme of yeeres therein granted.

9. *Tenant at will.*

A Tenant at will, is he, vnto whom lands or tenements are granted, to hold at the will of the Lord or leaſor, by whom they are granted. And this tenant may bee diſplaced or put out at any time, without further notice, at the Lords pleaſure; yea, although he hath tilled and ſowne his grounds. Yet in this caſe the law alloweth him free libertie of ingreſſe, egreſſe, and regreſſe, aſwell to take, cut, and carry away his corne when it is ripe; as to take and carry away his goods, and houſe-hold-ſtuffe, within conuenient time; without puniſhment of committing treſpaſſe, or otherwiſe; for that he knew not his Lords intention or time of entrance. But with tenant for terme of yeeres it is otherwiſe.

Diſplaced at pleaſure.

Libertie to take his corne and goods.

Like remedie for rent.

And the Lord or leaſor here hath the like remedie againſt this his tenant at will, for his rent, if it be behinde and vn-paid, as he hath againſt the tenant for terme of yeeres, laſt before mentioned.

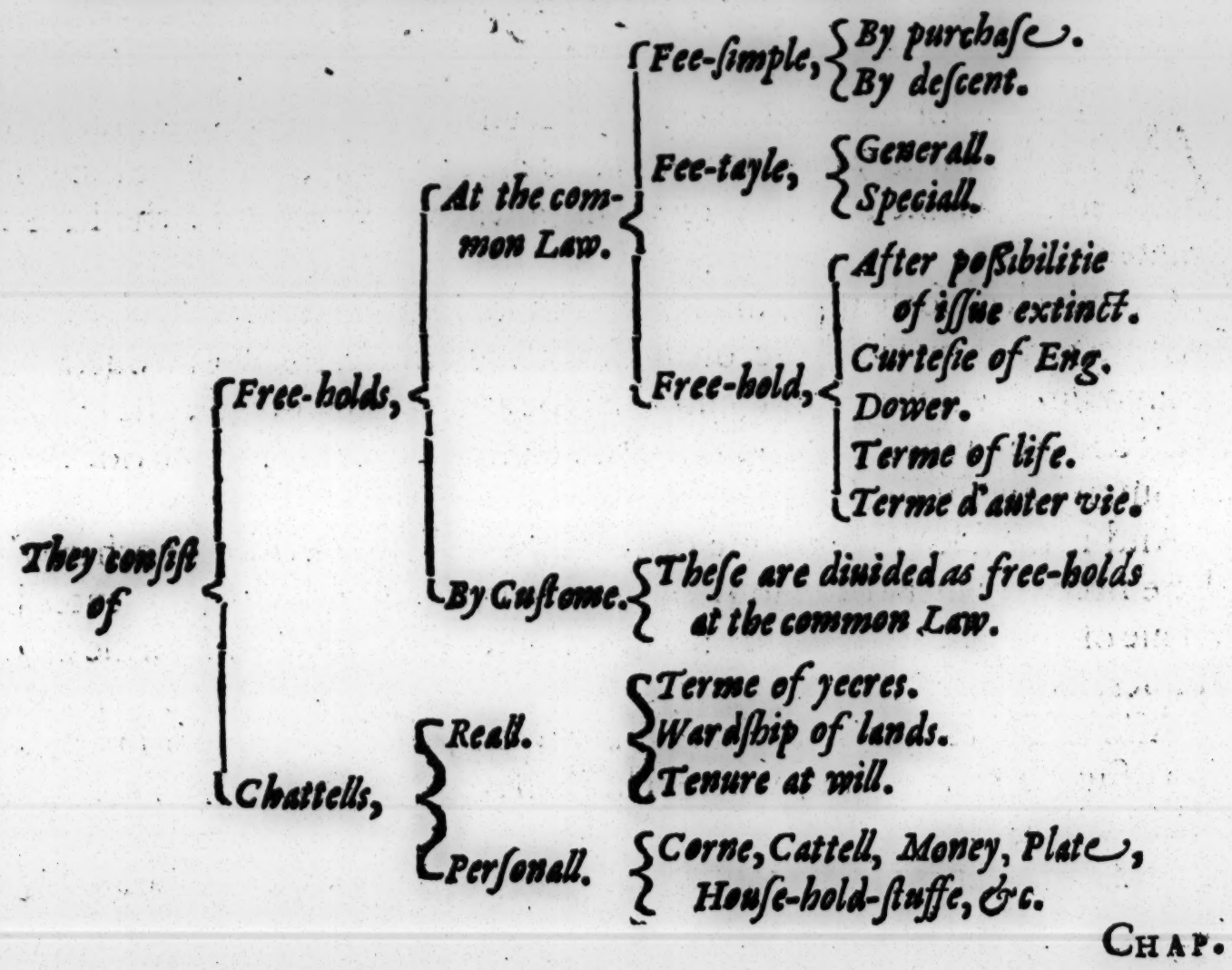
No action of Waſte.

And it is to be noted, that this tenant at will is not by the law charged or chargeable with reparations, as is the tenant for yeeres; and therefore no action of waſte lyeth againſt him; vnleſſe he wilfully committeth waſte, by pulling downe the buildings, or felling of timber, &c. In which caſe it is held, that the leaſor may bring his action of treſpaſſe, and recouer his loſſe ſuſtayned.

Treſpaſſe.

And thus much briefly concerning eſtates in land; whereof the two later kindes are termed Chattells reall; *and all moueable goods are called* Chattels perſonall; *as appeareth by this breuiate following.*

An ANALYSIS or briefe reſolution of all eſtates in generall.



CHAP.

CHAP. IIII.

Of the diuerſitie of Tenures, and their ſeuerall natures, with the ſeruices belonging.

HAuing informed our ſelues, as before, of the diuerſitie of eſtates (for all lands whatſoeuer, conſiſt of ſome of thoſe, formerly mentioned) let vs here, next, conſider of the ſeuerall tenures and ſeruices thereunto particularly belonging; which are maine and principall obſeruations to be had and vſed in ſuruey of a Mannor, and moſt meet and neceſſarie for a Surueyor to know and vnderſtand. Whereof in order as followeth.

1. *Knights ſeruice.*

THis tenure of Knights ſeruice includeth Homage and Fealtie, and commonly Eſcuage; and whoſo holdeth any lands or tenements by this ſeruice, is bound by the Lawes of this our Realme, to doe vnto his Lord Homage and Fealtie; being a ſeruice of the greateſt humilitie and reſpect, that can be performed by a tenant vnto his Lord: and for the moſt part, he is to pay Eſcuage, called *Scutagium*, that is, ſeruice of ſhield; which is to be aſſeſſed by authoritie of Parliament, as ſhall be hereafter declared. *Homage.*

When the tenant ſhall doe homage to his Lord, the Lord ſhall ſit, and the tenant kneeling downe before him, on both his knees, and holding both his hands betweene his Lords hands, ſhall ſay thus: I become your man, from henceforth, of life and member, and of earthly honor, and to you will be faithfull and true, and faith to you ſhall beare for the lands I hold of you (ſauing my faith which I owe and beare vnto our Soueraigne Lord the King) and then the Lord ſo ſitting, ſhall kiſſe him. *How the tenant ſhall doe homage*

Fealtie is as much to ſay, as *Fidelitas*, or fidelitie. In doing whereof, the tenant ſhall lay his right hand on a booke, and ſay thus: I will be vnto you my Lord, faithfull and true, and faith to you ſhall beare, for the lands and tenements which I claime to hold of you, and duly ſhall doe and performe vnto you the cuſtomes and ſeruices which I ought to doe, at the termes aſſigned, as helpe me God; *and then he ſhall kiſſe the booke.* And it is to be noted, that *Homage* muſt be done vnto the Lord himſelfe perſonally; but *Fealtie* may be made to the Steward of the Court, or to the Baylife thereof. Alſo, tenant for terme of life ſhall doe *Fealtie*, but not *Homage*. *How fealtie is to be done.*

As concerning Eſcuage: hee that holdeth his lands by a whole fee of Knights ſeruice; when the King goes in perſon to the warres, he is bound to be with him by the ſpace of fortie dayes, ſufficiently appointed for the warres: and he that holdeth by the moitie of the fee of Knights ſeruice, is bound by his tenure to be with the King in ſuch ſort as before, by the ſpace of twentie daies; and ſo proportionally, according to the quantitie and rate of his tenure. *What dutie belongeth to Eſcuage.*

Parliament. And it is to be vnderſtood, that after the Kings returne from the warres, a Parliament is vſually called; by authoritie whereof, it is aſſeſſed, what and how much, in money, euerie man that holdeth by a whole fee of Knights ſeruice (who was neyther perſonally, nor any man for him, with the King) ſhall pay vnto the Lord of whome he holdeth his land, by Eſcuage: and according to ſuch aſſeſſement, euerie tenant ſhall pay to his immediate Lord, after the rate and proportion of his tenure: and this money thus aſſeſſed, is called *Scutage*, or *Eſcuage*; for which, the Lord vnto whom the ſame is due, may diſtraine for non-payment thereof.

What Eſcuage to be paid.

Cuſtome. Yet ſome tenants by cuſtome, are not otherwiſe bound, but to pay a moitie or third part of what ſhall be aſſeſſed, as aforeſaid.

Eſcuage certaine And in ſome places the cuſtome is, that whatſoeuer be aſſeſſed by Parliament, yet their Eſcuage is certaine, and they pay neither more nor leſſe, but ſuch a ſumme of money, as fiue ſhillings, or the like; and this is called *Eſcuage certaine*: and this tenure is Socage tenure, and not Knights ſeruice; the *Eſcuage* whereof is alwayes vncertaine, and ſo called. And this *Eſcuage vncertaine* (belonging alwayes to Knights ſeruice) draweth vnto it Ward, Marriage, and Reliefe, as hereafter appeareth.

2. *Ward, Marriage, and Reliefe.*

AS formerly appeareth, Knights ſeruice (the tenure laſt before mentioned) draweth vnto it *Ward*, *Marriage*, and *Reliefe*; and therefore I hold it fitting here next to treat thereof. Wherefore, firſt it is to be vnderſtood, that if a man hold any lands or tenements by this tenure, and dieth, his heire male being within the age of one and twentie yeares; the Lord of whome thoſe lands are holden, ſhall haue the *Ward*, that is, the cuſtodie and keeping of thoſe lands ſo holden of him, to his owne vſe and behoofe, without account, vntill the heire come to the full age of one and twentie yeares: for it is intended by the Law, that vntill he attaine to that age, he is not fit or able to performe ſuch ſeruice, as by this tenure is required.

Ward.

Marriage. And if at the time of the death of ſuch tenant the heire be vnmarried, the Lord ſhall not onely haue the wardſhip of his bodie and lands, but the beſtowing of him in marriage.

Heire female. And if a tenant by Knights ſeruice die, and leaue an heire female, being of the age of foureteene yeares, or vpwards; then the Lord ſhall not haue the ward of ſuch heire, neyther of her bodie nor lands; becauſe a woman of that age may haue a husband able to performe the ſeruices required by this tenure.

The full age of a woman.

But if ſuch an heire be vnder that age of foureteene yeares, and vnmarried at the time of her anceſtors death, then ſhall the Lord haue the wardſhip of her lands ſo holden of him, till ſhe attaine to the age of ſixteene yeares; by force of an act of Parliament, in the Statute of *Weſtminſter* 1. *Cap.* 12.

And note here the great difference betweene the ages of Males and Females:

males: for the Female hath theſe ſeuerall ages appointed vnto her by the Law. Firſt, at ſeuen yeeres of age the Lord her father may diſtrayne his tenants for aide to marry her. Secondly, at nine yeeres of age ſhee is dowable. Thirdly, at twelue yeeres of age ſhee is held able to aſſent to matrimonie: Fourthly, at foureteene ſhe is able to haue her land, and ſhall be out of ward, (if ſhee be of this age at the death of her anceſtor.) Fiftly, at ſixteene ſhee ſhall be out of Ward, though ſhee were vnder foureteene yeeres of age at the death of her anceſtor: and ſixtly, at one and twentie yeeres ſhee is able to make alienations of her lands or tenements. But the Law limiteth to the Male, only two ages, that is, at fourteene yeeres to haue his lands holden in Socage: and at one and twentie to make alienations.

Diuerſitie of the ages of women.

Mens ages.

As concerning *Reliefe*, if a man hold his lands by Knights ſeruice and dyeth, his heire male being of the age of one and twentie yeeres, or his heire female of the age of fourteene yeeres, the Lord of whom ſuch lands are holden, ſhall haue *reliefe* of the heire.

Reliefe.

Alſo it is to be noted, that all Earles, Barons, or other the Kings tenants (holding of him in chiefe by Knights ſeruice) if they dye, their heire being of full age, as aforeſaid, they ought to pay the old reliefe for their inheritance, that is, the heire of an Earle for a whole Earledom an hundred pounds; the heire of a Baron for a whole Baronie a hundred markes; the heire of a Knight fiue pounds, and he that hath leſſe ſhall giue leſſe, according to the old cuſtome of Fees. And the like is to be vnderſtood and obſerued of all others, that hold ſuch lands immediately of any other Lord.

What reliefes are to be paid, and how.

And alſo a man may hold lands of a Lord by two Knights fees, and then the heire being of full age at the death of his anceſtor, ſhall pay to his Lord for reliefe ten pounds.

3. *Caſtle garde.*

IT is alſo to be vnderſtood, that a man may hold lands by Knights ſeruice, and not by eſcuage, nor pay eſcuage for the ſame: But hee may hold by *Caſtle garde*, which is to keepe a tower or ſome other place of his Lords Caſtle, on a reaſonable warning, when the Lord heareth of the approch of his Enemies.

Defined.

This is likewiſe Knights ſeruice, and draweth thereunto ward, marriage and reliefe, in all reſpects as the common Knights ſeruice doth, before mentioned.

4. *Grand Sergeantie.*

ALſo there is another kinde of tenure in Knights ſeruice, which is called *Grand Sergiantie*, and this is where a man hold any lands and tenements of the King, by ſome ſuch certayne ſeruices, as he ought to performe in proper perſon, as to beare the Kings Banner, or his Speare, or to conduct his Hoſt, or to be Sewer, Caruer, &c. and ſuch ſeruice is called *Grand Sergeantie*, that is, a high or great ſeruice, becauſe it is the moſt honorable and worthy ſeruice that is: for whoſo holdeth by eſcuage, is not tyed by

Defined.

by his tenure to performe any other more ſpeciall ſeruice, then another holding by eſcuage, but he that holdeth by *grand Sergeantie*, is tyed to performe ſome ſpeciall ſeruice to the King.

And if a man hold land of the King by *grand Sergeantie*, and dye; his heire being of full age, then ſhall his heire pay vnto the King, not only fiue pounds, as he that holdeth by eſcuage, but alſo the cleere yeerely value of ſuch lands, as he ſhall hold by *grand Sergeantie*.

Cornage. And alſo in the borders of *Scotland*, diuers hold their lands of the King by cornage, which is to blow a horne, to giue notice to the Countrey of the Enemies approch, which ſeruice is alſo a kinde of *grand Sergeantie*.

No tenure in grand ſergeantie, but of the King. And it is to be vnderſtood that none can hold by this tenure of *grand Sergeantie*, of any other Lord ſaue only of the King.

5. *Petie Sergeantie.*

Defined. PEtie Sergeantie is, where a man holdeth lands or tenements immediately of the King, and for his ſeruice in reſpect thereof, is bound to pay vnto the King yeerely, a Bow, a Speare, a Dagger, or ſome ſuch other ſmall thing belonging to the warre. And this ſeruice is in effect no other then *Socage*, becauſe the tenant is not tyed to performe any perſonall ſeruice, but to pay ſomewhat yeerely, as a rent is paid. Wherefore this ſeruice of *petie Sergeantie*, *No Knights ſeruice.* is no Knights ſeruice. Yet can it not be held of any other Lord ſaue the King only, aſwell as *grand Sergeantie*.

6. *Homage Anceſtrell.*

IF a man and his Anceſtors, whoſe heire he is, haue holden lands or tenements of another, and his anceſtors time out of minde, by homage, and haue done vnto him homage, this tenant thus holding, is called *tenant by homage anceſtrell*, by reaſon of the long continuance which hath beene by title of preſcription, aſwell concerning the tenancie in the bloud of the tenant, as concerning the Signiorie in the Lord.

This tenure extinct. And it is to be noted, that if this tenant by *homage anceſtrell*, ſhall at any time alien thoſe lands vnto another, although he immediately, or at any time after, purchaſe them againe, he ſhall no longer hold by this tenure, becauſe he hath diſcontinued, but ſhall from thenceforth hold it by the accuſtomed homage.

7. *Socage tenure.*

Defined. TEnure in Socage is, where a man holdeth lands or tenements of a Lord by certayne ſeruice, for all manner of ſeruices; ſo as the ſeruice be not Knights ſeruice: As where a man holdeth of his Lord by fealtie and certayne rent, for all manner of ſeruices; or elſe, where a man holdeth by homage, fealtie and certayne rent, for all manner of ſeruices; for homage by it ſelfe maketh not Knights ſeruice. Alſo a man may hold his lands by fealtie only, which is likewiſe *tenure in ſocage*. For euery tenure that is not tenure in Chiualrie, is *tenure in Socage*.

Theſe

Theſe tenants were tyed in ancient time euery of them with their ploughs by certayne dayes in the yeere, to plow and ſow their Lords demeanes, for which cauſe this tenure was called *Socagium*, or *ſeruicia ſocæ*, which is the ſame with *Caruca*, one Soke or one plow land. But now that ſeruice is by mutuall conſent, betweene the Lord and Tenant, in moſt places, conuerted to an annuall rent, yet the name of *Socage* ſtill remayneth. *Why ſo called.* *Conuerted to rent.*

Alſo if a man holdeth by eſcuage certayne, as is before ſpoken, he holdeth in effect but by ſocage. *Eſcuage certayne.*

And further, it is to be vnderſtood, that when *a tenant in ſocage* dyeth, the heire is to pay vnto the Lord, of whom thoſe lands are holden, a reliefe, that is to ſay, the value of one yeeres rent, beſides the yeerely rent, for the payment of which reliefe, the Lord may at his pleaſure immediately diſtraine. *Reliefe.*

8. *Franke Almoigne.*

THis tenant in *Franke Almoigne*, or free almes, is where an Eccleſiaſticall perſon holdeth lands of his Lord, in pure and perpetuall Almes, which tenure beganne in ancient time, thus: If a man being ſeiſed of certayne lands and tenements in his demeſne, as of Fee, ſhould thereof infeoffe an Abbot and his Couent, or a Prior and his Couent, or any other Eccleſiaſticall perſon, as a Deane of a Colledge, or Maſter of an Hoſpitall, or the like, to haue and to hold the ſame lands to them and their ſucceſſors for euer, in pure and perpetuall almes, or in franke almes, in theſe caſes the tenements ſhould be holden in *franke Almoigne*. By force of which tenure, thoſe tenants which hold lands thereby, were bound to make Oriſons and Prayers, and to doe other diuine ſeruices, for the ſoules of their Grantors and Feoffors, &c. and therefore diſcharged by the Law, to doe or performe any other profane or corporall ſeruice, as fealtie, or the like. *Defined.* *Seruice to be done.*

But it is now otherwiſe, ſince the Statute called *Quia emptores terrarum*, *An.* 18. ED. 1. So as now no man can hold in *franke Almoigne*, but by force of ſuch grants as were made before that Statute. *Otherwiſe ſince the ſtatute.*

9. *Burgage tenure.*

A Tenure in Burgage is, where an ancient Borough is, whereof the King is Lord, and they which haue tenements within the ſame Borough, hold of the King, by paying a certayne yeerely rent, which tenure in effect is but ſocage tenure. And the like is, where any other Lord ſpirituall or temporall is Lord of ſuch Borough. *Defined.*

And it is to be noted, that for the moſt part ſuch ancient Boroughs and Townes, haue diuers and ſundrie cuſtomes and vſages, which other Townes haue not. For ſome Boroughs haue a cuſtome that the yongeſt ſonne ſhall inherit before the eldeſt, which cuſtome is commonly called *Borough Engliſh*. And in ſome places the woman by the cuſtome of the Borough there, ſhall haue all ſuch lands and tenements in Dower, as her husband at any time during the couerture ſtood ſeiſed of. *Diuers cuſtomes.*

There are diuers other cuſtomes in *England*, which are contrarie to the courſe of the common law, which being probable and ſtanding with reaſon, are *Diuers cuſtomes contrary to the courſe of common Law.*

are good and effectuall, notwithstanding they are against the common law.

No custome without prescription. But no customes are allowable, but those, as haue beene vsed by prescription, or time out of minde.

10. *Ancient Demesne.*

Defined. THere is likewise another tenure, called *ancient demesne*, and the tenants who hold by this seruice, are Free-holders by Charter, and not by copie of Court roll, or by the verge after the custome of the Mannor, at the will of the Lord.

And these are such tenants as hold of those Mannors, which were Saint EDWARDS the King, or which were in the hands of King WILLIAM the Conqueror, which Mannors are called *the ancient Demesnes of the King, or the ancient demesnes of the Crowne of England.*

Quit of toll. And to such tenants as hold of those Mannors, the Law granteth many large priuileges and liberties, as to be quit of toll and passage, and such like impositions, vsually demanded and paid of and by other men, for their goods and cattell, bought and sold in Fayres and Markets by them; also to *Free of taxe.* be quit and free of taxe and tallage granted by Parliament, except it please the King to taxe ancient demeanes, when he thinketh fit, for great and vrgent occasions. And diuers other priuileges are belonging to this tenure, wherein I refer you to our Lawes.

And if such tenant be at any time distrayned, to doe and performe vnto their Lord any such other seruice or dutie, which they or their ancestors haue not beene accustomed to doe, they shall sue out a Writ, called a *Monstrauerunt*, directed to the Lord, commanding him that he distrayne them not to doe other seruices or customes, then they haue beene accustomed to doe. *A writ of Monstrauerunt.*

Doomes-day booke. And it is further to be vnderstood, that in the Exchequer there is a booke remayning, called *Doomes-day-booke*, which booke was made in the time of S. EDWARD *the King*, and all those lands which were in the seisin and in the hands of the said S. EDW. at the time of the making of the said booke, are ancient demesnes.

And thus much concerning the diuersitie of tenures and seruices. Now next let vs consider of the rents thereon vsually reserued, and the seuerall kindes thereof.

CHAP. V.

Of Rents, and their seuerall natures.

COnsidering that on euery tenure there is vsually some rent or other reserued: I hold it not vnfitting to say somewhat here concerning the same. And first, it is to be vnderstood, that as there are diuersitie of tenures, so likewise of rents; as one sort which is called a *Rent seruice*, another *Rent charge*, and a third *Rent seck*, or *Redditus siccus*, a drie rent. *Diuersitie of Rents.*

Rent seruice. As concerning *rent seruice*, it fitly hath the name, for that it is vsually tyed

tyed and knit to the tenure; and is, as it were, a ſeruice, whereby a man holdeth his lands or tenements, or at leaſt, when the rents are vnſeparably coupled and knit with the ſeruice. As for example; where the tenant holdeth his land of the Lord by fealtie and certaine rent, or by homage, fealtie, and by certaine rent, or by any other kind of ſeruice and certaine rent, this rent is called *rent ſeruice*.

And here is to be noted, that if at any time this *rent ſeruice* be behind and vnpaid, the Lord of whome the lands or tenements are holden, whether in fee ſimple, fee tayle, for terme of life, for yeares, or at will, may of common right enter and diſtraine for the rent, though there be no mention at all, nor clauſe of Diſtreſſe put in the Deed or Leaſe. *Diſtreſſe of common right.*

The nature of this *rent ſeruice* I ſay is to be coupled and knit to the tenure; and therefore, where no tenure is, there can be no *rent ſeruice*: wherefore, if at this day I be ſeiſed of lands or tenements in fee ſimple, and make a Deed of Feoffement thereof vnto another in fee ſimple, and reſerue by the ſame Deed a rent, this can be called no *rent ſeruice*; for that there can be now no tenure betweene the Feoffor and Feoffee. But it is otherwiſe of Feoffements made before the Statute of *Quia emptores terrarum*, formerly mentioned. For before the making of that Statute, if any man had made a Feoffement in fee ſimple, and had reſerued thereon vnto himſelfe a certaine rent, although it had beene without Deed, here had beene created a new tenure betweene the Feoffor and Feoffee, and the Feoffee muſt haue holden of the Feoffor, who by meanes thereof, might of common right haue diſtrained for ſuch rent: but ſince the time of that Statute, there can be no ſuch holding or tenure created or begun; and conſequently, no *rent ſeruice* can at this day be reſerued vpon any gift in fee ſimple; except in the Kings caſe; who being chiefe Lord of all, may, and euer might, giue lands to be holden of him. Anno 18. Ed. 1. *No rent ſeruice can be now reſerued on gift in fee.*

Thus it is apparant, that at this day no ſubiect can reſerue any *rent ſeruice* vnto himſelfe, vnleſſe the reuerſion of thoſe lands, ſo by him graunted, be ſtill in himſelfe: as where hee graunteth them in fee tayle, or maketh but a leaſe for terme of life, or for yeares, or elſe at will; for in all theſe caſes, the reuerſion of the fee ſimple remaineth ſtill in him: wherefore, if any rent be here reſerued, it is to be called *a rent ſeruice*; and of common right is diſtrainable, although there be no clauſe of Diſtreſſe compriſed in the Deed or Leaſe.

And if a man ſhall abſolutely and wholly graunt away in fee ſimple anie lands or tenements by him ſo holden, leauing no reuerſion thereof in himſelfe; and yet ſhall reſerue vnto himſelfe in his Graunt an annuall rent; with a clauſe of Diſtreſſe in his Deed indented, That it ſhall be lawfull for him to diſtraine for the ſame, if need require; this rent (in regard, that the land is therewith charged) is called *a rent charge*: But hee cannot diſtraine for this rent of common right, as before for the other, but onely by force and vertue of his Deed indented. *Rent charge.*

And if there be no ſuch clauſe of Diſtreſſe contained in the Deed, then is this rent ſo reſerued called *a rent ſeck*. *Rent ſeck.*

Alſo, if a man ſtanding ſeiſed of lands and tenements in fee ſimple, will graunt either by Indenture, or poll Deed, an yearely rent vnto another, iſſuing

ing out of the ſame lands, whether it be in fee ſimple, fee tayle, for terme of life, for yeares, or at will, with clauſe of Diſtreſſe; then this rent is called a *Rent-charge*, and he vnto whome this rent is graunted, may for default of payment enter and diſtraine.

And it is further to be vnderſtood, that if a man make a leaſe vnto another for terme of life, and reſerue thereon vnto himſelfe an yearely rent, and afterwards graunteth that rent vnto A.B. reſeruing the reuerſion of the lands vnto himſelfe; this rent is but a *Rent ſeck*: for that A. B. who hath the rent, hath nothing in reuerſion of the land.

Rent ſeck.

And if a man giueth lands and tenements in tayle, and reſerue to him and his heires a certaine rent; or elſe make a leaſe for terme of life, reſeruing certaine rent; if he graunt the reuerſion to another, and the tenant attorne accordingly, the whole rent and ſeruice ſhall paſſe by this word reuerſion, becauſe the rent and ſeruice in ſuch caſe be incident to the reuerſion, and paſſe by the graunt of the reuerſion; and here is the rent a *Rent-charge*. But if hee had graunted the rent onely, it had beene then a *Rent ſeck*.

Rent charge.

CHAP. VI.

Of Repriſes and Deductions.

AS we haue formerly vnderſtood, what ſeuerall rents, profits, and commodities may yearely ariſe or grow out of any Mannor to the Lord thereof; ſo is it as fitting to conſider, what Repriſes, Deductions, Payments, Charges, and Duties, may be yearely iſſuing or going out of any Mannor from the Lord thereof: For otherwiſe, in the concluſion of our Suruey, or in making a perfect Conſtat, or Particular, (ſuch duties not being repriſed) the true value of the Mannor may oftentimes ſeeme greater then in truth it is; which would tend much to our ſhame and diſcredit.

Theſe *Repriſes* and *Deductions* are neuer certaine, or in all Mannors alike; but in this more, and in that leſſe: yet in one and the ſame Mannor they are commonly the ſame, and vſually ſuch as theſe here following.

Repriſes are any manner of Rents, either in Money, Capons, Hennes, Pepper, Cummin ſeed, or the like, iſſuing and payd out of one Mannor to another: alſo Suits of Court, or annuall fines for the ſame; and the like may be iſſuing and payable to a Sherifes Turne or Hundred; alſo Penſions or Portions to Eccleſiaſticall liuings: likewiſe a rent may be iſſuing for way-leaue, or ſome particular Paſſage; alſo for Water-courſes, or placing of Pipes for conueyance of Water: likewiſe yearely Fees to Officers, as Stewards, Receiuers, Baylifes, Collectors, Keepers, &c. and alſo ſtypends, ſalaries, or annuities to Chaplins, or the like: All which are euer to be deducted and repriſed out of the totall value of a Mannor.

And hauing thus furniſhed and informed our ſelues, firſt of the Mathematicall *part of Surueay, by the three former Bookes; and thus farre of the* Legall, *as to know*

know what a Mannor is, and the ſeuerall parts thereof; and likewiſe of all eſtates in generall; and what Tenures, Seruices, and Rents are thereunto incident, appertaining, and belonging; let vs proceed in an orderly and formall courſe; ſuppoſing we are now to vndertake the ſuruey of a Mannor, which is to be performed as followeth.

CHAP. VII.

Obſeruations and courſes to be held and taken, before the beginning of a Suruey.

IT is firſt to be conſidered, for whome the buſineſſe we vndertake, is to be performed: if for the King, then are we to obtaine Commiſſion from his Maieſtie out of ſuch Court or Courts as is requiſite, according to the tenure of the lands to be ſurueyed, as the Exchequer, Duchie, &c. In declaring the forme of which Commiſſions, I need not ſpend time, for that they are vſuall, and of ordinarie courſe (in ſuch caſes) graunted out of thoſe Courts. Yet ſeeing, that for the moſt part, thoſe Commiſſions giue power to the Surueyor, by reference to certaine articles annexed; and according to the efficacie and force thereof, the power and authoritie of the Surueyor is limitted; it behooueth to haue thoſe articles as ample, full, & forcible, as you may deuiſe; not knowing with what people you are to deale (who often proue obſtinate) nor the nature, eſtate, or condition of Tenancie (for the moſt part variable.) Which articles let be theſe here following, or the like in effect.

Articles to be inquired of, and courſes to be obſerued and held by A. R. in this preſent Commiſſion named, for the better effecting and execution of his Maieſties ſeruice, in ſurueying of his Highneſſe Honors, Lordſhips, or Mannors of A. and B. in the Countie of C. and of all Caſtles, Houſes, Parkes, Meſuages, Lands, Tenements, and Hereditaments thereunto belonging and appertaining.

1. FIrſt, the ſaid A.R. is to enter into the ſaid Honors, Lordſhips, and Mannors, and all and ſingular other the premiſſes, and euery of them, and into euery part and parcell of them, and euery of them, and to make a ſuruey of the quantitie, qualitie, and yearely value thereof, and of euery part, parcell, and member thereof reſpectiuely.

2. Alſo the ſaid A.R. is to call before him all ſuch as now are or formerly haue bin Stewards, Baylifes, Reeues, or Collectors of all or any his Maieſties iſſues, rents, reuenewes, and profits within the premiſſes, and their deputies, and euery or any of

them, and to charge them on their oaths to deliuer in vnto him true and perfect Rentalls of all and euery their ſeuerall collections: and likewiſe to call before him all and euery ſuch perſon and perſons, as haue or are ſuſpected to haue any Euidences, Court Rolls, Rentalls, Bookes of Suruey, Couchars, Terrars, Eſcripts, Writings, or Mynuments whatſoeuer, touching or concerning the ſaid Mannors, Meſſuages, Lands, Tenements, and Hereditaments, and euery or any of them; and all and euery ſuch perſon and perſons to examine vpon their oathes concerning the ſame writings, and euery of them: and alſo to demand, require, and receiue of them, all and euery ſuch Bookes, Rentalls, and other Writings, as he ſhall ſo find to be in their hands or cuſtodie: And if any ſhall make refuſall of the deliuerie thereof, to certifie his or their name and names, and the reaſon of ſuch his or their refuſall, to the Lord Treaſurer of England, and Chancellor of the Exchequer, that ſpeedie and due courſes may be therein held and taken accordingly. But this is to be vnderſtood of Bookes and Writings not being in the hands of the preſent Steward or Stewards of any of the premiſſes, nor in the cuſtodie of any of his Maieſties Officers of his Highneſſe Courts of Record at Weſtminſter; whereof he is only to take and extract notes, for his better inſtruction and information concerning the premiſſes.

3. Likewiſe he is to inquire, what are the ſeuerall limits, butts, and bounds of all and ſinguler the premiſſes, and to expreſſe the ſame accordingly; and what Lord or Lords are conioyning or boundering thereon; & whether they or any of them haue or do intrude or incroach vpon or within the limits or boũds aforeſaid, or the liberties or priuiledges compriſed within the ſame.

4. Alſo, whether the premiſſes, or any part thereof, doth lye or extend into any other Mannor; and whether any other Mannors, Meſſuages, Lands, or Tenements do lye within the limits or bounds of the premiſſes; whoſe, and what they are; and to make perfect diſtinctions thereof particularly.

5. What Caſtles, and other Mannor or Manſion houſes his Maieſtie hath within the ſame; in what eſtate of reparations the ſame now are and be; and if decayed or waſted, by whome the ſame hath bin committed, & to what value; what demeſne lands now are, or heretofore haue bin, belonging or appertaining to the ſaid Houſes, and in whoſe tenure and occupation the ſame

now

now are; by what right or title they clayme or challenge to hold; what ſeuerall rents they pay in reſpect thereof; and what is the true quantitie, qualitie, and yearely value of the premiſſes.

6. What Forreſts, Parks, and Chaſes his Maieſtie hath within the premiſſes; what number and ſtore of Game are in them; what Officers are thereunto belonging; what Fees they receiue in reſpect thereof; in what eſtate of reparations, the Houſes, Lodges, Walls, Pales, and Fences are; what is the quantitie, qualitie, and yearely value thereof by the acre; what Iuiſtments, or what Cattell, as Oxen, Kyne, Horſes, or the like, are vſually depaſtured within the ſame; who hath the diſpoſall thereof; and what is the value of a Beaſt-gate there.

7. Alſo, what Moores, Marſhes, Heaths, Waſts, or Sheepe-walkes, his Maieſtie hath of, in, vpon, or belonging to the premiſſes; what are the ſeuerall quantities thereof; how many Sheepe may be kept on thoſe walks; and what is a Sheepe-gate worth.

8. He is alſo to inquire, what Free-holders there are within and belonging to the premiſſes; what Mannors, Meſſuages, Lands, or Tenements they hold thereof, and what are their ſeuerall quantities; and likewiſe, by what ſeuerall tenures, rents, and ſeruices they hold the ſame.

9. Alſo, what other eſtates there are; as tenants for terme of life, or liues, yeares, or at will; what cuſtomarie or copy-hold tenants, or what other tenants there are within the premiſſes; what lands they do ſeuerally hold, and the true quantitie, qualitie, and yearely value thereof ſeuerally, and what yearely rents they pay for the ſame.

10. Alſo, what are the ſeuerall cuſtomes concerning the cuſtomarie tenants; whether their fines vpon death or alienation be certaine, or incertaine, and arbitrable; and if certaine, what Fines they vſually pay on euery death or alienation of Lord or Tenant; and how, and in what manner, doe thoſe cuſtomarie lands deſcend after the death of an anceſtor.

11. What Reliefes, Heriots, Fines, or other duties are payd, or anſwerable, vpon or after the death or alienation of any Free-holder, Copy-holder, or other tenant within the premiſſes; how and by whom are they vſually collected and diſpoſed of; and what may be the value thereof in *Communibus annis*.

12. Whether any cuſtomarie tenants (whoſe lands are hariotable) haue ſeuered, aliened, diuided, or diſmembred the ſame, who hath the vſe and occupation thereof, and what are the ſeuerall quantities, qualities, and yeerely values of the ſame.

13. Alſo, what are all and euery the cuſtomes in generall of, within, or belonging to the premiſſes, and how, by what meanes or for what cauſe, may a copy-holder or cuſtomarie tenant, forfeit or loſe his cuſtomarie eſtate.

14. What Commons there are, of, within, or belonging to the premiſſes, whether ſtinted or vnſtinted; if ſtinted, then how, by what meanes, and according to what rate and proportion, how many beaſt-gates they contayne, the value of each beaſt-gate, and the quantitie, qualitie, and value of the whole.

15. What arable fields and meddowes there are, which lie in common, what are their ſeuerall names, and of the ſeuerall furlongs and wents therein contayned; alſo how and in what manner they are kept and vſed; whether is it lawfull for any tenant at his pleaſure to incloſe any part thereof, without leaue of the Lord; how are they imploied when the corne and graſſe is taken away, how ſtinted, and what is the eatage therof worth by the beaſt-gate, or ſheepe-gate, after the corne and graſſe is ſo taken off, as aforeſaid.

16. What woods or wood-grounds his Maieſtie hath within the premiſſes, what grounds haue beene heretofore wood, and now conuerted to other vſes, how long ſince & by whom, what waſtes and ſpoiles haue beene had or made of his Maieſties woods, how long ſince, by whom, and of what value; whether may any profit by pannage be made or rayſed, by, or within the ſame woods, and what the profit or value thereof may yeerely be.

17. Alſo what tenants there are within the premiſſes, who demiſe or let any part or parcell of their lands or tenements vnto vnder-tenants, either for their whole terme or any part thereof, and what fines and rents haue or doe they receiue for the ſame.

18. Likewiſe, what lands, tenements, rents, ſeruices, or other profits, are concealed or detayned from his Maieſtie, how long ſince, by whom, and what the yeerely value thereof is.

19. What lands, tenements, leaſes, or other eſtates of, or in the

the premiſſes, haue beene or are eſcheated, or forfeited to his Maieſty, by whom, when, for what cauſe, and in whoſe occupation the ſame now are, and what is the value thereof.

20. What fines, iſſues, amerciaments, perquiſites of Court, heriots, waifs, ſtrayes, felons goods, and other caſualties doe yeerly accrue and grow vnto his Maieſtie out of the premiſſes, by whom the ſame is, or hath beene collected, gathered, and receiued, and what is and hath beene the value thereof yeerely in *communibus annis*.

21. Alſo what incloſures and Incrochments haue beene heretofore made of, in, or vpon any of his Maieſties commons, waſtes, or other grounds, how long ſince, by whom, what rents are paid for the ſame, and what the yeerely value thereof is.

22. What Corne-mills, Fulling-mills, or other mills, his Maieſtie hath within the premiſſes, who hold the ſame, what rents they pay, what is the yeerly value thereof, what cuſtomes are thereunto belonging, and in what eſtate of reparations are all and euery thoſe Mills.

23. What Markets and Fayres are there within the premiſſes, on what dayes kept, what tolls are belonging to the ſame, by whom the ſame is collected and receiued, and what yeerely profit ariſeth thereby vnto his Maieſtie.

24. Alſo what Warrens, Fiſhings, Fowlings, Hawking, Hunting, or other Royalties his Maieſtie hath within the premiſſes, by whom the ſame is occupied or enioyed, what rents are yeerely paid for the ſame, and what is the yeerely value thereof.

25. What quarries of ſtone, mines of Tinne, Lead, Cole, or other mines his Maieſtie hath within the premiſſes, who hath the vſe and occupation thereof, what rents they pay for the ſame, and what the yeerely value is.

26. What Moſſes of peate or turffe, what Broome, Heath, Furze or Flagge, are within the premiſſes, belonging to his Maieſtie, what are the rents and yeerely values thereof.

27. What Aduowſons, Patronages, Free-gifts or preſentations of Parſonages, Vicarages, Chappells or Prebends, or what Impropriations, are appendant or belonging to the premiſſes, who is or are the preſent Incumbent or Incumbents, who hath the vſe of ſuch Impropriation, what rent is paid for the ſame, and what is the yeerely value thereof.

28. Whether any Tenant or other perſon or perſons whatſoeuer haue plowed vp, caſt downe, remoued or taken away any meere-ſtone, baulke, hedge-row, or land-ſhare, betweene the demeanes of the premiſſes, and any other meſſuages, lands, or tenements, or between any the freeholds, and the tenement or cuſtomary lands, or betweene any of the premiſſes, and the lands of other Lords, by whom ſuch offence was committed, and where, and in what place and places thoſe altered bounders ought to ſtand and remaine.

29. Alſo what Officers his Maieſtie hath within the premiſſes, what fees doe they yeerely receiue in reſpect thereof, what rents, deductions, repriſes, or other payments or ſummes of money, are yeerly paid, repriſed, or iſſuing out of his Maieſties reuennues of the premiſſes, and to whom, for what cauſe, and to what end and purpoſe are the ſame ſo paid.

30. And laſtly, the ſaid A. R. is to make all and euery other ſuch further and other inquirie and inquiſition of, for, and concerning all and euery ſuch matters and things whatſoeuer, as in his diſcretion ſhall be held fit and requiſite, for the better effecting and execution of his Maieſties ſeruice, in ſurueying of the premiſſes.

Theſe Articles or the like, being drawne and faire written in Parchment by the Surueyor (the commiſſion being to be taken out of the Exchequer) a briefe warrant is to be directed to one of the Remembrancers, and written vnder the Articles to this effect.

M. I. O. Theſe are to will and require you immediately to cauſe a Commiſsion to bee made, and directed to A. R. for the ſuruey of his Maieſties Honors, Lordſhips, and Mannors of A. and B. in the Countie of C. and of all Caſtles, Houſes, Parkes, Meſſuages, Lands, Tenements, and Hereditaments, thereunto belonging or appertayning; wherunto is to be annexed the abouementioned Articles. Whereof fayle you not: and theſe ſhall be your warrant in this behalfe. From the Court, &c.

Which

Which Warrant is to be ſigned by the Lord Treaſurer, or Chancellor of the Exchequer, and deliuered to the Remembrancer accordingly.

But if the buſineſſe vnder-taken, be not for the King, but for a priuate man, then in regard that a Surueyor hath no power by any authoritie of Surueyorſhip, to be granted vnto him by any ſuch priuate man, to miniſter an oath, or performe ſuch other duties as are requiſite, it is fitting either that the Steward of the Mannor, which is to be ſurueyed, ioyne with him, in calling a Court Baron, and Court of Suruey, to be there held (wherein the Steward is to giue the charge and to deliuer Articles, and miniſter oathes, alwell concerning the Court Baron as Court of Suruey.) Or otherwiſe, the Surueyor is to haue a commiſſion, grant, or deputation from the Lord of the Mannor vnder his hand and ſeale, of the office of Steward and Surueyor of his Mannors, Lands, and Tenements, for a certayne terme, or during pleaſure: And then may the Surueyor, of himſelfe execute all thoſe offices and duties fit and requiſite for a Steward and Surueyor, to doe and performe. Which commiſſion, grant, or deputation, let be thus, or to the like effect.

OMnibus ad quos hoc præſens ſcriptum peruenerit A. B. de C. Comit. E. Armig. ſalutem. Sciatis me præfat. A. B. tam pro ſincero amore & beneuolentia qua iamdudum affectus ſum, erga A. R. de cuius prouida circumſpectione, pia ſedulitate, ac ſingulari in hac parte prudentia merito plurimum confido, quam pro diuerſis alijs cauſis & conſiderationibus, ex mera & ſponta-
nea voluntate mea dediſſe & per præſentes conceſsiſſe eidem A. R. Officium Seneſchall. ſiue Seneſchalciam omnium & ſingulorum Domin. Maner. & hæreditament. meorum quorumcunq; in Comit. F. & cuſtod. ſiue officium tenendi omnes & omnimodas Cur. Baron. Letar. Viſ. franc. pleg. Dominior. & Maner. prædict. & eorum cuiuſlibet, ac gubernationem & ſuperuiſionem eorundem. Ac ipſum A. R. generalem ac capital. Seneſchall. ac Superuiſorem meum omnium Curiarum, Dominiorum, Manerorum, & hæreditament. meorum prædict. facio, conſtituo, & ordino per præſentes. Habend. tenend. gaudend. exercend. & occupand. Officia prædict. cum pertinentijs, à dat. præſentium durante bene placito meo. Mando inſuper vniuerſis & ſingulis Balliuis, Præpoſit. Firmarijs, tenentibus & occupatoribus meis præmiſſor. & eorum cuilibet, quod præfat. A. R. de tempore in tempus, aſsiſtentes ſint, obedientes, & auxiliantes in omnibus prout decet durant. termin. præd. In cuius rei teſtimonium huic præſenti ſcripto meo ſigillum meum appoſui. Dat. &c.

Or to the ſame purpoſe in Engliſh.

And being thus authoriſed, we may now proceede.

CHAP.

CHAP. VIII.

What courſes are firſt to be held in the beginning of a Suruey.

CONſidering how precious time is, and withall, how chargeable theſe imployments are to thoſe whome it concerne; it behoueth a Surueyor (reſpecting his credit and reputation) ſo to appoint and diſpoſe of his buſineſſe in an orderly courſe, as no time be idly loſt, or vainely ſpent therein. Wherefore, firſt let the Baylife of the Mannor be called, and a Warrant or Precept directed and deliuered vnto him, to ſummon as well a Court Baron (if need require) as a Court of Suruey; to this, or the like effect.

Branton. *A.R. Seneſchall. & Superuiſ. Manerij præd. Baliuo eiuſdem, ſalutem: Tibi præcipio pariter & mando, quod diligenter præmonere facias omnes tenentes infra Maner. prædictum, tam reſidentes quam non reſidentes, atq; omnes tenentes cuſtumarios Manerij prædicti, quod ſint coram me in hac parte ſufficienter deputato apud Branton prædictam, die Lunæ ſecundo die Septembr. proximè futuro poſt datum huius, Non ſolum ad faciend. ſectam ſuam ad Curiam Baron. & Superuiſ. ſed etiam ad producend. & oſtendend. omnes Literas, Chartas, Inſtrumenta, Indentur. copias Cur. Rotul. ac al. euidenc. vnde tenere vendicant ſeperal. terr. & tenement. ſuas de Manerio prædicto; & omnia alia quæ eis incumbent, & pertinebunt; & hæc nullatenus omittas, & habeas ibi hoc præceptum: Datum ſub ſigillo meo vicesimo quarto die Auguſti, Anno Regni &c.*

Or to the like effect in Engliſh.

Wherein let a conuenient time be limited, as fiue or ſixe daies at the leaſt after notice giuen for the tenants apparance, that they may the better prepare themſelues, and be the more inexcuſable, if they happen to make default.

Then are you to receiue from the Baylife all ſuch Rentalls as he hath concerning his whole collection, as well ſuch as are ancient, and of former times, as thoſe of his laſt collection; which you are diligently to compare together, noting the difference: and if the later be leſſer, then what decayes of rent there are, and how occaſioned; if greater, then what increment of rent, and whereon rayſed; which you are carefully to note and expreſſe, when you come to ingroſſe your Rentall.

Next would I haue you to reduce your Rentall to an Alphabeticall forme: wherein, vſe all the moderne Tenants names; not omitting the ancient; which will be a great helpe for the ſpeedie diſpatch of your entries; and the readie finding of any Tenants name, or rent, as you are to vſe them: which would be written thus, or in the like manner.

Atkinſon Thomas, late *Brownes*,——xx$^{s.}$
Armſtrong William, late *Tomlinſons*,——v$^{s.}$
Bennet Iohn, late *Brights*,————xv$^{s.}$
Branthwait Edward, late *Finches.*—xxij$^{s.}$

And in this ſort proceed Alphabetically with all the whole Rentall; which is much availeable, where manie Tenants are.

And hauing thus prepared your Rentall in a readineſſe, againſt you haue occaſion to vſe it; you are to ſpend the reſidue of the time, vntill the day appointed for your Court, in conferring with the Steward concerning the preſent eſtate of the Mannor; and in diligent view and ſearch of the Lords Euidences and Court Rolls; taking them orderly before you, and from yeare to yeare briefely to expreſſe in a Booke, for that purpoſe prouided, the ſeuerall Cuſtomes, Eſtates, Tenures, Rents, and Seruices, and all other remarkable things. So ſhall you be able fully to informe your ſelfe of the nature, qualitie, eſtate, and condition thereof; and to vnderſtand, what articles are now moſt fit and apt to be preſented vnto your Homage, or Iurie, to be inquired of, when you haue giuen them their charge; which you haue now alſo fit opportunitie to write in a readineſſe for them againſt that time. And theſe articles would I haue to be ſuch as are expreſſed in the laſt Chapter, or ſo manie thereof as you hold fitting and neceſſarie for the purpoſe; and (if need require) to adde and inſert ſuch others thereunto, as you ſhall find meet and requiſite; for as much as by the laſt article annexed to your Commiſſion, you haue power and authoritie to make ſuch further and other inquiſition, as in your diſcretion ſhall ſeeme fitting.

Alſo, now haue you conuenient time, either to ride or walke abroad, and to take a reſpectiue view of the ſituation and extent of the Mannor; whereby you ſhall be able to informe your ſelfe, where, how, and in what ſort you may with moſt conueniencie begin, continue, diſpoſe of, and performe your Inſtrumentall menſuration, either by your ſelfe or ſeruants, whome you imploy therein: wherein, for manie reſpects, I would haue nothing done or performed, before the firſt day of your Court be paſt; when as you haue read and made knowne your Commiſſion, and ſettled an orderly courſe with the tenants, for their attendance, aide, and aſſiſtance in that behalfe.

And thus, and in this like manner, may you ſpend the time to good purpoſe, till your Court day come.

CHAP.

CHAP. IX.

The order of keeping a Court of Suruey.

IF a Court Baron be kept with your Court of Suruey, as is euer moſt fitting, then are you firſt to enter the ſtile of the Court in this manner.

Branton. *CVria Baronis & Superuiſ. A. B. Armiger, ibidem tent. die Lunæ, videlicet ſecundo die Septem. Anno Regni* IACOBI, *Dei gratia Angliæ, Franciæ, & Hyberniæ Regis fidei defenſ. &c.* xiiij. *& Scotiæ* 50. *tent. per A. R. Seneſchallum & Superuiſ.*

After the ſtile of the Court thus entred, you ſhall cauſe the Bayliffe, who ſerueth the Court, to make Proclamation by crying once *Oyes*, and then ſhall you will him to ſay, thus; All manner of perſons, who were ſummoned to appeare here this day, to ſerue the Lord of the Mannor, for his Court now holden, draw neere and giue your attendance, and euery one anſwere to his name, as he ſhall be called, vpon the payne and perill that may fall thereon.

Then by your Rentall, call them all ſeuerally by their names, marking thoſe which are abſent to be amerced. Which done, cauſe the Bayliffe to make another *Oyes*, and willing them to draw neere, and keepe ſilence whilſt the Commiſſion be read; let the ſame be read vnto them, and likewiſe the Articles thereunto annexed, if it be for the King.

Then out of thoſe tenants which are preſent, make choiſe of the moſt ſufficient for your Iurie, wherein your beſt courſe is, formerly to informe your ſelfe, and to take ſpeciall notice, who are moſt fitting for your purpoſe, and to haue their names readie written in a paper by themſelues, which you may now thereby call accordingly. But being for the King, you haue alwayes a writ of aſſiſtance directed to the Sheriffe of the ſhire, requiring him to returne you a ſufficient Iurie: yet may you without him by vertue of your Commiſſion, impannell any Iurie at your owne pleaſure. Then direct the Fore-man of the Iurie, to lay his hand on the Booke, and ſweare him as followeth, or to the like purpoſe.

YOu ſhall diligently inquire and make true preſentment of all ſuch matters, as on the Lords behalfe of this Manner, ſhall be giuen you in charge, you ſhall neither for fauour, feare, affection, or other parciall reſpect whatſoeuer, forbeare to preſent what you ought to finde, or finde what you ought not to preſent, you ſhall herein keepe the Lords counſell, your owne, and your fellowes, and in all things according to a ſincere and vpright conſcience, you ſhall preſent the truth, the whole truth, and nothing

thing but the truth, as by euidence and your owne knowledge you ſhall be induced, to the beſt of your power, ſo helpe you God, and by the contents of the Booke, which he is to kiſſe.

And after the Fore-man is thus ſworne by himſelfe, then cauſe three or foure of the reſt of the Iurie, to lay their right hands together on the Booke, and giue them their oath, as followeth.

THe ſame oath which A. B. your Fore-man before you, for his part, hath made and taken, you and euery of you, for your parts ſhall truly keepe and performe to the vttermoſt of your powers, ſo helpe you God. And cauſe them ſeuerally to kiſſe the Booke.

And in like manner ſweare all the reſt. And all being ſworne, cauſe the Bayliffe to number them, as you reade their names. Then cauſe him alſo to make the third Proclamation, and ſay thus: All you that be here ſworne, draw neere and heare your charge, and all the reſt keepe ſilence.

Then make your exhortation, and deliuer the charge of a Court Baron, after the vſuall manner. Which being finiſhed, you are to addreſſe your ſpeech vnto them, concerning the preſent buſineſſe of Suruey, as occaſion ſhall be offered, whereof to preſcribe you any forme or preſident, were to little purpoſe, ſeeing it is to be framed and directed to ſuch ends and purpoſes, as the preſent cauſe requires, which you ſhall alwayes finde different and variable, and therefore I refer the ſame to your owne diſcretion, deeming you now able and fitting ſufficiently to performe the ſame in any kinde.

And then deliuer vnto them the Articles which you haue readie drawne, according to the directions of the laſt Chapter, which is their charge concerning the buſineſſe of Suruey, relating vnto them, that as they receiue theſe Articles (whereof they are to inquire) in writing, ſo are they to anſwere the ſame in writing vnder their hands and ſeales particularly by a day, now to be limitted and appointed, which for many reaſons is moſt fitting to be, about the time of your concluding the buſineſſe. Which day is to be expreſſed vnder their Articles, and your name ſubſcribed thereunto.

And now are you to take order, and giue ſpeciall directions vnto all the tenants for their attendance, aide, and aſſiſtance, in your inſtrumentall menſuration, appointing them by turnes, how, when, and where you are to vſe their helpe and aſſiſtance, wherein you are to deale with ſuch diſcretion, as you neither faile of their helpe, when occaſion ſerueth, nor oppreſſe them with grieuance by their ouer-much attendance.

And hauing thus farre proceeded, the reſt of this day may be ſpent in entering their ſeuerall deedes, euidences, and eſtates, in manner as ſhall be hereafter declared.

But before you diſcharge the tenants, you are to conſider (according to the number of them) in what time or how many dayes, you ſhall be able to enter their eſtates, and if they conſiſt of diuers Townеſhips, as large and ſpacious Lordſhips vſually doe; then your beſt courſe is to appoint them ſeue-

rall

rall dayes for their attendance, and bringing in of their euidence by ſeuerall Townеſhips, for it would be no leſſe troubleſome to your ſelfe, then diſtaſtefull to the tenants, to require their generall and daily attendance vntill the buſineſſe were wholly finiſhed.

And now may you aiourne the Court vnto the next day, (or ſuch other time as you thinke fitting) by cauſing the Bayliffe to make proclamation to that purpoſe, and the like from time to time, till you haue ended your buſineſſe.

The next day you may beginne your menſuration in the fields, either by your ſelfe or thoſe whom you imploy to that purpoſe, according to the inſtructions of the third Booke. But it were fitting for your owne part to be imployed in entering of the tenants eſtates, vntill you haue finiſhed, or you may ſpend ſuch time therein, as when the weather is not fitting to ſtirre abroad, or in the mornings and euenings, as you ſhall finde meeteſt for your purpoſe.

CHAP. X.

The order and manner of entering the Tenants euidence, and their ſeuerall eſtates.

FIrſt, it is to be conſidered, that moſt Mannors (as is formerly ſpoken) conſiſt of diuers Townеſhips or particular parts, and the tenancie of thoſe Townеſhips of diuers eſtates, as Free-holds, Copy-holds, &c. Wherefore, I hold it fitting and an orderly courſe, that not only euery of thoſe Townеſhips, but the ſeuerall eſtates therein, be entered and taken ſeuerally and particularly by themſelues, that is, all of one and the ſame Townеſhip and eſtate vnder one and the ſame title, for auoyding of confuſion. As, ſuppoſe you are to ſuruey the Mannor of *Branton*, which conſiſteth of theſe ſeuerall Townеſhips or parts, *Branton*, *Bodley*, and *Sutton*, and within thoſe Townеſhips, are diuers tenants, holding their lands by ſeuerall eſtates, as Free-hold, Copy-hold, &c. Then would I haue you make your ſeuerall entries vnder thoſe ſeuerall titles whereunto they properly belong, as vnder the title of *Branton townеſhip Free-hold*, enter all thoſe which are of that Townеſhip, and of that nature: and vnder the title of *Branton townеſhip Copy-hold*, enter all the Copy-holds of that townеſhip, and the like of all the reſt.

And theſe entries I would haue made in looſe ſheets of paper at large, keeping them alwayes ſorted, according to the ſeuerall Townеſhips and eſtates, till you haue finiſhed all your entries, and then to file them together orderly in a Booke, each Townеſhip following other.

In which ſeuerall entries obſerue this courſe. hauing written your title as before, in the head or top of the ſheet, then enter the tenants name, and the very words of grant, as they are in his Deede, Copy, or Leaſe, which is to be written from the margent the whole bredth of the ſheet, leauing only towards the right hand a ſpace, wherein is to be expreſſed the rents, and ſeruices, and in the margent alwayes expreſſe the tenure.

And

And conſidering, that in few or no Deedes, Euidences, Copies, or Leaſes, the lands are particularly expreſſed by particular names, cloſes, and quantities, as the tenant now holdeth the ſame, and as you ſhall find them in your inſtrumentall menſuration; hauing entred the effect of the Deed, Copie, or Leaſe, according to the purport thereof: it is fitting to queſtion with the tenant, what ſeuerall parcels he holdeth; for and vnder the ſeruices and rents contained in euerie Deed, Copie, or Leaſe particularly; as, what Meadow, what Arable, what Paſture, and their ſeuerall names and quantities, as he eſteemeth them; and if he know not what acres they containe (as moſt tenants will ſeeme ignorant thereof) let him expreſſe of his Meadow how many daies mowing, of his Arable how many daies plowing, and of his Paſture how many Beaſt-gates, and the like: for although it be not greatly materiall for theſe their giuen quantities, in reſpect you meaſure euery particular; yet this kind of entrie will ſerue you to good purpoſe, as ſhall hereafter appeare. Alſo, you are to queſtion him concerning his Paſtures, as well thoſe in ſeuerall, as the Paſtures in common, what euery Beaſt-gate is worth by the yeare in euery of them ſeuerally; whereof you may otherwiſe alſo informe your ſelfe, leſt you be deceiued. The forme of which entries let be in this manner.

BRANTON Towne-ſhip Free-hold.

Socage, ANTHONIE BORNE *holdeth freely to him and his heires for euer, by deed indented, bearing date* 14. die Ianuarij, Anno Reg. Reginæ Elizabeth. &c. 30. *made and graunted by and from* WILLIAM BATEMAN, *of &c. All that Meſuage or Tenement* (expreſſing the verie words of Graunt) *On which Graunt is there a deed of Feoffement of the ſame date, with liuerie of ſeiſin thereon paſt accordingly; by the yearely rents and ſeruices of* } Fealtie, & iij.s.

Particular.

	r.	a.
THe manſion-houſe, out-houſes, and ſcite, conſiſting of two Orchards, three Gardens, and two Yards or Garths, containing together		3.
A Cloſe of Meadow, called Broad Meade, containing		10.
Another called White-thorne cloſe, containing		8.
Meadow in the common Meadow, called Long meade, three parcels containing		12.
A Cloſe of Arable, called Bennets, containing		7.
Another of Arable, called the High Cloſe,		15.
Arable in the South field, in ſix parcels, which containe all together		4. 3.

	a.	r.
Arable in the North field, fiue parcels containing	6.	2.
One Cloſe of Paſture, called the Oxe Paſture, containing 30. *beaſt-gates, at* 13.s. 4.d. le gate,	gates. 30.	
Another called the Calfe Cloſe, containing 12. *beaſt-gates, at* 8.s. le gate,	gates. 12.	
In the great common Moore, at 2.s. 8.d. le gate,	gates. 20.	
In the common Cow-paſture, at 6.s. 8.d. le gate,	gates. 10.	
On the Downes depaſturing, for 150. *ſheepe, at* 3.d. le gate,	gates. 150.	
Common ſans ſtint on the Moores.		

And in this order, and vnder this Title, enter all the Free-holds within the Towne-ſhip of BRANTON: But withall, obſerue this courſe in all your entries; that theſe particulars, in the entring of them thus in your rough Booke, exceed not, nor extend paſt halfe the breadth of euerie ſheet or leafe, or little further, becauſe directly after the ſeuerall contents, before ſpecified, there is particularly to be expreſſed afterwards, the true quantities of euery parcell found by meaſure; and after that, the ſeuerall and particular yearely values thereof, as ſhall be hereafter ſhewne.

In like manner, let your tenants for life or liues, in euerie Towne-ſhip, be entred vnder their due and ſeuerall titles thus.

BODLEY Towne-ſhip for liues.

THOMAS HOCKLEY *holdeth by Indenture, bearing date* 23. die Nouembr. Anno Reg. Reginæ Elizabeth. 32. *made and graunted by and from A. B. All that Meſuage or Tenement* (vſing the words of Graunt) *for the terme of the naturall liues of the ſaid* THOMAS HOCKLEY *now aged* 50. *yeares; of* IANE *his wife now aged* 42. *yeares, and* IOHN *their ſonne now aged* 30. *yeares, ſucceſſiuely each after other: and payeth rent* per annum — XX.s.

Particular.

AND here write the particulars in forme as before.

Then vnder the foot of thoſe particulars make a briefe *Memorandum* of ſuch neceſſarie obſeruations as are to be noted, thus.

There is due vnto the Lord on the death of euery of them dying tenant in poſſeſſion, the beſt Beaſt *Nomine Heriot*. The tenant is to pay his rent quarterly, or within one and twentie dayes, on paine of xx.s. *Nomine pœnæ*; or within fortie dayes, on paine of forfeiture by prouiſo. To doe all manner of reparations (except great tymber.) Not to let the whole, or any part, without the

the Lords licence. The Lord warranteth the premiſſes againſt him and his heires.

And the like notes may be expreſſed vnder all your other entries.

Where note, that ſpeciall care is to be alwayes had in the entrance of theſe grants for liues, whether they are all ioynt purchaſers, as all named in the words of grant, or whether only granted to one during all their liues; wherein there is great difference; which is to bee noted, and the verie words of grant to be euer preciſely expreſſed in your entries as before.

As concerning your Copihold Tenants let them be entred thus.

SVTTON Town-ſhip Copi-holds.

W. B. *holdeth the Copie of Court roll bearing date* viij. die Iunij Anno Regni Regis IACOBI Angliæ iij. *of the ſurrender of* C. D. *One meſſuage or tenement* (according to the words of the Copie) *late* E. F. *and before that* G. H. *To him and his heires at the will of the Lord according to the cuſtome of the Manner. For which he payd fine* XL^{s}. *And payeth rent* per annum. } xxx^{s}.

Particular.

Then write your particular as before: and after that ſuch briefe Memorandums as you ſhall finde fitting, according to the former directions in that behalfe.

And the like courſe are you to holde with Tenants for terme of yeeres, and at will; and all the reſt: vntill you haue fully finiſhed all your entries. Then place all your leaues in order; and if you bee aſſured that all are entered (which will eaſily appeare by your Rentall, if you alwayes croſſe the ſame, as you enter them ſeuerally) then write a new Rentall according to this your rough Booke, and ioyne the ſame to the beginning of your Booke, and file all together; But let your Rentall bee alphabeticall; or elſe if your booke be verie large, make an alphabeticall Index or Table of all the Tenants names, to be placed in the beginning of the Booke, before the Rental, with numbers of reference to the number of leaues; whereby inſtantly you may turn.e to any Tenants name therein as you ſhall haue occaſion to vſe the ſame.

And thus haue you finiſhed with the Tenants concerning their ſeuerall entries; but haue not yet fully perfected your rough Book; whereby you may be able to ingroſſe the ſame. Wherefore proceed therein as is taught in the next.

CHAP. XI.

The meanes and order of perfecting the Booke of entries laſt mentioned, and the due placing therein of the ſeuerall contents of euerie particular found by meaſure through the whole Mannor, with the valuation thereof.

WE are now to ſuppoſe before wee beginne this worke, that not onely all the Inſtrumental menſuration throughout the whole Mannor is finiſhed; but alſo the firſt plot drawne; and the ſeuerall contents thereof caſt vp, and expreſſed therein; with the proper and particular numbers, ſeuerally belonging thereunto, according to the inſtructions of the Third Booke.

Which being effected, we are firſt to make an Index or alphabetical Table of al the Tenants names; wherunto is to be added the Lords and the Parſons, the one for Demeſnes, the other for gleabe; which Index is thus to bee compoſed. If one ſheet of paper will not ſerue, you are to take two, three or more, and with mouth-glue faſten them end to end, making thereof a long ſcrole or ſchedule of the bredth of the whole ſheet. And with a ſmall margent towards the left hand, let it from thence be ruled with blacke or red Inke ouerthwart the whole paper, of the diſtance of lines in ordinarie writing; betweene which lines downe by the ſame left margent write all the Tenants names each vnder other alphabetically, leauing all the reſt of the ruled Paper to be thus imployed.

Firſt, take your Field-booke, and beginning where you firſt beganne your worke in the fields, take all the numbers before you expreſſed in the margent thereof, as appeareth in *Chap.* 10.3. and noting to what Tenants name they belong in your Field-booke, againſt the ſame name place them in your Index betweene the ruled lines; whereby you ſhall ſpeedily expreſſe and reduce the ſeuerall numbers repreſenting the ſeuerall fields and cloſes throughout the whole Mannor againſt euery mans name in the Index to whom they properly belong.

Then take your common Field-booke (mentioned in *Chap.* 40.3. and with it performe the like; but make a ſtroke or other marke for diſtinction betweene theſe and the former numbers; ſo haue you alſo euerie mans particular parcels lying in the common fields expreſſed againſt his name. And thus is your Index perfect and fit to be imployed as followeth.

Take now your rough Booke of entries, and turne to the firſt Tenants particular therein entred; and looke in your Index what numbers are belonging to that Tenant; alſo take your Field-booke, and comparing thoſe numbers in your Index with the ſame in the margent of your Field-booke, you ſhall find therein the ſeuerall names of the fields and cloſes belonging to that Tenant; and the like names ſhall you find in your Booke of entries, in that Tenants particular: wherefore expreſſe thoſe ſeuerall numbers againſt euerie particular parcell in the margent of your entries, whereunto they properly

perly belong: and the like performe in all reſpects with your common field-booke, for thoſe parcels lying in the common fields. Then lay your rough plot before you, and finding thoſe ſeuerall numbers in your plot, note the ſeuerall content and quantitie of euery ſeuerall parcell of ground, expreſſed in the plot repreſented by thoſe numbers; and thoſe quantities expreſſe and write downe in figures particularly in your booke of entries, to euery parcell whereunto they belong next after the quantities deliuered by the tenant. As for the particular quantities in the common fields, you ſhall not find them in the plot, but in your common field-booke, according to the direction thereof in *Chap.* 40. 3. before recited; which let thereby be expreſſed accordingly.

And the like courſe in all reſpects is to be holden with all other the entries throughout your whole booke.

But it is to be conſidered, that one tenant may within this Mannor hold lands of ſeuerall eſtates, and by ſeuerall rents and ſeruices, as Free-hold, Copy-hold, Tenement Lands, &c. yet are they all compriſed and repreſented within and by thoſe numbers expreſſed in the Index. In ſuch caſe you are to compare theſe numbers with the particular names expreſſed in your field-booke, and thoſe with the entries; and you ſhall moſt eaſily diſtinguiſh the one from the other, and expreſſe and aſſigne to euery of them their due and proper number and quantitie, as before.

Alſo, it is here to be conſidered, that we haue not yet ſpoken of any means to expreſſe each mans particular quantitie in the common ſtinted paſtures, or ſheepe-walkes, whereof onely the generall quantitie is taken by meaſure, and expreſſed in the plot. Which to performe, worke thus.

Suppoſe there is in the Mannor a ſtinted Cow-paſture, wherein euery tenant hath a certaine number of Beaſt-gates, ſome more, and ſome leſſe, which are vſually rated and ſtinted, either according to their rents, or after their quantities of knowne grounds, or their parts in the common arable fields: and imagine this paſture is found to containe by meaſure 212. Acres, 3. Roods, 20. Perches; firſt, collect out of your entries all the number of gates in the ſame paſture; which added together, let containe in the whole 116. Then reduce your meaſured quantitie into the loweſt denomination, as Perches (according to the directions of *Chap.* 48. 3.) whereby you ſhall find the ſame to be 34060. Perches: and ſuppoſing the tenant, whoſe quantitie you ſeeke, hath in the ſame paſture 10. beaſt-gates; by the rule of proportion reaſon thus. If 116. the whole number of beaſt-gates, giue 34060. Perches, the whole quantitie, what giues 10. gates; and by increaſing 34060. by 10. and parting the product by 116. your anſwer will be 2936. Perches, and a ſmall quantitie more, the proportionall quantitie belonging to 10. gates; which reduced into acres (by *Chap.* 48. 3. before recited) is 18. Acres, 1. Rood, 16. Perches. And in like manner worke with all the reſt. And here is to be noted, that hauing thus gotten the proportionall quantitie belonging to a beaſt-gate, and truely vnderſtanding the value of a beaſt-gate, you ſhall be able at pleaſure, and moſt certainely, to expreſſe by the acre the true yearely value thereof.

And thus haue you perfected your entries, for the true and certaine quantities, according to measure; and now resteth the valuation.

The best, speedyest, and most certaine meanes for your valuation, in mine opinion, is thus: Let it first be considered, that all grounds generally consist in qualitie of these three kinds, Meadow, Arable, and Pasture; and supposing euery of these kinds likewise to consist of three sorts in value and goodnesse; as the first and best sort; the second and meane; and the third and worst sort: In your instrumentall mensuration, when you write in your field-booke the title of euery field or close, consider with your selfe, which of those three sorts the same field or close consisteth of; if of the best sort, expresse in some place of your title the figure 1. if of the second sort, the figure 2. and if of the third and worst sort, the figure 3. and hauing informed your selfe by the best meanes you can (which I hold not fitting here to relate) of the generall value, what the best sort of Meadow, Arable, and Pasture, is worth by the acre, and the like of the other sorts, throughout the whole Mannor; then, according to those rates, passe ouer your whole booke of entries, and value euery particular parcell by it selfe; which, by finding in each title of your field-booke of what sort they are, is most speedily and exactly performed. And thus are your entries thus farre perfected.

But yet, before we proceed to the ingrossing hereof, or rather before wee finish with the tenants concerning their entries, it is to be enquired and considered, what other profits and commodities, besides these lands and tenements, are demised and granted by the Lord to any tenant within the Mannor for yearely rent, or otherwise; which likewise are to be entred and expressed in your rough booke, with the rents and yearely values thereof: as Mynes of Tynne, Lead, Copper, Coale, &c. Quarries of Stone, Fishing, Fowling, Hawking, and Hunting, Iuistments, Herbage, and Pannage, free Warrens, Customarie workes, or Seruices, profits of Fayres and Markets, and mosses of Peat or Turfe; all or any of which, and the like, may be within a Mannor, and disposed and letten for yearely rents, which by no meanes are to be omitted. All which premisses, and the seuerall quantities, rents, and values thereof, are here to be summed vp, and their seuerall totals expressed.

Then are you to expresse the seuerall reprises issuing out of this Mannor, being such as are mentioned in the sixt Chapter of this Booke. All which being likewise summed vp, the totall thereof is to be deducted from the former value, and the cleare remainder expressed.

Next are you to consider, if any of those profits and commodities, last before named, or the like, are within this Mannor, and not letten by lease, or otherwise, for any certaine yearely rent; and if any such be, then are they to be here mentioned and expressed as casualties, and the yearely value thereof estimated, what they may or are likely to proue worth by the yeare.

Also, the names and quantities of the common Fields, common Meadowes, stinted Pastures, and all other vnstinted Commons, are here to be expressed; and of those vnstinted Commons, how they are accustomed, held, and occupied; whether peculiar to the Lord and tenants of this Mannor; or whether any other Lord or Lords, and their tenants, haue rake, escape,

eatage,

eatage,or other intereſt therein; and the butts,bounds,and limits thereof ſeuerally.

Alſo, what woods and vnderwoods are within the Mannor, and their ſeuerall values.

Then would there be entred an abſtract, in nature of a Cuſtome-roll, ſhewing briefely all the ancient cuſtomes of and belonging to the Mannor. And alſo a Suit-roll of all the free ſuitors, &c.

And laſtly, a true and perfect deſcription of all the out-bounds and limits of the whole Mannor.

And hauing thus perfected your rough Booke, you may now call your Iurie, and receiue their verdict, as in the next.

CHAP. XII.

The manner and order of receiuing the Juries verdict, and the courſes therein to be obſerued.

WHen you are drawing towards a concluſion of perfecting your Booke, according to the directions of the laſt Chapter; it were not amiſſe, that you haſten the Iurie, in perfecting their verdict, leſt you be forced to ſpend time idly in attendance for the ſame: Which when they haue effected, conſidering, that for the moſt part they are vnacquainted with matter of forme, though in effect and ſubſtance they may anſwer your deſire and expectation, according to the articles deliuered vnto them; you are to call them before you, and reading, examining, and comparing the articles, with their ſeuerall and particular anſweres thereunto, reduce the ſame into an orderly forme of an Inquiſition, obſeruing ſtill the ſubſtance of what they haue found and preſented: and then hauing read the ſame vnto them, with their approbation and allowance thereof, cauſe your Clarke to ingroſſe the ſame accordingly in parchment, and then let them againe conſider thereof; and hauing ſet their hands and ſeales thereunto, demaund of them, if they are mutually agreed on this their verdict; which when they haue acknowledged, receiue the ſame from the Fore-man, and diſmiſſe your Court.

And here haue you finiſhed what here need to be performed: and now may you leaue the Mannor of BRANTON; *and repairing homewards, may there perfect your plot, as you are formerly directed by the third Booke; and ingroſſe your Suruey, as is hereafter declared in the next.*

CHAP.

CHAP. XIII.

The forme and order of ingroſsing a Suruey.

TO preſcribe and direct one certaine and ſetled forme and courſe herein for all in generall, were impoſſible, in reſpect of the varietie of occaſions offered, according to the nature of the buſineſſe, and the diſpoſition of thoſe for whome the ſame is performed: wherefore, the performance hereof muſt mainely depend vpon the iudgement, skill, and diſcretion of the Surueyor. For mine owne part, I neuer yet for any two Lordſhips or Mannors limitted my ſelfe to one and the ſame forme; but euer framed my courſe as the cauſe required: as in one Mannor, where I find a commixture of other lands and tenements within the ſame, being holden of other Lords; here of neceſſitie muſt I abutt and bound euery ſeuerall particular thereof: but in another, which I find entire, I hold it needleſſe. Againe, I find in one Mannor diuers and ſeuerall Towne-ſhips and parts, and thoſe to conſiſt of ſeuerall eſtates and tenures; in ſuch caſe, theſe are to be ſeuerally diſtinguiſhed and diuided, according to their ſeuerall parts. Another ſhall you find ſole and entire, which is to be ordered accordingly: and many other ſuch like differences ſhal you find, which will miniſter occaſion to alter any ſetled forme.

Beſides, the will and diſpoſition of him by whom you are imployed, ſhall often cauſe you to alter your courſe: one perhaps approuing of the forme you vſe; another will haue it in the nature of your ancient Terrars; a third, in order of a Particular, or Conſtat; and a fourth, it may be, in a fourth forme; for *Quot homines, tot ſententiæ*. And againe, one, for his owne vnderſtanding, will haue it in Engliſh; and another, of better vnderſtanding, will require it in Latine. And certainely it were very requiſite, although your rough booke be drawne in Engliſh, that alwaies your ingroſſed booke be written in Latine; vnleſſe the contrarie be ſpecially required. Yet in theſe mine examples and directions following, I hold it moſt fitting to deliuer the ſame in Engliſh, for the better vnderſtanding of thoſe who haue moſt need; conſidering, that a reaſonable Surueyor may be lame of that legge.

But notwithſtanding ſuch varieties often happen; yet will we for a generalitie propoſe theſe rules and directions following; which I hold moſt meet and fitting to be obſerued and held in a formall and well ordered Suruey.

To which purpoſe, let vs now ſuppoſe wee are to ingroſſe a Suruey of the Mannor of BRANTON; according to the rough booke thereof, ſpecified in the tenth Chapter of this Booke; wherein firſt begin with the title, which let be thus, or to the like effect.

An exact and perfect Suruey and view of the Mannor of BRANTON, in the Countie of D. being parcell of the poſſeſſions of A. B. who holdeth the ſame of our Soueraigne Lord the King, as of his Mannor of G. in free and common Socage, and by the yearely rent of xiij.$^{s.}$ iiij.$^{d.}$ Had, made, and taken there, as well by Inquiſition, and the oathes of a ſufficient Iurie in that behalfe, as by the view and particular menſuration of all and euery the Meſſuages, Lands, and Tenements, of, within, and belonging to the ſame. *Anno Domini* 1616. *Annoque Regni Regis* IACOBI, *Angliæ, &c.* 14.

By *A. R.* Superuiſ.

NExt after this, or the like Title, in the following leafe, are you to write and expreſſe an Index or Alphabeticall Table of all the Tenants names (as hath beene formerly taught) with numbers of reference againſt each name, in what leafe or leaues of the Booke each Tenants particular is to be found: But notwithſtanding, this Index is to be placed in your Booke firſt and next after the Title; yet is it moſt conuenient and fitting, to collect and write the ſame (and alſo the Rentall next hereafter following) after the whole Booke be ingroſſed; before which time, you ſhall not know how to place your numbers of reference therein, according to the number of the leaues.

Then after this let next be placed a generall rentall of the whole Mannor, but to be diuided into ſuch townehips and parts, as your Booke is diuided into; wherein firſt expreſſe your rents of ſuch demeſnes, as are letten in leaſe, then the rents and ſeruices of your free-holds of inheritance. Thirdly, of your cuſtomarie or copy-hold tenants. Fourthly, of the tenants for life or liues. Fiftly, of thoſe for terme of yeeres: and ſixtly and laſtly, your tenants at will, wherein let euery of theſe beare their ſeuerall titles, and vnder the foot of each kinde, let the totall thereof be collected and expreſſed, and in the end or foot of the whole rentall, expreſſe firſt the totall of euery kinde, and after that the generall and totall ſumme of all together. And if any rents or other repriſes be iſſuing out of this Mannor, you may here expreſſe them particularly vnder the title of repriſes, which let be deducted out of the former totall, and expreſſe the cleere remainder.

And thus is your Rentall finiſhed; but to be collected and written (as I formerly noted) after the bodie and ſubſtance of the Booke be ingroſſed.

And

And your rentall being thus finiſhed, you may next place (if you pleaſe) the out-bounder of the whole Mannor; and if any of the townеſhips or parts thereof lye diſperſed and remote (as in many places you ſhall finde them perhaps twentie miles diſtant, from the chiefe and principall part of the Mannor; and ſometimes in another Countie) it were very fitting and neceſſarie, to expreſſe ſeuerally the ſeuerall out-bounders of thoſe townеſhips and parts. And it is to be noted, that in the expreſſing of theſe bounders, a maine and principall care is to be had, that you vſe, obſerue, and keepe the olde and ancient names of ſuch meeres, markes, and bounds, as haue beene anciently vſed and accuſtomed; for that innouation in this kinde is very dangerous for many cauſes; yet if you finde the ancient meeres, markes, and bounds, to be very few and ſlender; or any of them decayed and worne out of knowledge, you may adde as many moe as in diſcretion you ſhal finde cauſe; but by any meanes omit not, or leaue out any of thoſe which are ancient and noted bounds. If you thinke good, theſe bounders may be placed after, or in the ende of the booke; which being no matter of neceſſitie whether (ſo it bee had at all) I leaue to your diſcretion.

And now are you to begin with the body and ſubſtance of the booke; and firſt of all with the Mannor or manſion houſe, and the ſcite thereof; wherein you are to conſider, whether the ſame be in the Lords owne hands and occupation; or whether letten by Leaſe, or otherwiſe vnto any Tenant or Tenants, and to enter the ſame accordingly, as followeth vnder this Title.

BRANTON DEMESNES.

A. B. Eſquire is Lord of this Mannor, and hath at this preſent in his owne hands and occupation, the Mannor or manſion houſe with the ſcite thereof; and ſo much of the demeſnes thereunto belonging as are hereafter particularly expreſſed. Which Mannor with all and ſinguler the appurtenances, he holdeth of our Soueraigne Lord the King, as of his Mannor of G. in free and common Socage, and by the yeerely rent of } xiij^{s} iiij^{d}

Particulars.

THe Mannor or manſion houſe called Branton Hall, being faire-ly built with free ſtone, and all offices thereunto belonging, with two Stables, one Oxe-houſe, and a Doue-houſe; alſo the ſcite conſiſting of three faire gardens, two orchards, two courts, and three out-yards, lying all together betweene the high ſtreete of Branton ſouth, and the Oxe-paſture hereafter mentioned North; abutting Eaſt, on Long meddow, and Weſt on the ſcite of the Parſonage. And containeth together fiue Acres, two Roodes, and twenty Perches.	a. r. p. 5-2-20.

Valet per anum —— xx^{li}.

Then next vnto the houſe and ſcite, expreſſe the Parkes (if any be) with the number of Deere therein; what number of Aunteller, and what of raſcall Deere; alſo what number of beaſts may bee therein Iuiſted without preiudice to the game; and alſo what pannage; and theſe may you particularly abutt and bound as before if neede require; which is moſt eaſily and ſpeedily done, hauing before you the rough plot of the whole Mannor, and in the concluſion hereof expreſſe the quantitie and yeerely value as before.

Then after theſe Parkes, enter particularly all ſuch ſeuerall fields and cloſes of the demeanes, as the Lord hath in his owne vſe at the time of this Suruey; which you may particularly abutt and bound as before, and expreſſe the ſeuerall quantities, and values thereof: But herein for orders ſake it were fitting firſt to enter all the meddow grounds particularly each after other, then the incloſed arable grounds, and next the paſtures; and if any of the demeſnes are lying in the common fields, then to expreſſe them particularly with their quantities and values; alſo you are to expreſſe what woods the Lord hath; and what right or cuſtome the Tenants haue or claime therein, either for depaſturing or otherwiſe: and laſtly, what waſtes the Lord hath within the Mannor. And at the foote of this particular, expreſſe the totall quantitie and value thereof.

But it is to be noted, that all theſe particulars are to be collected (by help of the numbers in your Index) out of your field-booke for the names, and out of the rough plot for the ſeuerall quantities, for that it is needleſſe to enter theſe lands which are in the Lords hands into your rough booke of entries; and the like courſe alſo is to be holden for your Glebe lands.

And if any of the Demeſne Lands are in Leaſe, let them alſo bee entered vnder the former Title, in this manner.

C. D. holdeth by Indenture of Leaſe, bearing date the twentith day of Ianuary An. R. Regis Iacobi &c. ſecundo, made & granted by and from A. B. vnto the ſaid C. D. All thoſe lands, &c. (vſing the very words of graunt) *for the terme of one and twentie yeeres, commencing from and after the Feaſt of the Birth of our Lord God laſt paſt, before the date of the ſame Leaſe, for and by the paiment of the yeerely rent of* ——— } XXX^li.

Particular.

THen here enter the ſeuerall and particular Cloſes, which you may abutt and bound, as before, expreſſing the particular quantitie and value of euery ſeuerall Cloſe, and at the foot of the particular expreſſe the totall quantitie and value, and if the yeerely value exceede the rent reſerued, deduct the rent from the totall value, and expreſſe the cleere remainder, thus.

Valet ad demittend. ——— Xli.

And

And vnder this particular expreſſe a briefe memorandum of the ſeuerall couenants, clauſes, conditions, and prouiſoes in the leaſe contayned, after this manner.

The Tenant is to pay his rent quarterly, or within one and twentie dayes after euery Feaſt, on payne of forfeiture, by prouiſo to that purpoſe. He is to doe all manner of reparations (except great timber) not to let or ſet without licence of the Lord. The Lord maketh ſpeciall warranties againſt his father, himſelfe and his heires, &c.

And the like courſe is to be held for all other Leaſes, after the particulars expreſſed.

And here alſo vnder this title of Demeſnes, are you to enter all ſuch Mills, Mines of Tinne, Leade, Copper, Cole, &c. alſo quarries of Stone, Slate, and the like; alſo Fiſhing, Fowling, Hawking and Hunting; likewiſe Iuiſtments, Herbage, Pannage, free Warrens, cuſtomarie Workes or Seruices, profits of Fayres and Markets, and alſo moſſes of Peate or Turffe, and the like, as are letten and demiſed by the Lord, to any Tenants within the Mannor by leaſe for yeerely rent or otherwiſe: all which (being thus letten) are in the nature of Demeanes, and are to be particularly entred and expreſſed accordingly, with their ſeuerall Rents and the yeerely values thereof.

But all of thoſe laſt mentioned (excepting Mills) are to be ſeuered and diſtinguiſhed from the Demeſne lands, becauſe they are not matters of firme, ſtable, and certaine perpetuitie: For notwithſtanding, that during the termes of the ſeuerall leaſes thereof made and granted, the Tenants may be charged and bound to pay ſeuerall yeerely rents for the ſame, which for the time being are certaine; yet perhaps at the end and expiration of thoſe termes, they may be of little or no value at all; or on the other ſide of farre greater worth and value then now they are, as often happeneth by thoſe mines of Tinne, Leade, Copper, Coles, and the like. Wherefore notwithſtanding they are entered vnder this generall title of Demeſnes; yet for deſtinctions ſake, let them paſſe more particularly vnder this title of *Caſualties made certayne.*

And after all theſe demeſnes are thus entred and ingroſſed, make a briefe concluſion thereof vnder-neath the ſame, in this or the like manner.

Con-

Concluſion of the Demeſnes.

The Demeſnes of this Manner, conſiſteth of

Lands, Tenements, & Mills in the vſe of

The Lord

	a.	r.	p.
Quantitie	132	0	0.
	li.	s.	d.
Value	66	0	0.

The Tenants

No. of Tenants			5.
	a.	r.	p.
Quantitie	163	0	0.
	li.	s.	d.
Rent	54	6	8.
Value	81	10	0.
Ad demitt.	27	3	4.

Caſualties made certayne.

No. of Tenants			6.
	li.	s.	d.
Rent	25	8	6.
Value	94	18	0.
Ad demitt.	69	9	6.

And after the Demeſnes are thus entred and ingroſſed, then next vnto it place the Rectorie or Parſonage, and then the Vicarage (if any be) vnder the proper title thereunto belonging, after this manner.

The Rectorie of BRANTON.

A. B. *Clarke, being Parſon there, holdeth the Rectorie of the gift of the Lord of this Mannor* (if it be ſo, and if otherwiſe, expreſſe it accordingly) *who hath the gift, nomination and preſentation thereof, as in the right of this Mannor, as often as the ſame ſhall happen to be void, which is valued in the Kings bookes* per annum. } Lvi li.

Particular.

THe Parſonage or Manſion-houſe with the out-houſes belonging thereunto, as a Barne, Stable, Oxe-houſe, and a Doue-coate, with the ſcite thereof, conſiſting of two Gardens, an Orchard, and three out-Yards, which lye together betweene the high ſtreet of Branton *South, and a field called the Oxe-paſture North, abutting towards the Eaſt on the ſcite of this Mannor, and Weſt on a Lane there leading out of the high ſtreet into the Oxe-paſture aforeſaid, and contayneth together, one Acre and three Roodes.*

a.	r.	p.
1	3	0.

Valet per annum ——— iij li.

And in this ſort let euerie particular parcell of glebe-Land bee expreſſed with the buttes and bounds thereof, which by helpe of the plot and field-Booke lying before you (being directed thereunto by the numbers in your Index) is inſtantly and exactly performed: For theſe glebe-Lands; and the Demeſnes which are in the Lords hands, are neuer entred in your rough Booke of entries. Wherein is alwaies to be obſerued; that you expreſſe the true quantitie and yeerely value of euerie particular parcell; and in the foot of the particular, the totall quantitie and value as before. Yet is it not vſuall neither of theſe nor the Freeholds of Inheritance to expreſſe any value at all; which I will referre to your owne diſcretion, and the will and diſpoſition of thoſe by whom you are imployed. And in like maner are you to expreſſe the Vicarage if any ſuch bee.

And hauing thus finiſhed your Parſonage, Vicarage and glebe-Lands, proceede next vnto the Free-holds within this Towne-ſhippe; which are to bee entered and ingroſſed after this order, and vnder this title following.

BRANTON Free-holds.

Socage. *A.B. holdeth freely to him and his heires for euer by deed indented bearing date* xxviij. die Marcij Anno Regni Regis IACOBI Angliæ, &c. Sexto *made and granted by and from* C.D. *All that meſſuage or tenement* (expreſſing the verie words of grant) *By the yeerely rents and ſeruices of* — Fealtie & v.s.

Particular.

THe Manſion houſe, out-houſes and the ſcite thereof conſiſting of one Garden, two Orchards and three out yards, lying together betweene the high ſtreet of Branton *North, and the common field called the South-field South; abutting Eaſt on the Church-yard, and Weſt on a lane leading into the South field. And containeth three roodes and thirtie perches.* a. r. p. 0. 3. 30.

And thus proceed with euerie parcell belonging to this free-holder; which being finiſhed, at the foot of this particular expreſſe the quantitie and value thereof: But as concerning the valuation of Free-holds, vnleſſe it be ſpecially required, by reaſon of ſome purchaſe thereof to be made; or a poſſibilitie of eſcheat, or the like, you need not trouble your ſelfe therewith.

And hauing perfected your particular, expreſſe vnderneath the ſame, a briefe *Memorandum* of ſuch neceſſarie obſeruations as you ſhall find fitting, aſwell concerning the Tenants euidence, as what Heriots, Reliefes and other Duties and Seruices the Tenant ought to yeld, doe and performe vnto the Lord on euery death or alienation.

And

And in like manner vnder the ſame Title enter all other the free-holders within this townеſhip; after which, colle&t and expreſſe together their ſeuerall quantities in one totall ſumme, and likewiſe their values (if it bee required as before:) But in theſe and all others, as I haue formerly noted, I would alwaies haue an orderly courſe holden in placing the particular lands of one and the ſame nature and qualitie together, as firſt (after the houſe and ſcite) all the meddow grounds, &c.

And thus hauing entered and ingroſſed your free-holds of this townеſhip, let next be entered your copyholds or cuſtomarie Tenants, after this manner, and vnder this Title.

BRANTON Copy-holds.

A. B. *holdeth by copie of Court roll, bearing date* 26 Februarij Anno Regni Regis Iacobi Angliæ &c. 4^to. *of the ſurrender of* C. D. *All that meſſuage or tenement &c.* (vſing the very words of the Copie) *to him and his heyres at the will of the Lord, according to the cuſtome of the Mannor. For which he paid Fine on his admittance* vli. *And payeth rent for the ſame* per annum, ——— } xls.

Particular.

LEt your particulars here be entred in all reſpects as before with the ſeuerall buts and bounds thereof, expreſsing the quantitie and value of euery ſeuerall parcell: and in the foote of the whole particular expreſſe the totals as before; then out of the totall value (which admit to bee xli.) deduct the Rent, and expreſſe the remainder thus.

Valet in toto per annum xli.

Viz. ad dimittend viij li.

Then vnder this particular thus perfected, make (as before) a briefe Memorandum of ſuch neceſſarie obſeruations as are fitting, as what herio (if any be due on the death of the Tenant) what fines on death or alienation; and what other ſeruices the Tenant oweth, &c. And after this order and vnder this title enter all the reſt of the copy-holds within this townеſhip.

And the like courſe in all reſpects is to bee holden in entrie of the Tenants for life, or liues; for terme of yeeres; and thoſe at the Will of the Lord; whereof to make ſeuerall demonſtrations, and to deliuer ſeuerall examples, were but great labour to ſmal purpoſe, ſeeing they tend all to one and the ſame end.

Wherefore take this for a briefe and generall rule, that all lands whatſoeuer, and the tenancy thereof, conſiſt of one of theſe ſeuen kindes, which in euery Mannor where they are, are to be vſed as your ſeuerall titles, and ought to be placed in the ingroſſing of your Booke, each after other, as here they are expreſſed, viz.

1. Demeſnes.
2. Gleabe-lands.
3. Free-holds.
4. Cuſtomarie.
5. For liues.
6. For yeeres.
7. At will.

And hauing after this forme and order entred and ingroſſed the ſeuerall Lands and Tenements, lying within this Towne-ſhip of BRANTON, vnder the ſeuerall titles laſt before mentioned, collect your totall of euery kinde, and in the end of this Towne-ſhip make your concluſion to this or the like purpoſe following.

Con-

Concluſion of the Towne-ſhip of BRANTON.

This Towne-ſhip conſiſteth of

- *The Lord*
  - *Quantitie* — a. r. p. — 1320 — 0 — 0.
  - *Value* — li. s. d. — 660 — 0 — 0.
- *Demeſnes in the vſe of*
  - *The Tenants*
    - *Lands*
      - *No. of Tenants* — 5.
      - *Quantitie* — a. r. p. — 163 — 0 — 0.
      - *Rent* — li. s. d. — 54 — 6 — 8.
      - *Value* — 81 — 10 — 0.
      - *Ad demitt.* — 27 — 3 — 4.
    - *Caſualties made certayne,*
      - *No. of Tenants* — 6.
      - *Rent* — li. s. d. — 25 — 8 — 6.
      - *Value* — 94 — 18 — 0.
      - *Ad demitt.* — 69 — 9 — 6.
- *Gleabe lands* — *Quantitie* — a — 56 — 0 — 0.
- *Freeholds of inheritance,*
  - *No. Tenants.* — 7.
  - *Quantitie* — a. — 230 — r. — 2 — p. — 0.
  - *Rent* — s. d. — 13. 6. — ll. — 3. pepper.
- *Cuſtomarie lands,*
  - *No. Tenants.* — 16.
  - *Quantitie* — a. — 340 — r. — 3 — p. — 0.
  - *Rent* — 17 — 16 — 5.
  - *Value* — 152 — 13 — 4.
  - *Ad demitt.* — 134 — 16 — 11.
- *Tenement lands.*
  - *For liues*
    - *No. Tenants.* — 12.
    - *Quantitie* — a. r. p. — 432 — 2 — 0.
    - *Rent* — li. s. d. — 143 — 6 — 3.
    - *Value* — 220 — 0 — 0.
    - *Ad demitt.* — 76 — 13 — 9.
  - *For yeeres*
    - *No. Tenants.* — 23.
    - *Quantitie* — a. r. p. — 624 — 2 — 0.
    - *Rent* — li. s. d. — 156 — 13 — 9.
    - *Value* — 310 — 18 — 0.
    - *Ad demitt.* — 154 — 4 — 3.
  - *At will*
    - *No. Tenants.* — 8.
    - *Quantitie* — a. r. p. — 120 — 0 — 0.
    - *Rent* — li. s. d. — 42 — 5 — 6.
    - *Value* — 58 — 10 — 0.
    - *Ad demitt.* — 16 — 4 — 6.

And hauing thus finiſhed this Towne-ſhip, proceede in the like forme and order in all reſpects, with all other the townesſhips and ſeuerall parts of and belonging to the whole Mannor, obſeruing ſtill after euery townesſhip and part, to make ſuch or the like concluſion, as is laſt before ſpecified; and in the end of all a full and generall concluſion of the whole Mannor; not forgetting firſt to enter all repriſes, iſſuing, and going out of the ſame, which is to be deducted out of the whole value, as is before declared.

And after this concluſion thus perfected, you are to remember and expreſſe all ſuch neceſſarie obſeruations, as are fitting, according to the directions in that behalfe deliuered, in the latter end of the 11. Chapter of this Booke, and your worke is finiſhed.

Now might I here inlarge and amplifie this worke with many rules and examples, tending to theſe purpoſes, but preſuming that what I haue formerly deliuered (being well vnderſtood and practiſed) may ſufficiently ſerue a reaſonable capacitie; I will forbeare to peſter the practicioner in reading, or my ſelfe in writing of needleſſe varieties; and therefore will here conclude my labours, and expoſe them to thy good liking.

FINIS.

Mens workes haue faults, ſince ADAM *firſt offended,*
And thoſe in theſe, are thus to be amended.

ERRATA.

PAge 35 line 31. for CB. reade CD. p.38. lin. laſt, for cut out, r. cut. p. 42. l.9. for equiangles, r. equiangled. p.77. l.19. for FD. r. ED p. 110. l. 39. for draw the, r. draw to the. p.118. l.11. for 24. r. 240. p.127. l.17. for $\frac{4}{3}$ r. $\frac{3}{4}$. p.133. l.10. for $\frac{3}{2}$. or $\frac{5}{4}$. r. $\frac{1}{2}$. or $\frac{3}{4}$. p.146. l.13. for 15. r. $9\frac{1}{4}$. p.162. l.38. for O Q r. X. Q. p.204 l.16. for euery other ſuch, r. euery ſuch.

In the Diagrams.

Page 109. in Diagr. 109. place K. in the angle oppoſite to C. In Diagr. of Chap. 29. 3. neere vnto K. place O. oppoſite to N.

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K La Faille (J.C. de)
3
528. m. 8.
9

IOANNIS DELLA FAILLE ANTVERPIENSIS

E SOCIETATE IESV

In Academia Matritensi Collegij Imperialis
Regij Matheseos Professoris

THEOREMATA DE CENTRO GRAVITATIS PARTIVM CIRCVLI ET ELLIPSIS.

ANTVERPIÆ,
EX OFFICINA TYPOGRAPHICA
IOANNIS MEVRSII.
ANNO M. DC. XXXII.

MVSEVM
BRITAN
NICVM

POTENTISSIMO REGI CATHOLICO PHILIPPO IV. HISPANIARVM INDIARVMQ. MONARCHÆ.

DE Circuli grauitate demonſtrationes REX MAXIME peruigili ſtudio elucubratas, ad ſublime MAIESTATIS V. ſolium debito cultu proſtratus defero: qui orbem circuis poteſtate, grauitate moderaris. Quid enim Circulo ſimilius potentiâ tuâ? quid ponderi propius imperio tuo? Quod enim in cælo orbitæ ambitúſque, in orbe diffuſum imperium Veſtrum eſt, cuius, vt ſic dicam, circulo maria terráſque ambis, complecteríſque tot regna populóſque alio ſub ſole calentes. Potentiam magnificentia comitatur, nec aliter quàm ſol diurno re-

greſſu mundum illuminat, ita ſparſo per vniuerſum fulgore, Maieſtas V. luſtrat omnia fouétque. Nec aliter ipſa terræ moles grauitate ſuâ, quàm pondere & conſtantiâ regiminis tui quieſcit, omni pedum impulſu vacillatura, niſi vnum hoc momentum pertinacius eam loco aſſereret ſuo. Rectè igitur hoc inuentum tuis temporibus reſeruatum, quibus feliciſsimè magnitudinem tuam velut ſymbolum repræſentaret. Etſi enim Circulum è centro ſuo æquilibrare notiſsimum fuerit, quibus tamen ex punctis eius æquilibrent partes hactenus fuit ignoratum: & primus, ni fallor, in fragmenta diſſectum, ad ſeueram illam mechanicarum ſpeculationum trutinam expendi. Hieroni Syracuſarum Regi Archimedæa inuenta accepta ferimus: cuius tam felicis ingenij exuuiæ, ſi non totæ periere, certè minore ſui parte vitarunt Libitinam. TIBI REX MAXIME non inferiora debebunt poſteri, feciſti enim

enim vt multi per Te aggredi possent, quod sine Te vix audebant velle, testaríque naturam non in vnius Archimedis ingenio effœtam esse. Quibus sub prima auspicia Academiæ tuæ, hoc mathematico conatu præire volui, non exemplum daturus reliquis, sed id festinando assecuturus, vt nondum melioribus præuentus, ante æstimari mea possint, quàm doctiorum superuentu obscurari.

LECTORI S.

ECentro grauitatis quadratam ab Archimede parabolen nosti Amice Lector, eamq; ad propria deinde Geometrarum principia reuocatam. Hæc mihi occasio fuit cogitandi de centro grauitatis partium circuli, & an non aliqua ad quadraturam eius hinc pateret via, primitus inquirendi; quæ quàm firmo cum hoc centro sociata sit nexu, hoc opusculum percurrenti tibi palam fiet. Præsertim quòd reciproca quædam sit sequela, & problematicè inuento grauitatis centro quadretur circuli Sector, adeoque totus; ac vicissim quadrato circulo, partium eius grauitatis centrum reperiatur. Nimium quantum cum figuræ huius in quadratum metamorphosi doctorum virorum ingenia vano conatu luctata sint, dum alij per ignoratam hactenus dimetientis cum perimetro proportionem, alij per curuas quasdam lineas, cuiusmodi sunt helices & quadratrices, alij denique per lunulas id sunt aggressi; nemo tamen, quod sciam, hanc institit viam, vt à circuli grauitate ad explicandam eius aream proficisceretur. Annis abhinc vndecim cùm primum in Dolana sequanorum Academia Mathematicas disciplinas professus sum, hæc à me Theoremata fuêre excogitata, eaque tetigi verbo vno, sed demonstratione prætermissa, in iis quæ obiter de centro grauitatis auditores mei scripto exceperunt; Triennio post in Mechanicis Thesibus, quas typis vulgaui, eorum mentionem feci, nec ad id tempus apud scriptorem aliquem vestigium huius speculationis deprehendi; ab Archimede sanè nihil tactum, nec ab iis qui commentaria in ipsum edidere. Fridericus Commandinus & Lucas Valerius in circulo & ellipsi centra grauitatis & figuræ in idem conue-

conuenire punctum oſtenderunt, nempe quid ſine vllo demonſtrationis adminiculo facile quilibet admiſiſſet, hac tamen in parte non ſine laude ſcrupuloſæ Mathematicarum diſciplinarum integritati obſecuti. Maioris tamen moliminis fuit centra portionum vtriuſque figuræ inueſtigare; quod inuentum vt à nouitate ac ſpeculationis ſubtilitate commendari poſſit, ego tamen inuentionis modum impenſius ſum admiratus, qui longè alius eſt ab eo, quem primâ fronte demonſtrationes oſtendunt. Decreueram aliquando diſcurſus huius velut itinerarium ſcribere, vti pateret quâ viâ in hanc determinationem veniſſem, & opinione mea infinitis ſpeculationibus aditum aperuiſſet, ſed in iuſtum volumen excreuiſſet opus, multarum figurarum deſcriptione impeditum, quod quàm difficulter committi prælo potuiſſet, ſcio. Quî enim fieri aliter poſſet, vt (repudiatis tot futilibus libris qui magnitudinum dumtaxat dimenſiones & inſtrumentorum vſus iam toties ad nauſeam repetitos, obtrudunt) non fuiſſet inuentus aliquis, qui Apollonij Pergæi & Pappi Alexandrini opera, in doctorum virorum commodum iterato prælo ſubieciſſet? Mathematica multi ſciunt, Matheſim pauci. Aliud enim eſt noſſe propoſitiones aliquot, & nonnullas ex iis obuias elicere, caſu potius quàm certa aliqua diſcurrendi normâ, aliud ſcientiæ ipſius naturam ac indolem perſpectam habere, in eius ſe adyta penetrare, & ab vniuerſalibus inſtructum eſſe præceptis, quibus Theoremata ac Problemata innumera excogitandi, eademque demonſtrandi facilitas comparetur. Vt enim Pictorum vulgus prototypon ſæpè ſæpius exprimendo, quemdam pingendi vſum, nullam verò pictoriæ artis, quam optica ſuggerit ſcientiam adquirit, ita multi lectis Euclidis & aliorum Geometrarum

rum libris, eorum imitatione fingere propoſitiones aliquas ac demonſtrare ſolent, ipſam tamen ſecretiſſimam difficiliorum Theorematum ac problematum ſoluendi methodum prorſus ignorant. Antiqui ſanè Analyticen ſubtiliter inuenerunt, de qua libro ſeptimo diſertiſſimè Pappus, cuius beneficio nonnulla illius artis, ad eamque ſpectantium librorum rudera ad nos peruenêre, quæ iam viri doctiſſimi inſtauratum eunt; ſed quæ in hanc rem à nobis ſunt excogitata additaque ſuo tempore dabimus, pleraque tamen in ſubiecta materia, velut apertiſſima exercitationis arena expreſſa fuerant. Plurium adhuc magnitudinum de quibus nemo hactenus, centra grauitatis determinaui ſenſim edenda, quæ ſimul dare viſum non fuit, tum vt explorarem, quis de his ſpeculationibus doctorum virorum futurus ſit ſenſus, tum quod antiquorum more librum vno ſubiecto conſtare debere exiſtimem (quale ſunt circulus & ellipſis eiuſdem omnino eſſentiæ figuræ) nec quidquam adſtruendum putem, quod velut adſcititium ad fucum vel molem faciendam referri poſſit. Idcirco quantum nonnulli operæ ponunt vt ſcriptiones diducant, tantum mihi laborandum fuit, vt ampliſſimum argumentum in has anguſtias cogerem. Quod ſi viris ſcripſiſſem doctis, arctius contraxiſſem, ſed vel à leuiter geometriâ tinctis volui hæc percipi poſſe. Ideo ſicut pleraque accuratè ſunt demonſtrata, ita nimios ſcrupulos, ne elementa viderer ſcribere negligendos duxi.

IOAN-

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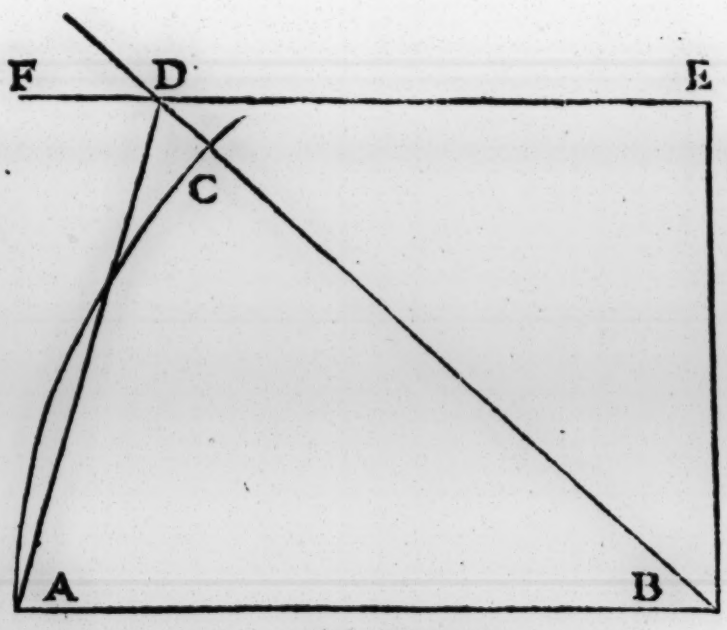
THEOREMATA

DE CENTRO GRAVITATIS PARTIVM CIRCVLI ET ELLIPSIS.

PROPOSITIO I. THEOREMA I.

Dato quolibet Sectore circuli, vnoq; eius latere producto, possibile est à termino alterius lateris, lineam ducere ad productum latus, vt illud, quod fiet triangulum rectilineum, Sectori sit æquale.

SIT datus circuli Sector A B C.



Dico posse duci A D, concurrentem cum latere B C in D, vt triangulum A B D æquale sit Sectori A B C.

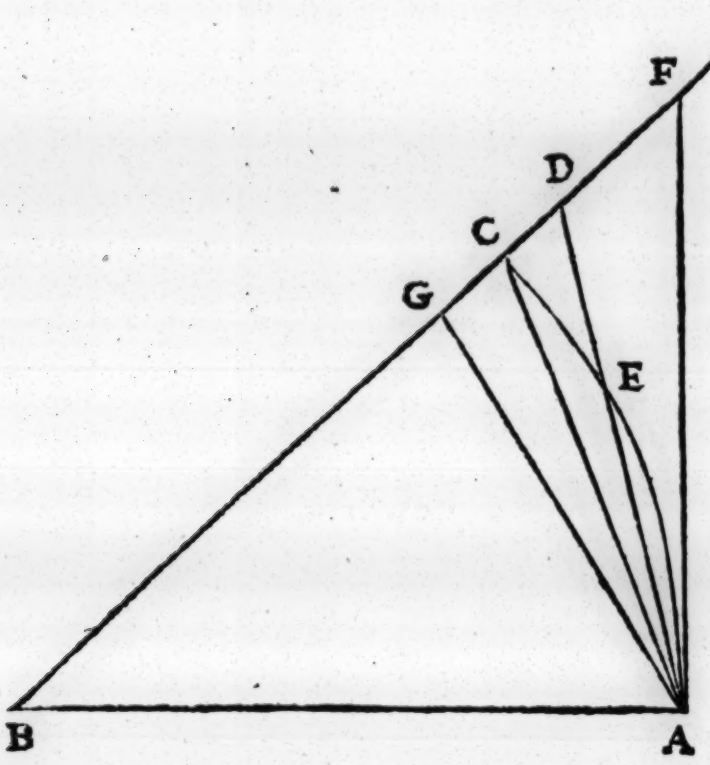
Nam per ea quæ ab Archimede demonstrata sunt, in libro de lineis spiralibus, possibile est arcui A C dare lineam rectam æqualem; sit illa B E, ducta perpendiculariter ad A B, ducatur E F parallela B A, secans B C in D, ducaturque A D.

Cum trianguli A B D altitudo sit æqualis arcui Sectoris, & basis A B æqualis semidiametro, erit triangulum æquale Sectori.

Igitur dato quolibet Sectore, &c. quod fuit demonstrandum.

PROPOSITIO II. THEOREMA II.

Si à latere Sectoris circuli, triangulum Sectori æquale descriptum fuerit vt prius, alterum trianguli latus circa communem cum Sectore angulum, Sectoris latere maius erit; & tertium latus eius arcum secabit.



SIt triangulum A D B Sectori B A E C æquale.

Dico latus B D maius esse latere B A.

Si enim latus B D non sit maius, cadet punctum D vel in punctum C, vel infra C in G; & vtroque casu debebit triangulum A D B, quod est pars Sectoris, toti Sectori esse æquale, quod fieri nequit, cadet ergo D vltra C.

Dico præterea lineam A D, secare arcum Sectoris in aliquo puncto E.

Ducatur enim A F tangens circulum in A.

Cum totus Sector B A E C, cadat intra triangulum A F B, erit illud maius Sectore, ergo linea A D quæ aufert triangulũ Sectori æquale, cadet intra A F, sed etiam vltra A C subtensam, vt iam ostensum est, ergo secabit arcum in aliquo puncto E.

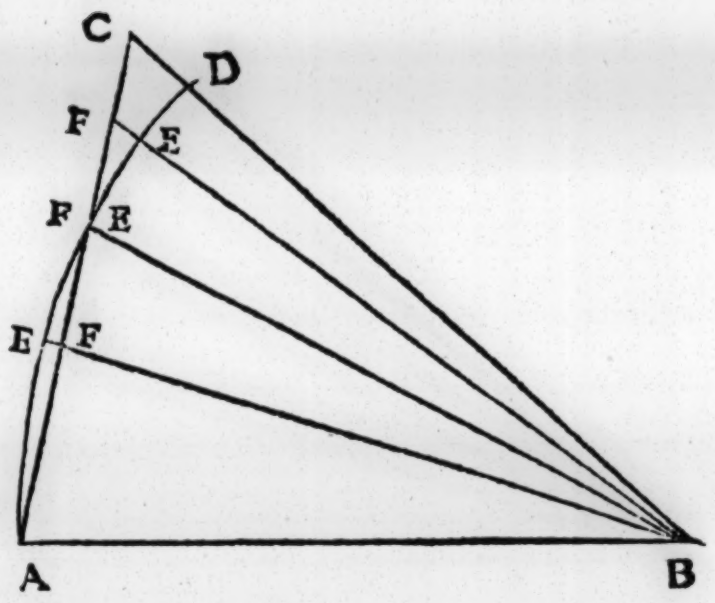
Igitur si à latere sectoris circuli, &c. quod demonstrare oportuit.

PROPOSITIO III. THEOREMA III.

Si triangulum quodpiam fuerit æquale Sectori circuli, à basi trianguli tamquam semidiametro, descripto vt supra; linea è centro Sectoris educta, arcum & oppositum trianguli latus ita secabit, vt minor proportio sit partis lateris

teris baſi vicinioris ad reliquam, quam partis arcus baſi vicinioris, ad reliquam.

SIt triangulum A C B æquale Sectori B A D, & ex centro B quæcumque linea ſecet arcum A D in E, & latus oppoſitum in F.



Dico minorem eſſe proportionem lineæ A F ad F C, quam arcus A E ad E D.

a Cum linea A C ſecet arcum A D in aliquo puncto, poteſt linea B F cadere in illud punctum, vel ſupra vel infra; quomodocumque vero cadat triangulum A F B, inter lineam B F & baſim poſitum, ſemper minus erit Sectore B A E, itidem inter baſim & lineam B E conſtituto, quod primùm demonſtro.

a 2. *huius.*

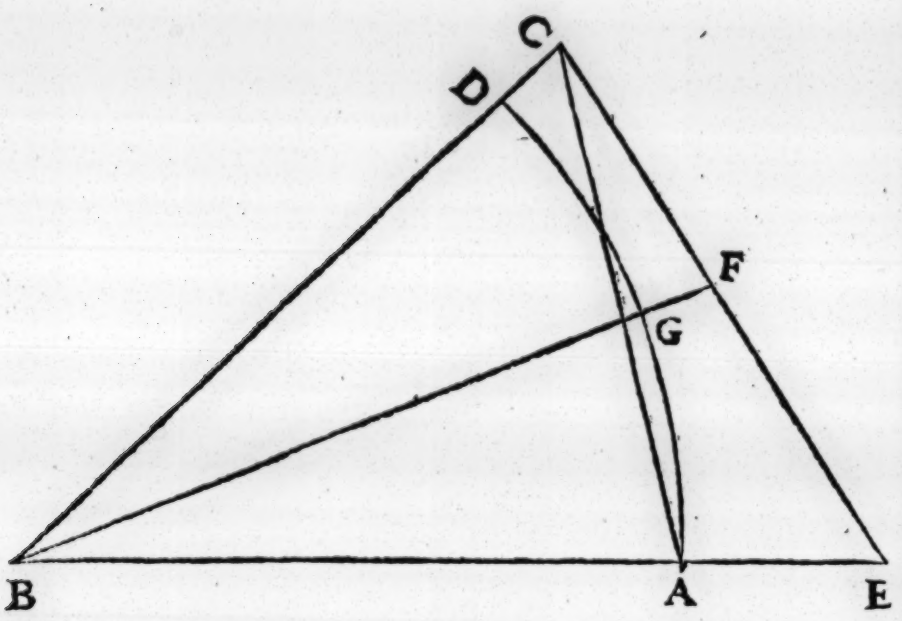
Cadat linea in punctum vbi A C arcum A D interſecat, manifeſtum eſt triangulum A F B, Sectori B A E inſcriptum, Sectore minus eſſe; ſi cadat infra interſectionem, cum B E ante ſecet lineam A C quam arcum, iterum conſtabit triangulum A F B, Sectore B A E minus eſſe. Si denique cadat ſupra interſectionem, cum Sector B E D iam cadat intra triangulum B F C, erit minor triangulo B F C, ſed totum triangulum A C B eſt æquale toti Sectori B A D, pars trianguli ſcilicet B F C eſt maior Sectore B E D, ergo reliqua pars trianguli, ſcilicet A F B, minor erit reliquâ parte Sectoris B A E, atque ita ſemper triangulum A F B, minus eſt Sectore B A E.

Hoc poſito minor erit proportio trianguli A F B ad Sectorem A E B, quam trianguli B F C ad Sectorem B E D; & permutando minor quoque proportio trianguli A F B ad triangulũ B F C, quam Sectoris B A E ad Sectorem B E D; ſed vt ſunt triangula, ita ſunt lineæ A F, F C, & vt ſe habent Sectores, ita ſe habent arcus A E, E D, ergo etiam minor erit proportio lineæ A F ad F C, quam arcus A E ad arcum A D.

Ergo ſi triangulum quodpiam fuerit, &c. quod propoſitum fuit demonſtrare.

PROPOSITIO IV. THEOREMA IV.

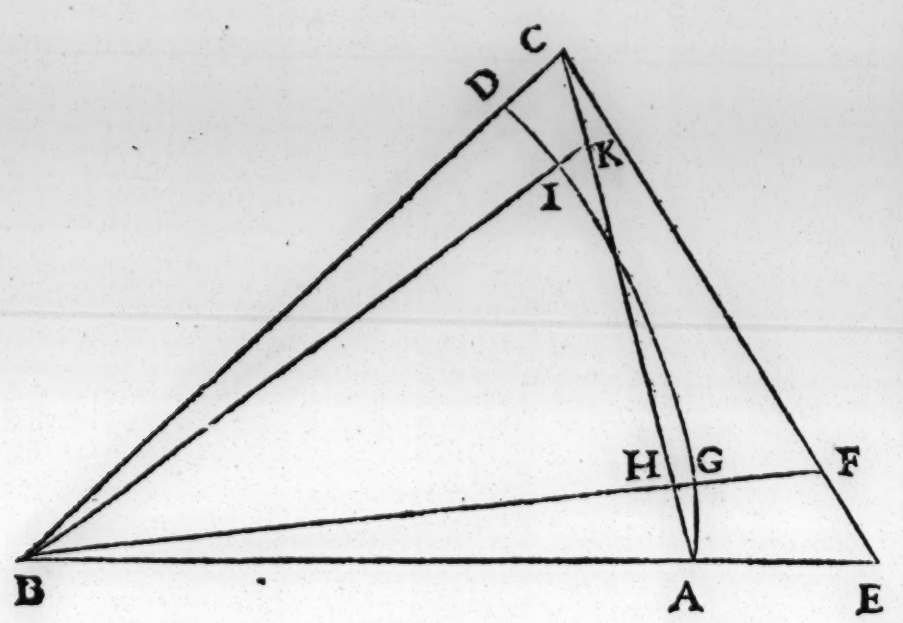
Si triangulum quodpiam vt ſupra, Sectori circuli æquale deſcriptum fuerit, & à trianguli vertice ducatur linea in baſim productam, triangulum priori ſimile conſtituens; quæ è centro euocabitur linea, ita ſecabit iam ductam lineam, vt pars inter baſim & lineam ductam, ad reliquam maiorem habeat proportionem, quam arcus baſi & ductæ iam lineæ interiectus, ad reliquum.



SIt triangulũ A B C æquale Sectori A B D, ducaturq; C E, vt triangulum A C B ſimile ſit triangulo E C B, item aliqua ex centro linea B F, ſecans vtcumq; E C in F, & arcũ A D in G.

Dico maiorem eſſe proportionem E F ad F C, quam ſit arcus A G ad G D.

Diuidat primò B F arcum A D bifariam in G, erit ſicut B E ad B C, ita E F ad F C; ſed E B maior eſt B C, cum B C maior ſit B A, & B A, B C, B E ſint tres continuæ proportionales, ob ſimilitudinem triangulorum, ergo E F maior eſt F C; ſed arcus A G æqualis eſt arcui G D, ergo maior proportio E F ad F C, quam arcus A G ad G D.



Secundò non diuidat B F bifariam arcum A D, ſed ſit verbi gratia A G arcus, minor arcu G D, & in G D ſumatur arcus D I, æqualis arcui A G, ducaturq́ue B I, quæ producta ſi opus fuerit ſecet A C in K; ſecet quoque linea B F lineam C A in H.

Cum

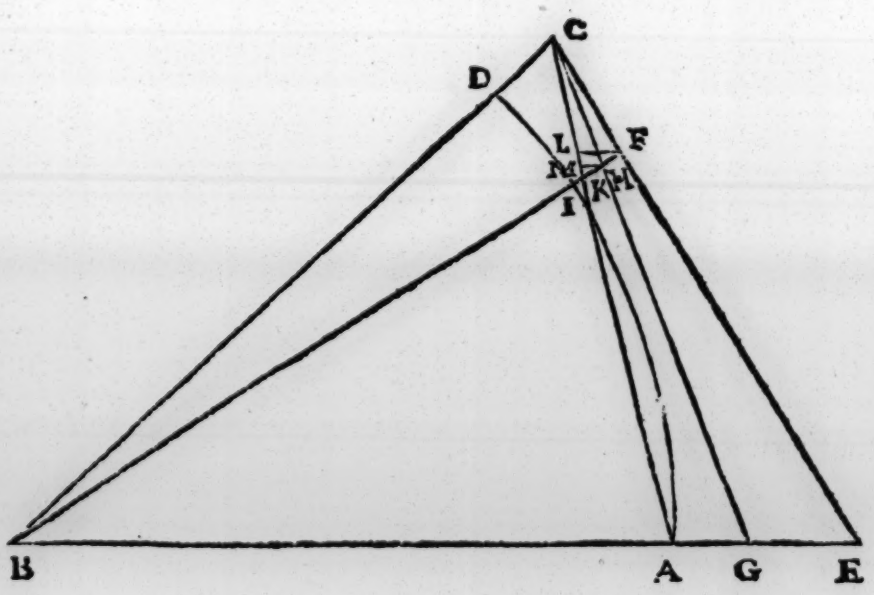
Cùm triangula B C A, B C E sint similia, & in iis similiter ductæ sint lineæ B F, B K; erit sicut C K ad K A, ita E F ad F C,[a] sed minor est proportio A K ad K C, quam arcus A I ad I D, ergo conuertendo maior erit proportio C K ad K A, quam arcus D I ad arcum I A. Sed vt C K ad K A, ita E F ad F C vt iam ostendimus, ergo maior proportio E F ad F C, quam arcus D I ad I A, id est A G ad G D. [a] 3. huius.

Idcirco si triangulum quodpiam vt supra, &c. quod fuit ostendendum.

PROPOSITIO V. THEOREMA V.

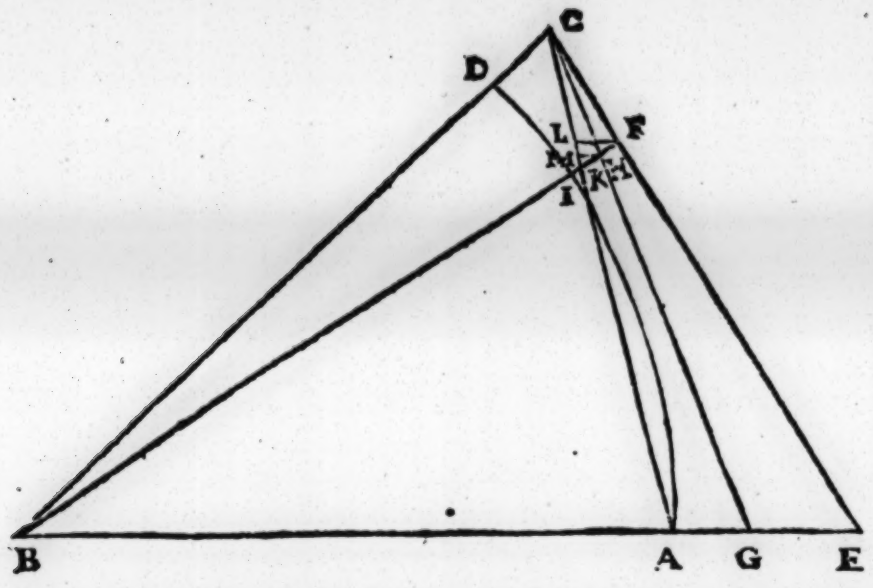
Si triangulum quodpiam vt supra, æquale Sectori circuli descriptum fuerit, & à trianguli vertice ducta linea in productam basim, triangulum priori simile constituens; si è centro euocetur linea aliqua, secans duas lineas angulo Sectoris oppositas, possibile est à vertice ducere lineam cadentem inter vtramque, quæ ab illa quæ è centro ducta est linea, secetur in eandem rationem, in quam Sectoris arcus diuiditur.

SEctori B A D æquale sit triangulum A C B, ductaque C E, vt triangula A C B, E C B inter se sint similia, è centro vtcumque ducatur B F, secans vtramque C A, C E in F & K.



Dico posse duci C G, inter C E & C A, secantem B F in H, vt sit sicut G H ad H C, ita arcus A I ad I D.

Sit enim C A secta in M, vt sicut arcus A I ad arcum I D, ita sit A M ad M C;[a] cum sit maior proportio arcus A I ad arcum I D, quam A K, ad K C, cadet punctum M supra punctum K; ducatur deinde F L parallela E B, erit A L ad L C, vt E F ad F C;[b] ergo cum maior sit proportio E F ad F C, quam arcus A I ad arcum I D, erit quoque maior pro- [a] 3. huius. [b] 4. huius.

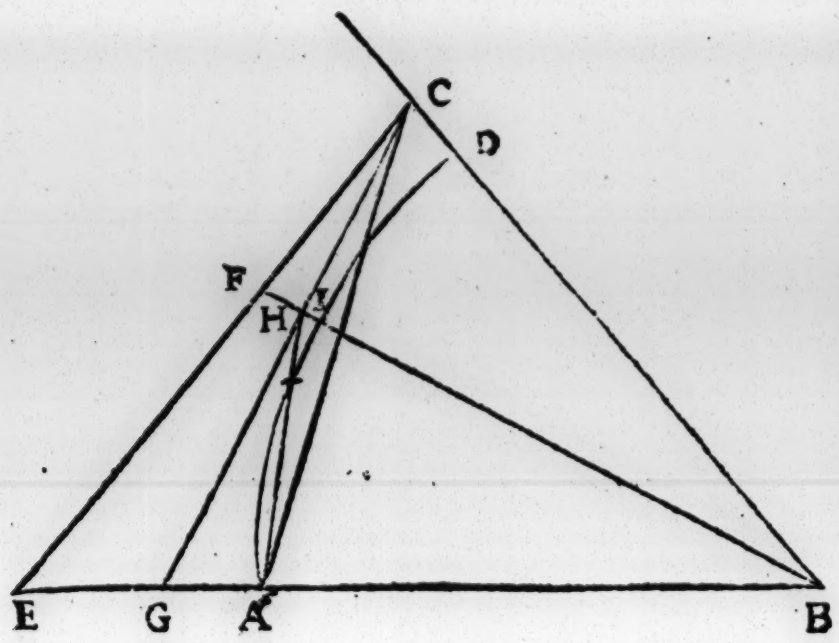


portio A L ad L C, quam arcus A I ad arcum I D, & conſequenter quam lineæ A M ad M C, cadetq; punctũ L ſupra punctum M. Ducatur M H parallela E B, ſecabit F K in aliquo puncto H, per quod ſi ex C ducatur C H G, erit G H ad H C, vt A M ad M C, id eſt vt arcus A I ad arcum I D.

Igitur ſi triangulum quod piam, &c. quod fuit demonſtrandum.

PROPOSITIO VI. THEOREMA VI.

Sectori B A D *vt ſupra æquale ſit triangulum* A C B, *& triangulum* E C B *ſimile triangulo* A C B, *ducta item vtcumque* B F, *&* C G *ſecans* B F *in* H, *ita vt ſicut angulus* G B H *ad angulum* H B C, *ita ſit linea* G H *ad* H C, *ſi ducatur* A H, *erit triangulum* A H B *Sectori* B A I *æquale.*



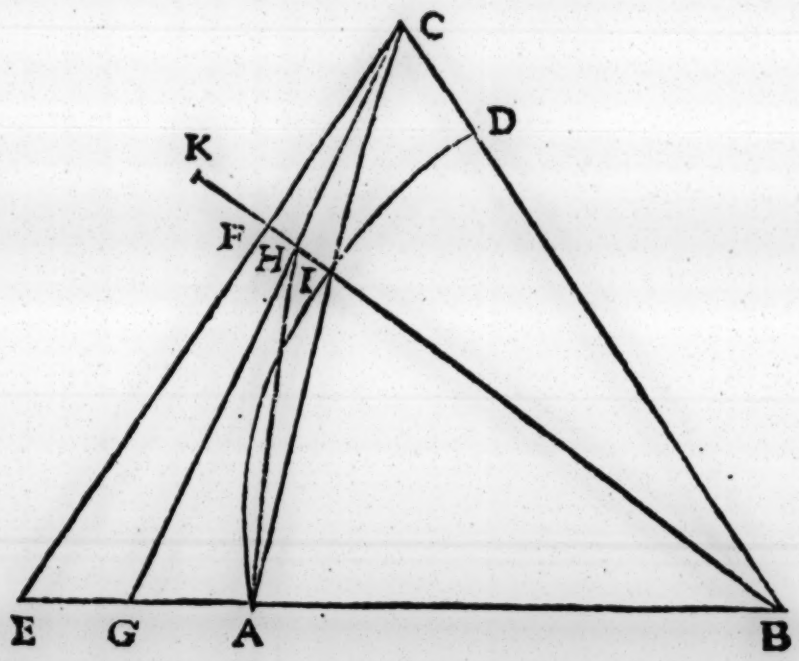
ESt enim vt linea G H ad H C, ita triangulum G H B ad triangulum H C B; & componendo vt G C ad G H, ita triangulum G C B ad G H B; ſed ita etiam ſe habet triangulũ G C A ablatum à maiori G C B, ad triangulum G H A ablatum à minori G H B, ergo reliquum A C B ad A I B, vt totum ad totum, id eſt vt C G ad H G; & vt arcus

cus AI ad arcum ID, ita GH ad HC, ergo componendo vt arcus AD ad arcum AI, ita GC ad GH, ergo Sector ADB ad Sectorem AIB, (qui ſe habent vt arcus) vt GC ad GH, ergo & Sector BAD ad Sectorem BAI, vt triangulum ACB ad triangulum AIB; ergo permutando vt Sector BAD ad triangulum ACB, ita Sector BAI ad triangulum AHB, ſed Sector ADB eſt æqualis triangulo ACB, ergo Sector BAI æqualis eſt triangulo AHB.

Igitur Sectori BAD vt ſupra æquale ſit, &c. quod oſtendere propoſitum fuit.

PROPOSITIO VII. THEOREMA VII.

Iiſdem poſitis quæ in præcedenti propoſitione; ſi fiat vt HB *ad* BC, *ita* BC *ad* BK, *Dico* BK *minorem fore* BE.



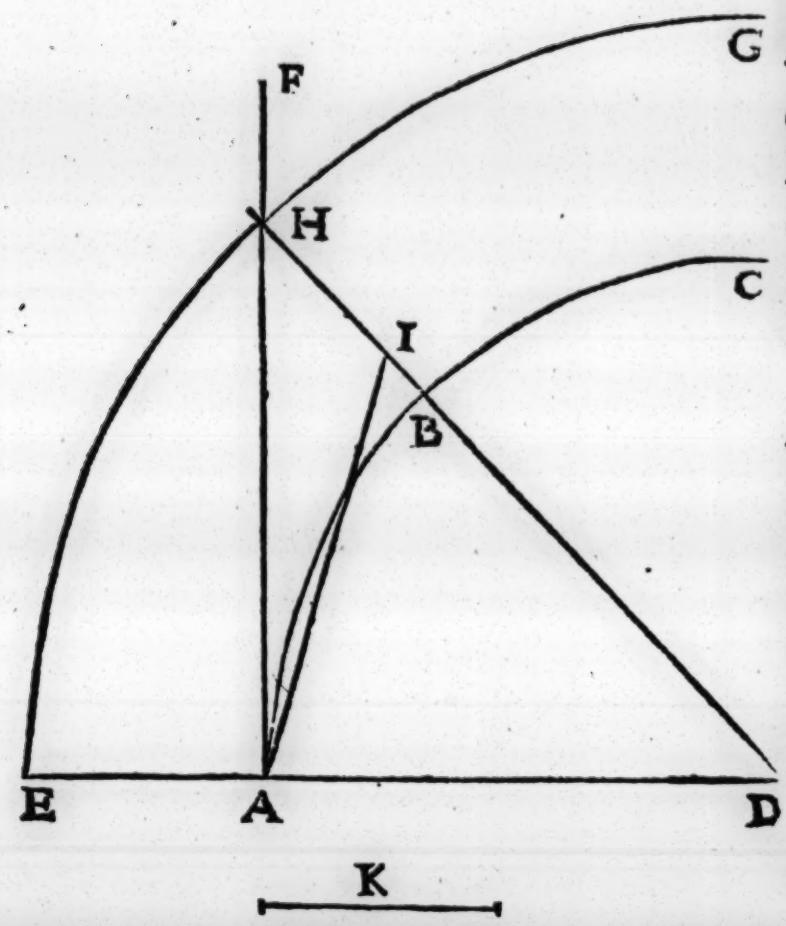
CVm enim ob ſimilitudinem triangulorum ECB, ACB, ſit vt AB ad BC, ita BC ad BE, rectãgulum ſub BA, BE, æquale erit quadrato BC; ſimiliter rectangulum ſub BH, BK, æquale erit quadrato BC, ergo rectangulum ſub BH, BK, rectangulo ſub BA, BE æquale erit; ergo vt BA ad BH, ita erit BK ad BE,[a] ſed AB eſt minor BH, ergo BK minor erit BE. [a] 2. *huius.*

Ergo iiſdem poſitis quæ in præcedenti propoſitione, &c. quod oportuit demonſtrare.

PRO-

PROPOSITIO VIII. PROBLEMA I.

Dato quolibet circulo, Sectorem assignare, cui si descriptum fuerit æquale triangulum vt supra, alterum trianguli latus circa communem angulum, superet latus Sectoris, interuallo quod minus sit quolibet dato.



DAtus sit circulus A B C, in eoq; designandus sit Sector, cui si æquale sit triangulum A I D, linea D I quę maior erit quam A D, superet A D interuallo, quod sit minus quolibet dato K.

Producatur D A, vt A E sit æqualis K, centroque D interuallo D E, describatur circulus E H G, & ex A erigatur perpendicularis A F, secans circulum E H G in H, ducaturq; H D, secans priorem circulum in B.

Dico Sectorem D A B, esse eum qui postulatur.

Cum enim A H tangat Sectorem in A, cadet A I intra A H, [a] secabitque D H, vltra B quidem sed intra H, igitur B I qui est excessus D I supra D B, minor erit B H, cum autem D E, D H sint æquales, item & ablatæ D A, D B, erunt reliquæ A E, B H æquales; sed E A ex constructione est æqualis K, ergo & B H erit æqualis K, ergo & B I minor K.

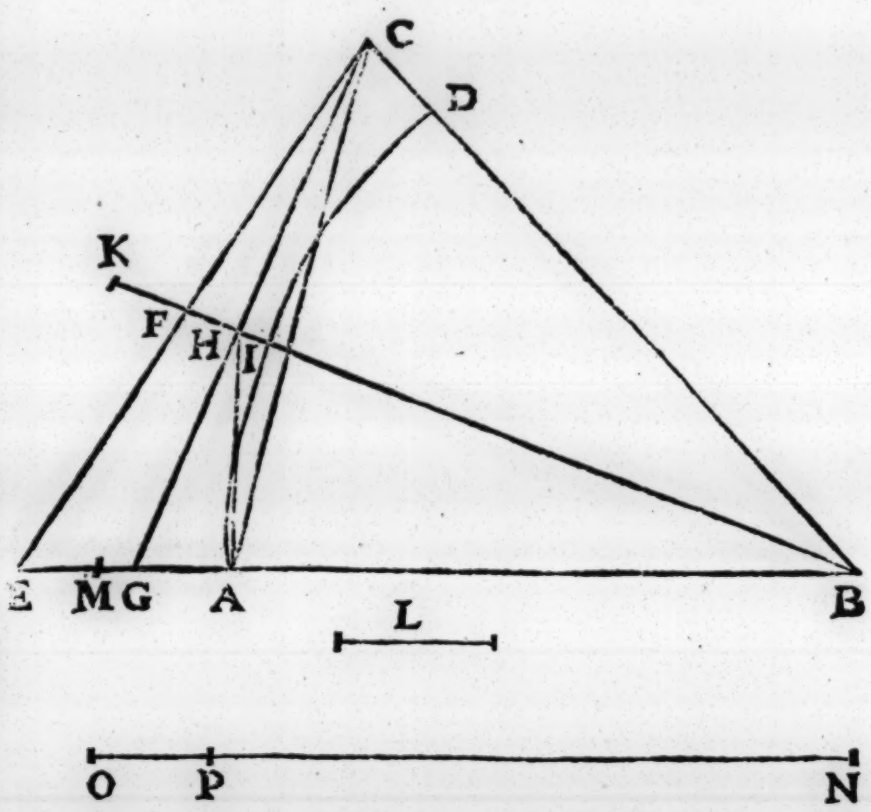
a 2. huius.

Itaque dato quolibet circulo, &c. quod fuit faciendum.

PRO-

PROPOSITIO IX. THEOREMA VIII.

Triangulum ABC *vt ſupra, æquale ſit Sectori* ADB, *& triangula* ABC EBC *inter ſe ſimilia, Dico ita duci poſſe* BF, *vt ductâ* CG, *quæ ita diuidatur in* H, *vt arcus* AD *in* I, *& ſumptâ duabus* BH, BC, *tertia proportionali* BK, *ſuperetur* BK *à linea* BE, *interuallo quod ſit minus quolibet dato* L.



AVferatur enim EM ab EA, quæ ſit minor dato interuallo L, & duabus lineis BM, BC, fiat tertia proportionalis NO, erit NO maior BA.

Cum enim BC ſit media inter EB, AB, item inter MB, NO, erunt rectangula EB, AB, & MB, MO, quadrato BC æqualia; & conſequenter inter ſe, eritque vt EB ad MB, ita NO ad AB, ſed EB maior eſt MB, ergo & NO maior AB.

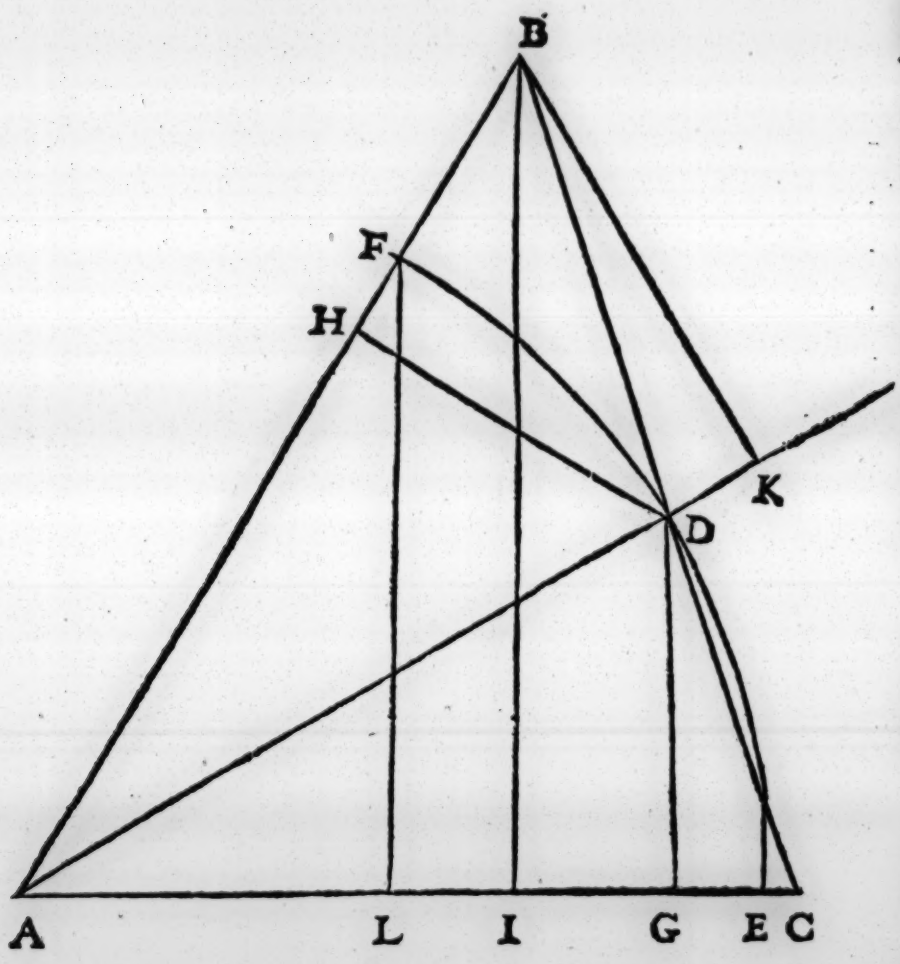
Auferatur deinde ex NO linea NP æqualis AB,[a] ſitque inuentus Sector AIB, vt poſito triangulo AHB, æquali Sectori AIB, linea HB ſuperet AB, minori interuallo quam NO ſuperat NP, id eſt AB; ductaque ſit CHG, & fiat vt BH ad BC, ita BC ad BK, ſuperabit BE lineam BK minori interuallo, quam ſit datum interuallum L. a 8. *huius.*

Cum enim BC ſit media inter BH BK, erit rectangulum ſub BH, BK, rectangulo ſub NO, BM æquale; ergo vt BH ad NO, ita BM ad BK; ſed BH minor eſt quam NO (cum ex conſtructione BH minori interuallo ſuperet NP, quam NO ſuperet eandem NP) ergo etiam BM minor eſt BK; ſed BK minor eſt BE, cum ergo BK oſtenſa ſit minor BE, & maior BM, ſuperabit BE lineam BK minori interuallo, quam ſit EM quo BE ſuperat BM, ſed EM minor eſt dato interuallo L, ergo BE ſuperat BK interuallo minori, quam ſit datum L.

Quapropter ſi triangulum ABC, &c. quod fuit demonſtrandum.

PROPOSITIO X. THEOREMA IX.

Si tres lineas ab vno puncto egredientes, & duos inæquales angulos comprehendentes, secuerit quarta quædam linea hoc pacto, vt partes quæ subtendunt angulos, eandem inter se proportionem habeant, quam anguli, media abscissarum erit minima, illa maior quæ minorem angulum claudit, maxima denique quæ cum media maiorem angulum comprehendit.



EX puncto A tres lineæ egrediantur, duos inæquales angulos comprehẽdentes, D A B maiorem, D A C minorem; illasq; secet linea C B hoc pacto, vt B D ad D C eandem habeat proportionem, quam angulus D A B, ad angulum D A C.

Dico primo D A mediam abscissarum, esse minorẽ C A, quæ cum illa angulum claudit.

Si enim D A non sit minor C A, erit æqualis vel maior; & primùm ponatur esse æqualis.

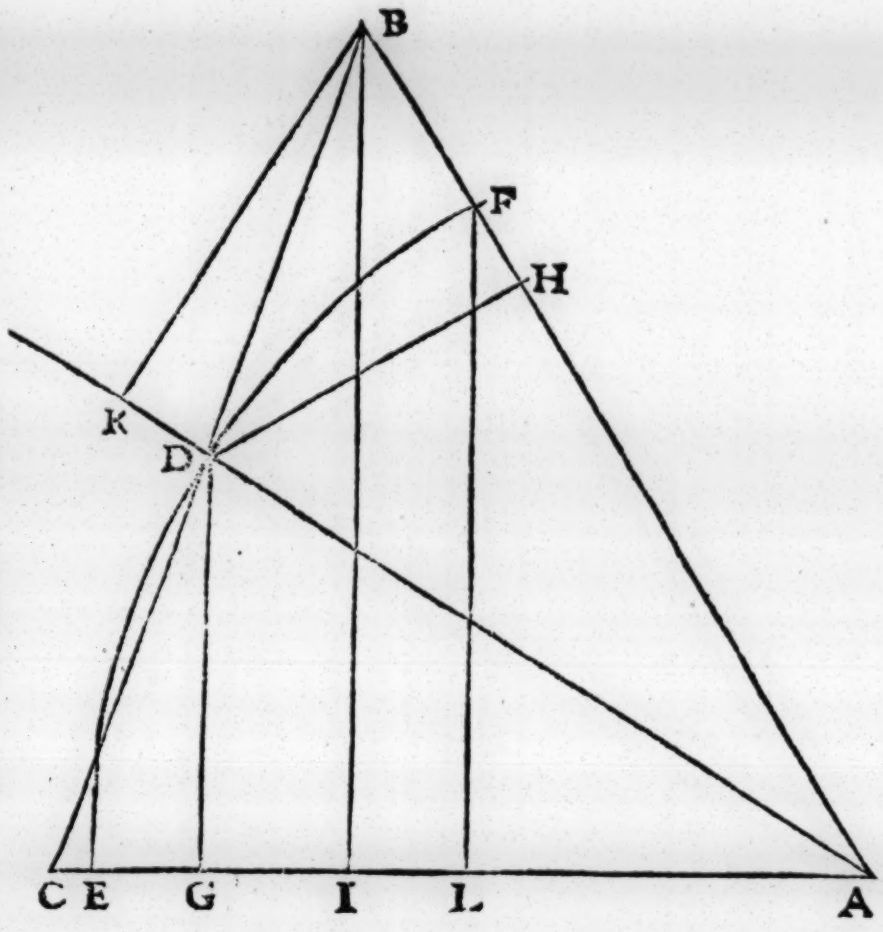
Centro A interuallo A D, describatur arcus E D F, ducanturque D H, D G, perpendiculares ad A B, A C; item B I, F L perpendiculares ad A C; & cùm angulus C D A sit acutus, vtpote angulus ad basim trianguli isosceli, non erit B D perpendicularis lineæ A D, ideoque dimittatur B K perpendicularis in A D productam, cadet enim extra B D, cùm angulus B D K sit acutus, & ad verticem vni illorum, qui sunt ad basim isosceli.

Cùm ex hypothesi A C, A D sint æquales, sintque bases triangulorum C B A, D B A, erunt illa triangula vt altitudines B I, B K; sed triangula sunt etiam vt B C ad B D, ergo erit etiam B I ad B K, vt B C ad B D,

sed

ſed cùm ſit vt B D ad D C, ita angulus B A D ad angulum D A C, ſeu quod idem eſt arcus F D ad arcum D E, erit etiam componendo vt B C ad D B, ita arcus F D E ad arcum D F, & conſequenter B I ad B K, vt arcus F D E ad arcum D F.

Iterum vt A B ad A F, ita B I ad F L, & vt B A ad A D, id eſt A F ita B K ad D H (ſunt enim triangula A B K A D H ſimilia) ergo vt B I ad F L, ita B K ad D H, ergo permutando vt B I ad B K, ita F L ad D H. ſed B I eſt ad B K, vt arcus F D E ad arcum D F, ergo etiam F L ad D H, vt arcus F D E ad D F,[a] quod eſt abſurdum; non igitur ſunt æquales A D, A C, ex quo hoc abſurdum conſequebatur.



a *Clauius de ſinibus Propoſ. 10. demonſtrat de arcubus & eorum chordis quod hîc in dimidiis arcubus & eorum chordis ſumitur.*

Sit ſecundò C A minor quàm A D, non poterit angulus C D A eſſe rectus, cùm enim latus D A ſit maius ex ſuppoſitione quàm C A, erit angulus D C A, maior angulo C D A, non erit igitur rectus C D A; demiſſa ſit perpendiculans B K in A D, quomodocumque cadat; cùm triangulum C B A, habeat baſim A C minorem A D baſi trianguli D B A, minor erit proportio trianguli C B A ad triangulum D B A, quàm altitudinis B I ad B K, (ſi enim trianguli C B A baſis eſſet æqualis, triangulum eſſet maius, haberetque eamdem proportionem ad triangulum D B A, quam altitudo ad altitudinem) ergo minor proportio B C ad B D, quàm B I ad B K, cùm B C ſit ad B D, vt triangulum ad triangulum, & conſequenter vt arcus F D E ad D F, qui ſunt vt B C ad B D; ſed F L eſt ad D H, vt B I ad B K, vt oſtendimus, ergo minor proportio arcus F D E, ad arcum F D, quàm linea F L ad D H, quod eſt abſurdum; non igitur minor eſt C A quàm A D, ex quo id conſequebatur.

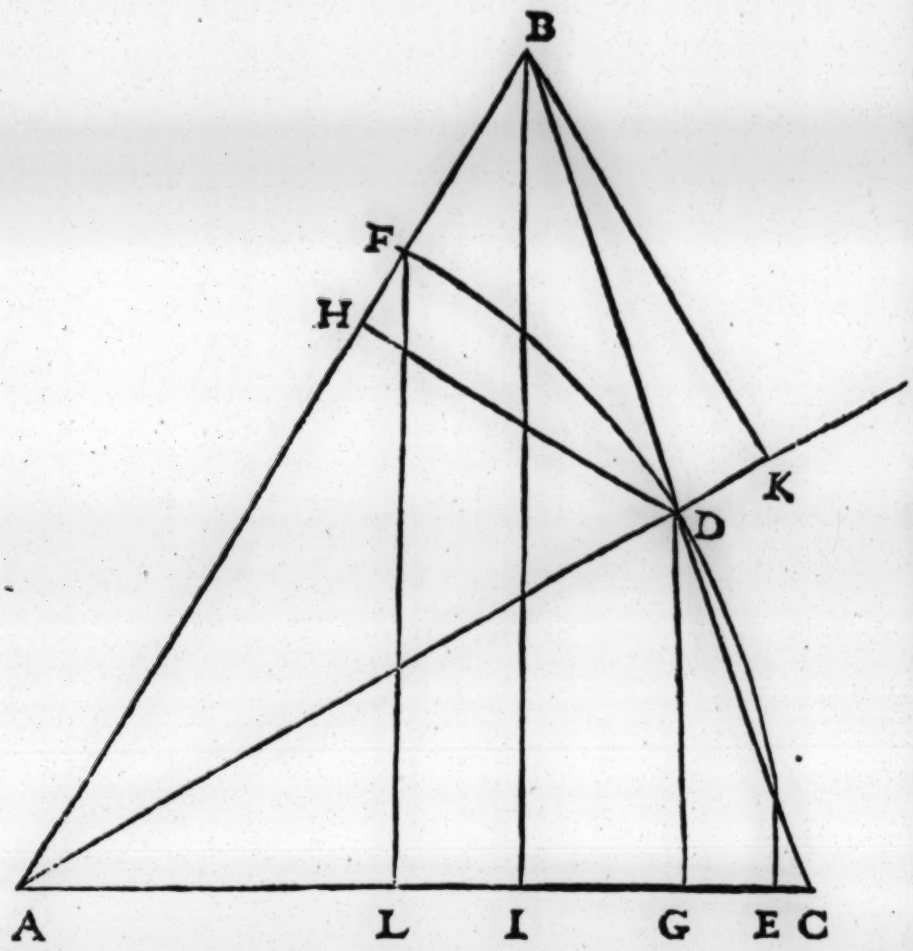
Dico ſecundò A C eſſe minorem A B.

Si hoc negetur erit vel æqualis vel maior.

Ponatur primò A C æqualis eſſe A B. Cùm in triangulis C D A, B D A, baſes A C, A B ſint æquales, erunt altitudines D G, D H, vt triangula; ſed etiam D C, D B ſunt vt triangula, & arcus D E, D F in eadem quoque

proportione; erit igit ur vt D G ad D H, ita arcus D E ad arcum D F; & permutando vt D G ad arcum D E, ita D H ad arcum D F,[b] quod eſt abſurdum; non igitur æquales erunt A B A C. è quo hoc inferebatur.

b *Clauius ibid.*



Ponatur ſecundò A B minor A C, erit minor proportio trianguli A B D ad triangulũ A D C, quàm altitudinis D H ad altitudinem D G, ergo & minor proportio B D ad D C, & conſequenter arcus F D ad arcum D E, quàm lineæ D H ad D G, & permutando minor proportio D F arcus ad lineam D H, quàm arcus D E ad lineam D G, quod eſt abſurdum; non igitur maior eſt A B quàm A C. ex quo id ſequebatur.

Ergo ſi tres lineas ab vno puncto, &c. quod fuit demonſtrandum.

PROPOSITIO XI. PROBLEMA II.

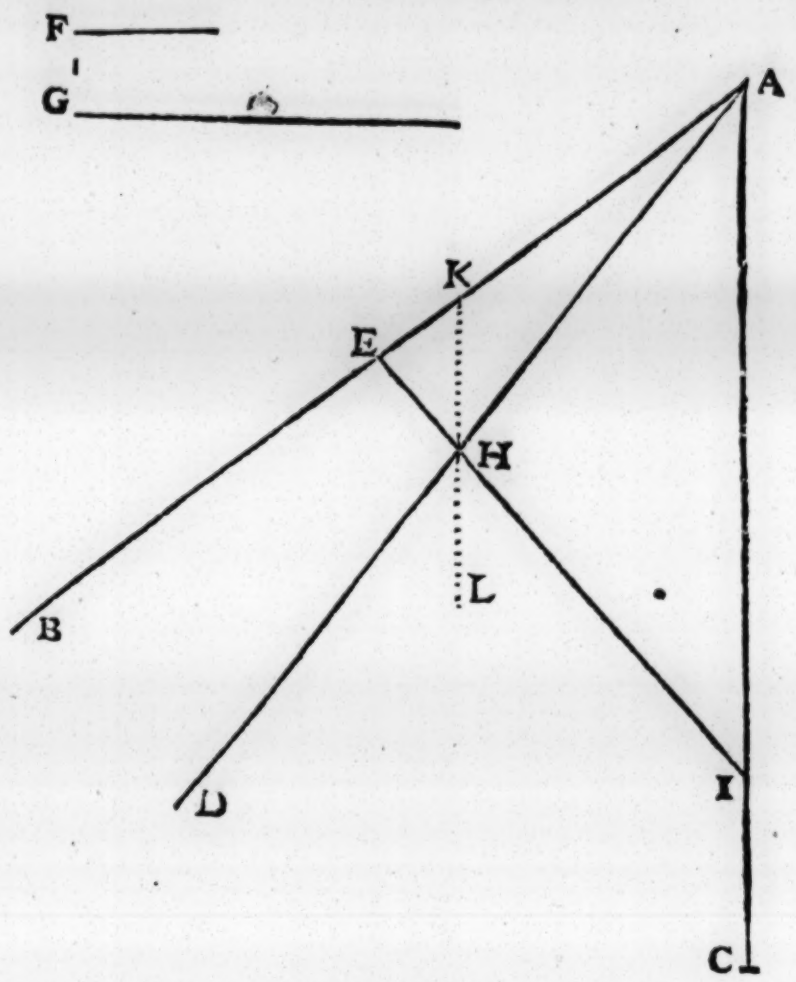
Duabus lineis indefinitis angulum quemuis comprehendentibus, eoq; per alteram indefinitam vtcumque diuiſo, ab aſſignato puncto in alterutra extremarum lineam ducere, terminatam ad alteram extremarum, quæ à media ita diuidatur, vt pars inter punctum datum & mediam intercepta, ad reliquam, datam habeat proportionem.

DVæ lineæ A B, A C, quemuis angulum comprehendant, eumque diuidat vtcumque A D. & ſit in alterutra extremarum puta A B, datum punctum E, dataque proportio F ad G.

Propoſitum eſt ducere E I, vt E H ad H I, datam habeat proportionem F ad G.

Diui-

Diuidatur E A in K, vt E K ad K A ſit vt F ad G, ducaturque K L parallela A C, quæ ſecet A D in aliquo puncto H, & per E & H ducatur linea E H I.



Dico E H ad H I eſſe, vt F ad G.

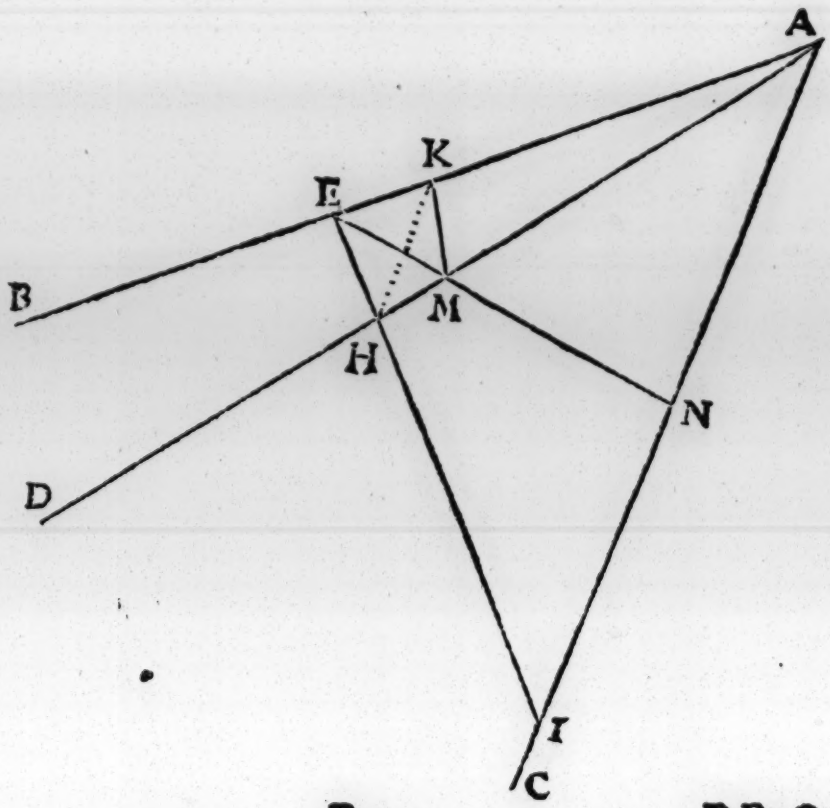
Cùm enim in triangulo A E I, linea K H ſit parallela A I, erit E H ad H I, vt E K ad K A, id eſt vt F ad G, ſic enim diuiſimus E A in K.

Igitur duabus lineis indefinitis, &c. quod fuit faciendum.

PROPOSITIO XII. THEOREMA X.

Iiſdem poſitis quæ ſuperiori problemate conſtruximus, non poterit ab eodem puncto alia duci linea, quæ ſimiliter diuidatur.

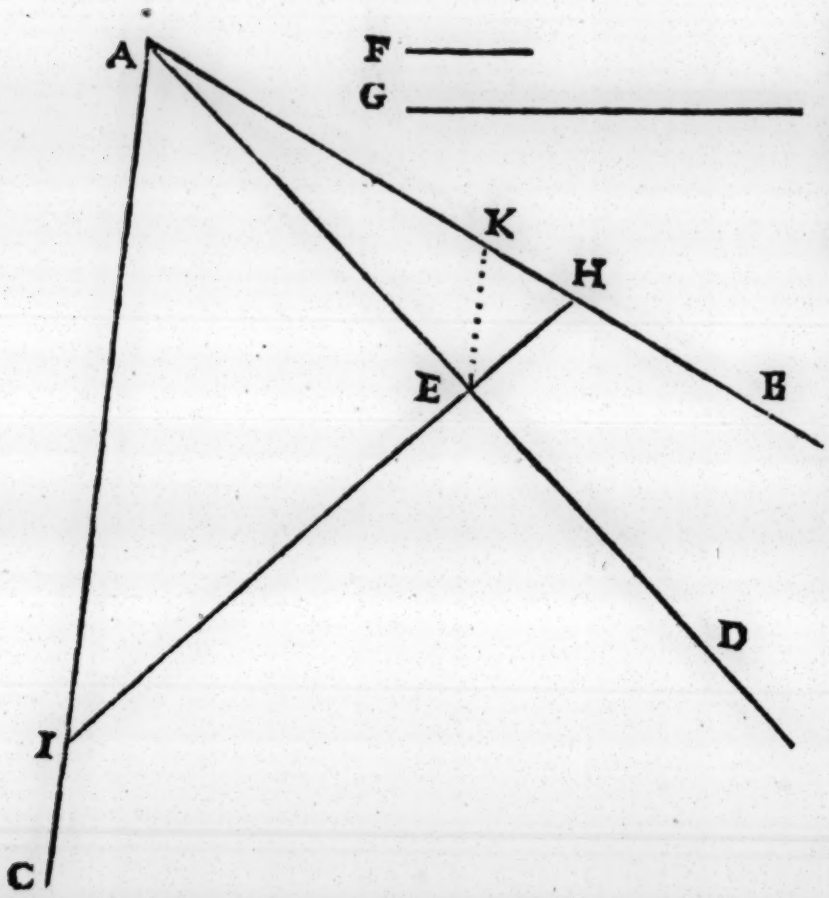
SI enim alia duci poſſit ſit illa E N, diuiſa in M, vt F ad G, erit etiam E M ad M N, vt E K ad K A, ergo K M erit parallela A C, ſed & K H eſt parallela A C, ergo K H, K M, ſunt inter ſe parallelæ, quod eſt abſurdum cùm conueniant in K; ergo non erit diuiſa E N ſimiliter vt E I, vnde id ſequebatur.



Igitur iiſdem poſitis quæ ſuperiori problemate, &c. quod oportuit demonſtrare.

PROPOSITIO XIII. PROBLEMA III.

Duabus lineis indefinitis quemuis angulum comprehendentibus, & illo vtcumque per aliam rectam diuiſo, per punctum in linea diuidente aſſignatum, lineam ducere, vtrimque ad extremas terminatam, vt pars definita ad reliquam, datam habeat proportionem.



LIneæ A B, A C, quemuis angulum comprehendant, quem diuidat vtcumque A D, in qua aſſignatum ſit punctum E.

Propoſitum eſt per E ducere lineam H I, ita vt E H ad H I datam habeat proportionem F ad G.

Ab E verſus A B ducatur linea E K, parallela A C, & fiat vt G ad F ita A K ad K H, & ducatur per H & E linea I E H;

Dico E H eſſe ad E I, vt F ad G.

Cùm enim in triangulo A I H, linea K E ſit parallela A C, erit A K ad K I, vt E H ad E I, id eſt vt F ad G. ſic enim fecimus A K ad K I.

Ergo duabus lineis indefinitis, &c. quod facere oportuit.

PROPOSITIO XIV. THEOREMA XI.

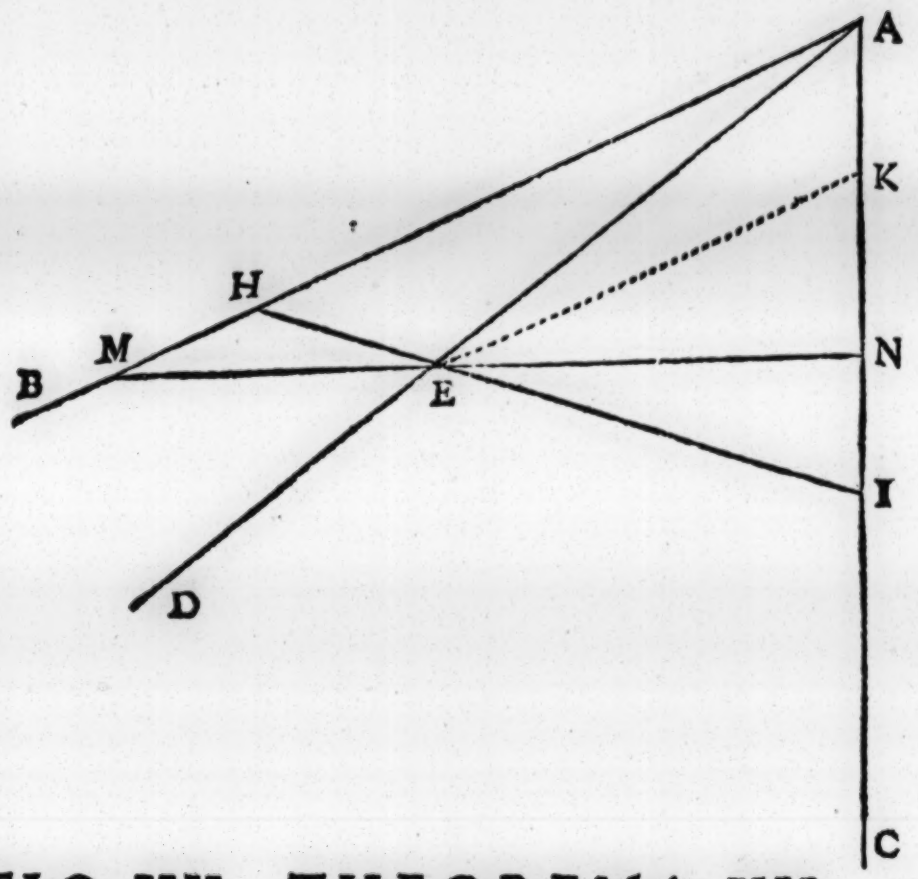
Iiſdem poſitis quæ in præcedenti problemate conſtruximus, non poterit per idem punctum alia duci linea, quæ ſimiliter poſitis partibus, ſimiliter diuidatur.

SI enim alia duci poſſit, ſit illa M N, per E tranſiens, & in E diuiſa, vt E M ſit ad E N, vt F ad G.

Cùm

Cùm in triangulo A N M, linea K E ſit parallela A M, erit vt M E ad E N, ita A K ad K N; ſed etiam ita eſt A K ad K I, quod fieri nequit, cùm maior ſit proportio A K ad K N, quàm ad K I.

Ergo iiſdem poſitis quæ in præcedenti, &c. quod fuit demonſtrandum.



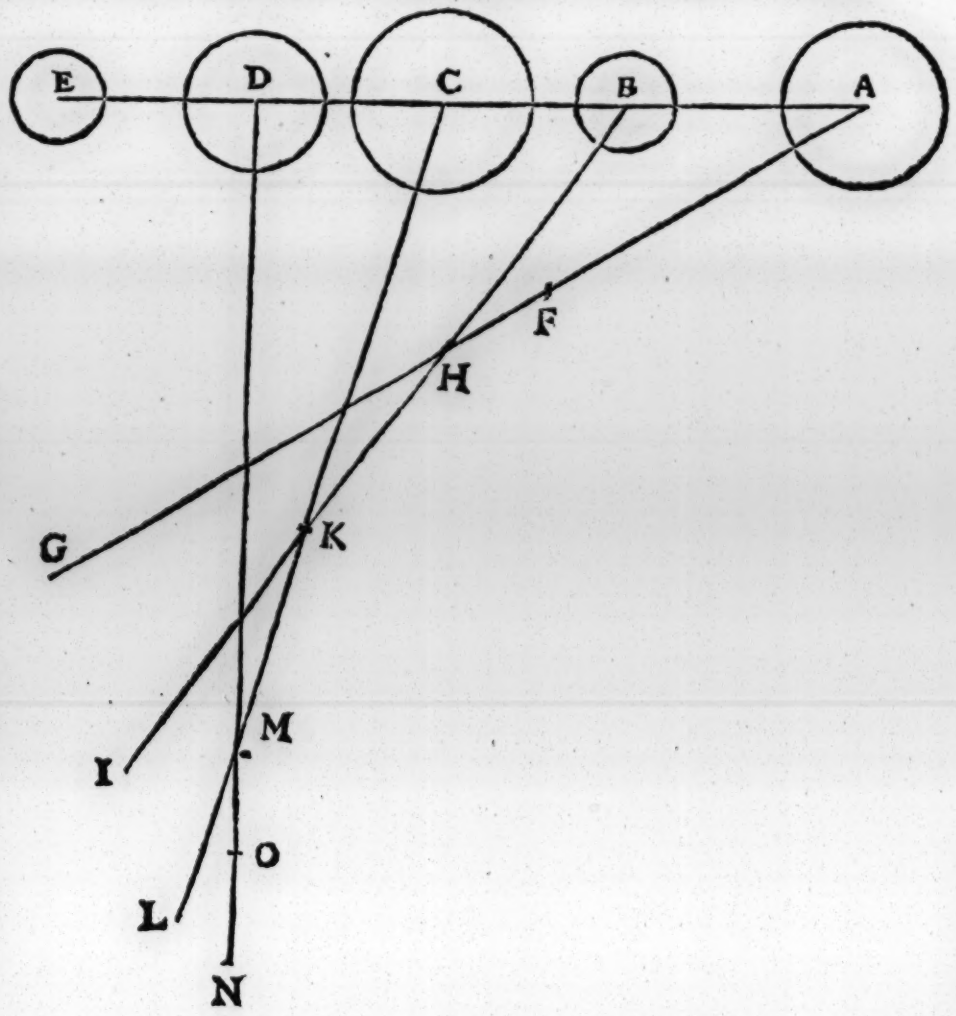
PROPOSITIO XV. THEOREMA XII.

Si quotcumque magnitudinum grauitatis centra in eadem recta linea poſita fuerint, magnitudinis ex omnibus compoſitæ grauitatis centrum, erit in eadem linea recta.

SInt quotcumque magnitudines A, B, C, D, E, earumque cẽtra grauitatis per eaſdem litteras ſignata, in eadem linea rectè poſita.

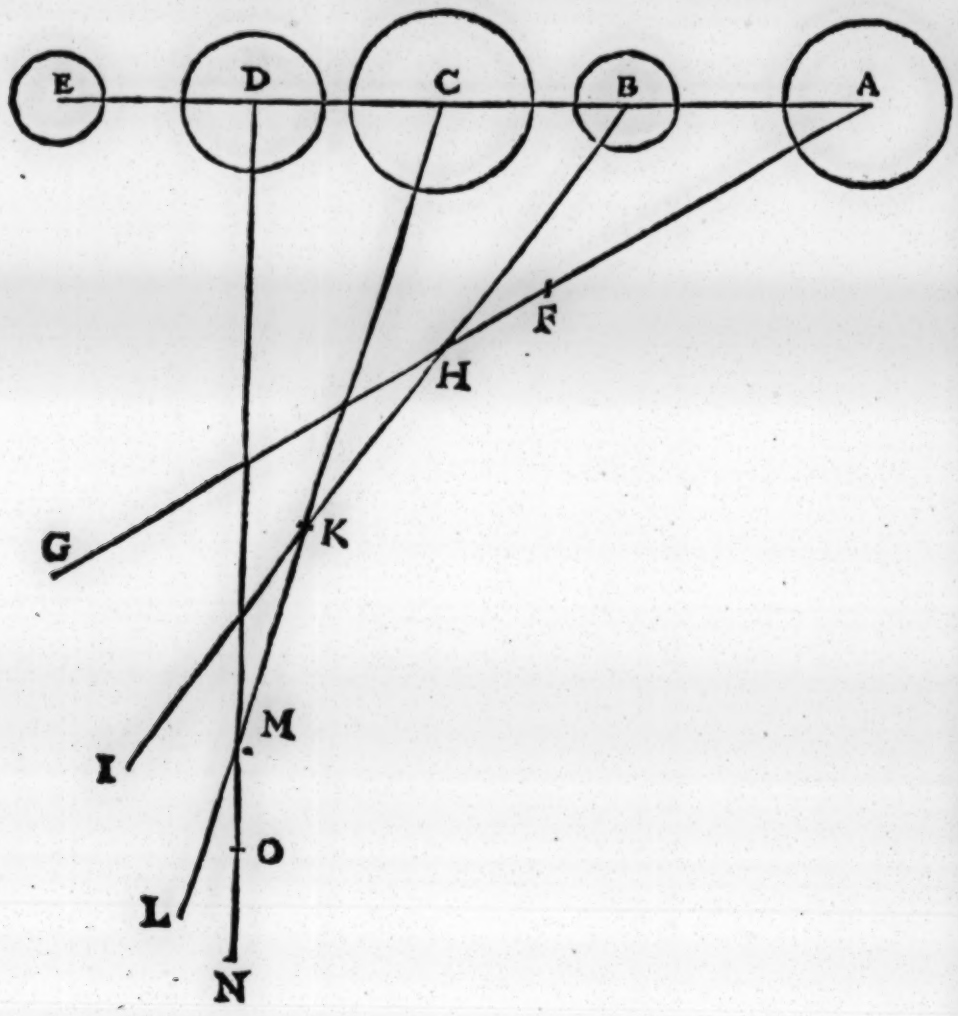
Dico commune grauitatis centrum in eadem recta reperiri.

Si enim fieri poſſit, ſit extra lineam in F, & per primæ magnitudinis centrum A, & F ducatur A F G,[a] erit in illa centrum grauitatis reliquarum magni-



[a] 8. *primi Archimedis de aquiponderantibus.*

tudi-



tudinum demptâ A, tamquam vnius, & reliquę partis, vltra punctum F in H. quod etiam erit extra lineam singularum centra continentem. Ducatur iterum per secundę magnitudinis centrum B, & punctum H, linea B H I; cùm H sit centrum totius reliquæ compositæ ex B, C, D, E, & B centrum partis, [b] erit in B I centrum reliquæ compositæ ex C D E, vltra punctum H in K.

b 8. *primi Archim. de æquip.*

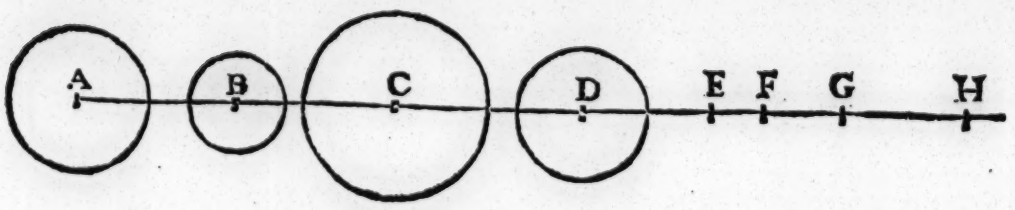
atque ita semper progrediendo, donec ad vltimam peruentum fuerit, reperietur centrum vltimæ magnitudinis E, in puncto aliquo O, extra lineam per singularum centra transeuntem, sed etiam E est centrum grauitatis vltimæ magnitudinis, ergo duo habebit centra grauitatis, [c] quod fieri nequit. Non erit itaque commune grauitatis centrum, extra lineam per omnium centra transeuntem, ex quo id impossibile sequitur.

c 1. *postulatum Lucæ Valerij de centro grauit.*

Quapropter si quotcumque magnitudinum, &c. quod fuit ostendendum.

PROPOSITIO XVI. THEOREMA XIII.

Si quotcumque magnitudinum grauitatis centra in eadem recta linea posita fuerint, magnitudinis ex omnibus compositæ grauitatis centrum, erit in eadem linea, intra centra extremarum magnitudinum.



SInt quotcumque magnitudines A, B, C, D, earumq; grauita-

uitatis centra, per eaſdem litteras ſignata, ſint in eadē linea recta A B.

Dico centrum grauitatis commune, intra centra A D extremarum magnitudinum, in eadem linea recta reperiri.

Si enim non ſit intra centra extremarum magnitudinum, cùm neceſſariò ſit in linea A B, erit in illa producta; ſit verò punctum E, nec refert verſus quam partem.

Cùm centrum totius ſit E, & vnius partis puta proximæ ſit D, centrum reliquæ, id eſt magnitudinis compoſitæ ex C, B, A,[a] erit vltra E in F. Similiter ſumpta magnitudine ex A, B, C, compoſita, cùm eius centrum ſit F, extra centra extremarum A, C, conſtitutum, proximæ verò partis centrum ſit C,[b] erit centrum reliquæ vltra F in aliquo puncto G, ac tandem reperietur centrum vltimæ magnitudinis in H, ſed illius centrum erat A, ergo duo habebit centra grauitatis,[c] quod eſt abſurdum: non erit itaque commune centrum totius compoſitæ magnitudinis, extra centra extremarum, ex quo id ſequitur.

Ideo ſi quotcumque magnitudinum, &c. quod oſtendere propoſuimus.

a 8. *primi Archim. de æquip.*

b 8. *primi Archim. de æquip.*

c 1. *poſtulatum Lucæ Valerij de centro grauit.*

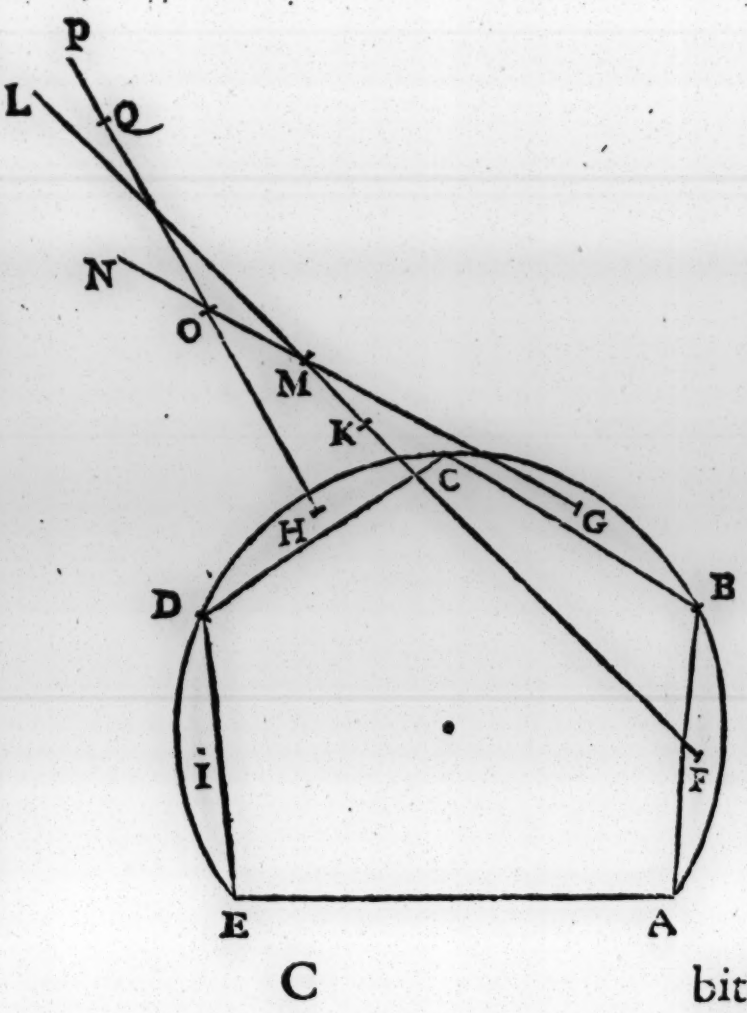
PROPOSITIO XVII. THEOREMA XIV.

Si ſegmento circuli vel ellipſis inſcripta fuerit figura quæcumque rectilinea, reliquorum ſegmentorum, demptâ figura rectilinea, commune grauitatis centrum, erit intra ſegmentum propoſitum.

SEgmento A C E inſcripta ſit quæcumque figura rectilinea A B C D E.

Dico commune grauitatis centrum, magnitudinis compoſitæ ex reſiduis ſegmentis A B, B C, C D, D E, eſſe intra ſegmentum A C E.

Centra grauitatis ſingulorum ſegmentorum reſiduorum, ſint ex ordine puncta F, G, H, I, quæ ſingula erunt intra ſua ſegmenta,[a] cùm ſint figuræ habentes ambitum in eaſdem partes cauum. Si commune centrum ſit extra ſegmentum A D E, vel in ambitu

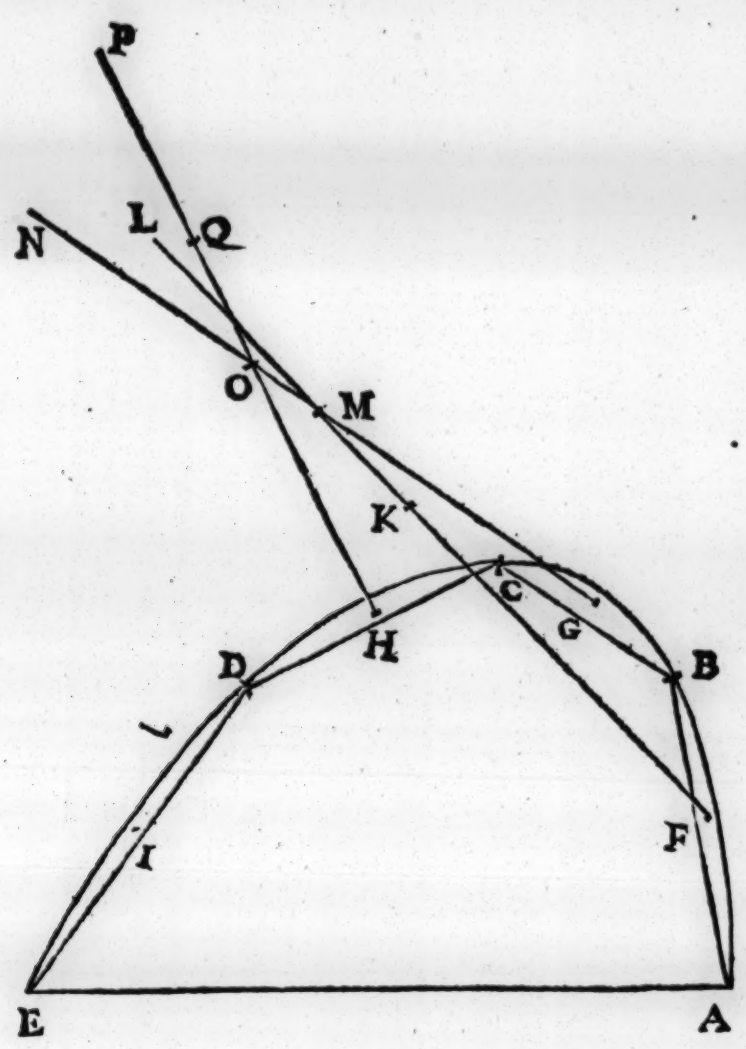


a 9. *petitio Archim. de æquipond.*

C

bitu figuræ, sit quotcumque punctum K. & à puncto F centro primi segmenti residui, ducatur F K L. cùm K sit centrum totius, & F centrum partis, [b] centrum reliquæ erit in F L, vltra K in aliquo puncto M, & consequenter extra segmentum ACE. Similiter cùm commune centrum magnitudinis compositæ ex residuis B C, C D, D E, sit extra segmentum A C E in M, ducta G M N, ex G centro partis, centrum reliquæ erit in G N, vltra M; atque ita progrediendo reperiemus centrum vltimi segmenti residui D E in Q, extra segmentum A D E, & consequenter extra ipsum segmentum D E, quod est absurdum; non erit ergo centrum magnitudinis compositæ extra segmentum, vel in ambitu vnde id sequitur.

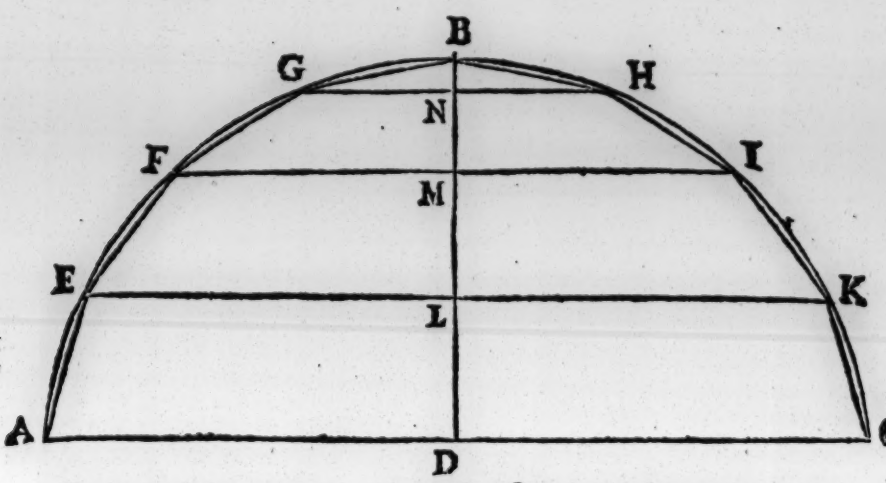
b 8. *primi Archim. de aquipond.*



Igitur si segmento circuli vel ellipsis, &c. quod fuit demonstrandum.

PROPOSITIO XVIII. THEOREMA XV.

Si segmento circuli vel ellipsis figura euidenter inscribatur, eius grauitatis centrum erit in diametro segmenti.



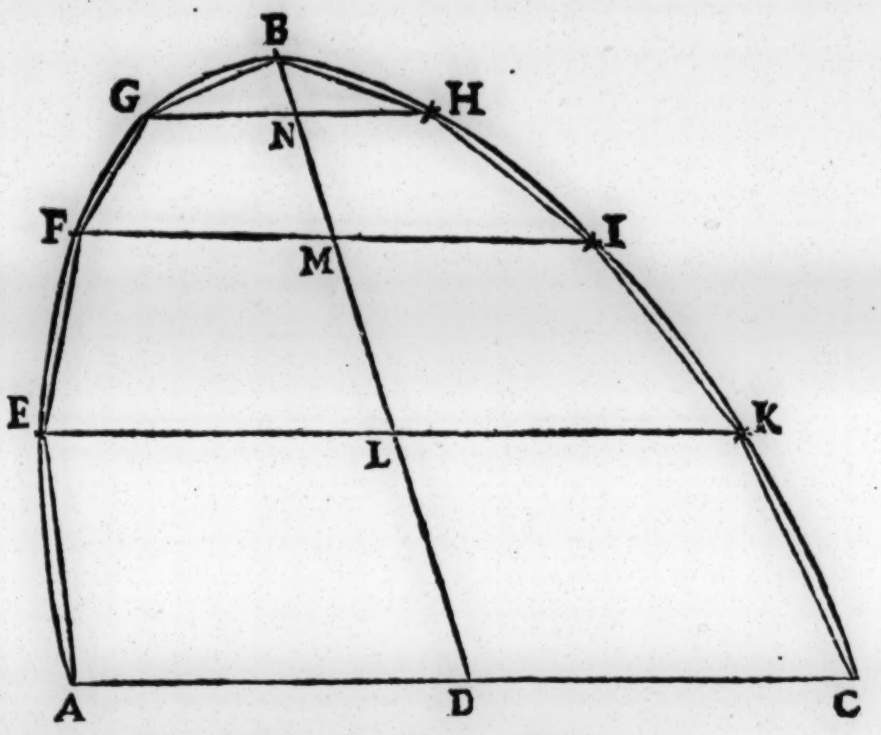
SIt segmentum circuli vel ellipsis ABC, diameter B D, figura euidenter inscripta A E F G B H I K C.

Dico centrum grauitatis eius esse in linea B D.

Coniungantur anguli figuræ lineis E K, F I, G H.

Quoniam Trapezij A E K C latera A C, E K, sunt parallela, bifariamque

riamque ſecantur in D & L,[a] centrum grauitatis ipſius eſt in LD, quæ eſt pars B D; eadem ratione centrum grauitatis Trapezij E F I K eſt in M L, id eſt B D, & Trapezij F G H I in N M id eſt B D, ad hæc quoque trianguli G B H centrum grauitatis eſt in B N, [b] ergo centrum grauitatis figuræ inſcriptæ, ſeu omnium partium ſimul ſumptarum eſt in B D.



a 15. *primi Archimedis de aequiponder.*

b 15. *huius.*

Igitur ſi ſegmento circuli, &c. quod fuit probandum.

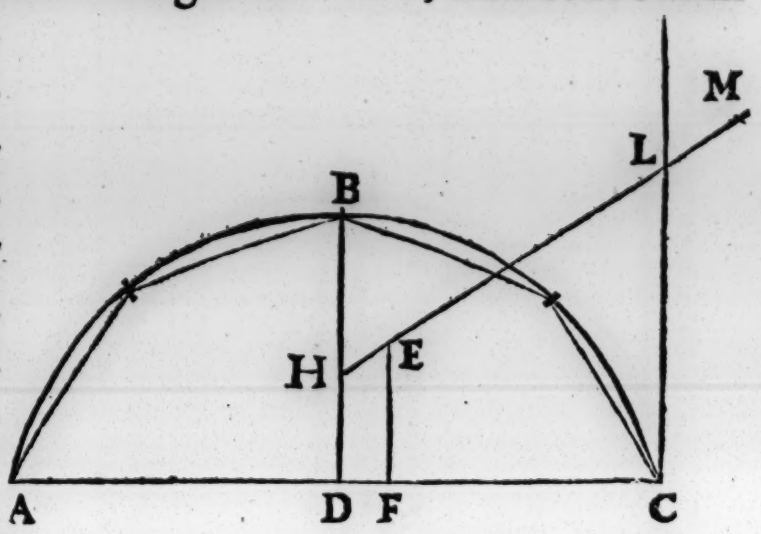
PROPOSITIO XIX. THEOREMA XVI.

Cuiuſcumque ſegmenti circuli vel ellipſis grauitatis centrum eſt in diametro ſegmenti.

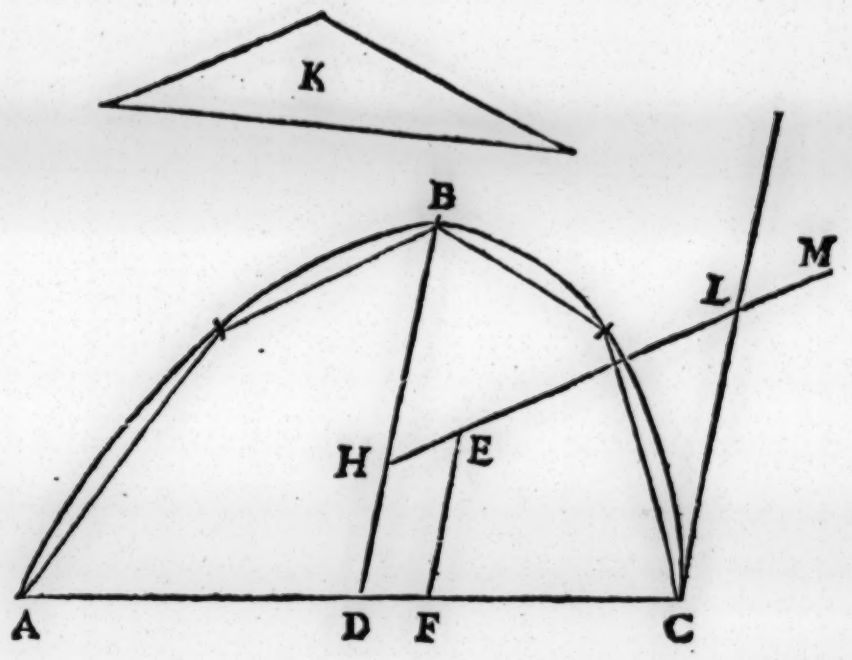
PRimò ſegmentum circuli vel ellipſis ſit A B C, ſemicirculo vel ſemiellipſi non maius, diameter eius B D.

Dico centrum grauitatis ipſius eſſe in B D.

Si enim non ſit, eſto punctum E & per ipſum ducatur E F, lineæ B D parallela, & inſcribatur ſegmento triangulum A B C, eamdem baſim & altitudinem habens cum ſegmento, & quam proportionem habet C F ad D F, hanc habeat triangulum A B C ad ſpatium K; inſcribatur euidenter ſegmento figura rectilinea, ita vt quæ reſidua erunt ſegmenta ſimul ſumpta, minora ſint ſpatio K; figuræ inſcriptæ grauitatis centrum eſt in B D, ſit igitur H, & iungatur H E, & producatur, & ducatur C L æquidiſtans ipſi B D, conſtat autem quod maiorem habeat proportionem figura ſegmen-

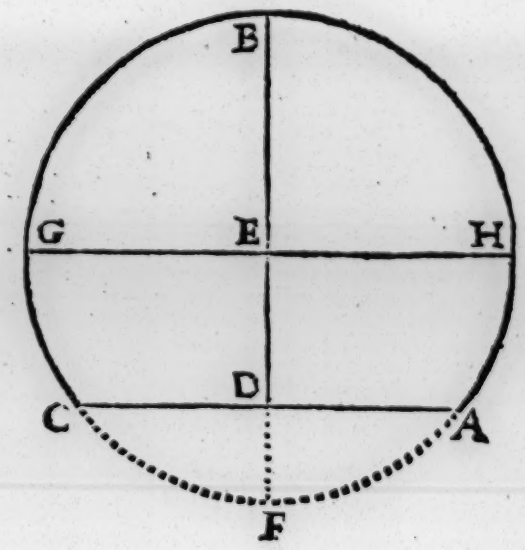


to inſcripta ad ea quæ reliqua ſunt ſegmenta, quàm A B C triangulum ad ſpatium K, quod eam habet quam C F ad D F, figura igitur inſcripta ad portiones relictas maiorem habet proportionem, quàm C F ad F D, hoc eſt E L ad E H. habeat itaque M E ad E H eam proportionem quam rectilinea figura ad portiones, quoniam igitur E eſt centrum totius ſegmenti, figuræ verò inſcriptæ centrum H, [a] conſtat quod magnitudinis compoſitæ ex reſiduis ſegmentis centrum grauitatis ſit in parte H E producta, quæ eam ad H E proportionem habeat, quam figura inſcripta ad reſidua ſegmenta, quæ ſit E M, erit igitur M centrum grauitatis magnitudinis compoſitæ ex reſiduis ſegmentis, extra ſegmentum, [b] quod fieri nequit, non eſt igitur centrum grauitatis ſegmenti circuli aut ellipſis, ſemicirculo aut ſemiellipſi non maioris, extra lineam B D ex quo id ſequitur.



a 8. *primi Archim. de æqui-ponder.*

Sit ſecundò ſegmentum circuli vel ellipſis A B C, ſemicirculo vel ſemiellipſi maius, compleatur figura & diameter producatur in F, ducatur etiam per centrum figuræ E linea G H, parallela C A baſi ſegmenti, totius figuræ G F H centrum grauitatis erit in E F per ea quæ iam diximus, ſed & partis C F A, erit in D F, [c] ergo & reſidui G C A H erit in E D, cum D F E D ſint partes eiuſdem lineæ E F; iterum centrum figuræ G B H eſt in B E per iam dicta & centrum figuræ G C A H, eſt in E D, ſed

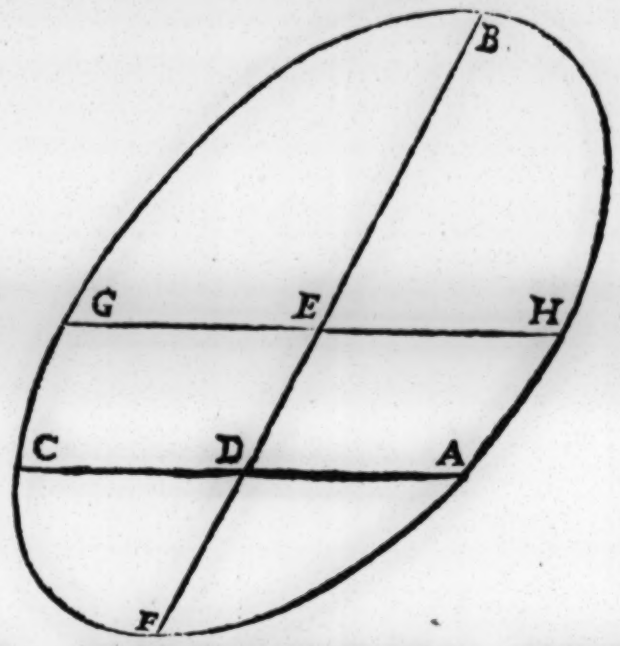


c 8. *primi Archim. de æqui-ponder.*

b 17. *huius.*

ſed B E E D componunt eamdem rectam, [d] ergo centrum totius ſegmenti C B A erit in B D.

Igitur cuiuſcumque ſegmenti circuli vel ellipſis, &c. quod fuit demonſtrandum.



d 15. *huius.*

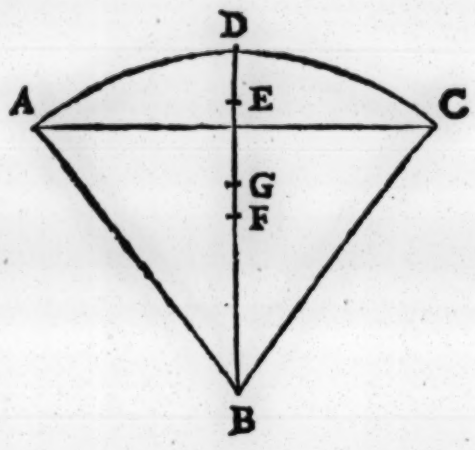
PROPOSITIO XX. THEOREMA XVII.

Cuiuſcumque Sectoris circuli grauitatis centrum, eſt in linea quæ illum ex centro bifariam ſecat.

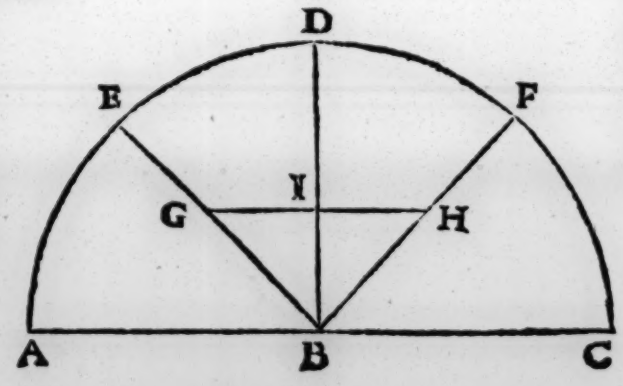
SIt primò Sector circuli A B C ſemicirculo minor, eumque linea B D ex centro B bifariam ſecet.

Dico centrũ grauitatis ipſius eſſe in B D.

Cùm enim linea B D angulum Sectoris bifariam ſecet, lineam quoque A C & arcum A D C bifariam ſecabit; itaque cùm A D bifariam ſecet baſim trianguli B A C, [a] erit in ea centrum grauitatis trianguli, item cùm bifariam ſecet arcum ſegmenti A D C, eiuſque baſim A C, [b] erit in ea centrũ grauitatis ſegmenti, [c] ergo in B D centrum grauitatis ex vtraque magnitudine compoſiti id eſt Sectoris, quod quoque intra Sectorem cadet.



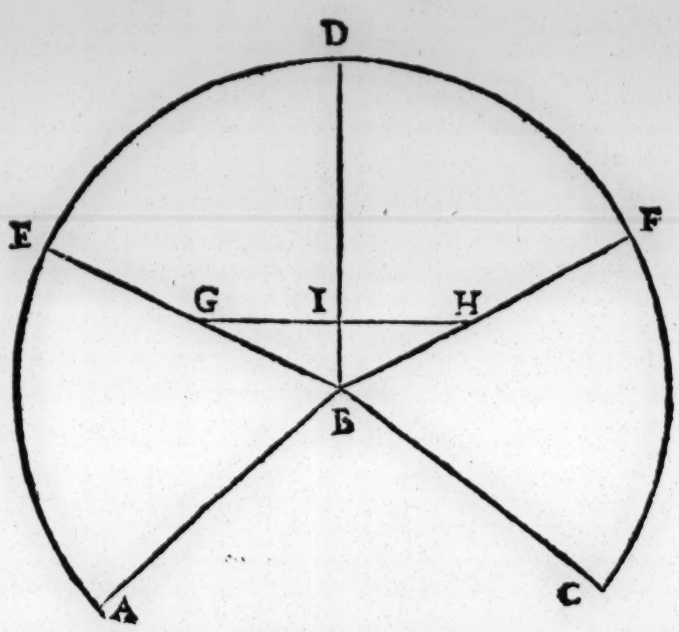
a 13. *primi Archim. de æquip.*



b 19. *huius*

c 15. *huius.*

Sit ſecundò ſemicirculus vel ſemicirculo maior A B C D, eumque linea B D ex centro bifariam ſecet, in duos Sectores A B D E, C B D F, erunt illi neceſſario ſemicirculo minores, diuidantur illi iterum bifariam per ſemidiametros B E, B F, centra grauitatis eorum erunt in illis ſemidiametris vt diximus, ſintque G & H, propter æqualita-

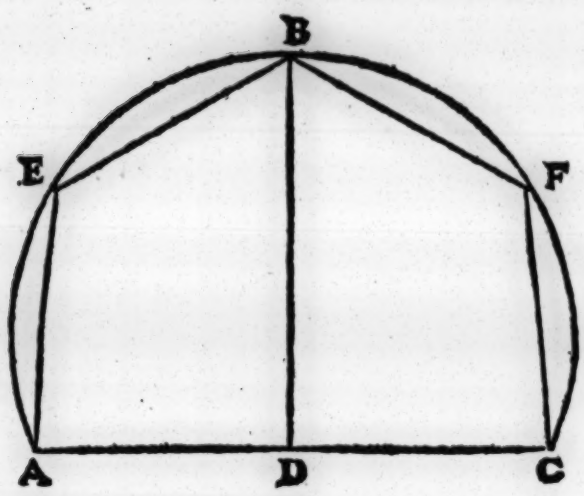


tem & ſimilitudinem Sectorum, erunt B G, B H æquales, & G H bifariam diuidetur à B D in I, & ob Sectorum æqualitatem, centrum commune ſeu totius Sectoris erit I, & conſequenter in B D, Sectorem bifariam ſecante.

Ergo cuiuſcumque Sectoris circuli, &c. quod fuit oſtendendum.

PROPOSITIO XXI. THEOREMA XVIII.

Si ſegmento circuli vel ellipſis figura euidenter inſcribatur, magnitudinis ex reſiduis ſegmentis compoſitæ grauitatis centrum, erit in ſegmenti diametro.



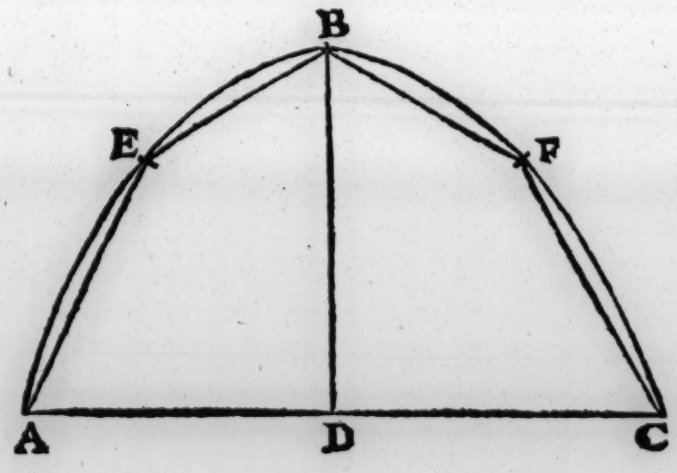
SIt ſegmentum circuli vel ellipſis A B C, figura euidenter inſcripta A E B F C, diameter ſegmenti B D.

Dico centrum grauitatis magnitudinis compoſitæ ex reſiduis ſegmentis A E, E B, B F, F C eſſe in linea B D.

a 19. *huius.*

b 18. *huius.*

c 8. *primi Archim. de æquip.*



[a] Cùm enim in B D ſit centrum grauitatis totius ſegmenti, [b] item & figuræ euidenter inſcriptæ, [c] erit in illa centrum reſidui, id eſt magnitudinis ex reliquis ſegmentis compoſitæ.

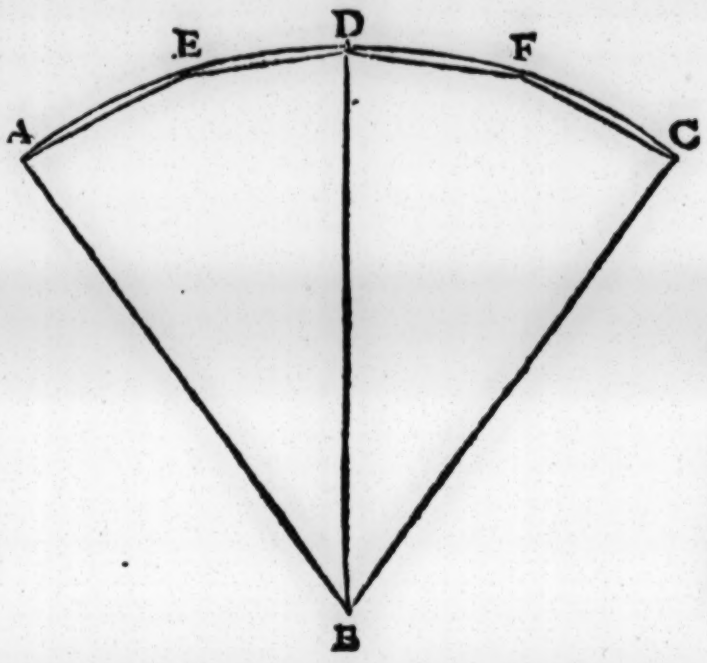
Ideo ſi ſegmento circuli vel ellipſis, &c. quod oportuit demonſtrare.

PROPOSITIO XXII. THEOREMA XIX.

Si Sectori circuli rectilinea figura fuerit euidenter inſcripta, centrum grauitatis figuræ inſcriptæ, erit in recta Sectorem bifariam ſecante.

Sector

SEctor circuli sit A B C D, figura euidenter inscripta A E D F C B, linea Sectorẽ bipartito secans B D.



Dico centrum grauitatis figuræ rectilineæ, quæ Sectori euidenter inscripta est, esse in linea B D.

[a] Cùm enim totius Sectoris grauitatis centrum sit in linea B D, [b] item & partis illius, quæ ex residuis segmentis componitur, [c] erit & reliquæ partis, id est figuræ rectilineæ, Sectori euidenter inscriptæ grauitatis centrũ in linea B D.

a 20. *huius*

b 17. *huius*

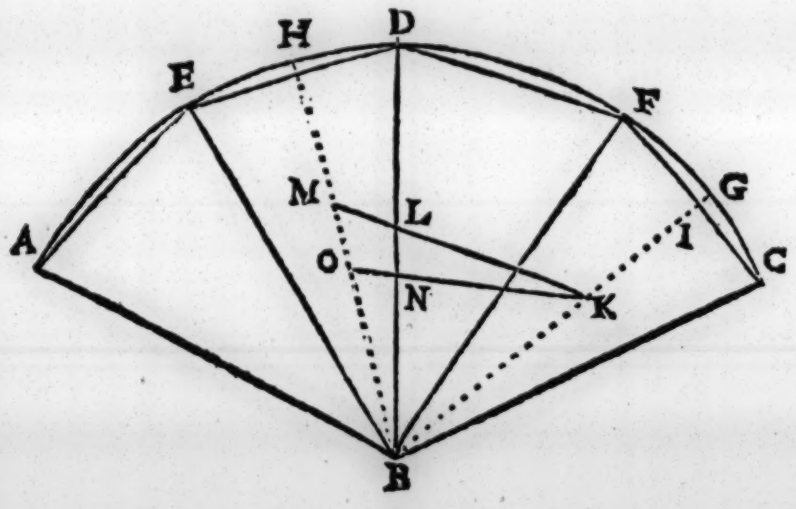
c 8. *primi Archim. de aquip.*

Quapropter si Sectori circuli, &c. quod fuit demonstrandum.

PROPOSITIO XXIII. PROBLEMA IV.

Figuræ rectilineæ Sectori circuli euidenter inscriptæ, grauitatis centrum inuenire.

SEctor circuli sit A B C D, & figuræ rectilineæ illi euidenter inscriptæ grauitatis centrum inueniendũ esto.

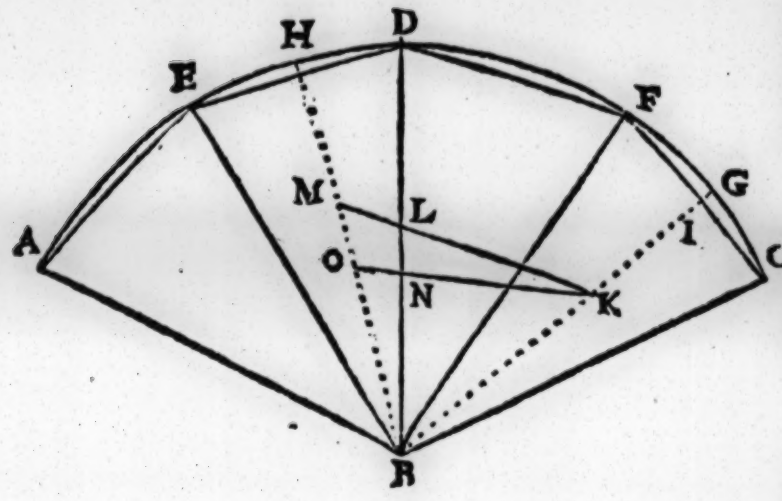


Cùm hæc figura ex triangulis isosceles componatur, sumatur vnum extremorum triangulorum puta B F C, & Sector illi respondens F B G C, diuidatur bifariam à linea B G; reliquum Sectorem cum inscripto sibi rectilineo bifariam secet B H, totum verò Sectorem cum inscripto sibi rectilineo bifariam diuidat B D. [a] Trianguli B F C repertum sit grauitatis cẽtrum K, [b] è puncto K ducatur K M, terminata ad B H, quæ ita diuidatur à B D in L, vt reciprocè M L ad L K sit, vt triangulum B F C ad

a *per* 14. *primi Archim. de aquiponderant.*

b *per* 11. *huius.*

reliquam



reliquam partem figurę euidenter inſcriptæ, proportio autem facile ex numero triangulorum colligetur.

Dico punctum L eſſe centrum grauitatis figuræ rectilineæ, Sectori euidenter inſcriptæ.

Si enim non ſit, eſto quoduis aliud punctum N, in linea BD, ducaturque K N O.

Cùm K ſit centrum vnius partis, ſeu trianguli B F C, centrum verò totius ſit N, centrum reliquæ partis erit O, (eſt enim in linea KN producta & in linea BH) erit igitur O N ad N K, vt triangulum B F C ad reliquam figuræ inſcriptæ partem, ſed ita quoque eſt ML ad LK, [d]quod tamen fieri nequit, vt nimirum K M & K O in L & N ſimiliter diuidantur, igitur punctum diuerſum à puncto L, non eſt centrum grauitatis figuræ Sectori euidenter inſcriptæ, ex quo abſurdum hoc ſequitur.

c 8. *primi Archim. de aquiponder.*

d 12. *huius*

Ergo figuræ rectilineæ, &c. centrum grauitatis inuenimus quod fuit faciendum.

PROPOSITIO XXIV. THEOREMA XX.

Cuiuſcumque Sectoris grauitatis centrum, minus diſtat à circumferentia, quam trianguli à lateribus Sectoris & ſubtenſa arcus comprehenſi.

SIt circuli Sector A B C D, triangulum verò B A C.

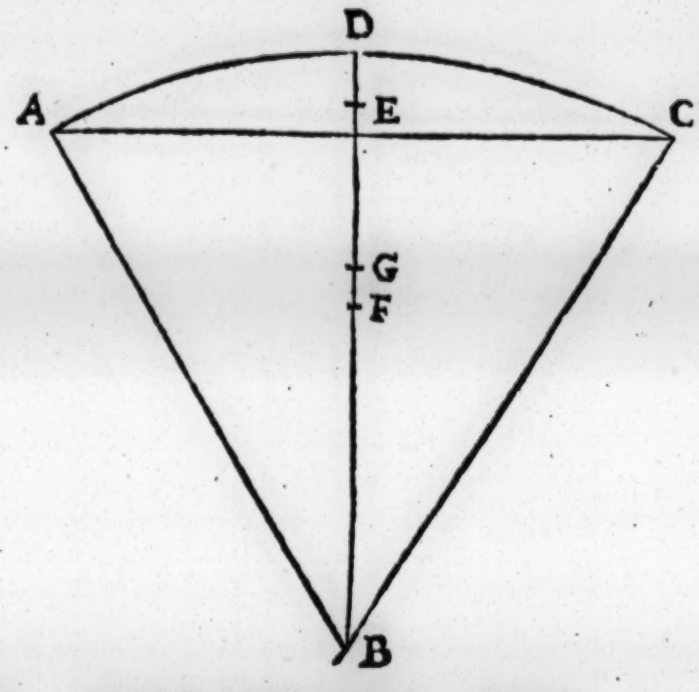
Dico centrum grauitatis Sectoris cadere ſupra centrum grauitatis trianguli.

Sit enim E centrum grauitatis ſegmenti A D C, centrum verò grauitatis trianguli F, cadet E ſupra F verſus circumferentiam, [a]cùm E ſit intra

a 9. *petit. Archim. de æquip.*

tra ſegmentum, b F verò intra triangulum, at centrum grauitatis totius, id eſt Sectoris, c cadit intra centra grauitatis partium, ſitque punctum G, conſtat punctum G cadere vltra punctum F, & vicinius eſſe arcui A D C.

Igitur cuiuſcumque Sectoris grauitatis centrum, &c. quod fuit oſtendendum.



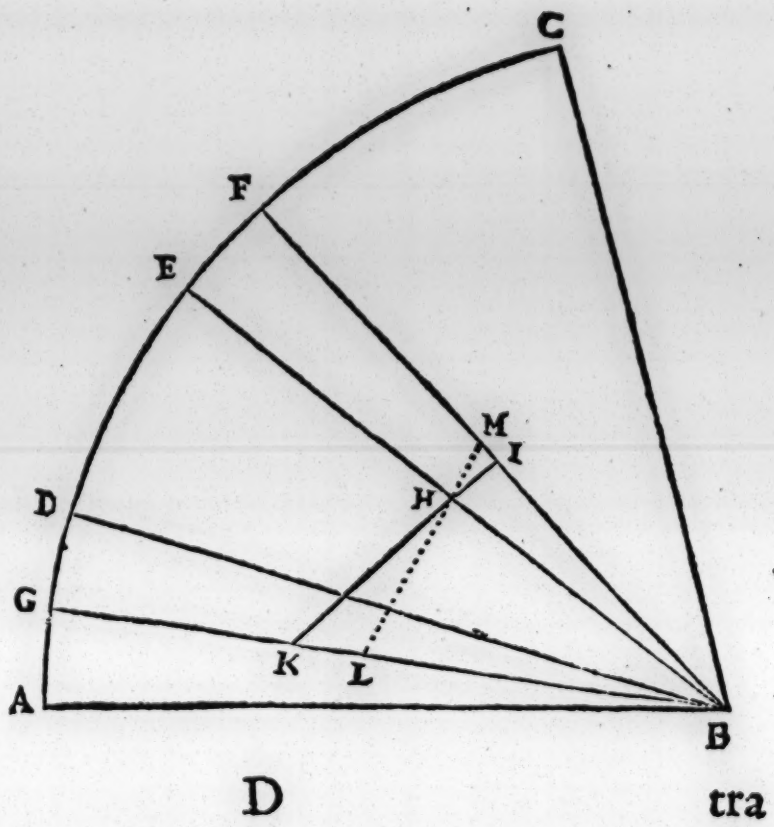
b 14. *primi Archim. de æquiponder.*
c 16. *huius.*

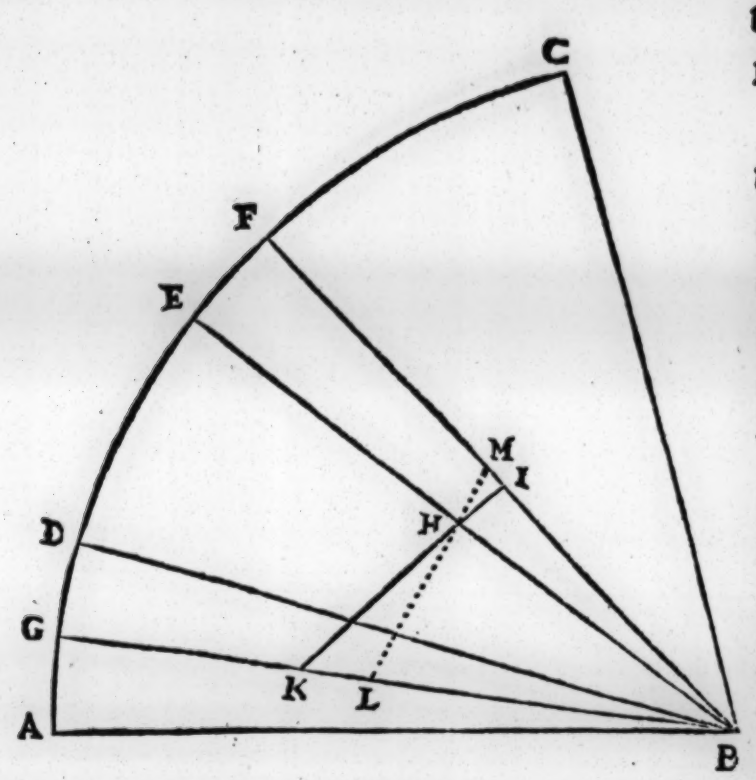
PROPOSITIO XXV. THEOREMA XXI.

Si datum fuerit centrum grauitatis Sectoris circuli, quì deinde in duos Sectores vtcumque diuiſus fuerit, & ipſi quoque Sectores per lineas è centro ductas iterum bifariam diuidantur; ſi ducatur linea per centrum grauitatis totius Sectoriſ, vtrimque terminata ad eas lineas, quæ alios Sectores bifariam ſecant, hac lege, vt diuiſa ad centrum grauitatis totius Sectoris, pars ad partem reciprocam habeat proportionem Sectoris ad Sectorem, extrema lineæ puncta erunt centra grauitatis partium.

DAtus circuli Sector ſit A B C D, diuiſus à linea B D, in duos Sectores A B D G, D B C F, quos etiam bifariam ſecent lineæ B G, B F; in linea verò B E, totum Sectorem biſecante ſit H, centrum grauitatis totius Sectoris, & per H ducta ſit linea I H K, vtrimque terminata in I & K, hac lege, vt ſit reciprocè quemadmodum angulus G B E, ad E B F, ita I H ad H K.

Dico pũcta I & K eſſe cen-





tra grauitatis ſuorum Sectorum.

Si enim non ſint, alterutrius Sectoris centrum erit diuerſum ab illis punctis; ſit ergo Sectoris ABDG centrum grauitatis L, & per puncta LH, id eſt centrum vnius partis & totius, ducatur linea, quæ occurrat BF in M, (occurret autem alioquin Sector DBCF nullum haberet grauitatis centrum, [a] cùm debeat eſſe in illarum linearum concurſu) erit M centrum grauitatis reliqui, ac proinde erit reciprocè vt Sector ABDG ad Sectorem DBCF, ita MH ad HL; ſed vt Sectores ita etiam ſe habent ipſorum anguli DBA, DBC, eorumque dimidij DBG, DBF, & vt angulus DBG ad DBF, ita IH ad IK, ergo etiam MH ad HL, vt IH ad IK, [b] quod oſtendimus fieri non poſſe; non igitur aliud punctum fuit centrum grauitatis alterutrius Sectoris, ex quo abſurdum ſequitur.

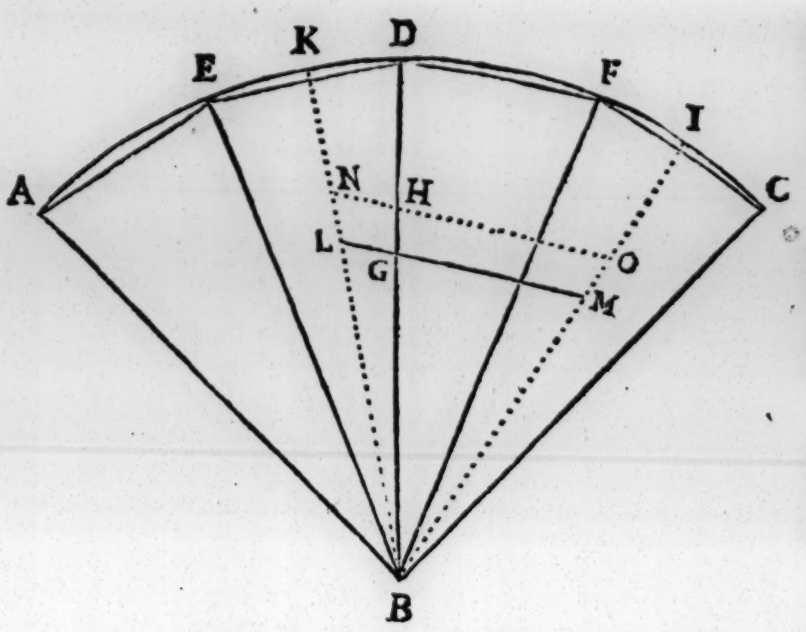
Quamobrem ſi datum fuerit, &c. quod demonſtrare voluimus.

a 8. primi Archim. de æquip. & 20. huius.

b 14. huius

PROPOSITIO XXVI. THEOREMA XXII.

Si Sectori circuli figura rectilinea euidenter fuerit inſcripta, centrum grauitatis ipſius cadet infra centrum grauitatis Sectoris.



SIt Sector circuli ABCD, illique euidenter inſcripta ſit figura AEDFCB.

Dico centrum grauitatis figuræ inſcriptæ, cadere infra centrum grauitatis Sectoris.

Si hoc non fiat, vel cum illo coincidet, vel cadet ſupra.

Ponatur primùm coincidere, & ſit G centrum grauitatis tum Sectoris tum figuræ inſcriptæ, ſumatur vnum triangulorum extremorum FBC, eiuſque angulus ad centrum bifariam ſecetur à BI,

à B I, reliqua etiam figuræ pars, dempto triangulo F B C, biſecetur à linea B K, & [a] per G ducatur L M, terminata ad lineas B I B K, quæ diuidatur ad G, vt reciprocè ſit L G ad G M, vt Sector B C I F ad Sectorem B A K F, erit etiam vt patet L G ad G M, vt triangulum B F C, ad reliquam partem figuræ euidenter inſcriptæ, & etiam vt angulus K B D ad D B I, vt ſæpius iam oſtenſum eſt. [b] Eruntque puncta L, M, centra grauitatis Sectorum A B F K, B C I F, eodemque diſcurſu quo id ſupra de Sectoribus oſtendimus, etiam demonſtrabitur L eſſe centrum figuræ euidenter inſcriptæ Sectori B A K F, & M eſſe centrum grauitatis trianguli B F C, igitur idem punctum M eſt centrum grauitatis trianguli B F C, & Sectoris B C I F, [c] quod eſt abſurdum; non igitur punctum G eſt centrum Sectoris & figuræ euidenter inſcriptæ vnde hoc ſequitur.

[a] 13. huius.

[b] 25. huius

[c] 24. huius

Cadat ſecundò centrum grauitatis figuræ inſcriptæ in H, ſupra G centrum Sectoris, & per H ducatur N O, parallela L M, erit N H ad H O, vt L G ad G M, id eſt reciprocè vt triangulum B F C ad reliquum figuræ inſcriptę dempto triangulo, cùm igitur H ſit centrum grauitatis totius figuræ inſcriptę, conſtat N & O eſſe centra partiũ, igitur O eſt centrum grauitatis trianguli B F C, caditq; ſupra M centrum grauitatis Sectoris B C I F, [d] quod fieri nequit, ergo centrũ grauitatis figuræ inſcriptæ, non cadit ſupra G centrum Sectoris, ex quo abſurdum illud conſequitur.

[d] 24. huius

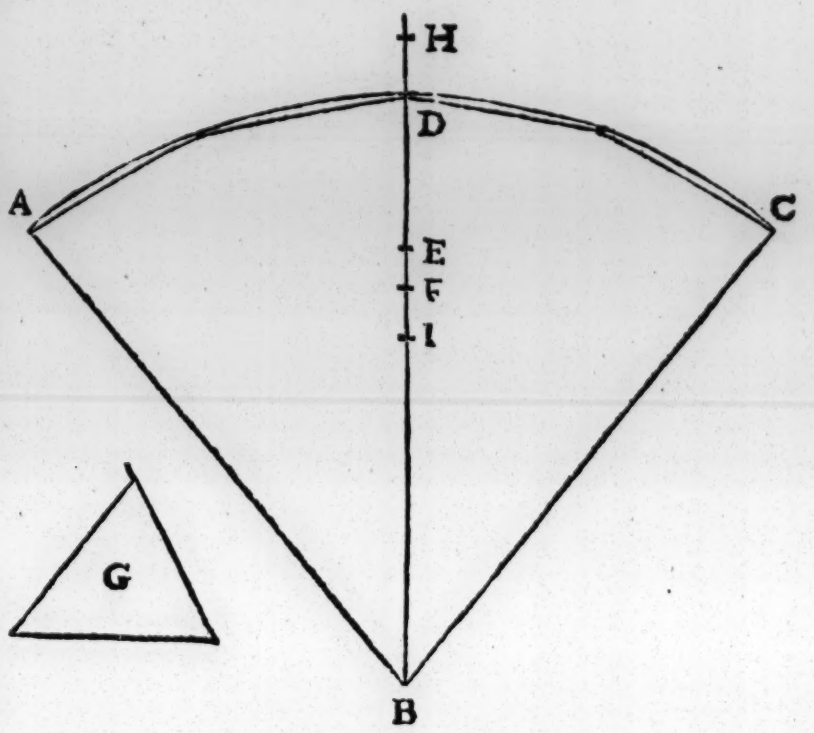
Ideo cùm centrum grauitatis figuræ inſcriptæ, non coincidat cum centro grauitatis Sectoris, nec cadat ſupra, neceſſe erit cadere infra.

Ergo ſi Sectori circuli, &c. quod fuit demonſtrandum.

PROPOSITIO XXVII. THEOREMA XXIII.

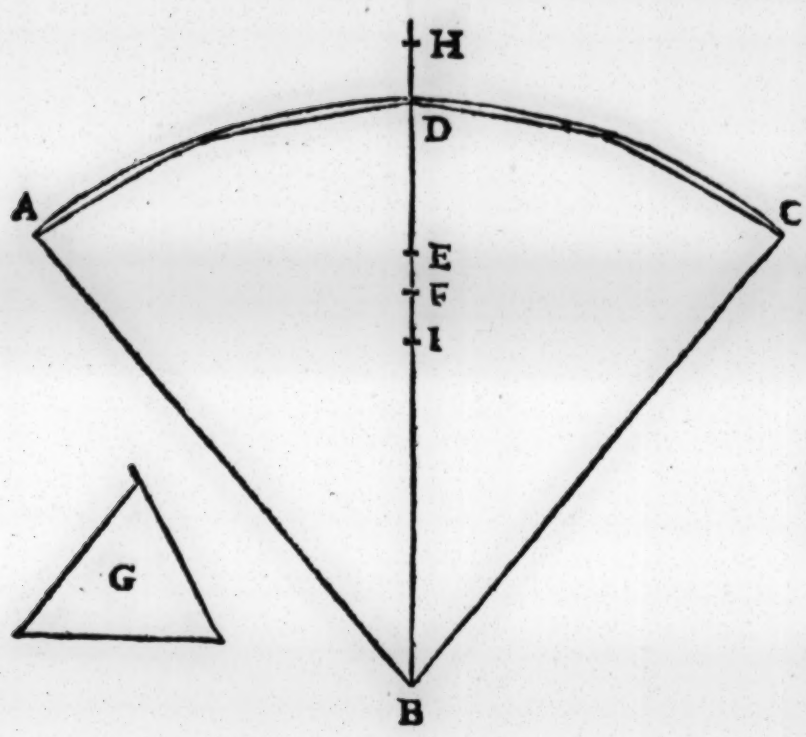
Poßibile eſt Sectori circuli figuram euidenter inſcribere, vt centrum grauitatis figuræ inſcriptæ, minus diſtet à centro grauitatis Sectoris, quolibet interuallo dato.

SIt Sector A B C D, centrum grauitatis eius E, interuallum datum E F, infra F; cadit enim centrum figuræ inſcriptæ infra centrum Sectoris.



Dico Sectori inſcribi poſſe euidenter figuram rectilineam, cuius centrum grauitatis minus diſtet ab E, quàm punctum F.

Sumatur aliquod ſpatium G, ad quod Sector maiorem habeat proportionẽ quàm



DF ad EF, & Sectori figura euidenter inſcribatur, donec ſpatium ex reſiduis ſegmentis demptâ inſcriptâ figurâ compoſitum, ſit minus ſpatio G, conſtat verò id fieri poſſe.

Dico centrum grauitatis illius figuræ euidenter inſcriptæ, cadere ſupra punctum F.

Si non cadat ſupra F, vel cadet in ipſum F, vel infra.

Ponamus primùm coincidere cum F; cùm E ſit centrum totius, & F partis ſeu figuræ inſcriptæ, erit centrum reliquæ partis id eſt magnitudinis ex reſiduis ſegmentis compoſitæ in FE, producta verſus E, in eoque puncto puta H, vt FE ad EH ſit, vt reſiduum ex ſegmentis ad inſcriptam figuram; quare etiam erit componendo vt HF ad FE, ita totus Sector ad reſiduum ex ſegmentis; cùm verò illud reſiduum ſit minus ſpatio G, erit maior proportio Sectoris ad reſiduum, quam Sectoris ad ſpatium G, ideoque maior proportio HF ad FE, quæ eſt vt Sector ad reſiduum, quam DF ad FE, quæ eſt vt Sector ad ſpatium G. maior ergo erit HF quàm DF, ergo H centrum grauitatis ſegmentorum reſiduorum, cadit extra arcum, & conſequenter extra
a17.*huius* ſegmentum ADC, [a] quod fieri nequit; ergo centrum grauitatis inſcriptæ figuræ non coincidit cum F, ex quo abſurdum ſequitur.

Cadat ſecundò centrum grauitatis infra F in I. Cùm E ſit centrum grauitatis totius id eſt Sectoris, & I centrum grauitatis figuræ inſcriptæ, centrum reliqui ſit H; vt prius, erit maior proportio Sectoris ad reſiduum ex ſegmentis compoſitum, quam DF ad DE, ergo quoque diuidendo maior proportio figuræ inſcriptæ, id eſt Sectoris demptis reſiduis ſegmentis, ad reſidua ſegmenta, quàm DE ad EF, multoque maior quàm DE ad EI; ſed vt inſcripta figura ad reſidua ſegmenta, ita HE ad EI, ergo maior proportio HE ad EI, quàm DE ad EI, ergo HE maior quàm DE, ergo H cadit extra Sectorem, centrum nimirum gra-
b17.*huius* uitatis reſiduorum ſegmentorum, ergo extra ſegmentum ADC, [b] quod fieri nequit; ergo nec inſcriptæ figuræ centrum grauitatis cadit infra F, ex quo id impoſſibile ſequitur.

c20.*huius* Cùm ergo inſcriptæ figuræ centrum, [c] quod neceſſariò cadit infra E Sectoris centrum, nec coincidat cum F, nec cadat infra F, cadet intra

ſpatium

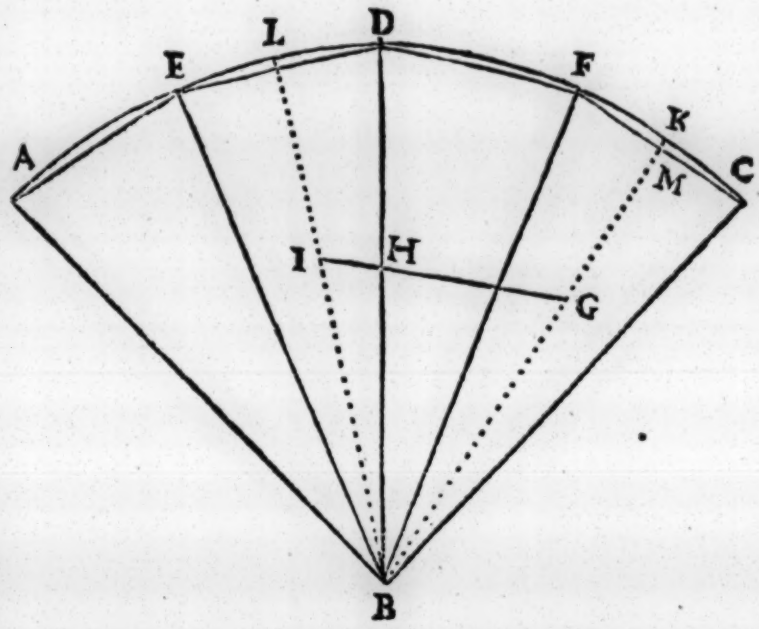
ſpatium F E, eritque vicinius centro Sectoris, quam ſit datum ſpatium F E.

Itaque poſſibile eſt Sectori circuli,&c. quod oportuit demonſtrare.

PROPOSITIO XXVIII. THEOREMA XXIV.

Omnis figuræ rectilineæ, Sectori euidenter inſcriptæ grauitatis centrum, minus abeſt à centro circuli, duabus tertiis ſemidiametri partibus.

SIt circuli Sector A B C D, figura rectilinea euidenter inſcripta A E D F C B.



Dico centrum grauitatis figuræ inſcriptæ, minus abeſſe à centro B, duabus tertiis ſemidiametri B D.

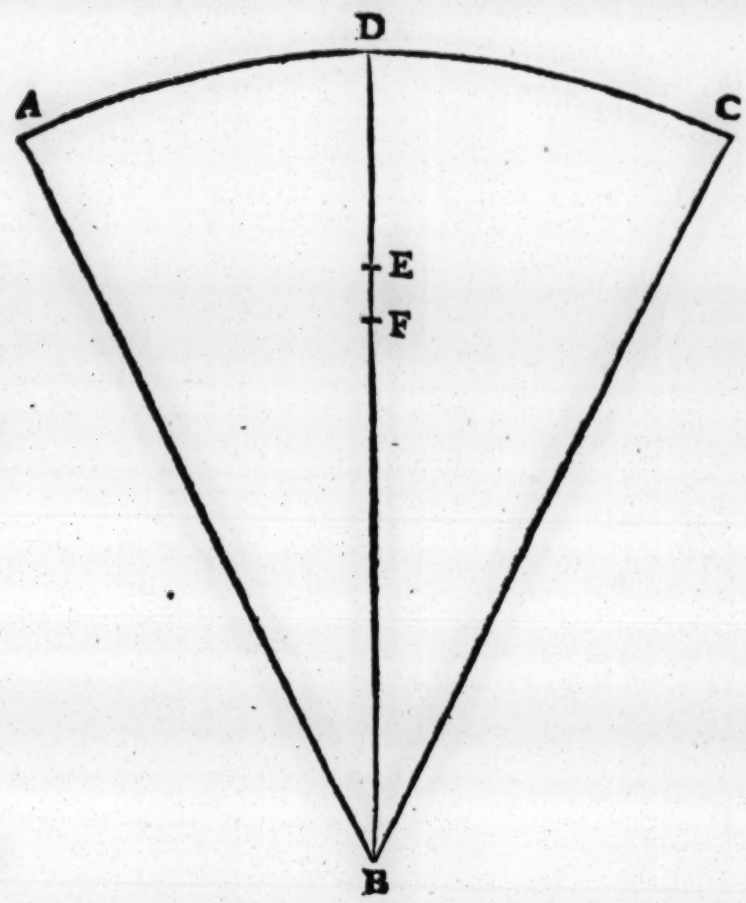
Sumatur enim triangulum B F C, & ex centro grauitatis ipſius G, inueniatur centrum grauitatis totius figuræ euidenter inſcriptæ, quod ſit H,& conſequenter reliquę partis dempto triangulo centrum erit I; lineâ B G K biſecante angulum trianguli B C F, & lineâ B H D, biſecante angulum totius figuræ inſcriptæ, lineâ denique B I L, biſecante reliquam figuræ partem,dempto triangulo B F C.erit itaque I H ad H G, vt triangulum B F C ad reliquam figuræ inſcriptæ partem, & conſequenter vt angulus F B C ad angulum F B A,& iterum vt eorum dimidij, id eſt F B K ad F B L, quibus cùm ſint æquales L B D, D B K, conſtat verò angulum L B D eſſe minorem angulo D B K (niſi figura inſcripta duobus tantum conſtet triangulis de quo ſtatim) [a]ergo B H minor erit B I, & B I minor B G, multo ergo minor erit B H quàm B G, ſed B G minor eſt duabus tertiis ſemidiametri partibus,cùm ſit ſubſeſquialtera B M, quæ ſemidiametro minor eſt, ergo & B H minor erit duabus tertiis ſemidiametri,quæ eſt interuallum ſeu diſtantia H, centri grauitatis figuræ inſcriptæ à centro B. [a] 10. *huius*

Si verò figura inſcripta duobus tantum conſtet triangulis, cùm illa ſint æqualia,erunt etiam B G,B I æquales,& angulus I B H æqualis H B G, & B H perpendicularis ad I G, ideoque minor B G, quæ minor eſt duabus tertiis ſemidiametri partibus,ergo & ipſa B H.

Quare omnis figuræ rectilineæ,&c. quod fuit oſtendendum.

PROPOSITIO XXIX. THEOREMA XXV.

Nullius Sectoris circuli grauitatis centrum, abeſt à centro circuli, interuallo quod excedat duas tertias ſemidiametri.



a 27. *huius*

b 28. *huius*

SIt Sector circuli A B C D.

Dico centrum grauitatis ipſius, non abeſſe à centro B, interuallo quod ſuperet duas tertias ſemidiametri B D.

Si enim fieri poſſit, ſint B F duæ tertiæ ſemidiametri, & E centrum grauitatis Sectoris, [a] poſſibile eſt Sectori figuram rectilineam euidenter inſcribere, cuius centrum grauitatis minus diſtet à centro grauitatis Sectoris, dato interuallo F E, & conſequenter magis abſit à centro B, interuallo B F, id eſt duabus tertiis ſemidiametri partibus, [b] quod fieri nequit; ergo nec centrum Sectoris poteſt à centro circuli [a] abeſſe maiori interuallo, quam duarum tertiarum ſemidiametri, ex quo impoſſibile illud ſequitur.

Igitur nullius Sectoris circuli, &c. quod fuit demonſtrandum.

PROPOSITIO XXX. THEOREMA XXVI.

Omnis Sectoris circuli grauitatis centrum, minus diſtat à centro ſui circuli, duabus tertiis ſemidiametri partibus.

SIt Sector circuli A B C D.

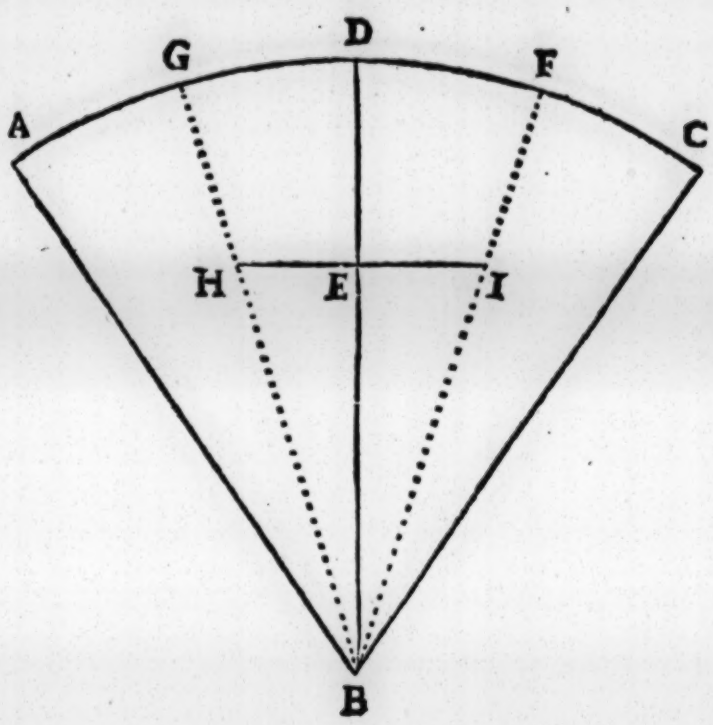
Dico centrum grauitatis ipſius minus abeſſe à centro circuli B, duabus tertiis ſemidiametri B D.

Si enim non abſit minus, cùm magis etiam abeſſe non poſſit, poterit abeſſe duabus tertiis ſemidiametri partibus. Sit igitur E, & B E duæ tertiæ ſemidiametri; [a] cùm B D Sectorem bifariam ſecet, ducantur etiam B F B G, quæ æquales partes bipartitò diuidant, ducaturque H I

a 20. *huius*

per

per E, perpendicularis ad BD, quæ lineis B F, B G occurrat in I & H, erunt H & I centra grauitatis partium, [b] nulla enim alia linea per E, bifariam diuidetur in E, terminata ad B F, B G; [c] cùm centra grauitatis partium ob earum æqualitatem debeant esse in linea per E bifariam secta, ergo H erit centrum grauitatis partis seu sectoris ABD, sed B H maior est B E, quæ est duarum tertiarum semidiametri, ergo centrum grauitatis Sectoris ABD, abest amplius duabus semidiametri partibus, [d] quod fieri non potest; impossibile igitur erit centrum grauitatis totius, abesse duabus tertiis semidiametri partibus, ex quo consequitur.



b 14. huius

c 20. huius & 25. huius.

d 29. huius

Ergo omnis Sectoris grauitatis centrũ, &c. quod fuit ostendendum.

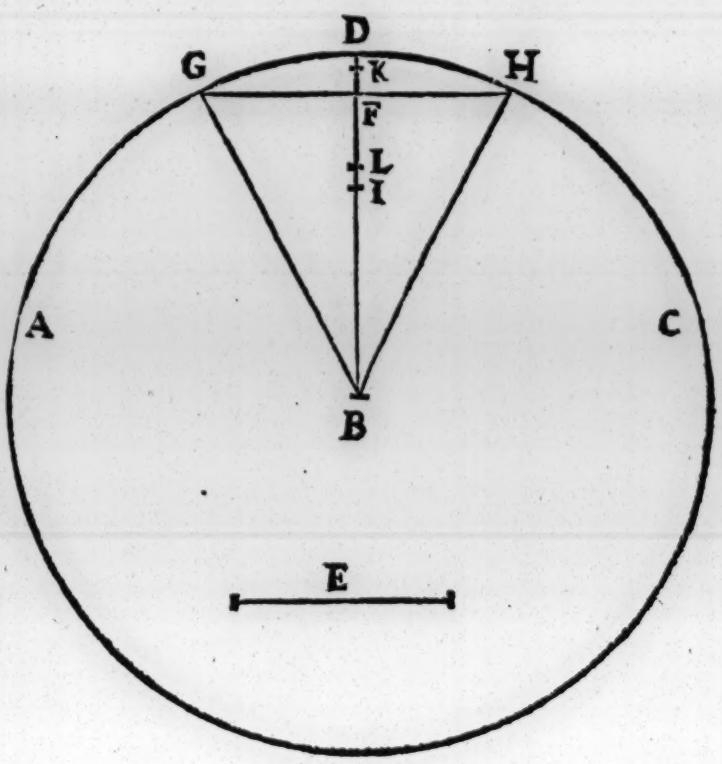
PROPOSITIO XXXI. PROBLEMA V.

A dato quolibet circulo auferre Sectorem, cuius centrum grauitatis magis distet à centro circuli, quolibet interuallo dato, quod minus sit duabus tertiis semidiametri partibus.

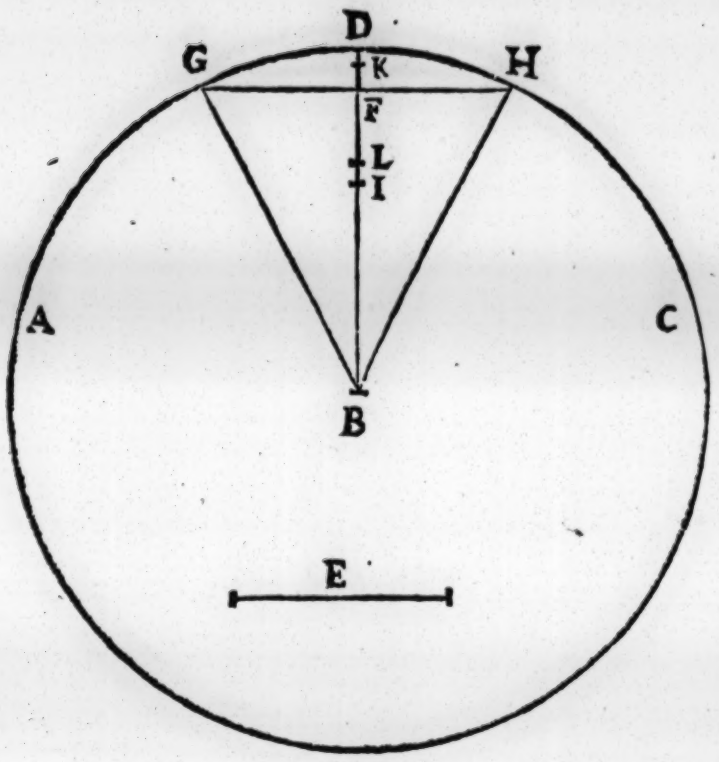
DAtus sit circulus ABC, cuius centum B, & data linea E, minor duabus tertiis partibus semidiametri.

Propositum sit auferre Sectorem, cuius centrum grauitatis magis distet à centro circuli B, dato interuallo E.

Cùm E sit minor subsesquialtera semidiametri dati circuli, à ducta qualibet semidiametro B D, auferatur B F sesquialtera lineę E, erit enim B F minor B D, per F ducatur G H, perpendicularis B D, se-



cans



a 9. *petitio Archim. de æquip.*
b 16. *huius.*

cans circulum in G & H, ducanturque BG, BH.

Dico GBHD esse Sectorem qui postulatur.

Sumatur in linea BD, quæ Sectorem bifariam diuidet vt constat, linea BI æqualis E, erit I centrum grauitatis trianguli BGH, cùm BI sint duæ tertiæ BF, sumatur quoque punctum K esse centrum grauitatis segmenti GDH, [a] quod erit intra DF, cùm I & K sint centra partium, [b] centrum totius erit intra IK, puta punctum L, quod magis distat à puncto B, interuallo BI, id est lineæ E.

Igitur à dato circulo, &c. quod fuit faciendum.

PROPOSITIO XXXII. THEOREMA XXVII.

Si quolibet Sectore circuli bifariam secto, à latere eius velut basi descriptum fuerit triangulum, dimidio Sectori æquale, communem cum Sectore eum habens angulum, qui ad centrum circuli est, deinde à vertice trianguli ad basim producta linea quæ simile priori triangulum constituat, centroq; Sectoris alius describatur Sector, eiusdem anguli cum dato Sectore, cuius latus sit sesquialterum eius lineæ, quæ inter centrum circuli & concursum lineæ à vertice ductæ ad productam basim, interiicitur; trianguli vertex erit centrum grauitatis Sectoris postremo descripti.

SEctor circuli sit BADC, bifariam sectus à linea BD, descriptumque triangulum AEB, æquale dimidio dicti Sectoris ADB, & sit vt AB ad BE, ita BE ad BF, ducaturque EF; seu ducatur EF, vt triangulum BEF, sit simile triangulo BEA; sumatur deinde BG sesquialtera BF, & centro B interuallo BG, describatur Sector BGIH.

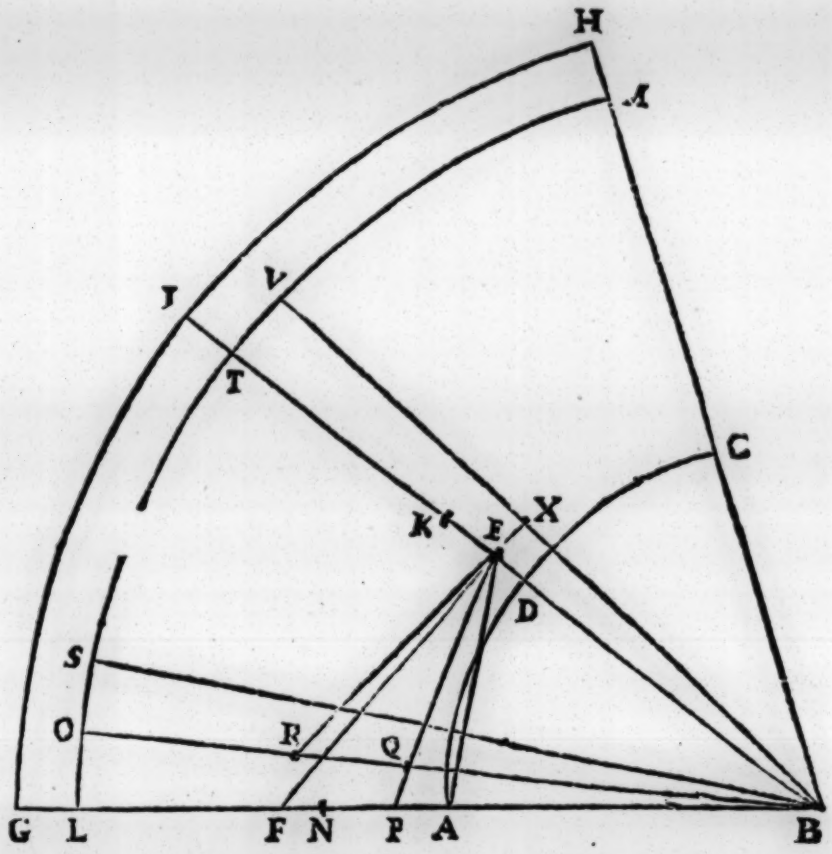
Dico

Dico punctum E esse centrum grauitatis Sectoris BHKI.

Si enim non sit; cùm sit in linea BK, cadet supra vel infra punctum E.

Cadat Primum supra E in K, & fiat vt BK ad BI, ita BE ad BL, & centro B interuallo BL describatur Sector BLM,[a] erit E centrum grauitatis Sectoris BLM, cùm Sectores BLM & BGIH sint figuræ similes, & puncta E & K in vtroque similiter ponantur, sitq; punctum K, ex hypothesi centrum grauitatis Sectoris BGIH.

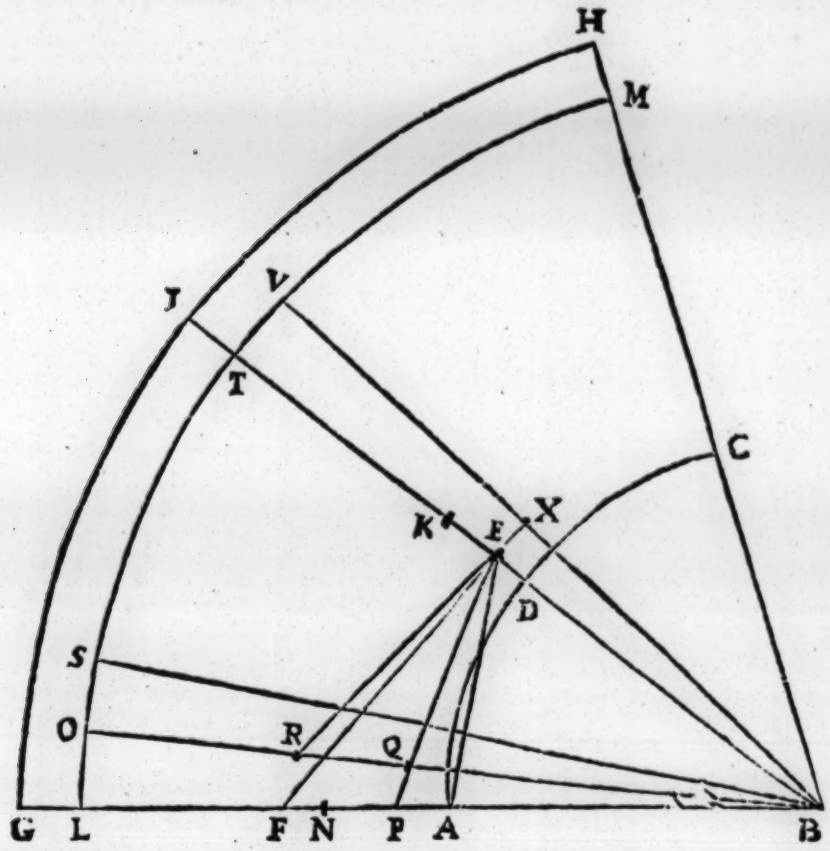
[a] 7. *petit. Archim. primi de æquipond.*



Diuidatur BL in N, vt BL sit sesquialtera BN, & iuxta ea quæ Propositione nona ostensa sunt, ducta sit BO, vt deinde ducta EB, diuidatur ad BO in Q hac ratione, vt sit PQ ad QE, vt angulus PBQ ad angulum QBE, & duabus lineis QB, BE, sumptâ tertiâ proportionali BR, excedat linea BF lineam BR minori interuallo, quam FN, quo BF superat BN, & consequenter sit maior quàm BN, sumatur quoque angulus LBS duplus anguli LBO, & angulus TBV æqualis angulo LBO, & per puncta R & E ducatur linea quæ occurrat BV in X.

Cùm lineæ QB, BE, BR sint tres continuæ proportionales ex constructione, erunt triangula BER, BEQ similia, & anguli BER, BQE æquales, ergo etiam anguli deinceps, id est BEX, BQP æquales erunt, ergo & triangula BEX, BPQ similia erunt, cùm præter angulos BEX, BPQ etiam angulos EBX, QBP æquales habeant, quare in triangulis BQP, BEX erit sicut PQ ad QB, ita EX ad EB, similiter etiam in triangulis BQE, BRE, erit sicut BQ ad QE, ita BE ad ER, sed PQ est ad QE vt angulus BPQ ad QBE, id est angulus LBO ad OBT, ergo etiam XE erit ad ER, vt angulus LBO ad angulum OBT, ergo etiam vt angulus LBS, qui est duplus anguli LBO, ad angulum SBM, qui est duplus anguli OBT, ergo erit XE ad ER reciprocè, vt Sector LBS ad Sectorem SBM, quia Sectores habent eamdem proportionem quam anguli; cùm verò E sit centrum grauitatis totius Sectoris BLM, & BO bifariam secet Sectorem LBS, & BN Sectorem SBM, partes totius Sectoris, & linea RX per E ducta, & vtrimque terminata ad BO, BN diuidatur ad punctum E, in recipro-

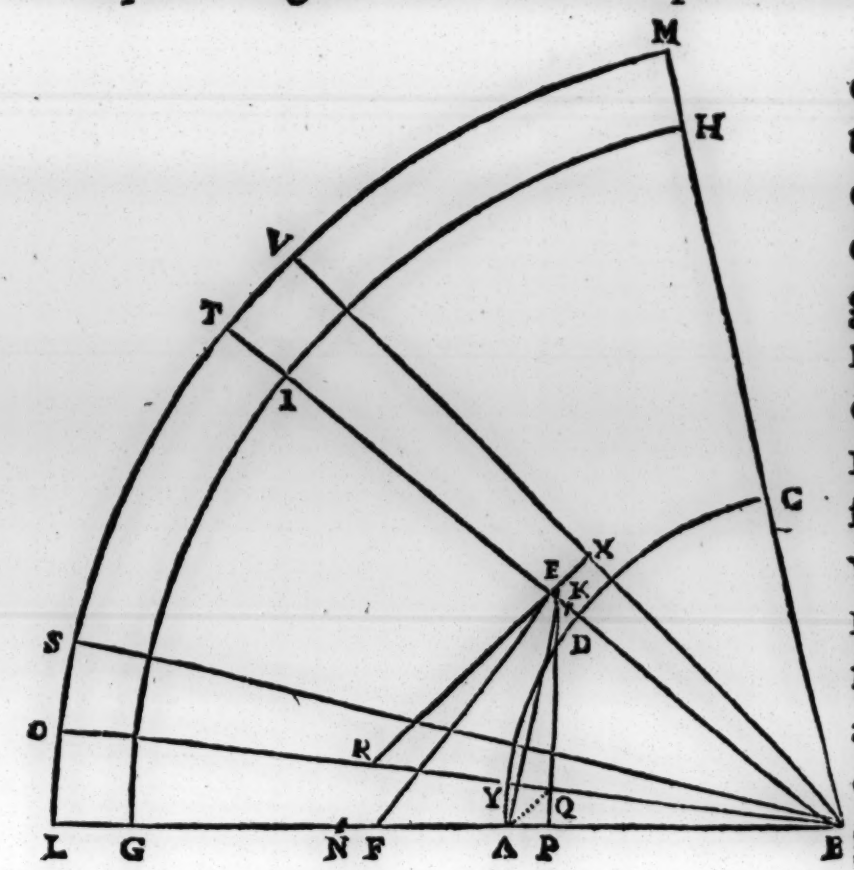
b 25.*huius* cam proportionem partium,[b]erunt R & X centra partium,ergo R erit centrum grauitatis Sectoris LBS ; ſed RB eſt maior BN duabus ſcilicet tertiis BL,ergo alicuius Sectoris centrum,plus duabus tertiis ſemidiametri partibus diſtat à



c 30.*huius* centro circuli, c quod fieri nequit; non igitur centrum grauitatis Sectoris BGHI, cadit ſupra E ex quo hoc abſurdum ſequitur.

Cadat ſecundò infra E in K, & fiat vt BK ad BI, ita BE ad BL, & centro B interuallo BL, deſcribatur Sector BLM, erit E centrum grauitatis Sectoris BLM, vt ſuprà oſtendimus.

Diuidatur BL in N, vt BL ſit ſeſquialtera BN,& iuxta propoſitionem 31.inuentus ſit Sector BLS, cuius centrum grauitatis R,maiori interuallo diſtet à centro circuli B,quam ſit interuallū lineæ BF,quod eſt minus BN,ſcilicet duabus tertiis ſemidiametri BL,ductaque ſit BRO,diuidet illa bifariam Sectorem BLS,fiat angulus TBV æqualis angulo LBO , ducaturque RE, donec occurrat BV in X.



d 8.*primi Archim. de æquip.*

e 20.*huius*

Quandoquidem E ſit centrum grauitatis totius Sectoris BLM, & R centrum grauitatis Sectoris BLS , [d] centrum grauitatis reliqui Sectoris SBM, erit in RE producta, [e] ſed etiam erit in BV, quæ illum bifariam ſecat,ergo in X; ergo erit vt EX ad ER, ita Sector BLS ad Sectorem BSM,& ſimiliter angulus LBS ad angulum SBM, eorumque dimidij, ſcilicet angulus LOB ad angulum OBT,

OBT, & angulus TBV ad angulum TBO, vt iam oſtendimus.

Fiat deinde vt BR ad BE, ita BE ad BQ; ducatur EQ, & producatur vſque in P, ducatur item AQ; ſimilia erunt triangula REB, EQB, & anguli REB, EQB æquales, ergo & in triangulis BEX, BQP anguli BEX, BQP qui prioribus ſunt deinceps, æquales erunt, ergo & triangula BEX, BQP ſimilia vt ante oſtendimus, eritque vt EX ad EB, ita PQ ad QB, & vt EB ad ER, ita QB ad QE, ergo ex æqualitate vt EX ad ER, ita PQ ad QE; ſed EX ad ER, vt angulus ABQ ad QBE, hoc eſt vt angulus LBO ad angulum OBT, f ergo triangulum AQB æquale Sectori BAY, ergo BQ maior BA; ſed EB eſt media proportionalis inter BQ, BR; item inter BA, BF, ergo rectangulum ſub BF, BA, æquale eſt rectangulo ſub BR, BQ, ergo erit vt BF ad BR, ita BQ ad BA, ſed BE eſt minor BF ex conſtructione, ergo erit BQ minor BA, ergo BQ ſimul erit maior & minor BA, quod fieri nequit; non cadet igitur centrum grauitatis Sectoris BGIH infra E, ex quo abſurdum conſequitur; ſed oſtenſum eſt etiam non cadere ſupra E, ergo erit ipſum punctum E, centrum grauitatis Sectoris BGIH.

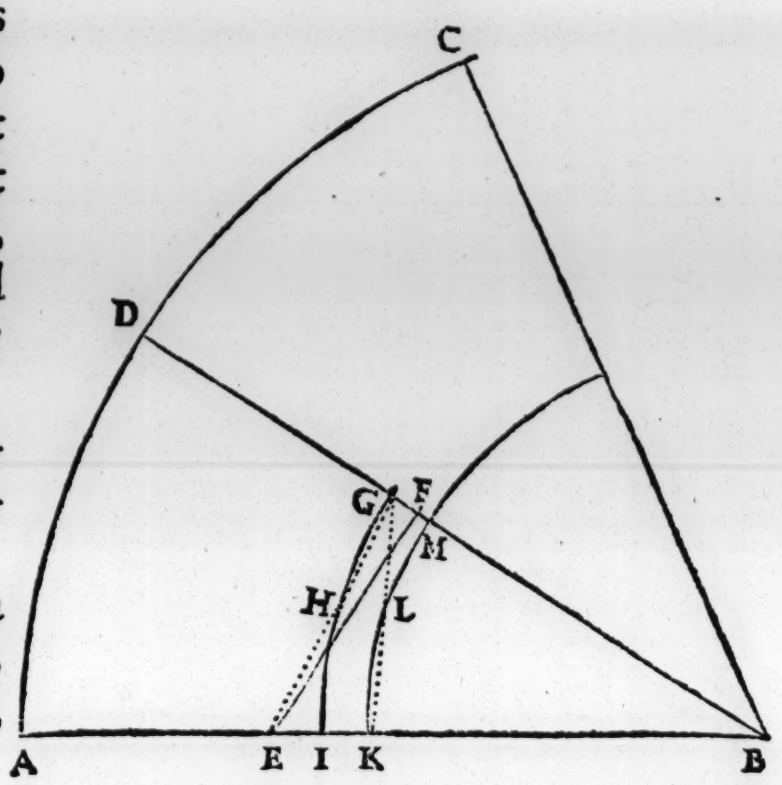
f 6. huius.

Igitur ſi quolibet Sectore circuli, &c. quod fuit demonſtrandum.

PROPOSITIO XXXIII. THEOREMA XXVIII.

Si quolibet Sectore circuli è centro bifariam diuiſo, ducta fuerit perpendicularis in lineam biſecantẽ, à puncto lateris quod duabus tertiis partibus ſemidiametri abſit à centro circuli, & fiat vt dimidius Sectoris arcus ad ſemidiametrum, ita perpendicularis ducta ad quartam quamdam lineam, è centro circuli in linea biſecante ſumendam, eius terminus eſt centrum grauitatis Sectoris propoſiti.

SIt datus Sector ABC, ſectus bifariam à ſemidiametro DB, & latere AB diuiſo in E, vt AB ſit ſeſquialtera BE, ducta ſit EF perpendiculans lineæ DB, & ponatur eſſe vt arcus DA ad ſemidiametrum AB, ita EF ad BG.

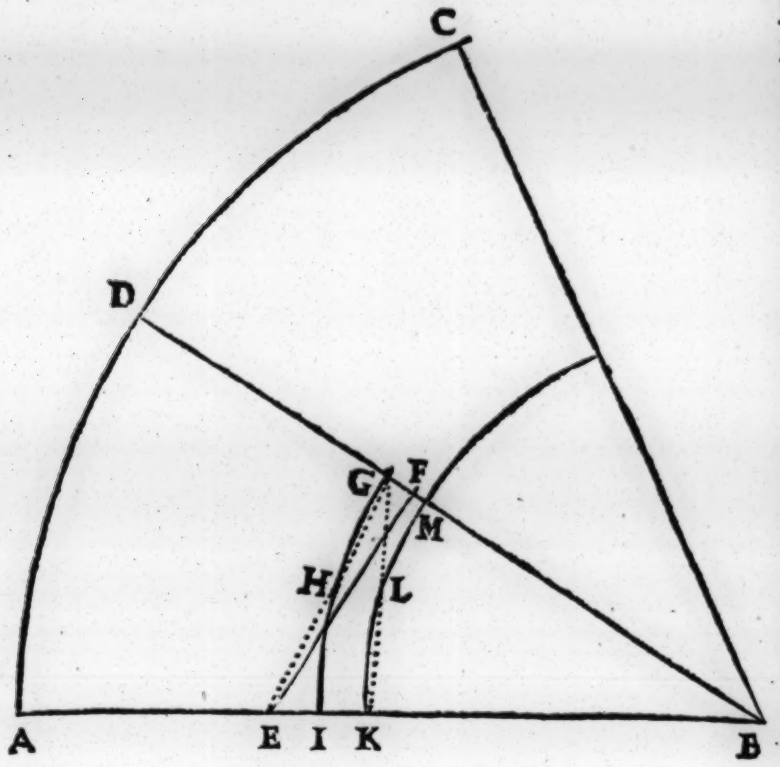


Dico punctum G eſſe centrum grauitatis Sectoris propoſiti.

Fiat enim vt BE ad BG, ita EG ad BK, ducanturq; GE GK; item centro B, interuallo BG,

deſcribatur arcus GHI, & eodem centro interuallo BK, deſcribatur arcus KLM, & producatur in N.

Cùm ſit vt arcus DA ad ſemidiametrũ AB, ita arcus GHI ad ſemidiametrum BG, & vt arcus DA ad lineam AB, ita EF ad BG, eadem erit proportio arcus GHI ad GB, quæ rectæ EF ad BG, igitur arcus GHI erit rectæ EF æqualis, ſed triangulum GEB habet baſim ſemidiametrum Sectoris BGHI, & altitudinem EF æqualem arcui GHI, ergo eſt æquale Sectori BGHI; item Sectores BGHI, BKLM, cùm ſint ſimiles, ſunt in duplicata ratione ſuarum ſemidiametrorum, cùm verò ſit vt BK ad BG, ita BG ad BE, erit Sector BKLM ad Sectorem BGHI, vt BK ad BE, & conuertendo vt BE ad BK, ita Sector BGHI ad BKLM, & permutando vt triangulum BGE ad Sectorem BGHI, ita triangulum BGK ad Sectorem BKLM; ſed triangulum BGE oſtenſum eſt Sectori BGHI æquale, ergo & triangulum BGK Sectori BKLM æquale. Cùm igitur Sector BKMN ſit bipartitò diuiſus à linea BM, & Sectori BKLM ſit æquale triangulum BGK, producto latere BM comprehenſum, ſitque triangulum BGE ſimile triangulo BGK, & BA ſeſquialtera BE, Sectoris BADC,[a] centrum grauitatis eſt punctum G.



[a] 32. huius

Ergo ſi quolibet Sectore circuli, &c. quod fuit demonſtrandum.

PROPOSITIO XXXIV. THEOREMA XXIX.

Dato quolibet Sectore circuli, è centro bifariam diuiſo, ſi fiat vt Sectoris arcus, ad duas tertias partes rectæ ſubtendentis arcum, ita ſemidiameter ad quartam quandam lineam è centro ſumendam, in ea quæ Sectorem bifariam ſecat, eius terminus erit centrũ grauitatis Sectoris propoſiti.

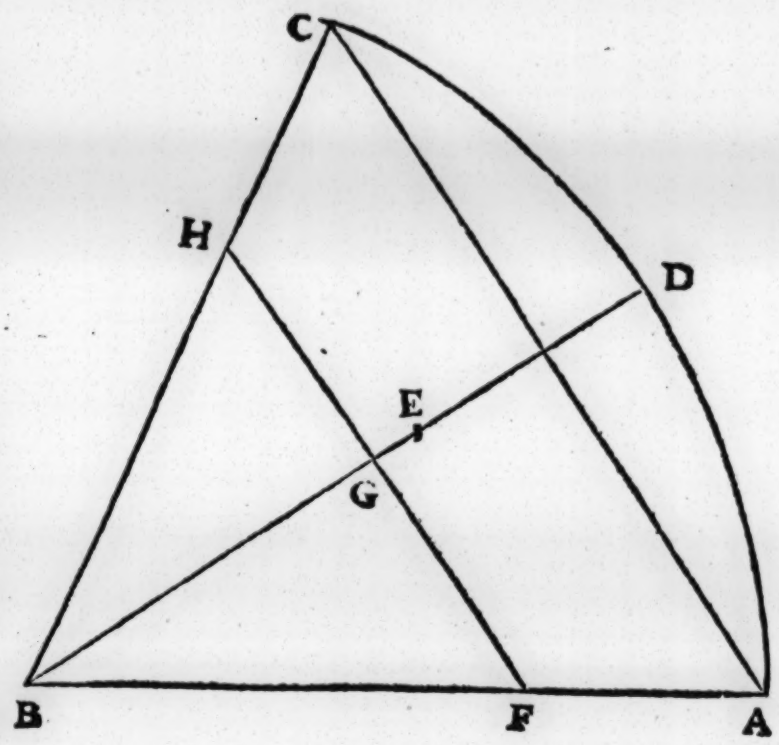
SIt ABCD Sector circuli, ductaque BD Sectorem biſecante, ſit veluti arcus ADC ad duas tertias ſubtenſæ AC, ita ſemidiameter BD ad quartam BE.

Dico

Dico E esse centrum grauitatis Sectoris propositi.

Diuidatur enim AB in F, vt sit AB sesquialtera FB, & ducatur FH parallela AC.

Cùm BF sit subsesquialtera BA, & sit vt BF ad BA, ita FH ad AC, erit quoque FH duarum tertiarum AC, quare erit vt arcus ADC ad FH, ita AB ad BE, & vt DA dimidium arcus ADC, ad FG dimidiam FH, ita AB ad BE, ergo conuertendo vt arcus AD ad rectam AB, ita FG perpendicularis lineæ BD, ad lineam BE; cùm igitur AB sit sesquialtera FB, per præcedentem[a] constabit E esse centrum grauitatis Sectoris propositi.



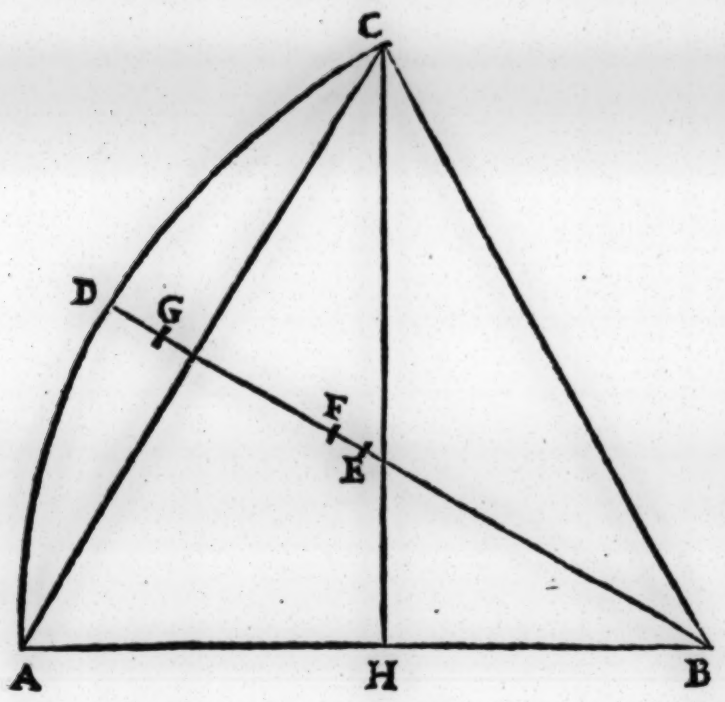
[a] 33. huius.

Igitur dato quolibet Sectore circuli, &c. quod fuit ostendendum.

PROPOSITIO XXXV. THEOREMA XXX.

Si datum fuerit quodlibet segmentum circuli semicirculo minus, ductisq; semidiametris quæ segmenti arcui insistunt; inuentis deinde centris grauitatis Sectoris, & trianguli à Sectoris lateribus & segmenti subtensa comprehensi, si linea vtraque connectens centra grauitatis inuenta, versus centrum Sectoris producta ita augeatur, vt augmentum ad compositam ex augmento & inuenta iam linea, quæ centra connectit, eam habeat proportionem, quam perpendicularis ab extremo arcus Sectoris ducta in Sectoris latus, ad segmenti arcum, erit illius augmenti extremum, centrum grauitatis dati segmenti circuli.

SIt ADC segmentum semicirculo minus, ductisque ad centrum circuli cuius est segmentum lineis AB, CB, item positis, F quidem centro grauitatis Sectoris ABCD, at puncto E centro grauitatis trianguli ABC, per quæ ducta linea EF ita producta sit in G, vt quemadmodum CH perpendiculans in AB, ad arcum ADC, ita sit GF ad totam GE.



Dico punctum G, esse centrum grauitatis segmenti ADC.

a 8. *primi Archim. de æquiponder.*

a Cùm enim linea EF ducatur per centrum totius, id est Sectoris; & vnius partis, id est trianguli; erit in illa versus F producta centrum reliquæ partis, id est segmenti ADC; in eoque puncto puta G, vt linea EF ad FG, habeat reciprocam proportionem partium; sed illam habet EF ad FG; nam vt est perpendiculans CH ad arcum ADC, ita est triangulum ABC ad Sectorem ADBC; (Sector enim æqualis est triangulo sub basi æquali AB, & altitudine quæ æqualis sit arcui ADC) ergo triangulum & Sector sunt in proportione GE ad GF, & diuidendo vt triangulum ABC ad segmentum ADC, ita FG reciprocè ad FE. b Cùm itaque F sit centrum grauitatis totius, centrum verò partis, id est trianguli sit E, erit G centrum grauitatis reliqui, id est segmenti ADC.

b 8. *primi Archim. de æquip.*

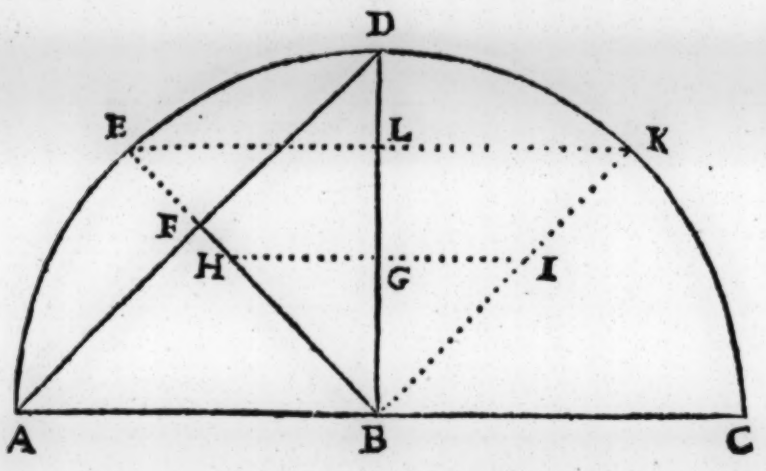
Ergo si datum fuerit quodlibet, &c. quod fuit ostendendum.

PROPOSITIO XXXVI. THEOREMA XXXI.

Si datus fuerit semicirculus, aut circuli portio semicirculo maior, parte circumferentiæ & duabus semidiametris comprehensa, & sicut dimidius arcus, ad duas tertias partes suæ subtensæ, ita perpendicularis è centro circuli in dictam subtensam, ad quartam quampiam lineam, sumendam à centro, in linea totam portionem è centro bisecante, in eius termino erit centrum grauitatis semicirculi, aut portionis propositæ.

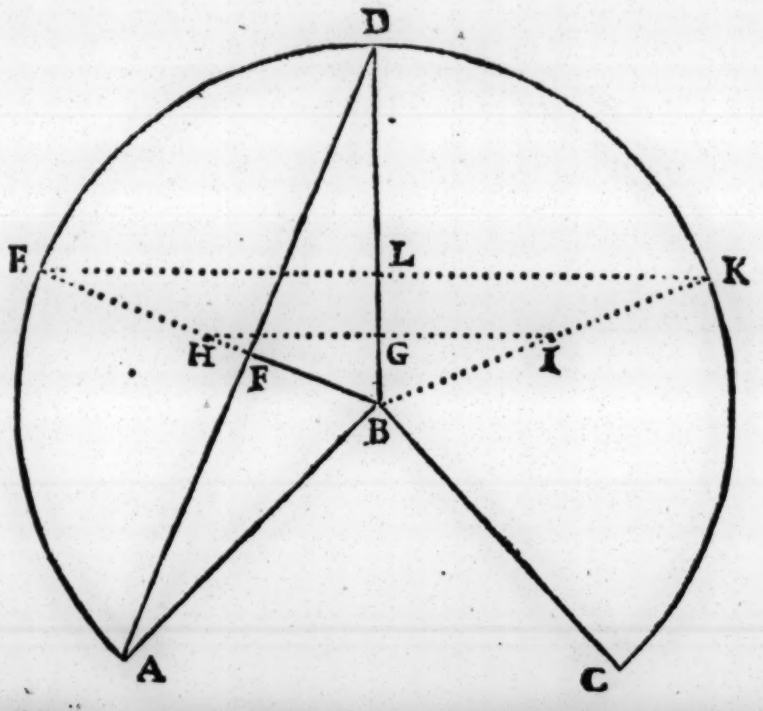
Sit

SIt datus ſemicirculus, aut portio maior vt eam iam deſcripſimus ADC, quam bipartitò ſecet BD, ductaque AD, ſubtenſa dimidij arcus AED, in eamque perpendicularis BF; ſi ponatur eſſe vt arcus AED, ad duas tertias partes ſuæ ſubtenſæ AD, ita BF ad BG.



Dico punctum G eſſe centrum grauitatis ſemicirculi, aut portionis propoſitæ.

Producatur BF in E, quæ biſecabit Sectorẽ ABDE, ſimiliter reliquum Sectoris dimidium, ſcilicet BDKC bifariam ſecet BK, ducatur EK, & per punctum G ducatur HI, parallela EK, occurrens lineis BE, BK, in H & I.



Cùm BD totam portionem bifariam ſecet, & lineæ BE, BK reliquas partes, erit arcus EDK dimidio arcui portionis æqualis, & EK æqualis AD, & conſequenter linea BL æqualis BF; ergo vt BF ad BG, ita BL ad BG, id eſt vt arcus AED, ad duas tertias ſubtenſæ AD; at in triangulo BEK, linea FI eſt baſi parallela, ergo latera in eamdem proportionem diuidit, ſcilicet in proportionem lineæ BL ad LG; erunt igitur EB ad BH, & BK ad BI, ſicut arcus ſingulorum Sectorum, ad duas tertias ſuæ ſubtenſæ, [a] ergo puncta H & I centra grauitatis Sectorum ABDE, CBDK, [b] ergo centrum totius in HI, ſed etiam eſt in BD, ergo in G.

[a] 34. *huius*

[b] 20. *huius*

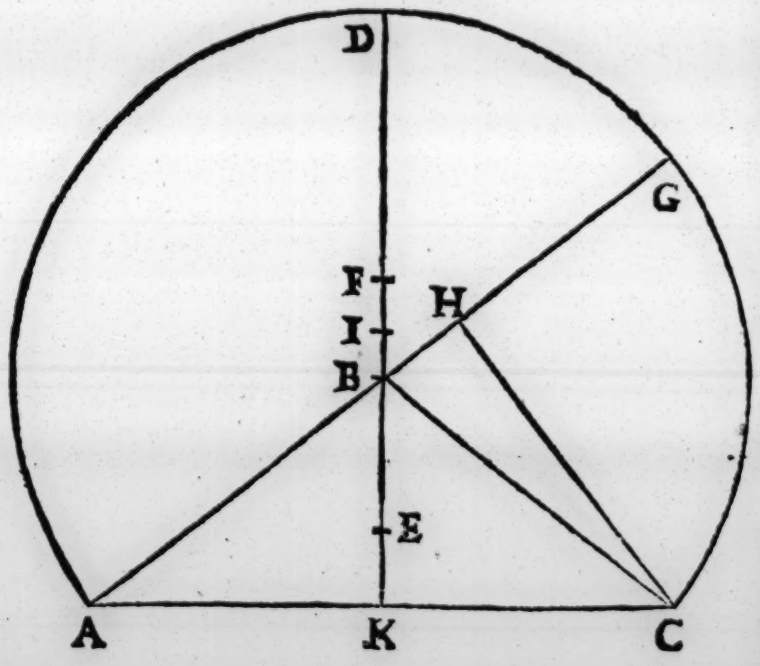
Igitur ſi datus fuerit ſemicirculus, &c. quod demonſtrare oportuit.

PROPO-

PROPOSITIO XXXVII. THEOREMA XXXII.

Si datum fuerit ſegmentum quoddam ſemicirculo maius, ductiſq; à baſis extremis duabus ad centrum lineis, factiq; cum baſe trianguli, & reliquæ portionis grauitatis centra fuerint data; producto vno trianguli latere, & in illud demiſſa perpendiculari, ex oppoſito baſis extremo, ſi linea centra grauitatis trianguli, & reliquæ portionis connectens, ita diuiſa fuerit, vt quam proportionem habet ſegmenti arcus, ad perpendicularem iam ductam, eam habeant partes lineæ, hac lege, vt maior pars centro grauitatis trianguli adiaceat; hoc diuiſionis punctum, erit centrum grauitatis ſegmenti propoſiti.

SIT ADC ſegmentum ſemicirculo maius, ductiſque AB, CB ſemidiametris, trianguli ABC grauitatis centrum ſit E, reliqui verò Sectoris grauitatis centrum ſit punctum F, producatur AB in G, ductaque ad eam perpendiculari CH, ponatur linea EF dicta grauitatis centra coniungens, ita diuiſa in I, vt quam proportionem habet arcus ſegmenti ad perpendicularem CH, eam habeat EI ad IF.



Dico punctum I, eſſe centrũ grauitatis dicti ſegmenti.

a 16. huius

[a] Neceſſe enim eſt, vt centrum grauitatis ſegmẽti propoſiti, ſit in FE connectente centra partium, b in eoque puncto, quod lineam diuidit reciprocè in rationem partium; at triangulum ad reſiduam portionem illam habet proportionem, quam CH ad arcum ſegmenti, (illa enim portio æqualis eſt triangulo rectangulo, ſub ſemidiametro & altitudine quæ ſit dicto arcui æqualis) in eamdemque poſuimus reciprocè diuiſam lineam FE, ergo I erit centrum grauitatis ſegmenti propoſiti.

b 6. & 7. primi Archimedis de æquiponder.

Itaque ſi fuerit datum quoddam ſegmentum, &c. quod fuit demonſtrandum.

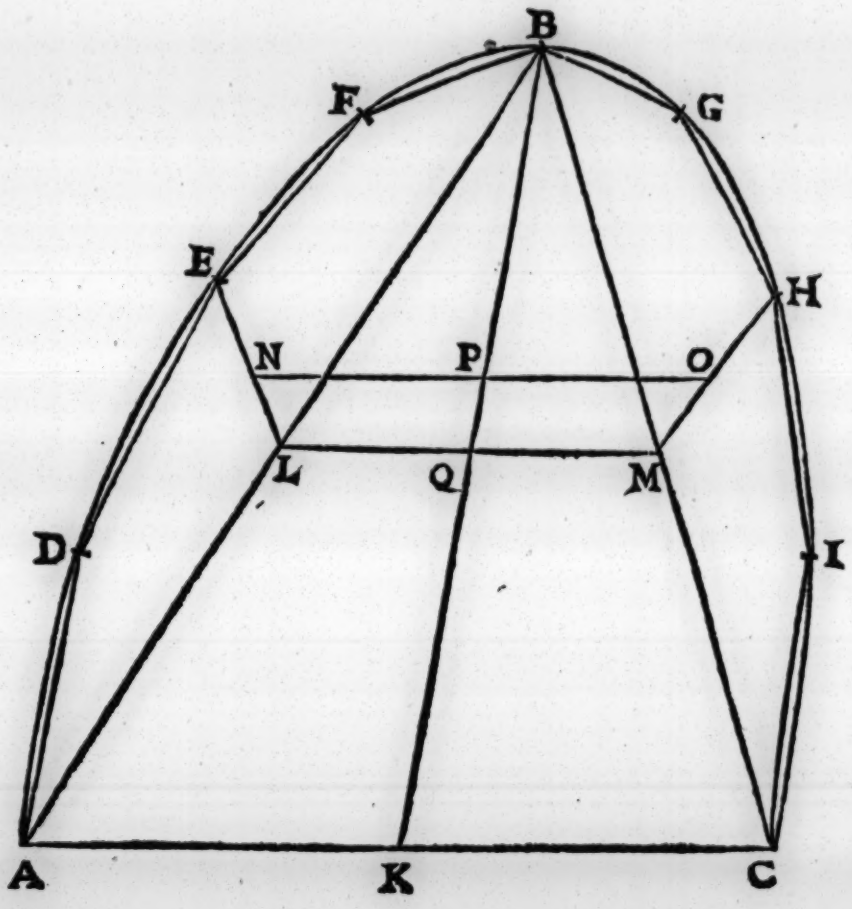
PROPO-

PROPOSITIO XXXVIII. THEOREMA XXXIII.

Si ſegmento circuli vel ellipſis, figura rectilinea euidenter inſcripta fuerit, ſegmentorum reliquorum demptâ figurâ inſcriptâ, commune grauitatis centrum, minus aberit à ſegmenti vertice, dimidio lineæ, quæ à vertice ad mediam baſim ducitur.

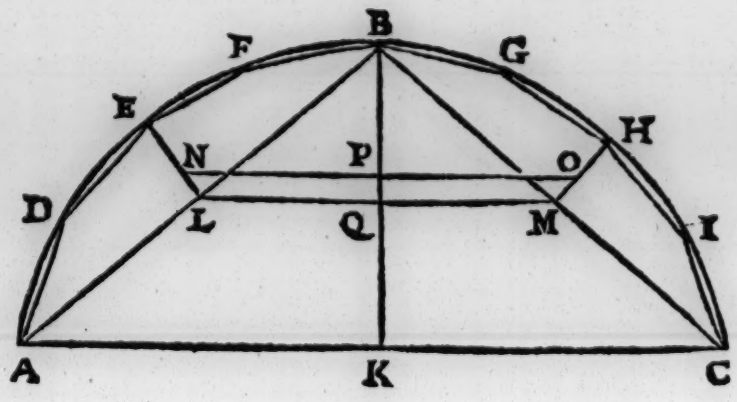
SEgmento ABC, euidenter inſcripta ſit figura ADEFBGHIC.

Dico magnitudinis ex reliquis ſegmentis, demptâ figurâ inſcriptâ compoſitæ, grauitatis centrum, (a quod eſt in diametro BK) minus abeſſe à vertice B, dimidio lineæ BK. a 18. *huius.*



b 21. *huius*

Ducantur AB, CB, ſegmentoque AEB euidenter inſcripta erit figura ADEFB, & ſegmento CHB figura CIHGB, quare ductis EL, HM diametris, b erunt centra grauitatum magnitudinū compoſitarū ex ſegmentis reſiduis, demptis figuris ADEFB, CIHGB, in lineis EL, HM. ſit N centrum grauitatis, magnitudinis compoſitæ ex ſegmentis portionis AEB, & ſumatur O centrum grauitatis, magnitudinis cōpoſitæ ex ſegmentis portionis BHC, ductaque NO, c cùm centrum grauitatis ex omnibus compoſitæ ſit in NO, & d ſit præterea in BK, erit in P, quæ communis vtriuſq; lineæ interſectio eſt; ducatur deinde LM, cadet illa infra NO, & cùm lineas AB, BC ſecet bifariam in L & M, lineam quoque BK bipartitò ſecabit in Q, quapropter cùm P cadat ſupra Q, punctum P, centrum grauitatis



c

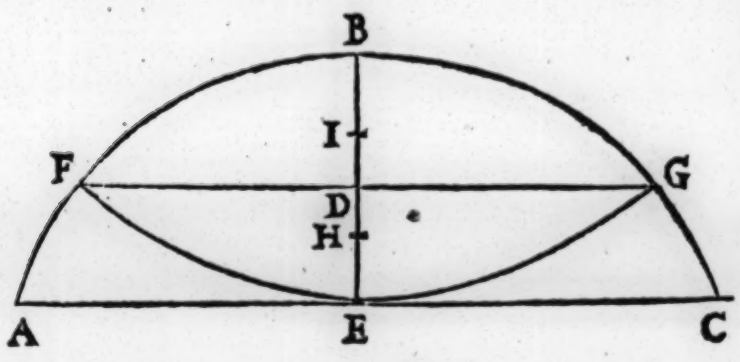
c 21. *huius.*

uitatis magnitudinis ex omnibus ſegmentis compoſitæ, aberit à vertice B, interuallo BP, quod minus eſt BQ, dimidio lineæ BK.

Igitur ſi ſegmento circuli vel ellipſis, &c. quod fuit demonſtrandũ.

PROPOSITIO XXXIX. THEOREMA XXXIV.

Cuiuſlibet ſegmẽti circuli vel ellipſis grauitatis centrum, plus diſtat à ſegmenti vertice, dimidio diametri ſegmenti.



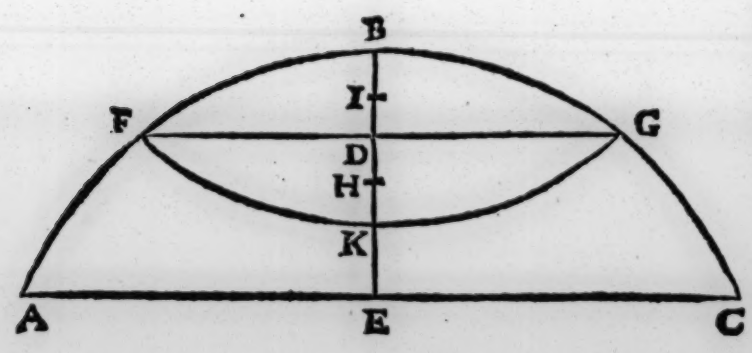
a 19. huius.

SIt A B C ſegmentum circuli vel ellipſis, & linea B E ſegmenti diameter.

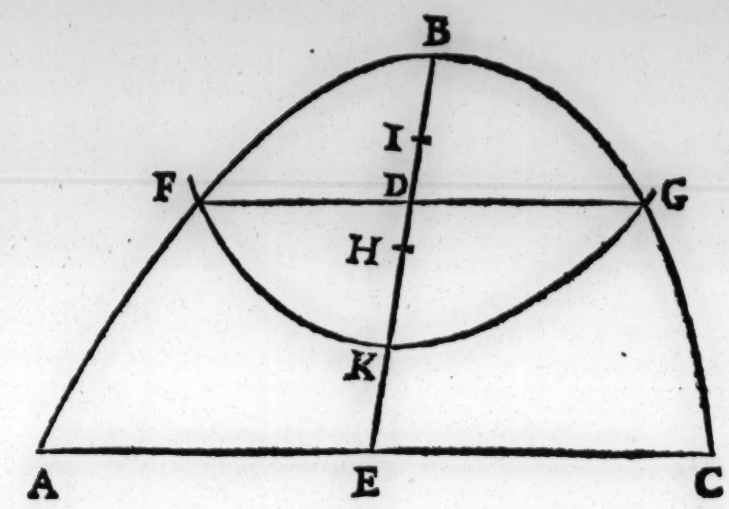
Dico centrum grauitatis ſegmẽti, [a] quod eſt in diametro BE, plus dimidio B E, abeſſe à vertice B.

Si enim id verum non ſit, vel aberit dimidio lineæ BE, vel interuallo quod dimidio minus ſit, à ſegmenti vertice.

Primò ponatur abeſſe dimidio BE, ſitque punctum D; ducatur per D linea F G, parallela A C, occurrens vtrimque arcui ſegmenti in F & G, & per puncta F E G, deſcribatur ſegmentum circuli vel ellipſis, æquale & ſimile ſegmento F B G, totumque cadet intra figuram A E G C, quæ omnia ex geometria euidentia ſunt. [b] Segmenti quoque F E G centrum grauitatis erit in D E, & ſit H, centrum item grauitatis ſegmenti F B G ſit I, [c] erunt D H & D I æquales, [d] ideoq; magnitudinis compoſitæ ex ſegmẽtis F B G, F E G, commune grauitatis centrum erit D, ob æqualitatem & ſimilitudinem ſegmentorum, & ex D æquiponderabunt;



b 19. huius



c 2. poſtulatum Luca Valerij.
d 1. petitio Archim. de æquip.

bunt; additâ igitur magnitudine, quæ cum ſegmento FEG, componit figuram AFCG,[e] inclinabit pondus ad partes ſegmenti FEG, tota enim compoſita magnitudo AFCG, ponderat ad partes ſegmenti FEG;[f] nam cum totius ſegmenti ABC, centrum grauitatis ſit in BE, item & ablatæ partis EBG ſit in eadem linea, etiam reliquæ figuræ AFGC, erit in eadem linea,[g] ſed etiam eſt intra ipſammet figuram AEGC, ergo erit intra lineam DE, ideoque tam ſegmentum FEG, quàm hoc quod additum eſt ſimul ſumpta, ponderant ex DE, cùm igitur inclinetur pondus, non erit D centrum grauitatis ſegmenti ABC.

e 2. *petitio Archim. de æquipond.*

f 8. *primi Archimedis de æquiponder.*

g 9. *petitio æquipond. Archim.*

Ponatur ſecundò centrum D, abeſſe minori interuallo, quam ſit dimidium BE, & ſumatur DK æqualis DB, & per FKG deſcribatur ſegmẽtum circuli vel ellipſis vt ſuprà, quod amplius cadet intra figuram AFGC, & inſtitutâ vt prius argumẽtatione, idem abſurdum conſequetur.

Idcirco cùm centrum grauitatis ſegmenti ABC, nec dimidio lineæ BE, nec interuallo dimidio minori abeſſe poſſit, cadet vltra dimidium lineæ BE, ſeu diametri ſegmenti, initio ſumpto à vertice.

Quare cuiuſlibet ſegmenti, &c. quod oſtendere oportuit.

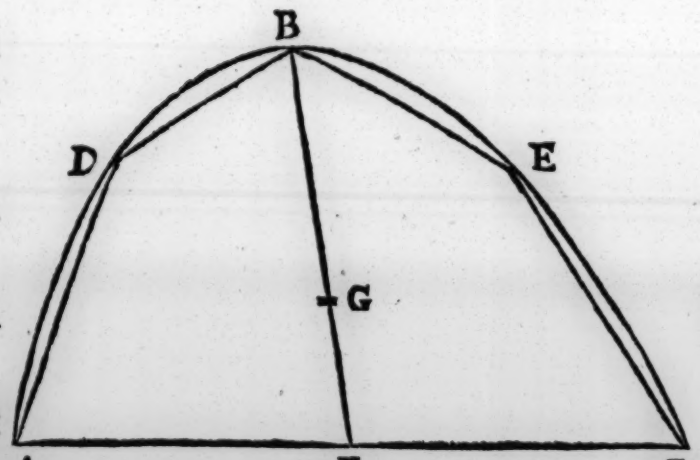
PROPOSITIO XL. THEOREMA XXXV.

Centrum grauitatis figuræ, ſegmento circuli vel ellipſis euidenter inſcriptæ, non poteſt idem eſſe, cum centro grauitatis ipſius ſegmenti.

SIt ABC ſegmentum circuli vel ellipſis, & figura ADBEC, illi euidenter inſcripta.

Dico vtriuſque idem eſſe non poſſe grauitatis centrum.

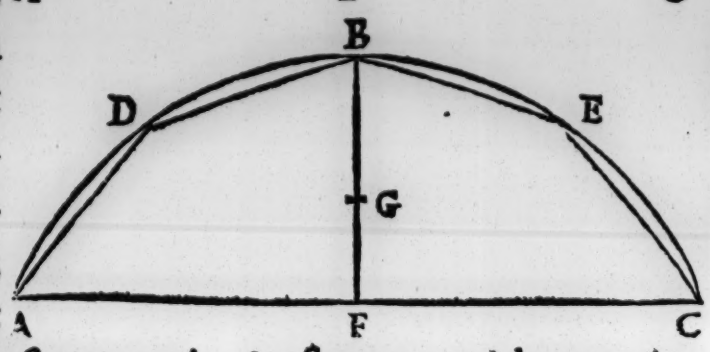
Si enim fieri poſſit, [a] cùm ſit in diametro BF, ſit punctum G.



a 18. & 19. *huius.*

Cùm G ſit centrum totius, id eſt ſegmenti; & partis, id eſt figuræ euidenter inſcriptæ, [b] erit etiam centrum grauitatis reliqui, id eſt magnitudinis ex reſiduis ſegmentis compoſitæ. ergo BG erit [c] minor dimidio BF, & ſimul [d] maior, quod fieri nequit; ergo nec hoc vnde ſequitur, ſcilicet idem eſſe ſegmenti, & figuræ euidenter inſcriptæ grauitatis centrum.

b 18. *primi Solidorum Lucæ Valerij.*



c 33. *huius*

d 39. *huius*

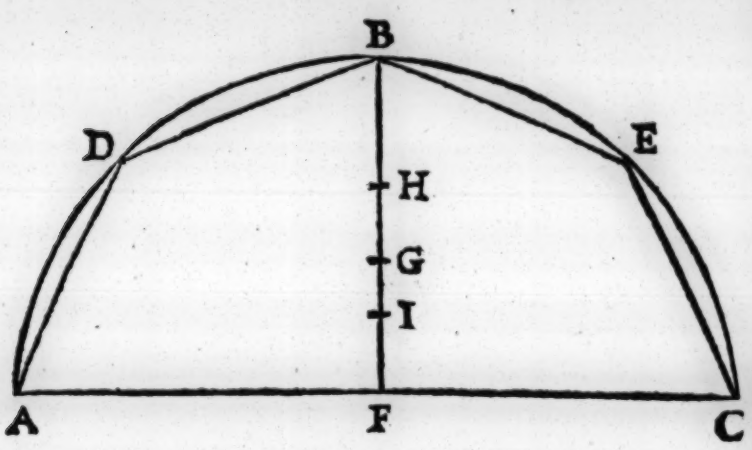
Ergo centrum grauitatis figuræ, &c. quod fuit demonſtrandum.

PROPOSITIO XLI. THEOREMA XXXVI.

Si ſegmento circuli vel ellipſis, figura rectilinea fuerit euidenter inſcripta, totius ſegmenti grauitatis centrum, erit vertici ſegmenti propinquius, quàm centrum figuræ inſcriptæ.

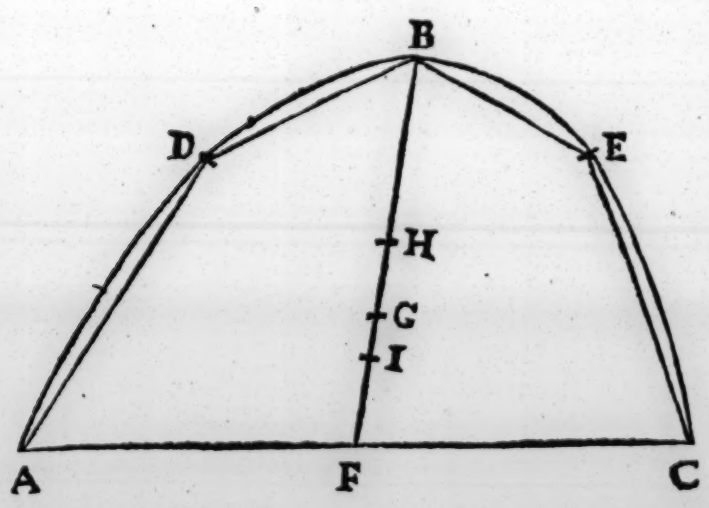
SIt ABC ſegmentum circuli vel ellipſis, & ADBEC figura rectilinea, ſegmento euidenter inſcripta.

Dico centrum grauitatis ſegmenti, minus diſtare à vertice, quam centrum grauitatis figuræ euidenter inſcriptæ.



a 18. & 19. *huius.*

b 40. *huius*



c 8. *primi Archim. de æquip.*

d 38. *huius*

e 39. *huius*

[a] Cùm vtriuſque magnitudinis centrum grauitatis ſit in BF, ſegmenti diametro, [b] nec idem poſſit eſſe vtriuſque centrum; ſit G centrum ſegmenti, H verò centrum magnitudinis, compoſitæ ex reſiduis ſegmentis, demptâ figurâ inſcriptâ. Cùm G ſit centrum totius, & H centrum partis, [c] neceſſe eſt centrum reliquæ partis cadere ad alteram partem G, id eſt in aliquod punctum I, [d] & cùm H ſit vicinius vertici quàm G, [e] debebit I eſſe remotius à vertice quàm G.

Itaque ſi ſegmento circuli vel ellipſis, &c. quod oportuit demonſtrare.

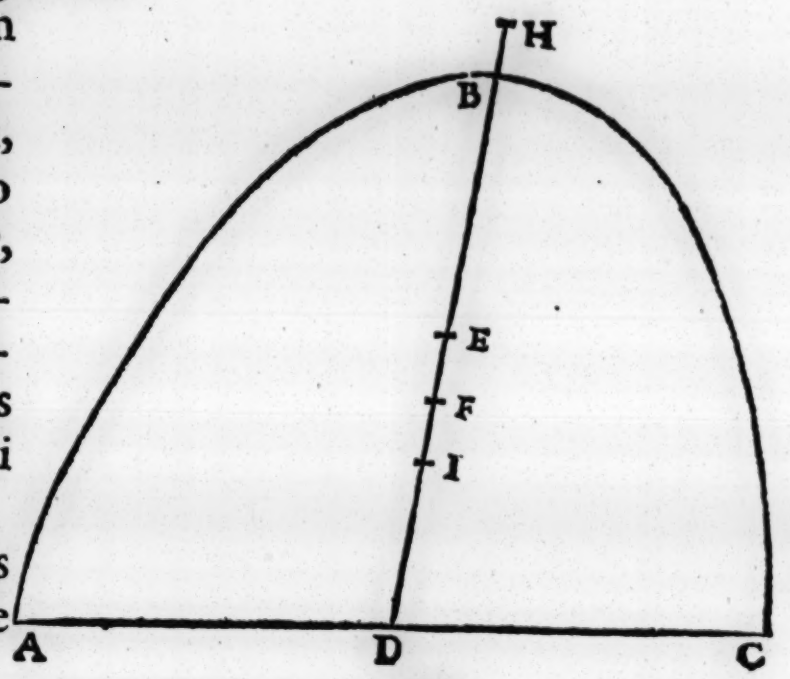
PRO-

PROPOSITIO XLII. THEOREMA XXXVII.

Dato ſegmento circuli vel ellipſis, poteſt figura rectilinea euidenter inſcribi, vt linea quæ inter centrum grauitatis ſegmenti & centrum inſcriptæ figuræ interiicitur, ſit quacumque lineâ rectâ datâ minor.

SEgmentum circuli vel ellipſis ſit ABC, centrum grauitatis eius E, interuallum datum EF, ita vt F ſit infra, [a] cùm centrum grauitatis figuræ cadat infra centrum ſegmenti. a 41. *huius*

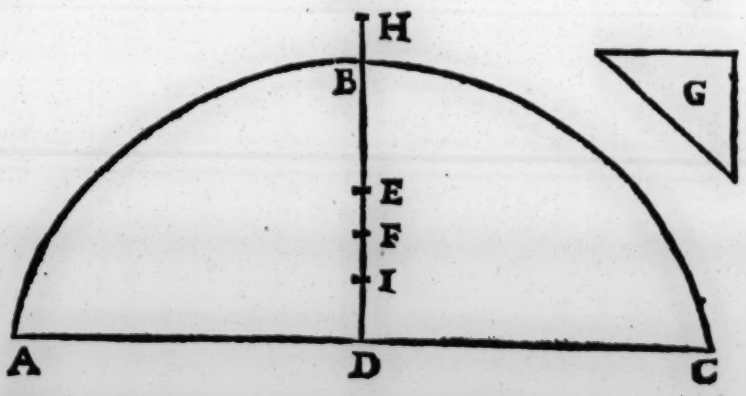
Sumatur aliquod ſpatium G, ad quod ſegmentum maiorem habeat proportionem, quàm BF ad EF, & ſegmento figura euidenter inſcribatur, donec ſpatium ex reſiduis ſegmentis, dempta inſcripta figura compoſitum, ſit minus ſpatio G, conſtat verò id fieri poſſe.



Dico centrum grauitatis illius figuræ inſcriptæ, cadere ſupra F.

Si non cadat ſupra F, vel cadet in ipſum F, vel infra.

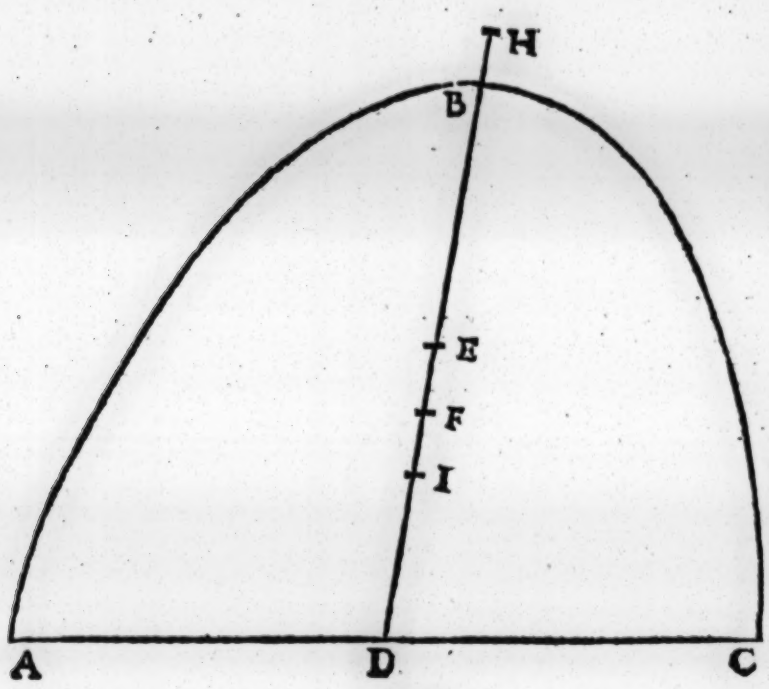
Ponamus primò coincidere cum F; cum E ſit centrum totius ſegmenti, & F partis, ſeu figuræ euidenter inſcriptæ, [b] erit centrum reliquæ partis, id eſt magnitudinis ex reſiduis ſegmentis compoſitę, in FE, producta verſus E, in eoque puncto puta H, vt FE, ad EH ſit, vt reſiduum ex ſegmentis, ad inſcriptam figuram; quare etiam componendo erit vt HF ad FE, ita totum ſegmentum, ad reſiduum ex ſegmentis; cùm verò illud reſiduum iuxta conſtructionem ſit minus ſpatio G, erit maior proportio ſegmenti ad reſiduum, quàm ſegmenti ad ſpatium G, ideoque maior proportio HF ad FE, quæ eſt vt ſegmentum ad reſiduum, quàm BF ad FE; ergo H centrum grauitatis ſpatiorum reſiduorum, cadit extra ſegmentum ABC, [c] quod fieri nequit. igitur centrum grauitatis inſcriptæ non coincidit cum F, ex quo abſurdum conſequitur.

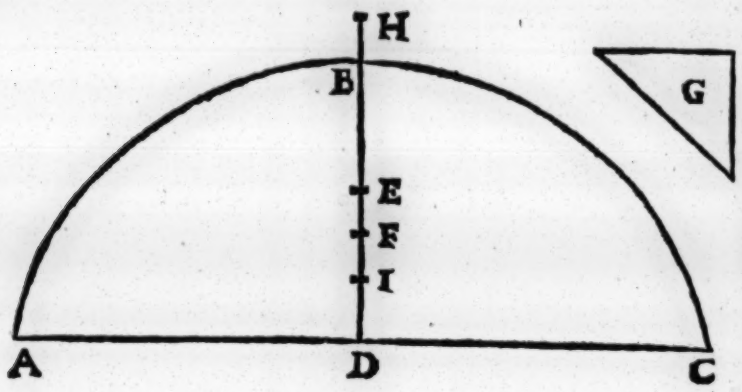


b 8. *primi Archim. de æquip.*

c 17. *huius*

Cadat ſecundò centrum grauitatis infra F in I; cùm E ſit centrum grauitatis totius, id eſt ſegmenti; & I centrum grauitatis figuræ inſcriptæ, centrum reliqui, id eſt magnitudinis ex reſiduis ſegmentis compoſitæ ſit H; vt prius erit maior proportio ſegmenti ad reſiduum, quàm BF ad BE, ergo quoque diuidendo, maior erit proportio figuræ inſcriptæ ad reſidua ſegmenta, quàm B E ad E F, multoque maior quàm BE ad E I, ſed vt inſcripta figura ad reſidua ſegmenta, ita HE ad E I, ergo maior proportio HE ad E I, quàm BE ad E I, ergo HE maior B E, ergo H cadit extra ſegmentum, centrum nimirum grauitatis reſiduorum ſegmentorum, d quod fieri nequit; ergo nec inſcriptæ figuræ centrum grauitatis cadit infra F, ex quo id impoſſibile ſequitur.





d 17. huius

Cùm ergo inſcriptæ figuræ centrum grauitatis, quod neceſſariò cadit infra E centrum ſegmenti, non coincidat cum F, nec cadat infra F, cadet intra ſpatium F E, eritque propinquius centro ſegmẽti, quàm ſit datum ſpatium F E.

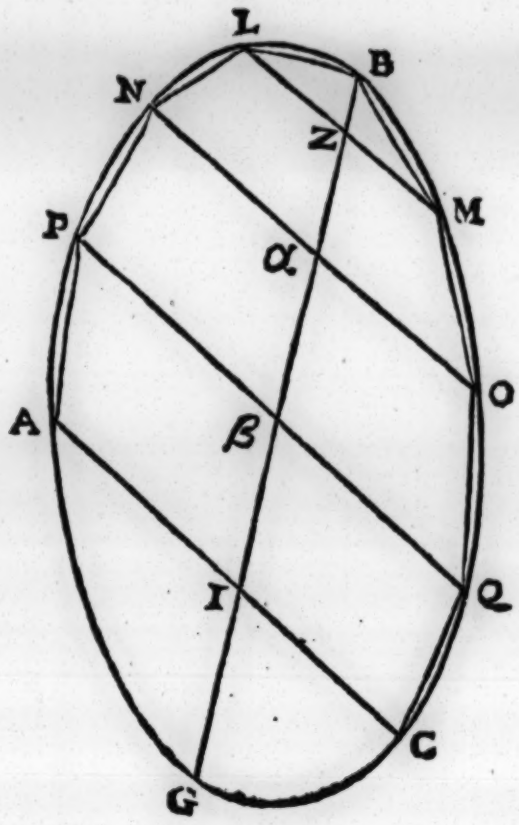
Quare poſſibile eſt dato ſegmento, &c. quod fuit demonſtrandum.

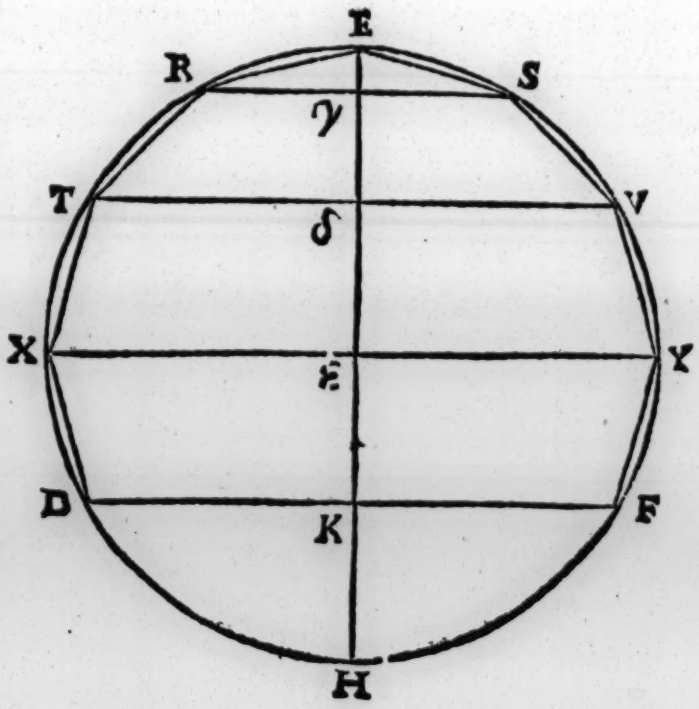
PROPOSITIO XLIII. THEOREMA XXXVIII.

Si dentur duo ſegmenta, vnum ellipſis alterum circuli, & quam proportionem habet ſegmentum ellipſis ad totam ellipſim, eamdem habeat ſegmentum circuli ad totum circulum; & in vtroque rectilineæ figuræ euidenter deſcribantur, quarum latera ſint multitudine æqualia, figurarum centra grauitatis, ſimiliter ſecabunt ſegmentorum diametros.

Sit

SIt ABC ſegmentum ellipſis, DEF ſegmentum circuli, & ſit ſegmentum ellipſis ad totam ellipſim ABCG, vt ſegmentum circuli ad totum circulum DEFH, & ſegmento quidem ellipſis, figura APNLBMOQE euidenter inſcripta ſit, ſegmento verò circuli, figura DXTRESVXF.





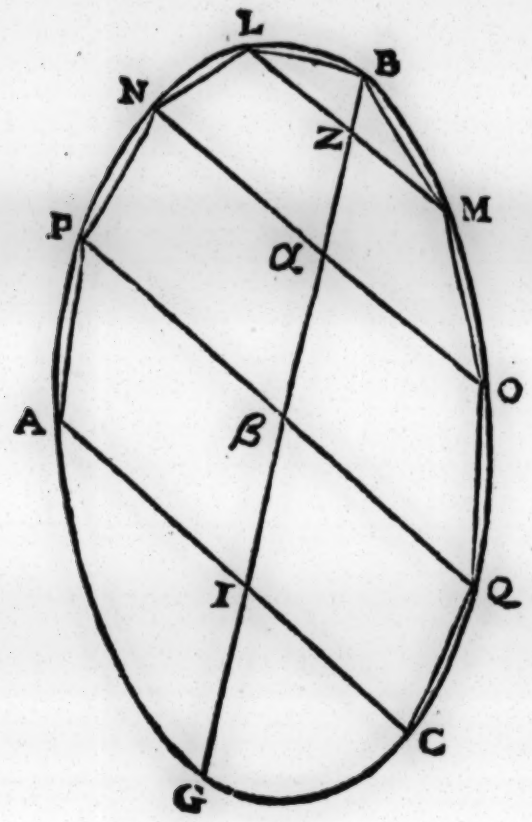
Dico centra grauitatis vtriuſque figuræ inſcriptæ, ſimiliter ſecare diametros ſegmentorum BI, EK.

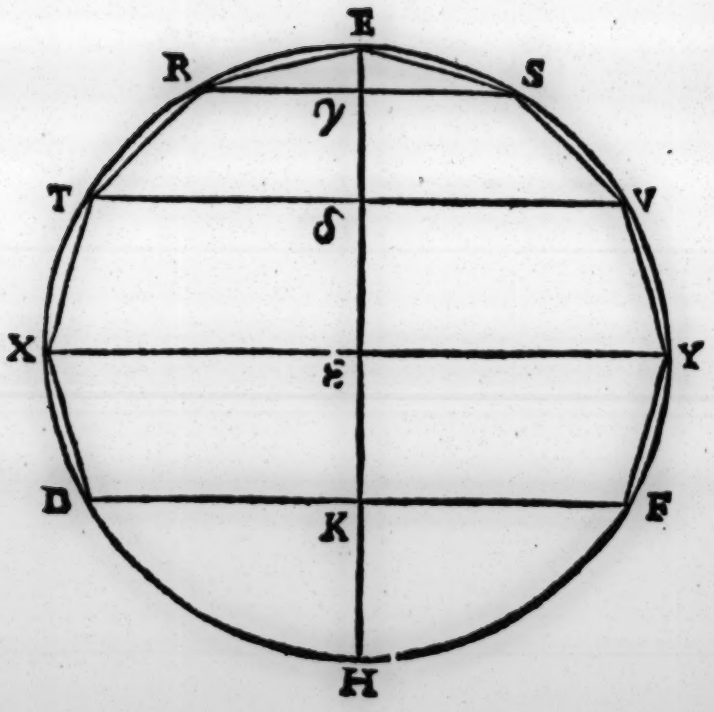
Iungantur LM, NO, PQ, in ellipſi; & RS, TV, XY in circulo.

Iam ſuppono ex Conicis, ductas lineas inter ſe parallelas eſſe, & ad diametros bifariam diuidi; & parallelas in ſegmento ellipſis, cum reſpondentibus comparatas, inter ſe eamdem habere proportionem, quam illæ quæ in ſegmento circuli reſpondent illis, quæ in ellipſi inter ſe comparatæ ſunt; denique ipſas ſegmentorum diametros, ad totas ſuarum figurarum diametros, eamdem habere proportionem, eò quòd ſegmentorum areæ, ad totas ſuarum figurarum areas, eamdem proportionem habeant. Hæc enim omnia ex Conicis demonſtrationibus, apud geometras nota ſunt, & ea hîc demonſtrare ab inſtituto alienum foret.

His ſuppoſitis propoſitum demonſtrabo. [a] Trapeziorum APQC, DXYF centra grauitatis, erunt in Iβ, Kε, rectis lineis ſimiliter poſita, cùm eamdem habeant proportionem AC, PQ, quam DF, XY. Similiter

[a] 15. *primi Archim. de æquipond.*





liter etiam trapeziorum PNOQ, XTVY centra grauitatis, ſimiliter diuident lineas βα, εδ; denique & in trapeziis reliquis, centra grauitatis ſimiliter diuident ſuas lineas, eadem de cauſa; ſunt autem & triangulorum LPM, RES centra grauitatis, in lineis BZ, Eγ ſimiliter poſita; habent autem eamdem proportionem trapezia & triangula, vt ſibi reſpondent in ſegmentis propoſitis circuli & ellipſis, quare ſi duorum primorum trapeziorum in ſegmento ellipſis, commune centrum grauitatis inuentum fuerit; itidem duorum trapeziorum primorum in ſegmento circuli, lineæ αI in ellipſi & δK in circulo ſimiliter diuidentur, atque ita progreſſione facta ad reliqua, totius figuræ rectilineæ, in ſegmento ellipſis ABC inſcriptæ, centrum grauitatis ſimiliter diuidet diametrum BI, ſicut figuræ in ſegmento circuli inſcriptæ grauitatis centrum, diuidet lineam EK.

Igitur ſi dentur duo ſegmenta, &c. quod fuit demonſtrandum.

PROPO-

PROPOSITIO XLIV. THEOREMA XXXIX.

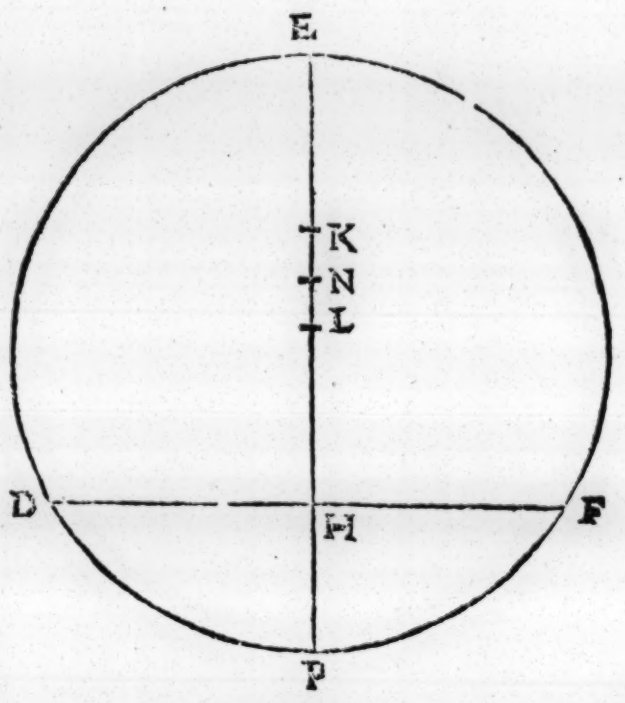
Si duo ſegmenta data fuerint, vnum ellipſis alterum circuli, & quam proportionem habet ſegmentum ellipſis ad totam ellipſim, eamdem habeat ſegmentum circuli ad totum circulum; centra grauitatis in eamdem proportionem diuident earum diametros.

SInt duo ſegmenta ABC, DEF, qualia propoſitio poſtulat, eorum diametri BG, EH, centra grauitatis I & K.

Dico eamdem eſſe proportionem BI ad IG, quæ eſt EK ad EH.

Si enim non ſit eadem proportio, erit maior vel minor.

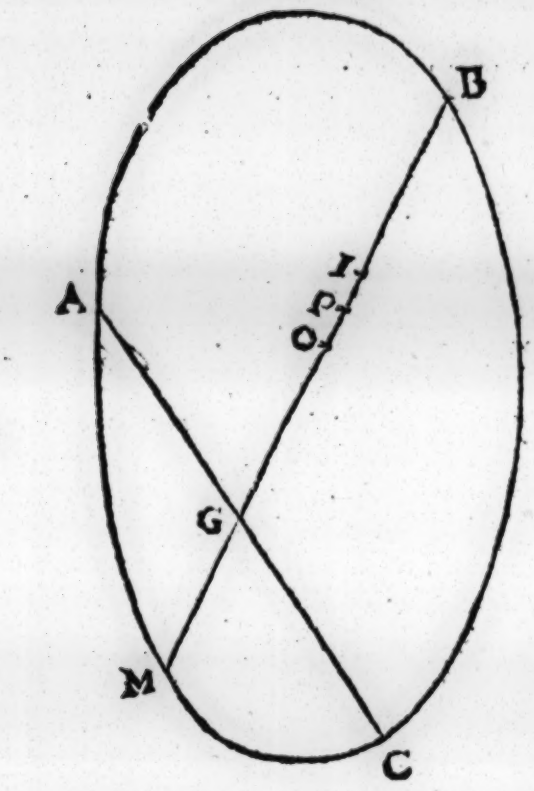
Sit primò maior proportio BI ad IG, quàm EK ad KH, & diuidatur EH in proportionem BI ad BG, in aliquo puncto L, cadet illud neceſſariò infra punctum K. [a] Inſcribatur ſegmento circuli DEF, euidenter figura rectilinea, cuius centrum grauitatis N, minus abſit à puncto K, interuallo KL, & ſicut diuiſa eſt EH in N, ita diuidatur BG in aliquo puncto O, cadet punctum O ſupra punctum I, nam maior eſt proportio EN ad NH, quàm EL ad LH, id eſt BI ad IG; deinde inſcribatur ſegmento ellipſis ABC euidenter figura rectilinea totidem laterum, eius centrum grauitatis erit O, [b] cadetque ſupra centrum grauitatis ſegmenti, [c] quod fieri nequit; vnde nec maior erit proportio BI ad IG, quàm EK ad KH, vnde id ſequitur.



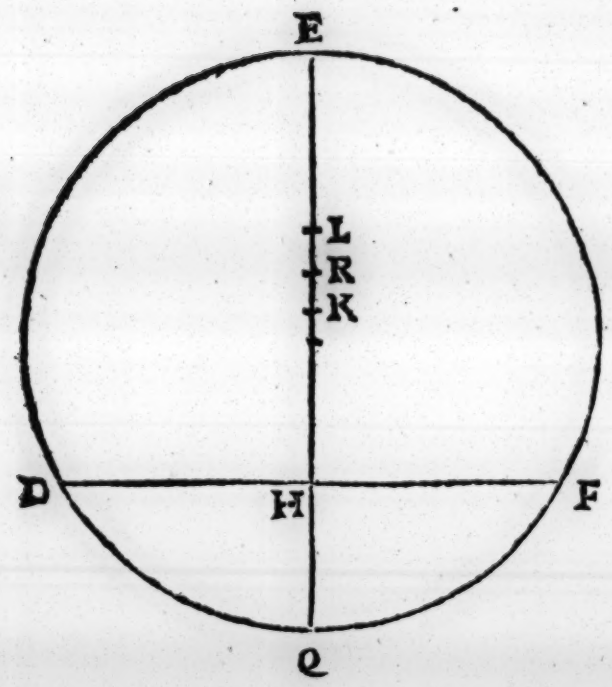
a 42. *huius*

b 43. *huius*

c 41. *huius*



d 42. huius



e 43. huius

Sit ſecundò minor proportio BI ad IG, quàm EK ad KH, & diuidatur EH in proportionem BI ad BG, in aliquo puncto L, cadet illud neceſſariò ſupra punctum K, & tum diuidatur BG in O, vt ſit BO ad OG, ſicut EK ad KH, cadet O neceſſariò infra I. [d] inſcribatur ſegmento AB C euidenter figura rectilinea, cuius centrum grauitatis P, minus abſit à puncto I, interuallo OI, & ſicut diuiſa eſt BG in P, ita diuidatur EH in R, cadet R ſupra K; nam minor eſt proportio BP ad PG, quàm BO ad OG, id eſt EK ad KH, deinde inſcribatur ſegmento circuli DE F, euidenter figura rectilinea totidem laterum, eius centrum grauitatis erit R, [e] cadetque ſupra centrum grauitatis ſegmenti, quod fieri nequit; vnde nec minor erit proportio BI ad IG, quàm EK ad KH, vnde id ſequebatur.

Idcirco cùm proportio BI ad IG, nec maior nec minor ſit proportione EK ad EH, erit eadem.

Ideo ſi duo ſegmenta data fuerint, &c. quod oſtendere oportuit.

PROPO-

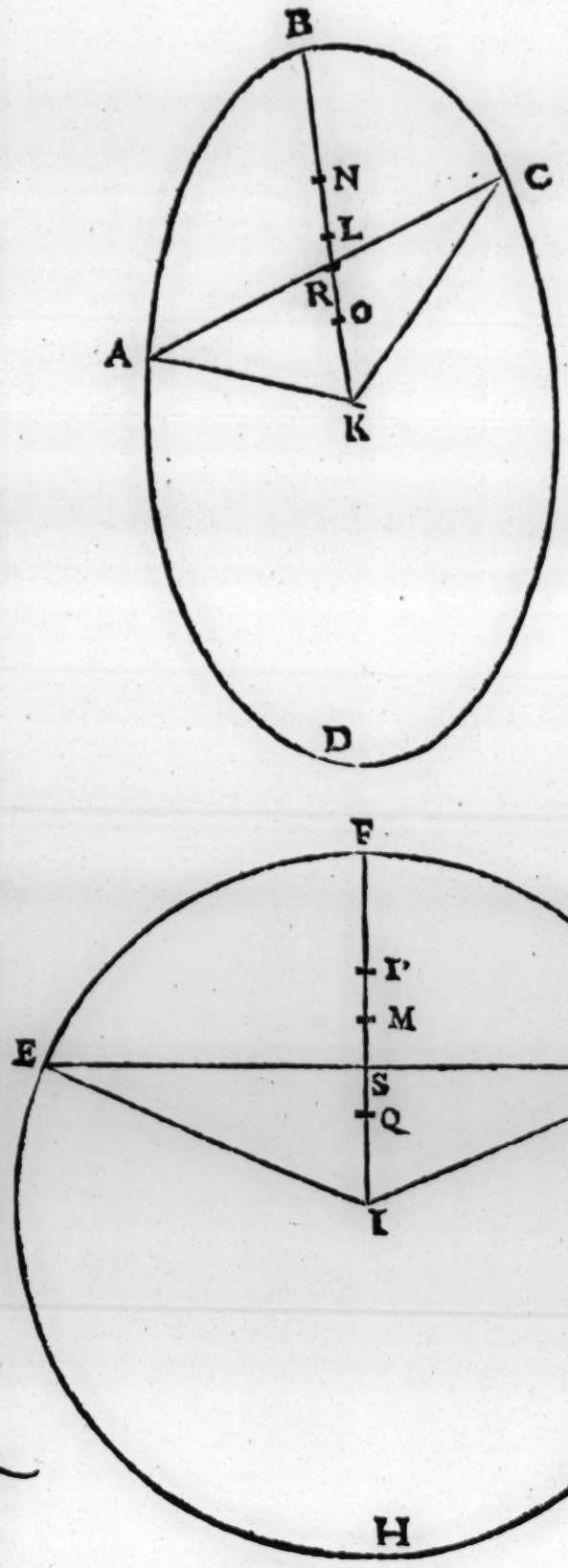
PROPOSITIO XLV. THEOREMA XL.

Si fuerint duo Sectores, vnus ellipticus alter circularis, dimidiis ſuis figuris minores, æquales, vel maiores; & quam proportionem habet vnus Sector ad ſuam figuram, eamdem habeat alter Sector ad ſuam, centrum grauitatis ipſorum, in eamdem rationem diuidet ſemidiametros illas, quæ Sectores bifariam ſecant.

SInt primò Sectores ſemiellipſi aut ſemicirculo minores ABCK, EFGI, eoſque bifariam ſecent ſemidiametri KB, IF.

Dico ſemidiametros illas, à centris grauitatis diuidi in eamdem rationem.

Ducantur AC, EG; quòd AC EG ſecent ſimiliter ſemidiametros in R & S, & ſegmenta ABC, EFG ſingula ad reliqua triangula, & totas figuras, eamdem habeant proportionem, ex Conicis demonſtrationibus notum eſt. ponantur N & P eſſe centra grauitatis ſegmentorum; puncta verò O & Q eſſe centra grauitatis triangulorum ([a] erunt autem N & P in ſemidiametris KB, IF, baſes & ſegmenta biſecantibus, [b] & O & Q in iiſdem ſemidiametris, quæ baſes triangulorum AKC, EIG bifariam diuidunt) [c] erit BN ad NR, vt FP ad PS. ditem KO ad OR, vt IQ ad QS, & cum N, O, ſint centra partium Sectoris elliptici, P & Q verò centra partium Sectoris circularis, totorum Sectorum

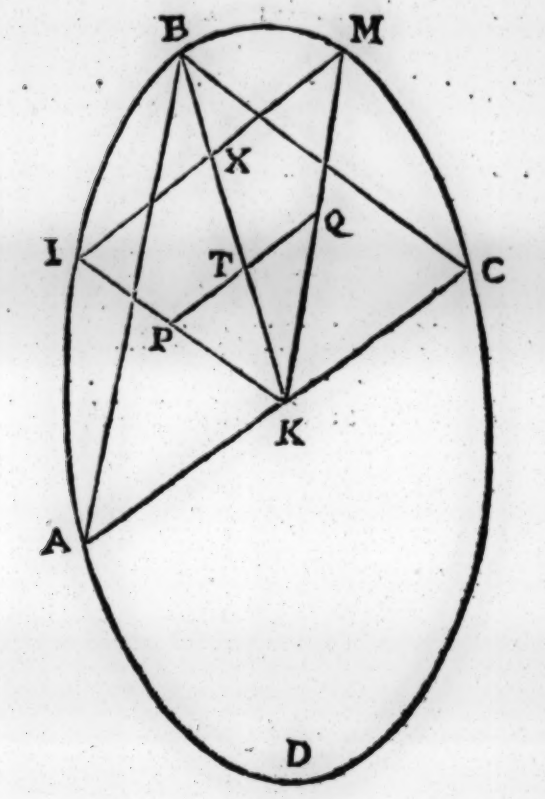


a 19. *huius.*

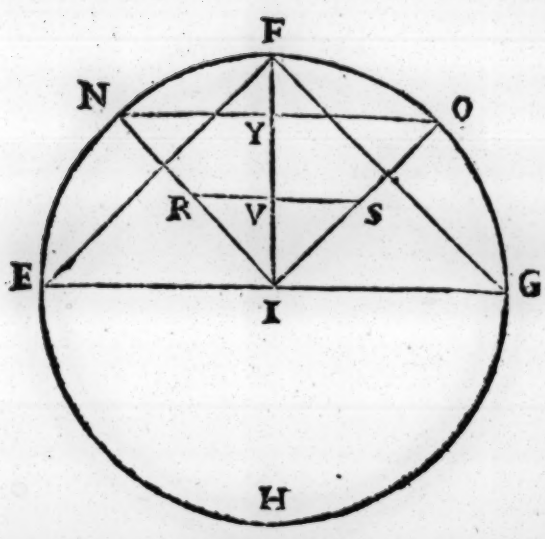
b 13. *primi Archim. de æquip.*

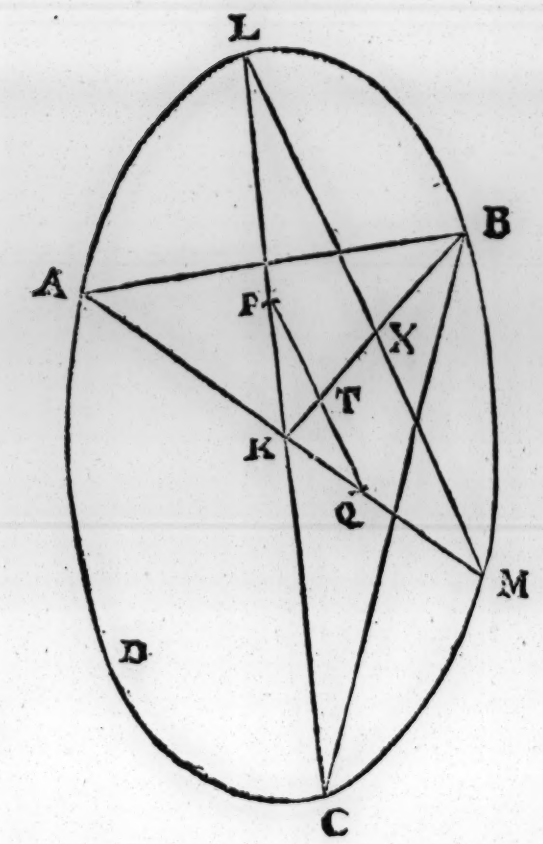
c 44. *huius*

d *corollar.* 14. *primi Archim. de æquip.*



e 6. & 7. *primi æquiponder. Archim.*





f *ex iam dictis.*

g 1. *petitio aquipond. Archim.*

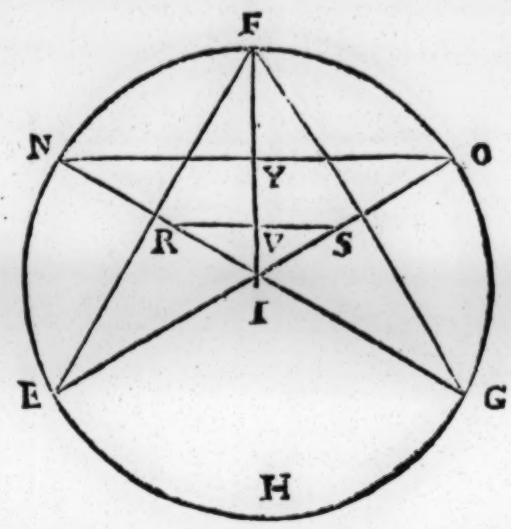
centra erunt in NO, PQ, & consequenter in semidiametris Sectores bifariam secantibus; sint L & M, e erunt reciprocè OL ad LN, & QM ad MP, sicut segmenta ad triangula, quæ cùm vtrobique in eadem sint proportione, erit OL ad LN, vt QM ad MP. ideoque manifestum est puncta N, Q, L, O, ita diuidere semidiametrũ BK, vt puncta P, S, M, Q, diuidunt semidiametrum FI, & consequenter puncta L & M, quæ sunt centra grauitatis Sectorum, semidiametros in eamdem rationẽ secare.

Sint secundo ambo mediæ parti suarum figurarum æquales aut maiores, ABCK ellipticus, EFGI circularis, eosque bifariam secent semidiametri BK, IF, factosq; Sectores, in ellipsi quidem ALBK, CMBK, in circulo verò ENFI, GOFI, bifariam secent semidiametri KL, KM, IN, IO, inuentaque sint centra dictorum Sectorum, in quas toti Sectores propositi diuisi sunt, (quos medietate figurarum suarum minores esse necesse erit) sintque centra grauitatis, in ellipsi quidem P, Q, in circulo verò R, S. (erunt enim in illis semidiametris vt iam ostendimus) ducanturque LM, PQ, NO, RS, quæ secent BK quidem in X & T, at FI in Y, V.

Lineas LM, NO bifariam diuidi à semidiametris BK, FI, ex conicis patet; sed cùm toti Sectores ad suas figuras eamdem habeant proportionem, etiam ipsorum dimidij ad suas figuras eamdem habebunt proportionem, f ideoque inuenta grauitatis centra A, Q; R, S. diuident omnes illas semidiametros in eamdem proportionem, & PQ erit parallela LM, & RS parallela NO, & tam PQ quàm RS, ad semidiametros BK, FI bifariam diuidentur, cum LM, NO ipsis parallelæ, bipartitò secentur. g erit quoque in ellipsi,

ob

ob Sectorum æqualitatem, commune centrum grauitatis amborum Sectorum, punctum T, id est totius Sectoris ABCK, & in circulo punctum V, commune duorum Sectorum centrum, id est totius Sectoris EFGH, sed BK diuiditur in X, quemadmodum FI in Y, vt constat ex conicis, & XK in T, sicut YI in V, cum YT ad TK, sit vt LP ad PK, & YV sit ad VI, vt NR ad RI, [h] sint autem LK in P, & NI in R similiter sectæ, ergo erit etiam tota BK in T, centro grauitatis Sectoris, ita secta, sicut FI in V, itidem centro grauitatis Sectoris.



h *ex iam dictis.*

Igitur si fuerint duo Sectores, &c. quod fuit demonstrandum.

COROLLARIA.

I.

INuentâ circuli quadraturâ, dabitur centrum grauitatis cuiuslibet Sectoris, minoris & maioris semicirculo, ipsius semicirculi, & segmentorum semicirculo maiorum & minorum, si quidem proportio arcus ipsorum, ad totam peripheriam sit cognita; & hæc de ellipticis figuris, quæ illis respondent, etiam intelligantur.

II.

Dato cuiuslibet istarum partium grauitatis centro, ipsius etiam quadratura erit obuia; dabuntur enim tres lineæ rectæ, quibus sumpta quarta proportionalis, sit arcui toti, aut dimidio figuræ propositæ æqualis. quâ detectâ à nobis connexione, inter quadraturam circuli & centra grauitatis partium eius, simul nouam aperuimus viam quadrandi circulum; si nimirum centra grauitatis partium, independenter à quadratura quærantur.

III.

Lunularum quarumuis, à quibuslibet duabus circumferentiæ portionibus comprehensarum; similiter spatiorum inter duas lineas parallelas, vel non parallelas interceptorum; Arbelorum quoque & scalprorum, seu securicularum, & quarumuis figurarum, vel ex arcubus circuli, vel arcubus & rectis lineis compositarum, dabuntur centra grauitatis; quæ omnes subtensis rectis lineis sub arcubus, in segmenta circulorum, & rectilineas figuras resoluentur. Et hæc in ellipsi quoque locum habent.

IV.

Sed & ad corporum quorumdam, quæ nos hactenus latuêre, grauitatis centra reperienda, hæc licebit transferre. Dabuntur enim centra grauitatis partium cylindrorum, dictis iam figuris velut basibus insistentium, bases oppositas parallelas habentium. Similiter & partium conorum per verticem Sectorum, talibus basibus siue circularibus siue ellipticis insistentium. Dabitur enim axis grauitatis, in cylindris quidem per centra grauitatis oppositarum basium parallelarum, in conis verò ductus ad centrum grauitatis basis per verticem, in cuius medietate centrum grauitatis partium cylindricarum, in quarta verò parte quæ basi adiacet, partium conicarum reperietur; iuxta ea quæ Fridericus Commandinus, & Lucas Valerius de centris grauitatis solidorum tradidêre.

R.P.PRO-

R. P. PROVINCIALIS Societatis IESV in Prouincia Toletana Facultas.

MICHAEL PACHECVS Societatis IESV, in Prouincia Toletana Præpositus Prouincialis, potestate ad id mihi facta à Reuerendo admodum Patre nostro MVTIO VITELLESCHO, Præposito Generali, facultatem facio, vt liber inscriptus: IOANNIS DELLA FAILLE ANTVERPIENSIS, E SOCIETATE IESV, IN ACADEMIA MATRITENSI COLLEGII IMPERIALIS, REGII MATHESEOS PROFESSORIS, THEOREMATA DE CENTRO GRAVITATIS PARTIVM CIRCVLI ET ELLIPSIS, eiusdem Societatis grauium doctorumque hominum iudicio approbatus, typis mandetur. In quorum fidem has litteras manu nostra subscriptas, & sigillo nostro munitas dedimus. Matriti, in nostro Imperiali Collegio x. Kal. Decembr. ∞ IↃC XXXI.

MICHAEL PACHECO.

GVILIELMVS DE VVAEL Societatis IESV per Flandro-Belgium Præpositus Prouincialis IOANNI MEVRSIO Typographo Sal.

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GVILIELMVS DE WAEL.

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Signat.

I. COOLS.

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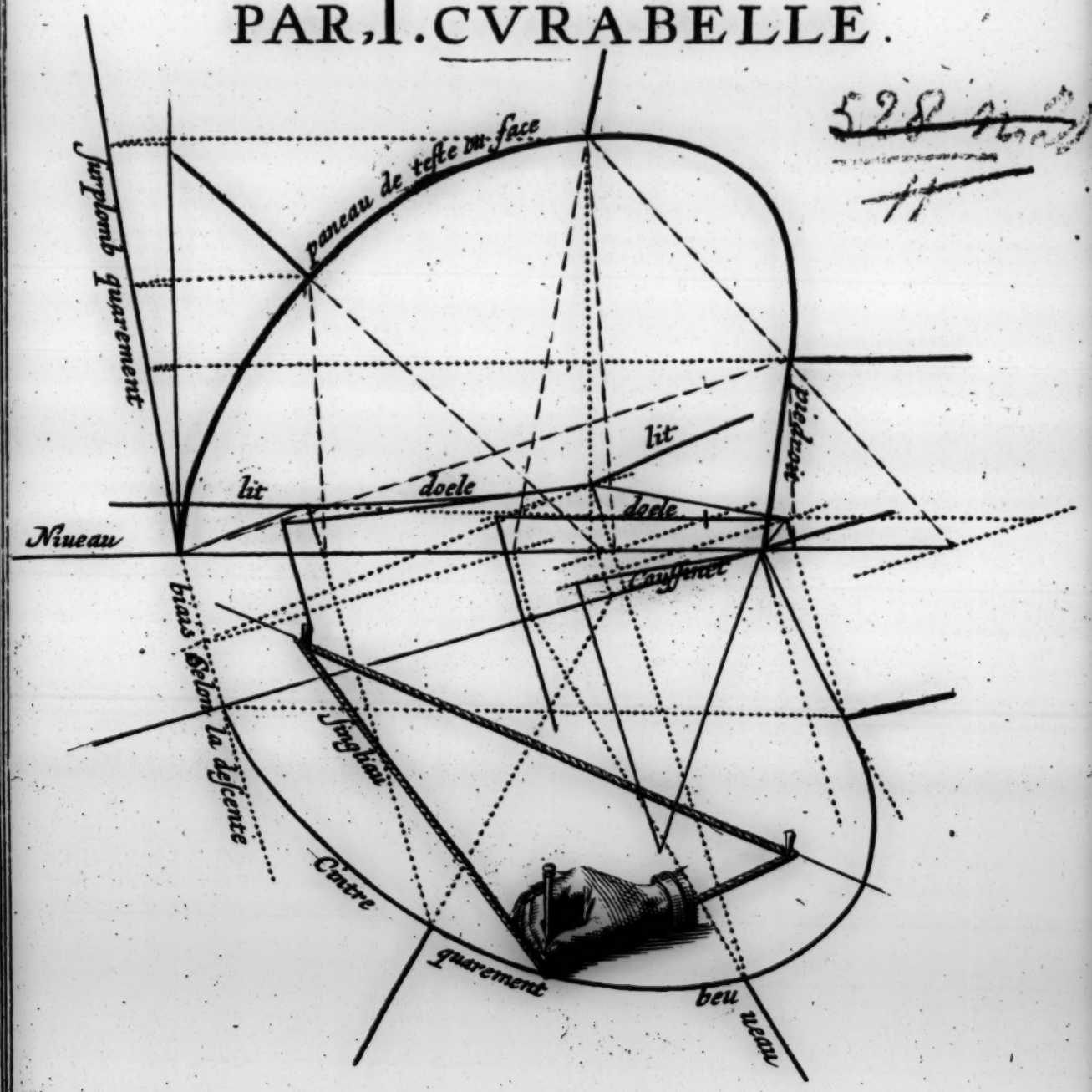
DV S.R DESARGVES
PAR, I. CVRABELLE.



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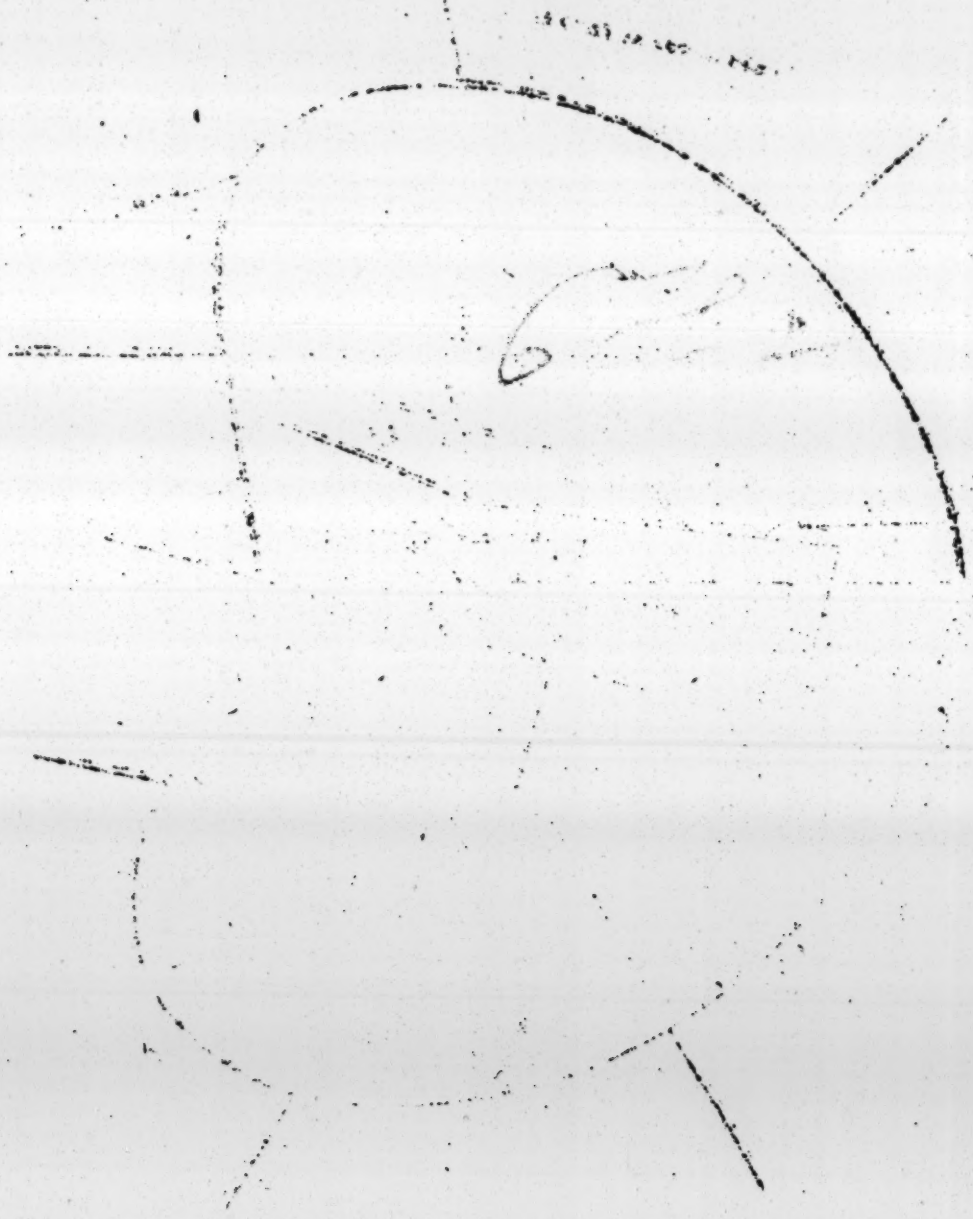


ADVERTISSEMENT AV LECTEVR.

VOicy le premier eſſay, & comme vn eſchantillon des fruits que mon eſprit par la culture des ſciences a produit, depuis que i'ay commencé à operer par le moyen de la raiſon, iuſques à maintenant. Ie les offre, comme les premices de mes eſtudes & labeurs, à la ſouueraine bonté, ſource inépuiſable de tous les excellens biens & dons parfaits qui découlent du Ciel icy bas, à laquelle doiuent retourner, comme à leur principe & par droict de reconnoiſſance les benefices que nous receuons de ſa main liberale; Si vous y rencontrez quelque choſe de bon, nous ſommes obligez vous & moy de le raporter à la gloire de Dieu, pour laquelle nous deuons agir inceſſamment; que ſi au contraire vous y remarquez quelque defaut, ſouuenez-vous que la nature ne nous a pas tellement accomply, ny departy ſi abondamment ſes faueurs, qu'elle ne nous ait touſiours laiſſé le charactere d'homme; Chacun pourra ſauourer de ces fruits que ie luy preſente, ſelon ſon gouſt, ou ſelon ſon beſoin; Ie ſcay bien que vous ne treuuerez point cét ouurage parſemé & diapré des fleurs de l'Eloquence; Le ſubiect que ie manie ne demande pas vn ſtyle fleuriſſant & curieuſement elabouré: ie laiſſe la politeſſe du diſcours aux Orateurs du temps qui s'eſtudient plus à perſuader en chatoüillant les aureilles auec des paroles mignardes & fardées, que par la force des raiſons: auſſi eſt-ce vne des principales parties de l'Art qu'ils profeſſent, & ce qui leur acquiert vne haute reputation parmy le vulgaire. Ie me contente de parler intelligiblement; ce m'eſt aſſez auec cela que ie ne commette point d'incongruité, exprimant nettement mes penſées en la matiere que i'ay à traiter, eſtant le plus ſouuent contraint d'vſer tant de mots cognus par les artiſans, que de certaine maniere de parler, qui bleſſent les aureilles delicates de ceux qui affectent le bien dire, ce que cette ſcience meſpriſe comme choſe inutile pour eſtablir & verifier ſes demonſtrations ſur quoy elle ſe fonde principalement. Au reſte ie vous prie de croire, que l'occaſion qui m'a meu de donner au public ce Liuret auparauant mon cours d'Architecture, n'eſt point vn deſir de paroiſtre; ny que ie ſois homme qui me plaiſe à ſindiquer & cenſurer les œuures d'autruy, mon humeur n'eſt point critique: moins encore que ie pretende acquerir quelque vaine gloire par cette voye, puis que ie vous prie derechef la referer à Dieu: Quant à l'intereſt que i'y eſpere, ie croy que chacun ſçait qu'en telle matiere il eſt de peu de conſequence. Mais le principal ſubiet & le plus puiſſant eſguillon qui m'a incité à donner au public ce petit Ouurage eſt ſans doute le maintien & la deffenſe de la préciſion & verité, de l'amour de laquelle ie ſuis ardemment eſpris, & embraſſé d'vne cordiale affection tous ceux qui la profeſſent, les ſuppliant de m'honnorer de leurs ſentimens, au preiudice meſme de mon propre intereſt, auquel ie ne ſuis point attaché, ſinon entant que veritable: A Dieu.

Extraict du priuilege du Roy.

PAr grace & priuilege du Roy il eſt permis à IACQVES CVRABELLE, de faire imprimer, & vendre par tel Imprimeur & Libraire que bon luy ſemblera, vn *Cours d'Architecture* par luy compoſé, diuiſé en quatre Tomes; Le premier deſquels contient, *La Stereotomie, ou ſection des ſolides, appliquée à la coupe des pierres*, Et *ſon Appendix des quadrans, tant par rayon d'incidence, que de fraction & reflexion.* Le ſecond comprend *l'Optique vniuerſelle, tant Theorique que pratique, auec ſon Appendix de l'Echometrie, & Muſique.* Le troiſieſme eſt, *Des organes vniuerſels pour les poids & mouuemens, tant des choſes ſolides qu'humides; & ſon Appendix des ſpiritales hydroliques.* Et le quatrieſme, *Les Ornemens & proportions d'Architecture, tant neceſſaires que decorables; auec vn Appendix de l'Architecture militaire, & Annotations ſur la Ciuile.* De plus, vn *Examen des Oeuures du ſieur Deſargues Lyonnois*; Et ce durant le temps de vingt années conſecutiues, à commencer du iour que chacun Liure ſera acheué d'imprimer pour la premiere fois; auec deffenſes à tous Libraires', Imprimeurs & autres perſonnes de quelle qualité ou condition qu'ils ſoient, de les copier, ny faire imprimer en tout, ny en partie, ny par aucun déguiſement que ce ſoit, ny les vendre & diſtribuer ſans permiſſion de l'Autheur, à peine de ſix mille liures d'amande aux contreuenans, & confiſcation de tous les exemplaires; comme auſſi de tous deſpens, dommages & intereſts; Et voulons que foy ſoit adiouſtée au preſent extraict, & tenu pour ſignifié, ainſi qu'il eſt plus au long contenu audit Priuilege. Donné à Paris, le 4. Decembre 1643.

Signé, LOVIS.

Et plus bas,

De par le Roy, la Reyne Regente ſa mere preſente,

DE GVENEGAVD.

Et ſeellé du grand Sceau en cire jaune.

Acheué d'imprimer pour la premiere fois le 29. Decembre 1643.

SOMMAIRE DES MATIERES CONTENVES au preſent Examen des Oeuures du ſieur Deſargues. premierement ſur ſon Broüillon de la coupe des pierres; & ſur ſon Liure de meſme matiere, imprimé l'an 1643.

PREMIERE PARTIE.

QVe la maniere de traict dudit ſieur par ſa ſous-eſsieu, eſt la meſme choſe que la ſouſtylaire des quadrans, & n'eſt que maniere imitée. page 10.

Que la traditiue amandée eſt triplement plus generale, & fait ce que la maniere dudit ſieur ne ſçauroit faire. 11.

Que le nom de plan en l'Architecture eſt plus general & plus propre que celuy de chemin, 12.

Que la Geometrie a des plans terminez contre le propoſé dudit ſieur. 13.

Qu'en l'Architecture, tant theorique que pratique, l'on ſuppoſe meſmes plans qu'en la Geometrie contre le propoſé dudit ſieur. 13.

Que le nom de biais ou obliquité eſt plus general & propre que le nom de routes. 14.

Que parpain eſt plus general que ſeconde teſte. 15.

Qu'en l'ancienne maniere les noms ſont beaucoup plus ſignificatifs, generaux & propres, que dans les eſcrits dudit ſieur. 15.

Que la demonſtration dudit ſieur, de prendre les beueaux ſur l'ouurage, eſt fauſſe, & mal expliquée pour ſon operation. 15.

Que toute la prolixité des lignes de ſon operation de ſous-eſsieu ſe peut faire auec deux lignes ſeulement, outre le traict quarré. 16.

Faute audit ſieur d'auoir exercé ſon traict ſur de faux arcs defectueux, & faits ſans regle ny proportion. 18.

Maniere generale pour deſcrire tous arcs rampans de deux ouuertures de compas. 20.

Faute en l'arc ſurbaiſſé de la planche 80. du liure dudit ſieur, 22

Autre maniere generale pour deſcrire arcs rampans la hauteur du ſommet eſtant donnée. 23.

Autre maniere generale, pour deſcrire arcs rampans, la hauteur du ſommet eſtant auſsi donnée. 24.

Prolixité dudit ſieur en la conſtruction d'vn angle ſolide. 25.

Faute audit ſieur de vouloir introduire vne maniere ſi cõtrainte, & tant embroüillée & incognuë,

au preiudice d'vne generale, expeditiue & cognuë, comme eſt l'ancienne. 26.

Que l'application du traict dudit ſieur ſur tant de plans de differents ſubiects eſtoit neceſſaire, ou ſuperflue: ſi elle eſt ſuperflue, elle ne ſert qu'à groſsir ſon Liure; & ſi elle eſt neceſſaire, elle ne contient pas la generalité publiée. 26.

Cas amandé dans l'ancienne maniere. 27.

Maniere de reduire le biais ſelon la deſcente ou rampe. 28.

Maniere de reduire les deſcentes ou rampes à niueau, & n'auoir qu'vn talus ſimple ou ſurplomb. 30.

Demonſtration de deux diuerſes manieres de traict generales, pour toutes ouuertures requiſes ſur mur droit, appliquées à vn os en deſcente biaiſe auec talus, par la traditiue amandée. 33.

Raiſonnement & maniere de trouuer les paneaux de lict, par les deux diuerſes manieres generales. 33.

Cas amandé dans l'ancienne maniere. 36.

Remarques des courbes dudit ſieur, qui ſe coupent dans la clef, faiſant les voutes defectueuſes. 36.

Prolixité des figures du Liure dudit ſieur, enſemble le contraire de ſes promeſſes. 37.

Faute audit ſieur de n'auoir donné des manieres pour faire les ſurbaiſſemens des courbes, touchant vn point donné, ce qui eſtoit extrement neceſſaire, comme eſtant le fondemẽt ſurquoy l'on trauaille. 38.

Que les arcs ſurbaiſſez & touchant vn point de hauteur donnée, ſont touſiours l'vne des trois ſectiõs coniques. 38.

Que la pretenduë maniere dudit ſieur pour trouuer les paneaux des trompes, eſt de l'ancienne. 38.

Que la trompe dudit ſieur planche 81. eſt fauſſe, la deſirant en talus, le prenant au pied de ſa demonſtration. 41.

Demonſtration d'vne trompe ſur le coin ſelon l'ancienne. 43.

Cõmentation ſur l'ancienne, 45.

Que les coupes de la trompe dudit ſieur propoſée pour eſpargner place, s'eſtendent plus, ou auſsi loin qu'autrement. 47.

Que la maniere dudit ſieur de trouuer paneaux, & de deux lignes ſeulement, la preparation eſtant faite, eſt la maniere de trouuer aux quadrans l'inclination des lignes horaires à l'axe. 47.

Defy dudit ſieur à vn particulier. 47.

Que la concluſion, ny le propoſé du defy, n'eſt pas de grand art, 47

Manque aux deux planches, 96. & 97. 48.

SECONDE PARTIE.

FAute à la planche 97- demõſtrant vne ſeconde maniere de faire la preparation. 49.

Que les manieres particulieres

dudit ſieur ſont de l'ancienne, 51.
Erreurs en l'vne des pretendues manieres particuliers dud. ſieur. 52
Demonſtration d'vne trompe biaiſe d'angles inégaux, & ſurbaiſſée, faite par les deux diuerſes manieres generales de la traditiue commentée & amandée. 55.
Faute audit ſieur de n'auoir narré, ny donné les moyens de creuſer les doelles de ſes trompes, qui demeurent imparfaites; comme auſſi les courbes des faces de ſes trompes qui ſont fauſſes & defectueuſes, 55.
Que les manieres exercées par porfil dudit ſieur, ſont de l'ancienne. 57.
Que ſa maniere de la premiere & ſeconde teſte ſe pratique ſelon l'ancienne. 57.
Que la maniere vniuerſelle de la trompe 105. *eſt ſi peu vniuerſelle, que ſi elle eſtoit en talus, elle ſeroit fauſſe.* 57.
Que les paroles dudit ſieur ſont fauſſes de dire, que ſa pretendue maniere ſoit plus generale que l'ancienne. 58.
Que ledit ſieur s'explique fort mal; & qu'il y a eu beaucoup de temps à perdre pour faire le preſent examen. 59.
Que tous les reſtes des porfils dudit ſieur ſont de meſmes manieres, exercées ſur trois differãs points fixes; ce que fait auſſi l'ancienne. 59.
Deux manquemens remarquez ſur ledit ſieur; l'vn pour n'auoir demonſtré les cherches ralongées; L'autre de n'auoir demonſtré diſtinctement de reduire les biais à niueau donnez ſelon la deſcente, 60.
Que l'ancienne maniere pratique les meſmes porfils, & en trois diuers points fixes requis, & qu'elle n'a beſoin d'vne teſte cognuë comme le pratique ledit ſieur, 60.
Sommaire deduction des fautes du Liure de l'art des traicts du pere Derand Ieſuiſte, demonſtrée ſelon l'ancienne, imprimé en 1643. *page.* 61.
Que c'eſt par ledit ſieur Deſargues improprement nommer eſſieu la ſection de deux plans ſeulement. 63.
Que les manieres par porfils dudit ſieur ſont de l'ancienne, & aux lieux où il l'a voulu plus eſtẽdre que l'ancienne, il y a des abſurditez, 64.
Concluſion de l'Examen de la coupe des pierres par deux remarques, où ſont ſommairemẽt deduites les fautes du liure dudit ſieur, 64.
Que ceux qui veulent donner regles des choſes qu'il ne cognoiſſent que ſuperficiellement, tombent infailliblement dans des erreurs, ſubiets à cenſure, 65.
Fin de l'Examen de la coupe des pierres.

Sommaire de l'Examen de ſa Perſpectiue.

QVe la maniere de Perſpectiue dudit ſieur, n'eſt autre que celle des anciens, ſinon deux exceptions defectueuſes, 66.

Demonſtration pour approcher tant que voudrez le vray éloignement, qui par meſme moyen fait voir l'originaire rapport des deux eſchelles, ſçauoir de front & d'éloignement. 68.

Demonſtration de la conſtruction & rapport des deux eſchelles, & ſans ſortir du champ de l'ouurage ſelon le ſieur Aleaume, qui fait voir que la maniere vniuerſelle dudit ſieur n'eſt pas nouuelle, mais copiée. 69.

Matiere riſible des propoſitions dudit ſieur touchant la Perſpectiue. 69 & 70.

Sur le petit Liuret de Perſpectiue.

Qv'il y a erreur aux demonſtrations des planches 114. 115. *& leurs ſuiuantes.* 70.

Demonſtration d'vne regle de Perſpectiue fondamentale, conduiſant les demonſtrations dudit ſieur à l'abſurdité. 74. & 75.

Reſtabliſſement à la ruptare du compas optique, contre le propoſé dudit ſieur. 76.

Sur les quadrans dudit Sieur.

Qve la maniere dudit ſieur pour traçer quadrans, vn ſtyle ou axe eſtant placé, n'eſt pas vniuerſelle, comme il la propoſe, n'eſtant que l'vne des manieres ordinaires. 78.

Que ſur le plan de l'Equinoctial c'eſt improprement nommer par ledit ſieur, vne ligne accidentelle, ligne de ſix heures; & qu'il pouuoit auſſi bien nommer ligne de douze heures. 78.

Sur ſa maniere de plaçer le ſtyle.

Qve la maniere pretendue dudit ſieur, de plaçer le ſtyle n'eſt pas nouuelle, & n'eſt que l'antitheſe de la maniere de Morolic. 79.

Que la maniere dudit ſieur de plaçer ſtyle eſt fauſſe. Ce qui a fait que Morolic ne l'a appliquée à plaçer ſtyle, mais ſuiure la maniere vſitée par le moyen du pole. 80.

FIN.

EXAMEN
DES
OEVVRES
DV SIEVR
DESARGVES,
LYONNOIS.

PREMIERE PARTIE.

CONTENANT L'EXAMEN DE SON broüillon de la coupe des pierres, imprimé en 1640. & de ſon Liure de meſmes matieres, imprimé en 1643. depuis le commencement d'iceluy Liure iuſques à la planche 97.

IL y a certains eſprits leſquels ont vne ſi bonne eſtime d'eux meſmes, & ſont tellement aueuglez de l'amour qu'ils ſe portent, que s'ils font rencontre d'vne petite mouche, ils s'imaginent incontinent, & meſme le publient, qu'ils ont treuué vn gros Elephant; en ſorte, que ne s'aperçeuans point qu'ils ſont dans l'erreur; ils taſchent encor de tromper l'imagination des autres, fondez ou ſur leur intereſt particulier; ou emportez du deſir de vaine gloire qui les chatoüille à merueilles; ou quelquefois agitez tout enſemble de ces deux paſſions qui ſont extremement violentes de leur nature.

Le ſieur Deſargues s'eſtant auiſé ſur vn pan de mur ou face droite, auquel on deſire perçer quelque ouuerture ou entrée *Que la meſ-mere de trait*

dudit ſieur par ſa ſous-eſſieu, eſt la meſme choſe que la ſouſtylaire des quadrans, & n'eſt que maniere imitée.

oblique, au reſpect de la perpendicule, ou du niueau d'iceluy mur, ou de tous les deux, d'y plaçer vne ligne ſemblable à celle que l'on place aux quadrans declinants, ou declinants inclinez, qui eſt la plus oblique à l'axe, que l'on nomme ſouſtylaire, ou ſoubs l'axe du monde, & que ledit ſieur nomme en ſes operations *la ſous-eſſieu*: puis au reſpect de la perpendiculaire à ladite ſous-eſſieu, plaçer vne autre ligne faiſant angle auec icelle, ſemblable à l'angle que la ſouſtylaire fait auec ſon ſtyle ou axe; & eſtant la plus proche, & contre l'axe, ledit ſieur la nomme *contre-eſſieu*. Cedit angle eſt auſſi ſemblable à l'angle de la ſous-eſſieu & eſſieu, à ſçauoir leſdites ouuertures, ou portes; eſtant ſuppoſées paralleles à l'axe ou eſſieu; en conſequence de cecy ledit ſieur s'eſcrie à haute voix *ἑυρηκα, I'ay trouué*. Celuy qui le premier s'eſt auiſé de deſcrire, & ſans * defaut la ſouſtylaire au quadrant declinant, ou declinant incliné, & d'y plaçer le ſtyle à angle droict; & partant auoir l'angle qu'iceluy ſtyle fait auec ſa ſouſtylaire, n'a pas fait tel cry, ſçachant biẽ d'où il le prenoit, & iuſques à quelles limites cette maniere s'eſtendoit; auſſi ne l'a t'il cottée pour maniere generale aux fins de traçer quadrans ſur toutes ſortes de faces droites données. Si ledit ſieur euſt eu vn commun eſprit auſſi bien qu'vne commune maniere auec l'Auteur, il n'euſt pas donné ny publié cette maniere * copiée pour generale ſur toute face droite donnée; veu que elle eſt ſi peu generale, que ſans ſortir d'vne deſcente de caue, où il l'a appliquée en ſon broüillon de l'année 1640. & en la plus part de ſon Liure de la coupe des pierres, imprimé l'année 1643. ſi icelle deſcente eſtoit ébraſée des deux coſtez, ou d'vn ſeulement; (ce qui arriue aſſez ordinairement) & la clef touſiours parallele à l'eſſieu, ladite maniere generale receuroit conuulſion, & en faudroit recourir à la traditiue & ancienne maniere. Ne voila pas en donner à garder d'importance à la bonne foy du Lecteur? car en ſon dit Broüillon au bas de l'eſcrit de la premiere fueille, il dit: *Cette maniere icy de traict, dont la figure d'vne porte en la face plate d'vn mur à talus pour vne deſcente biaiſe ayant l'arc rampant, ou tous les ioints ſont en ligne droite: & où ceux des faces ou teſtes ne tendent pas tous à vn meſme point, contient en vn ſeul exemple generalement tout ce qui ſe peut rencontrer en quelconque ouuerture, dont les ioints ſont en ligne droite; & n'y a qu'vn ſeul & meſme traict pour toutes, &c.* Ce qu'il cõfirme dãs l'eſcrit de ſon Liure cy deſſus dit en la fueil. 18.

* *I'appelle ſans defaut operant ſur vne vraye eſleuation & precis equinoctial; mais ledit ſieur a operé ſur vn faux arc rãpant, fait à veuë de nez, & ſans regle, ainſi que l'on verra en ſuite.*

* *I'appelle copiée n'eſtãt autre choſe que la ſouſtylaire des quadrans cy deuant dits: dont il y a tant de nombre d'années que nous l'auons en vſage.*

par cēs mots. *Mais i'ay iugé que cette façon-la d'enſeigner* (parlant de la traditiue) *n'eſtoit pas la meilleure pour inſtruire promptement & pleinement de cette matiere ; & i'en ay fait vne vniuerſelle & methodique à mon aduis autant qu'il ſe peut.* Ledit ſieur deuoit ſçauoir que depuis le temps que l'on pratique les traits Geometriques en la coupe des pierres, qu'il y a eu en l'Architecture des perſonnes aſſez ſçauantes, pour ſur vne ſurface droite où l'on deſiroit faire quelques ouuertures, y plaçer vne ſous-eſſieu, & ſon angle auec l'eſſieu, ſi ils l'euſſent iugée autant generale, expeditiue & methodique que la traditiue. Pour la generalité i'y ay reſpondu cy deuant, & feray voir en ſuite, par vrayes demonſtrations & methode, appliquées à vne deſcēte rampante, biaiſe en talus, dōt les coupes ne tendēt à meſme cētre, deſcrit au frōt de ce liuret, & la ſemblable maniere appliquée en vn os de ſubiection cy deſſus demonſtré dans ce liuret (mais en l'os il y a deux differentes manieres) que la traditiue amandée eſt generale ; & fait non ſeulemēt tout ce que la maniere dudit ſieur fait, mais encore tout ce qu'elle ne peut faire, qui eſt bien en plus grand nombre que ce qu'elle fait ; car en toutes les ouuertures qui ſe font par la maniere dudit ſieur, il y aura deux exceptions pour chacune, ſçauoir quand leſdites ouuertures ſeront ébraſées des deux coſtez, & quand elles ne ſeront ébraſées que d'vn coſté comme eſt dit cy deuant ; & partant la maniere ancienne amandée ſera triplement plus generale que la ſienne ; Et ne ſeruira rien de dire que telles manieres d'ouuertures ſont du genre des trompes, car cela eſt faux, d'autant qu'en toutes trompes les areſtes des licts de doelles tendent à meſme centre ; & en ces portes cy dites elles n'y tendent pas : & partant il faut qu'il demeure d'accord de ce que deſſus, n'ayant dit l'exception, ny changement pour chacun lict. Quant à ce qui eſt de l'expeditiō & methode, chacun peut voir comme il y a moins de lignes ; & comme il y a plus de vray ſemblance, & recognoiſſance de la choſe que l'on deſire faire en l'vne qu'en l'autre. Mais pour le refuter auec ordre, ie paſſeray à l'examen de toutes & chacune des parties du Liure cy deuant dit, & en demonſtreray clairement les erreurs, ſoit en l'eſſentiel, ou en la ſuperfluité, ou en la iactance.

Que la traditiue amandée eſt triplement plus generale & fait ce que la maniere dudit ſieur ne ſçauroit faire.

Et ie diuiſeray cet Examen en deux parties, dont la 1. finira en la plāche 97. où finiſſent diuerſes pieces de trait par ſa pretenduë maniere generale : & la ſeconde partie finira où finit ſon Liure.

Premierement, ie diray que depuis la premiere fueille iuſques à la cinquante ſixiéme, ce ſont tous differents eſcrits ſimples, employez partie à reciter ce qu'il cõuient entendre, diſtinguer, & faire; partie à dõner l'explication & intelligence des termes & nõs vſitez en l'Art, & ſubſtituer en la place d'iceux des nouueaux noms; partie auſſi à loüer ſa methode, & ſupprimer l'ancienne: à quoy ie ne reſponds, ſinon à quelques noms impropres fort briefuement, mon deſſein n'eſtant pas de m'eſcrimer des armes d'Vlyſſe, mais de combattre de bonne guerre les conſequences, effets, & figures de tout le liure par vraye demonſtration; car en matiere de Geometrie * oſté l'explication de la figure les paroles, comme auſſi les eſcrits ſont de nul effet & vigueur.

* *Tous les traits de la coupe des pierres ſont conſtruits par la Geometrie des plans ou des ſolides.*

Tout le commencemẽt de ſon Liure depuis la premiere planche iuſques à la planche 11. eſt employé à demonſtrer partie des choſes cy deuant dites; comme de demonſtrer par figures tant quelques eſleuatiõs de trait, que les noms & figures de quelques pierres: auſſi les bẽueaux & la maniere de les prendre ſur l'ouurage, enſemble les baptizer & leur donner des noms aſſez barbares & impropres; moins expreſſifs & ſignificatifs de la choſe, que ceux de l'ancienne receus en vſage; comme par exemple, pourquoy nommer *chemin* quelques aires, ſoit de niueau, ou penchant, pluſtoſt que plan, puis que ſi le plan ou aire eſtoit penchant au reſpect du niueau de quelque mur, où l'on voulût faire quelque porte (comme il arriue bien ſouuent) alors l'operation par ſon plan de chemin ſeroit faulſe, veu que de ſuppoſer imaginairemẽt ſon chemin de niueau en face ainſi qu'il ſera contraint, & il ne le ſera pas, de neceſſité il faudra le ſuppoſant, que ce ſoient des Anges qui marchent ſur ſon chemin imaginaire; car les hommes ne peuuent cheminer ſur l'air, que par vne vertu ſurnaturelle & du tout miraculeuſe. Donc cela eſt improprement nommé *plan de chemin*, puis qu'il y a communement des plans qui ne ſont qu'imaginaires, & que chemin eſt vne choſe reelle. Et partant dans l'ancienne cela meſme eſt plus proprement appellé plan, d'autant que plan n'eſt rien autre, que ſuperficie, & ne contient en ſoy aucune matiere; & que l'on le peut approprier ou imaginer ſur vne aire où chemin, ou ſur quelque autre matiere plane; mais auſſi que l'on le peut imaginer n'y eſtre pas, mais auoir quelque autre diſpoſition; & pour cette conſideration iceluy plan ne change ny ſa qualité ny ſon nom. Et par conſequent doit eſtre,

Que le nom de plan en l'Architecture eſt plus general & plus propre que chemin.

plus proprement & generalement nommé plan que chemin.

De plus, pourquoy *assiete de villes, maisons, ou portes, &c.* pluſtoſt que plan de villes, maiſons, ou portes ?&c. Ledit ſieur en deduit ſes raiſons dans les eſcrits de ſon Liure, fueillet 7. & 8. & dit en ſubſtance, *Qu'en la Geometrie Theorique on nomme plan vne grandeur ſans fin ; ſans entendre par ce mot de plan iamais aucune ſorte de figure ; & que deſlors qu'il y a quelque figure à nommer, ou plate, ou autre, on vſe d'vn autre mot que celuy de plan ; comme par exemple du mot de triangle, de quarré, &c. Et qu'en l'Architecture practique l'on entend le nom de plan autrement, & eſtre à niueau & terminé ; comme par exemple, le plan d'vne ville, ou porte, &c. qui eſt à niueau, & terminé par la figure de la ville, ou porte.* Mais il eſt bien aiſé de luy prouuer le contraire, & qu'en la Geometrie Theorique auſſi bien qu'en l'Architecture Pratique l'on qualifie de plan vne figure plane terminée, auſſi biẽ que ſi elle ne l'eſtoit pas, teſmoin la huictieſme definition du premier d'Euclide, qui eſt. Ἐπίπεδος δὲ γωνία ἐστὶν, ἡ ἐν ἐπιπέδῳ δύο γραμμῶν ἁλλήλων, καὶ μὴ ἐπ' εὐθείας κειμένων πρὸς ἁλλήλας τῶν γραμμῶν κλίσις. *Angle plan eſt l'inclination de deux lignes l'vne à l'autre ſe touchant en vn plan nõ directement ;* Ce qui eſtoit neceſſaire afin de faire diſtinction entre les angles & figures planes, & les angles & figures ſolides de l'vnziéme d'Euclide. Donc la Geometrie a des plans terminés.

Que le nom de plan en l'Architecture tant Theorique que Pratique eſt plus general & plus propre que le nom d'aſsiete.

Il eſtoit auſſi beſoin à la Geometrie de ſuppoſer des plans infinimẽt grands, d'autant que cela eſt neceſſaire à l'imaginatiõ du Geometre, d'auoir des plans infinis pour receuoir toute figure deſirée, & de telle grandeur qu'il voudra. Et en l'Architecture Theorique l'on ſuppoſe meſmes plans de capacité infinie, conuenable à receuoir toutes les figures & deſſeins, & de telle grandeur que l'Architecture en aura beſoin, car autrement elle ſeroit gehennée & bornée ; mais cela ne conclud pas que l'on ne ſuppoſe des plans terminez, & ſelon le beſoin comme il eſt dit cy deuant. Donc en l'Architecture Theorique l'on entend par le nom de plan la meſme choſe que dans la Geometrie.

Et en l'Architecture Pratique où il y a commencemẽt ou fondement de maiſon, ou ville, &c. Si l'on conſidere la matiere, l'on dira, les fondements de maiſon, ou ville, &c. & ſi l'on conſidere la figure du fondement de la maiſon, ou ville &c. qui eſt abſtracte de la matiere, l'on dira le plan de la maiſon, ou ville ; & d'autant que telles figures ſont obiets de l'imagination & enfantez

Qu'en l'Architecture tant Theorique que pratique, l'on ſuppoſe meſ-

me plan qu'e la Geometrie.

par icelle; auſſi l'imagination les reçoit, & les rēpreſentē ſur le papier, ou par le diſcours, & non pas les matieres. L'eſprit de l'homme ſe figure toute ſorte de deſſeins & images ſans qu'il ait aucunement beſoin de matiere, & lors que les fondemens planes d'vne maiſon ou ville luy ſont repreſentez, il reçoit ſeulement la figure, & non pas les pierres ou matieres d'iceux fondemens. Et c'eſt ainſi qu'on ne cōſidere pas en vne ſtatuë l'eſtofe ou la matiere de quoy elle eſt faite; mais bien la diſpoſition, le deſſein, la belle forme qu'elle a, & qui n'eſt attachée qu'à la ſuperficie de la matiere, c'eſt ce que l'on regarde ſoigneuſement & auec vn ſingulier plaiſir. Et ſi vous deſirez que l'on conſidere les pierres ou matieres, il ſera bien plus à propos de ſe tranſporter pour cét effet en leurs carrieres, où elles repoſent comme en leur lieu naturel, & ſont aſſorties de leurs plus cōuenables proprietez & quantité, que non pas de les conſiderer en vn fondemēt de ville, ou maiſon, &c. De plus, l'obiet de l'Architecture Pratique eſt de former les matieres. Et quād l'on obiecteroit que l'Architecture Pratique a pour obiet les matieres propres pour effectuer; ce n'eſt pas dire qu'elle n'aura pas des figures precedentes dās l'eſprit pour former icelles matieres; & qu'elle n'vſe des meſmes intelligēces & manieres de parler & d'entendre, que l'Architecture Theorique, car pour auoir meſme intelligence & maniere de parler & d'entendre, ce n'eſt pas auoir meſme ſpeculation. Il eſt donc vray qu'en l'Architecture Pratique l'on entend par le nom de plan meſme choſe qu'en l'Architecture Theorique; & il enſuiura par contre-regle qu'en l'Architecture Pratique l'on entend par le nom de plan terminé de quelque figure, meſme choſe que dans la Geometrie.

De plus, eſtant verifié cy deſſus, que c'eſt la figure ſuperficielle ſeulement du fondement, ou autrement de maiſon, ou ville, &c. que l'on conſidere, & non les pierres, ou matieres, cette diſpoſition de figure plane & ſuperficielle d'vne maiſon, ou ville, &c. ſera plus * generalement & proprement nommée plan, qu'aſſiete, d'autant que aſſiete eſt Phyſique, & plan eſt Metaphyſic.

* *La generalité a eſté verifiée cy deuant contre le nom de chemin.*

Dauantage pourquoy dire pluſtoſt la route que le biais, ou obliquité Metaphyſique? la route ſuppoſe vn chemin reel où l'on paſſe; ainſi l'entend ledit ſieur, c'eſt pourquoy où l'on ne paſſera, ny cheminera iamais, cela eſt impropremēt appellé route: aux deſcentes & montées l'on ne cheminera iamais ſur ces plans de routes à niueau; l'on ne cheminera iamais auſſi par les plans des os, & autres telles fermetures, d'autant que ces plans ſont ſeulement

Que le nom de biais ou obliquité eſt plus general & propre que le nom de route.

imaginaires & metaphyſics ; & par conſequent tels biais & obliquitez ſont de beaucoup & plus generalement & proprement nommez biais,ou obliquitez,qui ont qualité de plan,que routes,tant de niueau que de chemin ; D'icy il s'enſuit,que les beueaux qu'il nomme beueau de route niuelée,beueau de route au chemin ; beueau de face & de chemin ; beueau de faces & de routes,ſont improprement nommés pour les raiſons cy-deſſus ; auſſi *route* eſt vn ancien mot Gaulois, qui n'eſt pratiqué & receu qu'entre les chartiers,à qui conuient auſſi l'vſage des eſſieux,& ſous-eſſieux quand il y a trop grande charge.

De plus, Pourquoy dire vne demie tour eſtre vn berceau de bout? il y a autant de raiſon de dire,qu'vn berceau eſt vne demie tour couchée; mais l'vn & l'autre eſt impropre, puis qu'ils ont chacun leurs noms particuliers & ſignificatifs par leſquels on les peut diſtinguer.

Que Parpain eſt plus general que ſeconde teſte.

Pourquoy dire encore le Parpain ou derriere d'vn mur eſtre ſeconde teſte, & que la premiere teſte eſt en la face? il y peut auoir en la face ſeule deux teſtes comme aux clefs des trompes ſur le coing, portes tant ſur le coing, que dans l'angle, & partant elles demeureront ſans Parpain: ledit ſieur deuoit donc dire, qu'aux clefs ſuſdites il y auroit vne troiſiéme teſte ; Et l'excuſe de dire, que les deux teſtes de face ſe font l'vne comme l'autre par la maniere de faire la premiere teſte, on luy repliquera,que le Parpain ou derriere ſe peut auſſi faire par la maniere de la premiere teſte;mais neantmoins cela n'exclud pas qu'il n'y ait pluſieurs teſtes ou faces.

Qu'en l'ancienne maniere les noms ſont beaucoup plus ſignificatifs, generaux & propres qu'ez eſcrits dudit ſieur.

Et partant ie concluray,qu'en l'ancienne maniere les termes & noms des choſes ſont beaucoup plus ſignificatifs, propres & vniuerſels qu'ils ne ſont dans les eſcrits dudit ſieur. Cela eſt ordinaire & naturel à ceux qui veulent vſer de fard & déguiſement, de renuerſer & changer tout ce qu'ils peuuent, eſperant que l'on ne recognoiſtra plus rien par le moyen de cette deſtruction;mais on les voit ordinairement decheus & fruſtrez de leur attente; & ce mauuais procedé leur apporte autant de blaſme & de confuſion parmy les ſçauants & experts en la cognoiſſance des choſes, qu'ils eſperoient en moiſſonner de loüanges & de gloire.

Que la demonſtration dudit ſieur de prendre

Mais paſſons du mot à la choſe. En la planche 8.& 9. il monſtre la maniere qu'il entend pour prendre les beueaux, comme les beueaux H B G, ſuiuant les routes à niueau d'vn bras, & l'a-

les beueaux sur l'ouurage, est faulse, & mal expliquée pour son operation

reste du pied droit de l'autre; & ainsi HBD de la planche 10. suit aussi les routes au chemin; & l'areste du pied droit, & de ses beueaux ainsi pris sur l'ouurage, sçauoir beueau des faces, & chemin, & beueau de face & de route simplemẽt sans autre exceptiõ. Il en fait vne operatiõ en la planche 11. laquelle, pour la bien loüer, ne vaut riendu tout, car en cete operatiõ il faut que le beueau H B G soit pris quarrément au mur, & non selon le biais du plan, ny de l'obliquité de l'areste du pied droit, & ce beueau H B G est faire la mesme chose qu'en l'ancienne mettre le talus, & de là auoir les plombs selon le talus, ou areste des faces de pied droit; comme aussi le talus selon le biais ou areste de tableau des pieds droits (s'entẽd de porte) ainsi que ie le demõstreray cy apres. Mais cõme il s'est mieux expliqué en l'escrit de son broüillon, cy deuãt dit, fueillet 2. ligne 29. où il nõme le plan imaginaire, suiuant lequel l'on place le beueau; *Le plan droit aux faces & niueau*; c'est pourquoy ie mets cette faute au rang des legeres; & qui seroit neantmoins bien lourde à ceux qui l'exerceroient, suiuant qu'il l'a demonstrée, tant en ses escrits qu'és planches cy-dessus dites.

Mais c'est beaucoup s'arrester sur des escrits; il m'ennuye que ie ne sois venu aux demonstrations; neantmoins auparauant ie vous diray, que depuis la planche 12. iusques à la planche 24. les escrits & les planches sont employez à demonstrer la preparation & construction de plaçer, tant la sous-esieu que grand esieu, qui n'est autre chose faire, que trouuer la sousstylaire, & l'angle du style d'vn quadrant, ainsi qu'il a esté dit cy deuant, & qui se peut faire auec deux lignes simplement, outre le traict quarré, autant, ou plus intelligiblement que ledit sieur ne l'a fait, auec toutes les planches & escrits susdits: joint à cela qu'il n'estoit besoin de son plan de niueau, ny du beueau de face & niueau, & que c'est assez d'vne ligne perpendiculaire seulement. Voyez ie vous prie, si ce n'est pas abuser entierement du temps & de la patience du Lecteur.

Que toute la prolixité des lignes de son operation de sous-esieu se peut faire auec 2. lignes outre le trait quarré.

Vous remarquerez de plus, qu'en la figure d'embas en la planche 24. qui enseigne de faire trouuer au talus les aresles des faces des pieds droits à plomb, que l'angle P A E n'est autre chose que le biais selon la descente, ou lict de dessous de la premiere Retombée, ainsi qu'en la figure d'enhaut (ce que l'Autheur laisse à deuiner à ceux qu'il pretent instruire;) d'autant que l'on n'est pas asseuré qu'iceluy angle P A F figure d'embas serue en mes-

me

me fonction que l'angle PAF figure d'enhaut, n'en ayant esté rien dit en aucun lieu par ledit sieur; mais comme tels tableaux de portes taluans ou surplombans, ne se mettent en pratique pour leurs defectuositez & incommoditez, cette regle est comme superfluë.

Maniere inutile & superflue dudit sieur, de construire le trait, la sous-esseu & es-sieu estant hors l'épeur: & estoit plus necessaire de donner les cherches pour creuser les doelles où les portes demeureront à pan.

Les planches és escrits 25. & 26. sont employées à faire voir, cõme la preparation contenuë és 12. planches cy deuant dites, se place sur vn proiect. La planche & escrit 27. est la preparation hors l'ouurage, qui est le mesme que si sur le proiect d'vne porte biaise l'õ ne se vouluſt pas seruir du biais, ou plan de lict des doelles qui sont dans le proiect de la porte; mais d'vne ligne parallele à iceluy biais, & qui seroit à costé, l'on ne laisseroit pour cela de construire le traict de la porte, se seruant d'icelle ligne parallele au biais & à costé, en mesme fonction, que si elle estoit dans le proiect de la porte, sçauoir de prendre tousiours les desrobemens, ou auances des plans des pierres, par des lignes perpendiculaires à icelle parallele au biais, conformément à la maniere ancienne; mais telles propositions sont superfluës, d'autant qu'icelle ligne se peut mettre dans le proiect de la porte, & que dedans, ou à costé operant par mesme maniere, il suit tousiours semblable consequence. Mais il estoit bien plus necessaire d'enseigner à faire ce qu'il supose, que de construire choses inutiles, comme enseigner de faire les cherches quarément, pour creuser les doelles des portes qu'il conuient faire; car ses portes seront fermées à pan, puis qu'il ne donne la maniere de trouuer les cherches, comme est dit cy deuant. Il me dira qu'elle est marquée par le deuant de la teste, voila qui est bien; mais il n'a pas enseigné que quãd on viendra à faire la doelle, il faudra que l'ouurier soit obligé de tenir sa cherche parallele à la teste, & de plus biaise, selon le parement; & que si on ne la creuse selon cette obligation, l'effet sera faux, & par ainsi il a laisse à deuiner & faire à l'ouurier ce qu'il n'a peu entendre. De plus quand il auroit enseigné ce que dessus; cette maniere toutesfois est defectueuse, estant trop penible, & d'obligation, & subiette à faillir par les ouuriers. Ie demonstreray cy apres par la maniere generale & de la traditiue amandée appliquée à vn os en descente biaise en talus; la methode de construire les cherches, & sur l'arc droit ou quarément, qui est la maniere requise & la plus facile aux ouuriers, n'estant d'aucune obligation.

Les planchēs & eſcrits depuis la planche 28. iuſques à 45. ſont employés à enſeigner la conſtruction du trait dudit ſieur, & appliqués à vne deſcente biaiſe, & en talus auec ſes circonſtances, comme de treuuer les beueaux du cintre * droit, ou quarément faire les paneaux de lict & doelles, tant par trait que confirmation du cintre droit; & eſt la meſme choſe que la deſcente rampante biaiſe, auec le talus, miſe au iour en ſon broüillon l'année 1640. & partant toutes les planches cy deuant ne ſont que redites; il y a ſeulement quelques déguiſemens de nouueau, comme de traçer les paneaux à part, ainſi que quelquefois l'õ fait par l'ancienne, & les confirmer par le cintre droit. Et la prolixité des 17. eſcrits & planches cy deſſus, ou des 4. de ſon broüillon vous paroiſtra cy apres par l'vne des manieres generales, appliquées à vn os cy deuant dit.

** Ie dis cintre, & non arc droit, car ledit ſieur n'a fait l'arc, mais ſeulement des pãs droits pour chaque doelle.*

Prolixité dudit ſieur.

De plus, vous remarquerés l'erreur dudit ſieur, qui a exercé ſa maniere ſur de fauſſes poſitions, faute de les auoir ſceu, & peu demonſtrer à conſtruire, ce qui eſt faire le ſemblable, que faute d'auoir ſceu, ou peu ſçauoir la maniere de cognoiſtre l'eleuation d'vn lieu, l'on en ait ſuppoſé vne à veuë de nez & conduite de la main ſeulement, puis ſur cette fauſſe poſition exercé la regle de traçer quadrant par la ſouſtylaire & angles d'icelle auec ſon axe, fait & parfait ladite horloge par ſa regle infaillible, il ne laiſſera pas d'enſuiure que le quadran ſera faux, s'il n'arriuoit que par extraordinaire hazard l'on euſt rencontré à poſer la vraye eſleuation du lieu. Et c'eſt par ladite cognoiſſance de l'eſleuation du lieu, qu'il faut cómmencer ledit quadrant, autrement la regle de traçer quadrants (quoy que vraye) produira vn effet faux. Ledit ſieur par ſa maniere qui eſt (comme cy deuant a eſté dit) le meſme que trouuer la ſouſtylaire & l'angle de l'axe auec icelle, a exercé ſon trait ſur la poſitiõ d'vn faux arc; *il faloit commencer par le commencement, & demonſtrer de deſcrire, tant iceux arcs rampans, qu'à niueau & ſurbaiſſé; tant à diſcretion qu'à hauteur & point donné, & par regle infaillible, qui n'eſt pas de petite conſequence; puis ſelon cette vraye poſition, en exercant le trait par vraye regle, l'effet n'en peut eſtre que precis, & de tout point accomply: mais au lieu de ce faire & n'en ſçachant point de regle, il le deſigne ſur le papier à veuë d'œil, & conduite de la main; & quoy que petit l'on ne laiſſe pas d'aperçeuoir le defaut; comme par exemple en toutes les fueilles

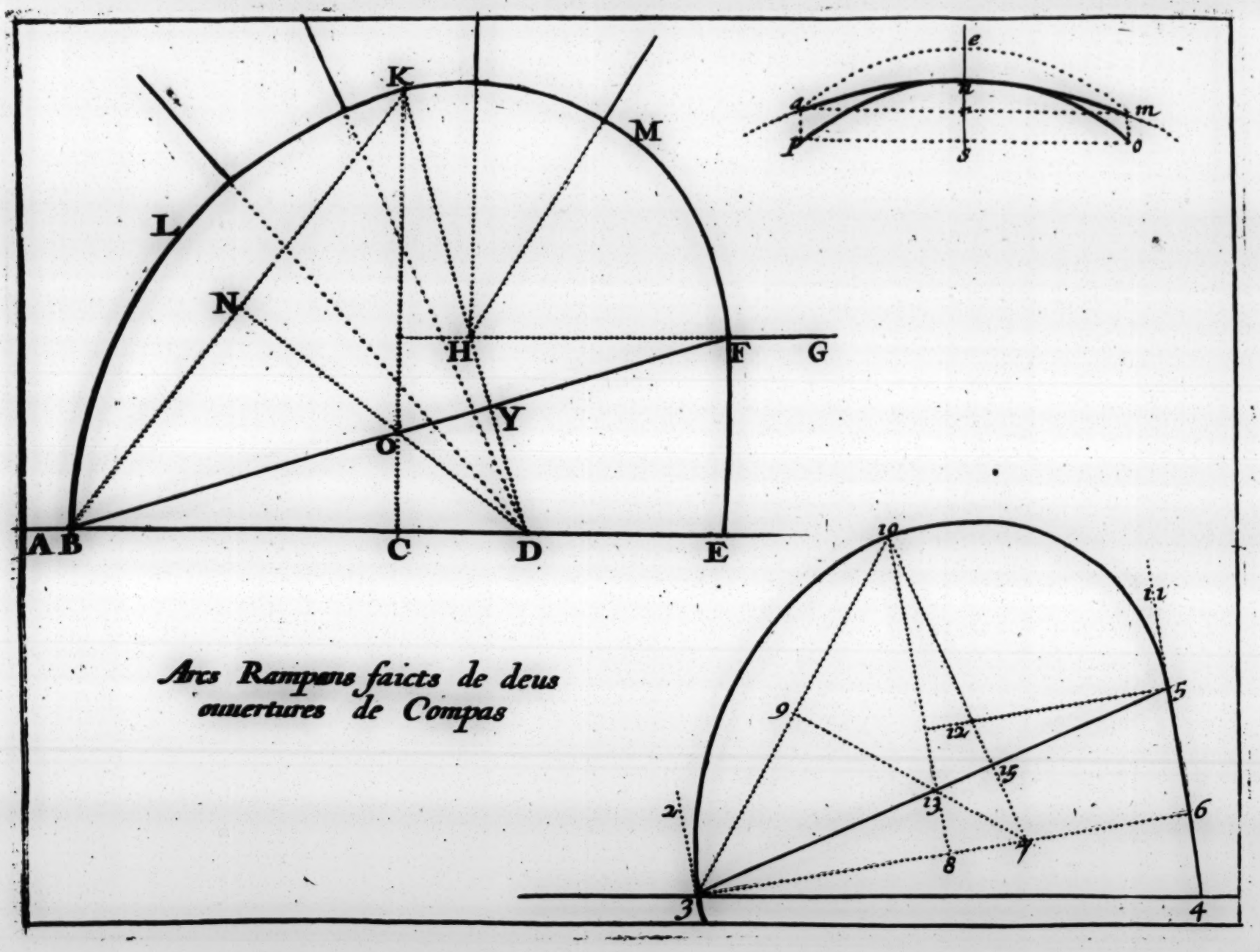
Faute dudit ſieur d'auoir exercé ſon trait ſur de faux arcs, defectueux, & faits ſans regle ny proportion.

L'arc rampant de ſon broüillon & autres en diuers endroits de ſon liure ſont faux.

de ſon brouillon, le tableau ou areſte C P coupe la courbe de deſſus, au lieu de la toucher, & partant icelle courbe cane & entre dans le tableau & fait angle & iaret auec iceluy. Il y a auſſi defaut au tableau D Q qui a coupé la courbe.& ne la touche pas au point Q, mais luy fait vn angle ou iaret. De plus, outre ce defaut toute la courbe eſt difforme & mal proportionée, comme ayant eſté faite ſans aucune regle ny proportion,& pareillemẽt toutes celles de ſon liure;Donc l'on peut conclure,qu'aux effets & grands arcs de ſept à huict toiſes, ou plus s'il eſt requis, la faute ſera extrememẽt ſenſible, puis que ſur vn petit papier elle eſt tant remarquable. Mais de plus,quelle methode faut-il tenir aux effets pour deſſeigner ces grands arcs? il n'en va pas de meſme que ſur vn petit papier; & ce qui a eſté pratiqué iuſques à preſent pour telle maniere d'arc, n'a eſté qu'à taſtons & ſans regles; c'eſt pourquoy venant à propos,i'en demonſtreray quelques-vns qui ſont des premieres cognoiſſances de ma Stereotomie, premier Tome du cours d'Architecture;& commenceray par ceux dont les hauteurs des ſommets des arcs ne ſont aſſubieties.

Maniere generale pour descrire tout arc rampant de deux ouuertures de compas.

Premierement, ie mets pour regle generale, qu'estant quelque rampe donnée comme B F,& l'areste des faces du Tableau F E,soit menée la ligne de la rampe B F,& soit diuisée en



Arcs Rampans faicts de deus ouuertures de Compas

deux au point O,& au point O soit esleuée vne ligne, autant grande qu'il sera necessaire & parallele à FE, comme K C; puis soit fait O K esgal à O F, puis menée la ligne B K diuisée en deux en N, & du point N & par O mener la ligne N D coupant la ligne B E qui est perpendiculaire à F E comme en D, & du point D faire le cercle B L K: plus faut mener F H perpendiculaire à F E,& conduire la ligne K D, coupant F H comme en H,& du point H vous descrirez le cercle K M F, ce qui se confirmera par O C égal à O Y, & les angles O Y K & O C B esgaux chacun à vn droit; & iceux cercles cy dits se toucheront au

au point K,& ne se couperont point,partant il n'y aura nul angle ny iaret. De plus ils toucheront la ligne F E en F,& sa parallele en B; donc iceux arcs rampans ne feront aucun angle ny iaret, ce qui est requis,& leur courbe B L K M F n'est que de deux sortes de cercles ou cherches,ce qui apporte beaucoup d'expedition & de facilité pour l'ouurage. Et puis quand mesme iceux arcs seroient de vingt toises de grandeur,ou plus, il est demonstré aux premieres cognoissances de ma Stereotomie, de construire icelles cherches en vn lieu de deux pieds d'espace seulement, sans qu'il soit besoin d'autres places, & qui seront autant & plus precises,qu'auec les singliots & places de vingt toises, ainsi que l'on pratique ordinairement; ce qui est vn moyen extremement expeditif & abregé dans l'Art de massonerie. Et pour les coupes & beueaux, ils sont faciles à trouuer la cherche estant donnée. La figure monstre que les coupes depuis K iusques en B se tirent du point D, & depuis K iusques en F elles se tirent du point H.

Maniere de faire la cherche composée de deux cherches.

Et pour la clef qui est composée de deux cherches,l'on la construira ainsi lors qui n'y a point d'épeurs ny places pour les traçer. Soit la grande cherche *a n m*,& soit la petite mise dessus, comme *a e m*, & soit diuisé *a m* en deux en *n*, puis soit faite la coupe *e s*, ou perpendiculaire à la cherche *a n m*; puis mener la droite *a m*, qui sera diuisée en deux par *e s*,comme en *r*; soit fait *n s* egal à *e r*, & par le point *s* soit menée la ligne *p s o*,& soit *s o* égal à *r m*,comme aussi *s p* égal à *r a*; puis par *p* & *o* tirer la petite cherche *p n o* qui touchera la grande cherche *a n m* en *n*, & partant la cherche *p n m*, composée des deux cherches,sera la cherche de la clef, & sera mise du costé & endroit qu'il est requis,sçauoir *n* en K. Et pour les coupes elles se feront de chacune portion des bouts des cherches.

La mesme maniere cy-deuant dite pour arc rampant, aplique aux ouuertures dont l'areste des faces de pied droit est oblique au plomb.

En outre, si l'areste en face des pieds droits estoit panchante, comme aux biais en talus, & par exẽple 11. 4. au respect d'vne ligne de niueau 3. 4. l'on procedera par la maniere generale cy deuant dicte, faisant tant 12.5. que 3.6. perpendiculaires à 11. 4. & 13. au milieu de 3. 5. & 13. 10. esgal à 13. 5. & parallele à 11. 4. puis 9. au milieu de 3. 10. & 9. 13. coupant 3. 6. comme en 7. & 7. 10. coupant 12. 5. comme en 12. alors 7. sera le centre du cercle 3. 10. & 12. sera le centre du petit cercle 10. 5. & la confirmation sera pareille à l'arc rampant cy deuant dit, & iceux

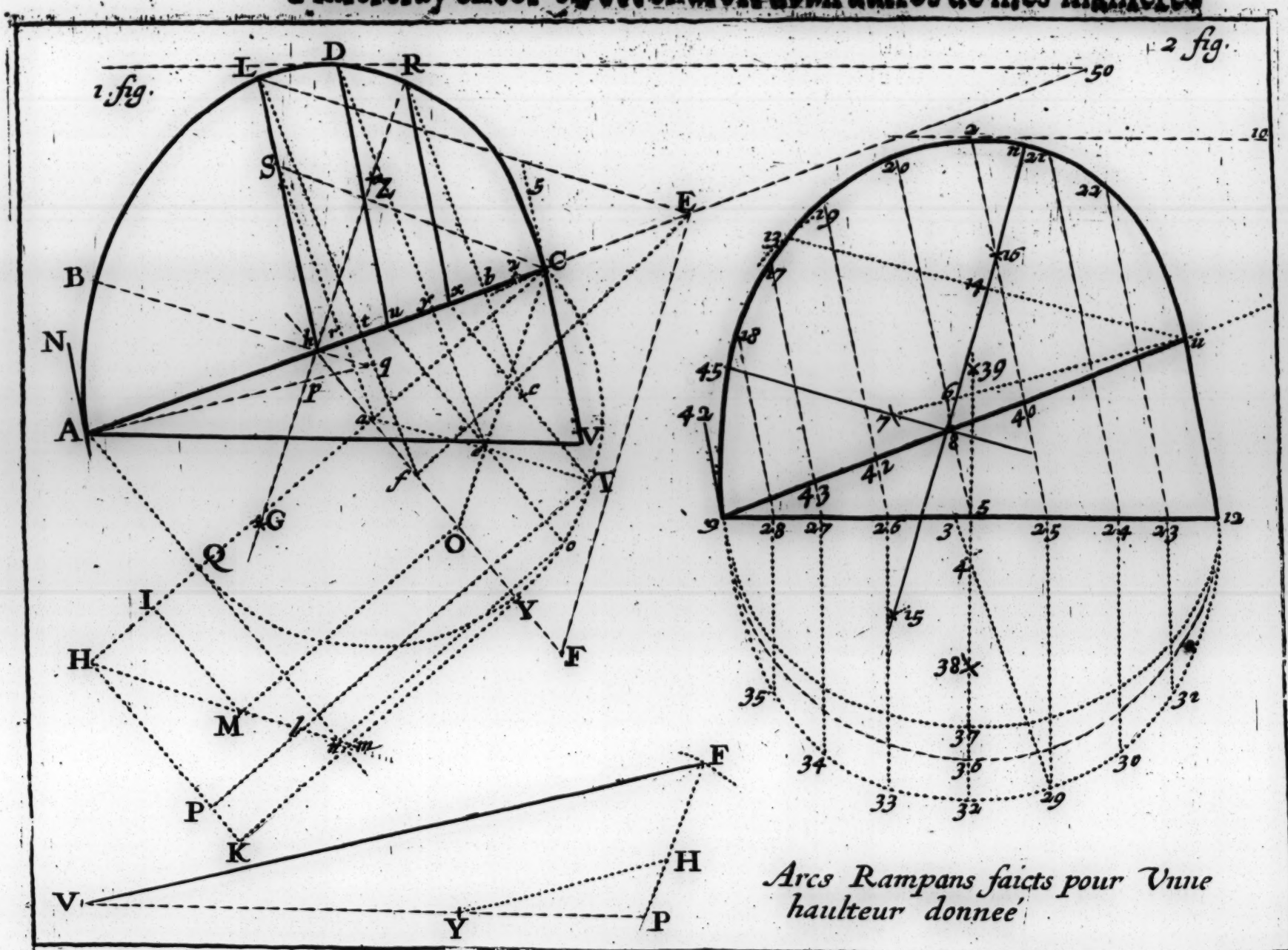
deux cẽrcles ſe toucheront en 10. & toucheront 2. 3. ẽn 3. & 11. 4. en 5. ce qui eſt requis.

Dans les premices de ma Stereotomie l'on verra differentes manieres de deſcrire arcs rampans, comme ſi 2. 3. & 11. 4. tendoient en vn centre, & la hauteur donnée & non donnée, il ſe verra la maniere de faire les ances de panier d'vne hauteur donnée, & de deux ouuertures de compas, les arcs furbaiſſez & touchans vn point donné à coſté.

Faute en l'arc ſurbaiſſé de la planche 80 du Liure dudit ſieur.

Comme l'arc de la planche 80. du Liure dudit ſieur, où le point *g.* eſt donné, & doit eſtre le plus haut de l'arc, & neantmoins en ladite planche il ne l'eſt pas, mais cét arc eſt fait à veuë de nez ſans regle ny proportion, non plus que ceux de ſes deſcentes rampantes cy deuant dites; mais ie vous demonſtreray dans ma Stereotomie cy dite, qu'icelle courbe eſt touſiours l'vne des trois ſections coniques, & ſe verra quantité d'autres cas iuſques à preſent inconus, qui feront merueilleuſement expeditifs en l'Art de maſſonerie.

I'inſereray encor en cét endroit deux autres de mes manieres



Arcs Rampans faicts pour Vnne haulteur donneé

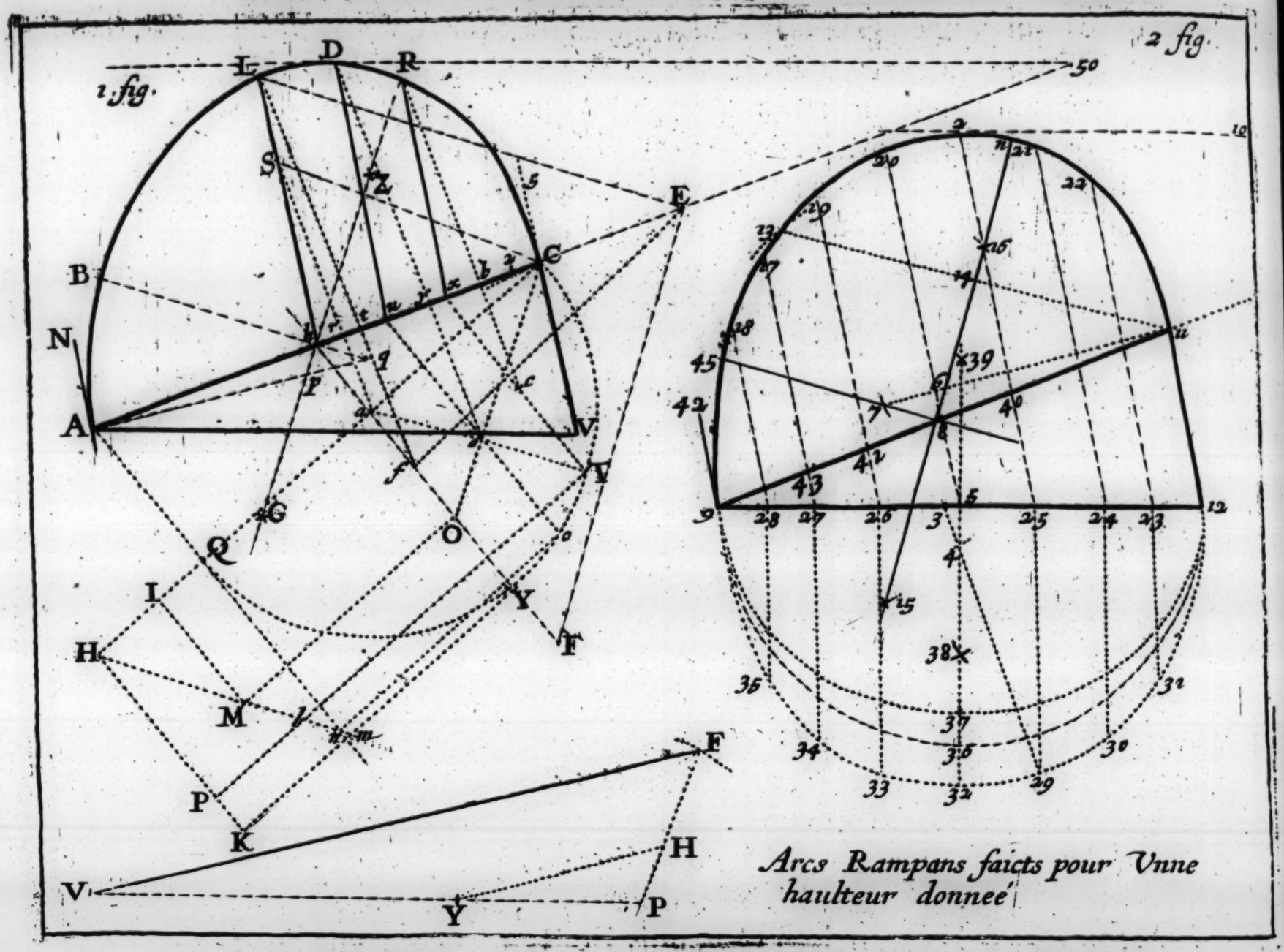
de descrire arcs rampans, & à vne hauteur de sommité donnée, pour s'en seruir au besoin cy apres. Et commenceray par la premiere figure qui est la plus speculatiue, disant. Soit A V vne ligne de niueau, comme aussi vne ouuerture donnée, & V C soit l'areste aux faces des pieds droits à plomb, ou panchans: à plõb, quand le mur ou face n'a point de surplomb, ou talus, ou autremẽt d'obliquitez au plan de niueau: & panchant quand ledit mur a de l'obliquité au plan de niueau. Soit V C la hauteur de la rampe; de plus soit L*50. vne hauteur donnée, dont l'on desire que la sommité de l'arc touche icelle L 50. soit paralelle, ou non à A V: soit menée vne ligne de A passant en C, & continuée tant qu'elle coupe la ligne L 50. comme en 50. maintenant il faut auoir vne place à part, pour descrire vn triangle, comme celuy de la presente figure V P F, & faire V Y esgal à A 50. ou à quelque partie d'icelle, si vous n'auez assez de place, cõme en la presente figure V Y est* le tiers de A 50. & partant commencant par le tiers, l'on operera touſiours par tiers; donc vous ferez Y P. aussi le tiers de C 50. puis vous prendrez le tiers de A C que mettrez au point P & descrirés vne portion de cercle, comme icy F, puis d'vn point en icelle, comme en F, menerez la ligne V F & F P, plus du point Y menerez la ligne Y H parallele à V F, coupant F P, comme en H, puis ferez C *u* triple de P H, & du point *u* menerez *u* D, parallele à C V, coupant L 50. comme en D, qui sera le lieu où la courbe à descrire touchera la ligne L 50. cela fait, vous menerez de *u*, vne ligne biaise à discretion, au respect de A C, & soit *u o*, & sa parallele A Q *n*, puis de C menerez vne ligne C H perpendiculaire à *u o*, & diuiserez Q C en deux, comme en *a*, & par *a* passerez la ligne *h a* Y, parallele à *u o*, & le point *h* sera le milieu de A C, & du point *a* descrirés le demy cercle C Y Q, coupant *u o*, comme en *o*. De plus du point D menez D *y e* perpendiculaire à A C, & coupant *u o*, comme en *e*, & soit H Q, esgal à *y e*, & soit menée *o n*, parallele à H C, qui coupera A *o n*, comme en *n*, puis soit menée la ligne H *n m*. De plus soit H I Q, esgal à *a r*, perpendiculaire à A C, & soit I M parallele à A Q *n*, coupant H *n*, comme en M, puis soit M O parallele à H *a*, & soit mené O C, puis de *a* soit mené *a* T perpendiculaire à O C, & coupant le cercle C Y Q, comme en T, soit mené T *x*, parallele à *u o*, puis *x* R parallele à C V & T P, parallele à H C, & faire *b c* perpendiculaire à A C, & esgal à P L, puis mener *c b*

Autre maniere generale pour descrire arcs rãpans, la hauteur du sommet estant donnée.

**Si icelle L 50. arriuoit parallele à A C, alors L toucheroit L 50. & ne seroit besoin du triangle cy apres deduit,*

** Vous suiurez le discours & non les figures que le graueur a assez mal imitées de son original.*

R coupans X R, comme en R, plus mener la ligne R *h* G, & ſa perpendiculaire B *h q*, faiſant A *q* eſgal à R *h*, & mener la ligne A



q, coupant R G en *p*, ferez A *p*. eſgal à *h* B, puis du point B, & ſur R G, marquerez les repaires du ſingliot Z, G, & chacun eſgal à *h* R, lequel ſingliot aura pour longueur Z G, & deux fois *h* R, & l'accommoderez, & deſcrirez l'arc rampant ADC, cõme la figure premiere du deuant du Liure vous le demonſtre, & ſerez aſſeuré d'auoir deſcrit vne courbe de ſuite, qui touchera C V, & ſa parallele N A, aux points donnez A & C, & touchera auſſi L 50. en D, ce qui eſt requis.

Autre maniere plus mechanique de deſcrire

La ſeconde figure eſt vne autre maniere de deſcrire ſemblables arcs, mais plus mechaniquement, que ie deduiray briefuement, vous aduertiſſant ſeulement de treuuer la longueur 11. 40. comme a eſté cy deuant demonſtré de trouuer *u* C, par le triangle V F P,

VFP, puis par 40. menerés 2.40.25. parallele à 11. 12. comme aussi 19.26. & 17.27. & 18.28. puis ferez 25.29. perpendiculaires, à 9. 12. comme aussi 26. 33. & 27.34. & 28. 35. en apres ferez 25.29. esgal à 2. 40. puis ferez 24. 4. esgal à 5. 12. & menerez 29. 4. coupant 9. 12. comme au point 3. puis ferez 5. 32. esgal à 29. 3. aussi du point 12. & sur 32. & 39. ferez les repaires du singliot, comme en 39. & 38. chacun esgal à 5. 32. & d'iceux repaires & auec le singliot, & de mesme qu'il est dit cy deuant, vous descrirez la demie ouale 9. 32. 12. plus ferez 33. 26. esgal à 41. 19. & 27. 34. esgal à 17. 43. & ainsi de 18. puis auec vn reglet flexible, & par les points 9. 45. 18. 17. 19. marquerez vne courbe; & du point 8. ferez 8. & 13. esgal à 8. 11. puis menerez la ligne 13. 11. & à icelle vne perpendiculaire 8. *n*, & menerez la ligne 8. 45. parallele à 13. 11. plus ferez 11. 6. esgal à 8. 45. & menerez 11. 6. tãt qu'elle coupe 45. 8. comme en 7. apres ferez 8. *n*. esgal à 11. 7. plus du point 45. & sur la ligne *n* 15. marquerez les repaires du singliot 16. & 15. & chacun esgal à 8. *n*. puis d'iceux repaires 16. & 15. & auec le singliot de mesure & maniere cy deuant dites, vous descrirez l'arc rampant 9. 2. 11. touchant 11. 12. & sa parallele 42. 6. aux points 9. 11. & touchera aussi la ligne 2. 10. en 2. que l'on demandoit.

arc rampant la hauteur du sommet estãt donnée.

Ie deduiray plus au long plusieurs autres particularitez d'icelles figures dans ma Stereotomie, que ie n'ay pas icy voulu mettre, tant à cause de la briefueté que ie me suis proposé dans cet ouurage; qu'à raison aussi que ie les traicte là amplement en leur lieu: Reprenons maintenant nos brisées.

Tous ses escrits & planches depuis 46. iusques à 52. & depuis 84. iusques à 97. ne sont employez qu'à construire vn beueau quarément, tant pour portes que pour trompes; ce qui n'est que faire vn angle solide, trois angles plans estant donnez; comme l'angle de la doelle, l'angle de la teste, & l'angle du lict, qui doiuent construire vn angle solide, & iceux trois angles plans sont cognus. Et partant les petits escholiers d'Euclide en construirõt vn angle solide par la 23. de l'vnziéme. Et bien qu'icelle 23. construise l'angle solide par le moyen de faire les angles plans donnez en triangle isocele, l'on peut aussi facilement construire l'angle solide, par le moyen de faire les angles plans dõnez en triangle rectangle, & à chaque fois deux. Ce qui se pouuoit demostrer en vne seule & simple figure de demy ligne, sans se seruir de tant

Prolixité dudit sieur en la construction d'vn angle solide.

de diſcours inutiles, qui ne font que redire la meſme choſe en diuerſes paroles ; mais que voulez vous ? celuy qui a quelque feſtin à faire eſt obligé à ſeruir pluſieurs plats, & neantmoins n'ayant peu par indigence auoir qu'vne ou deux ſortes de viandes ; il faut de neceſſité qu'il preſente pluſieurs fois la meſme viande apres l'auoir deſguiſée, autant qu'il luy a eſté poſſible.

Faute audit ſieur de vouloir introduire vne maniere ſi contrainte, tãt embrouillée & inconnuë, au preiudice d'vne generale, expeditiue & cognue comme eſt l'ancienne.

De plus, depuis la planche 53. iuſques à 59. ſont demonſtrées diuerſes manieres & façons de plaçer, tant l'eſſieu que ſous eſſieu, tant à vne porte droite ſans talus, qu'auec talus ; puis à vne porte biaiſe ſimplemẽt, plus auec talus ; à vne deſcente ſimple, puis auec talus ; cõme auſſi à vne deſcente biaiſe ſans talus, puis auec talus, leſquelles façons de faire ſe trouuerõt diuerſes, & les moyenemẽt experts qui en ſçauroiẽt l'vne, ne ſçauroiẽt pas pour cela l'autre, mais trouueront toutes ces diuerſes façons & preparations autant & plus difficiles, comme tout l'effet de la choſe meſme par l'ancienne methode. Et à le bien prendre, l'obliquité de la ſous-eſſieu renuerſe tellement l'obiect de l'épeur du trait, que l'on deſire faire, qu'on n'y recognoiſt quaſi plus rien qu'à rebours ou de trauers ; ce qui cauſe ſans doute vn bouleuerſement d'intelligence aux mediocres eſprits, qui ont ordinairement à pratiquer & mettre en œuure tels traits. Et puis, il y a plus de temps & plus de lignes à conſiderer, & beaucoup moins generales que dans l'ancienne maniere amandée, comme i'ay cy deuant remarqué ; pourquoy donc l'introduire au preiudice d'vne meilleure ; & non pas ſeulement l'introduire, mais la publier à haute voix, comme la plus parfaite & la plus generale en ce qu'elle contient, ainſi que i'ay monſtré au commencement de cette premiere Partie ?

Que l'application du trait dudit ſieur, ſur tant de planches de differents ſubiets eſtoit neceſſaire, ou ſuperflue ; ne ſeruant qu'à groſſir ſon Liure ſi elle eſt ſuperflue ; & ſi elle eſt neceſſaire,

Depuis la planche & leurs eſcrits 60. iuſques à 66. il monſtre la façon de faire par ladite methode diuers os, comme os ſimple & ſans talus ; os auec talus ; os biais ; os biais auec talus ; os en deſcente ; os en deſcente & biais auec talus, toutes leſquelles choſes derogent beaucoup à la pretenduë vniuerſalité de ſa maniere, où il dit qu'il n'en faloit qu'vn ſeul exemple pour tous, comme il eſt cy deuant remarqué ; & neantmoins il ſera contraint d'auoüer, qu'il a exercé ſa methode ſur tant de differents ſubiets & planches cy deuant cotées, ou pour groſſir ſon Liure, ou pour ce qu'il eſtoit neceſſaire de l'exercer & demonſtrer autant de fois qu'il y a de differens ſubiets : ſi c'eſt pour groſſir ſon Liure, c'eſt tromper le Lecteur ; ſi c'eſt qu'il eſtoit beſoin de l'exercer ſur tant de diffe-

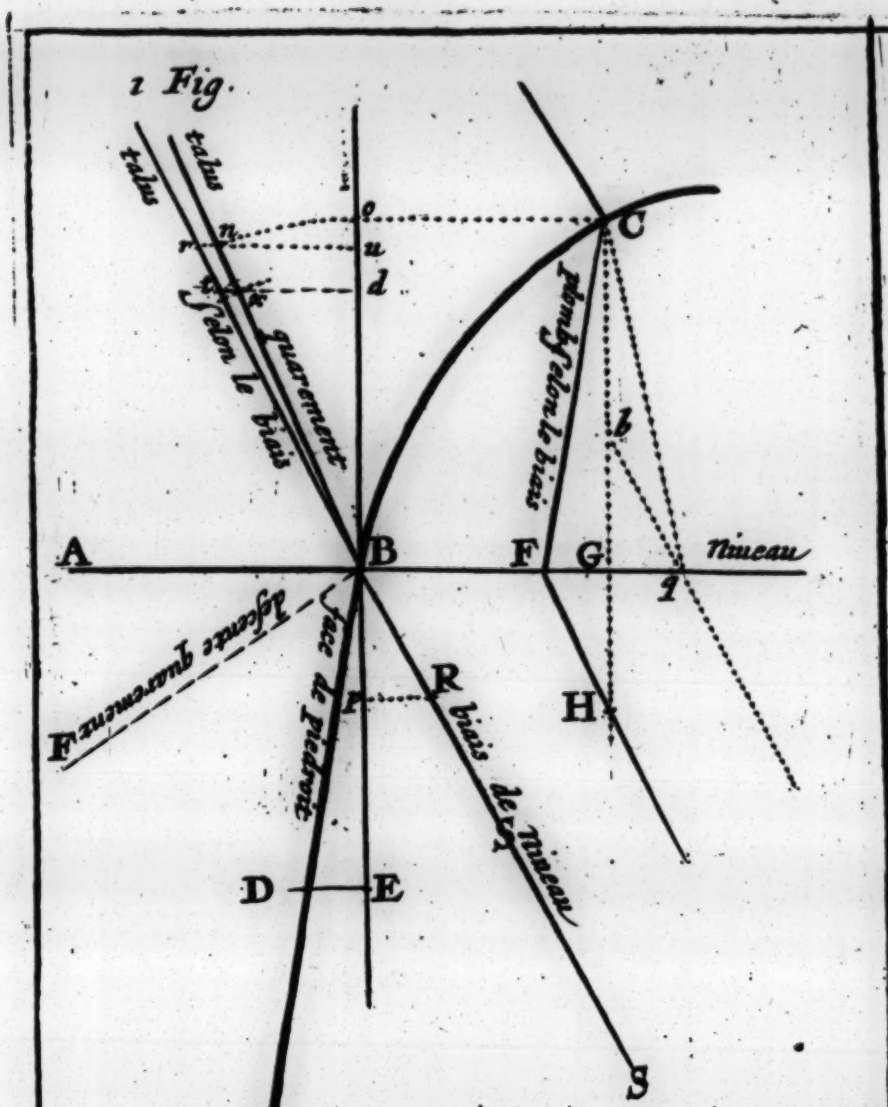
rents subiets; elle ne contient donc pas la generalité publiée; ce qui est constant par les raisons irrefragables deduites au commencement de ce Liuret.

elle ne cõtient la generalité pretendue.

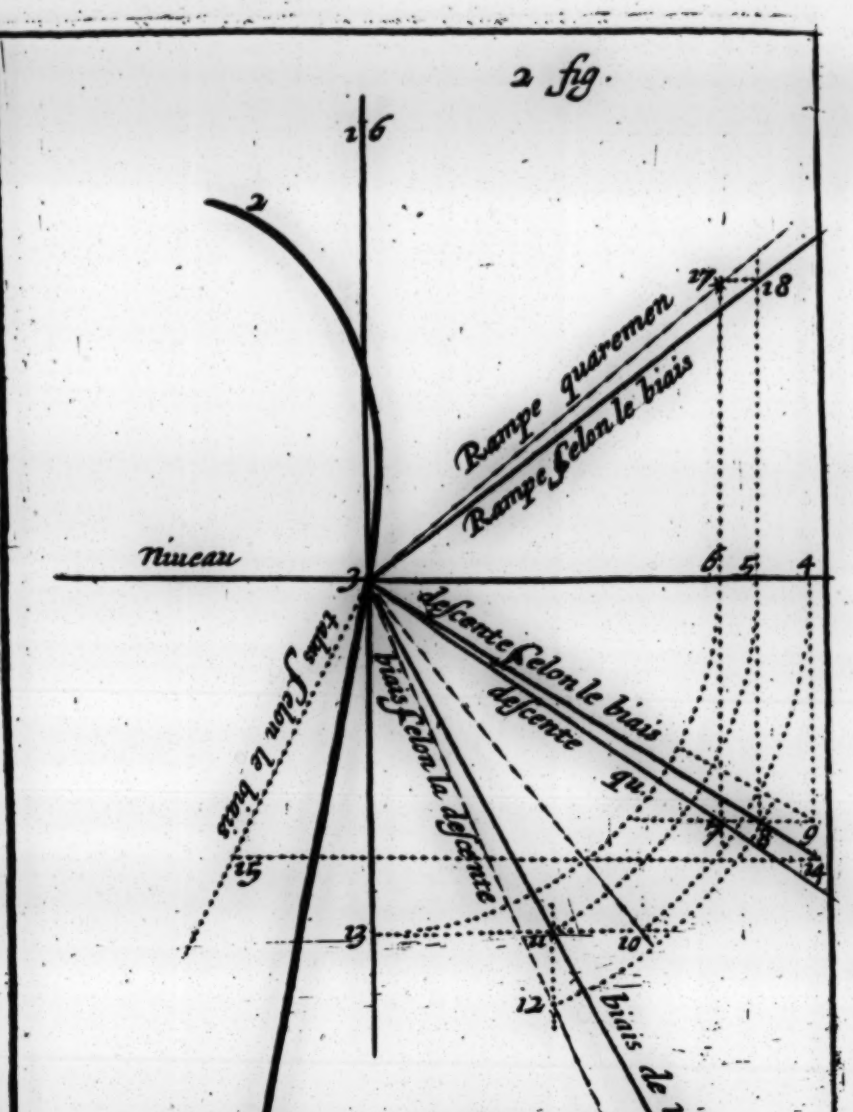
Mais puisque il vient à propos de parler des os, ie vous en donneray & demonstreray vn; par deux diuerses façons generales de la traditiue amandée dont ie me sers, qui est de ma Stereotomie, desquels vn chacun pourra librement prendre celles qu'il luy plaira, veu que l'vne est la confirmation & preuue de l'autre; c'est pourquoy il est bon que les curieux les sçachent toutes deux; mais aux effets & épeurs des traits, il ne sera besoin de s'en seruir que d'vne, si ce n'est en quelque point de doute, pour contenter l'esprit, où l'vne verifiera l'autre, & par beaucoup moins de lignes, qu'il n'est porté dans la planche; & ie vous asseure sans vanterie, que cette seule demonstration suffira pour construire toutes sortes d'ouuertures desirées sur vn mur droit, auec beaucoup moins d'embaras & beaucoup plus de facilité, qu'en la methode pretenduë generale dudit sieur, ainsi qu'il se verra cy apres. Il n'y sera parlé d'essieu, ny sous-essieu, ny de plan de route à niueau, ny de route au chemin, ny de semblables noms, autant barbares & impropres qu'ils sont nouuëaux; comme ie l'ay fait voir cy deuant: Ie me seruiray des manieres de parler propres, receuës & approuuées depuis tant de siecles passés, iusques à present. Premierement, ie vous declareray la methode que i'y tiens, & les choses qu'il faut auparauant cognoistre. Vous sçaurez qu'en toute descente ou montée ie pose en mon épeur la descente ou montée estre à niueau, qui est s'imaginer que le plan de la descẽte ou montée, est le plan de niueau; & de là s'exempter de la subiection de la rampe ou descente, & n'auoir qu'vn talus ou surplomb, au lieu & place d'vne descente ou montée, & si elle est biaise & en talus; vous n'aurez par la methode cy dite, qu'vn biais surplombant; ou taluent; ce qui est commun & facile aux ouuriers, & se conceura beaucoup mieux par la figure ou exemple, que par le discours. Il ne reste que de sçauoir rendre le biais selon la descente ou montée à niueau, puis que l'on conçoit iceluy plan de descente ou montée estre le plan de niueau, ce qu'il faut sçauoir precedẽment, & est l'vne des premieres cognoissances; puis se seruir d'iceluy biais à niueau, cõme on se sert à la construction ordinaire des simples biais qui sont réellement à niueau. Dõc pour cognoistre & reduire les biais selon la descente ou montée à niueau: vous en aurez icy vn exemple par la presente figure.

Propositionde construire vn os par les deux diuerses manieres generales de la traditiue amandée.

Cas amandé dans l'ancienne maniere.



pour Reduire les plombs des taluts ou surplombs selon le biais et les taluts ou surplombs selon le biais



pour Reduire les Rampes et descentes selon le biais et Rendre icelle a niueau

Maniere de reduire le biais selon la descente ou rampe.

Soit en la deuxiéme figure 3. 4. lignē de niuēau, & 16. 13. sa perpendiculaire: encore soit 4. 3. 11. le biais donné selon le niueau, soit aussi 3. 7. la descente donnée quarément au respect du niueau 34. estant donc question de dōner le biais selon icelle descente 3. 7. soit pris quelque point à discretion sur le biais de niueau; comme 11. & soit menée 11. 13. parallele. à 3. 4. & du point 3. soient descrites les portiōs du cercle 13. 6. & 11. 5. puis de 6. & 5. soit, & perpendiculairement à 3. & 4. menées les lignes 6. 7. & 5. 8. & ou 6. 7. coupera la descente quarément 3. 7. comme en 7. l'on menera 7. 9. paralleles à 3. 4. qui coupera la ligne 5. 8. comme en 8. & du centre 3. & au point huict soit descrite la portion de cercle 8. 12. coupant la ligne 11. 12. qui est parallele à 16. 13. comme en 12. & soit menée la ligne 3. 12. qui est le biais selon la descente dont l'on se seruira aux occurrences.

Aussi

Aussi ménant la ligne de 3.& 8.point cy deuant trouué, l'on aura la descente selon le biais, ou proprement la couche du coussinet, au respect du niueau 3.4. auquel niueau si l'on met le talus selon le biais trouué, par la maniere cy dessous, comme 3.15.& que 15.14. soit parallele au niueau 3.4. alors 3.15.14. sera le coussinet.

Maniere de reduire la descente selon le biais.

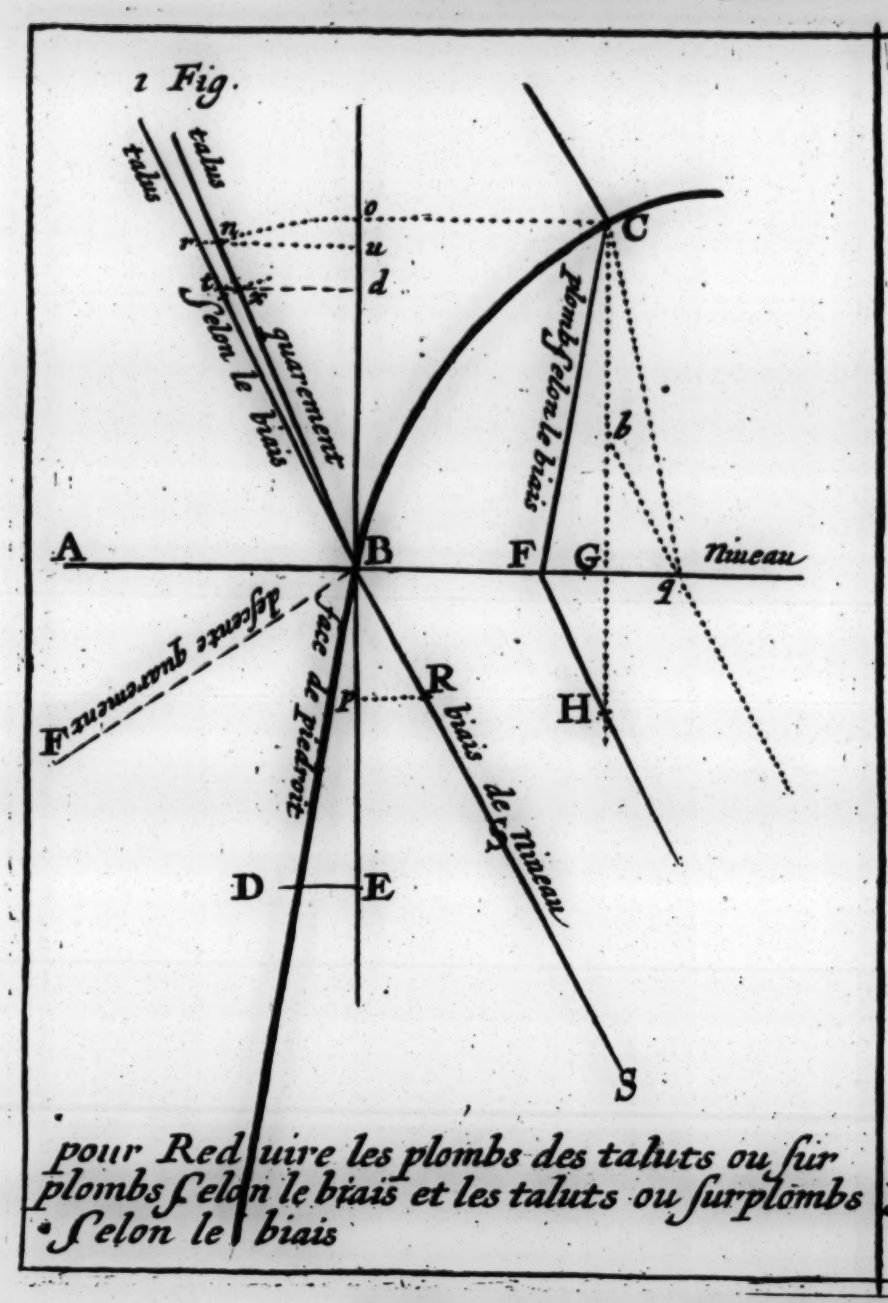
Et par mesme regle si 3.17. estoit vne rampe donnée quarément, & que l'on voulut auoir la rampe selon le biais estant 3.11. biais, donné quarément & à niueau, operant comme a esté dit cy deuant, descriuant les portions de cercles 13.6. & 11.5. puis de 6.& 5. esleuer des perpendiculaires à 3.4. & touſiours où icelle perpendiculaire du point 6. venant du point 13. ou icelle coupera la rampe comme en 17. mener 17.18. paralleles à 3.4.& coupant la ligne 5.18. comme en 18. faut mener 3.18. qui sera la rampe selon le biais.

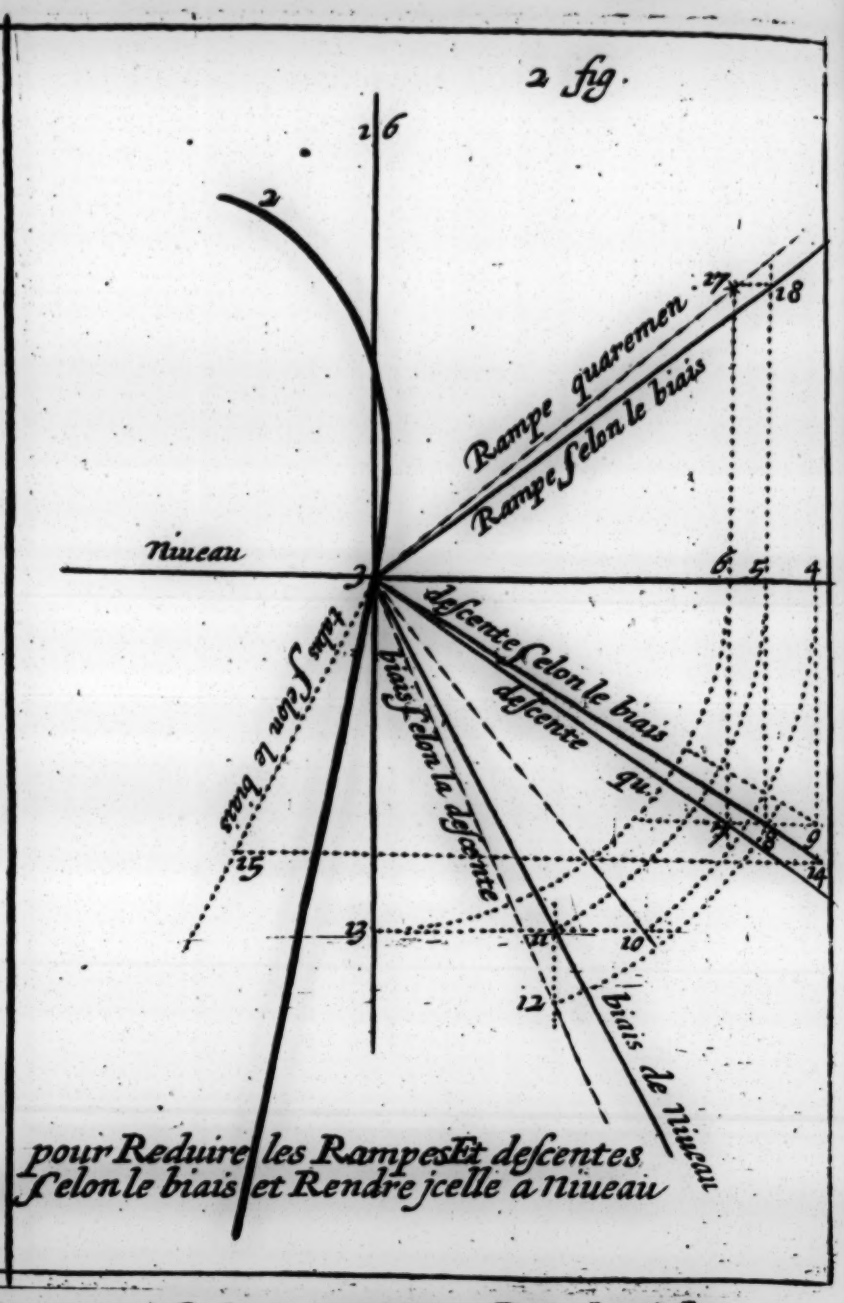
De mesme suite, il sera besoin de sçauoir que des talus ou surplombs donnés quarément, soient reduits selon le biais, comme en la figure premiere la ligne B *n*, est talus donné quarément au respect du niueau A G, & pour le reduire selon le biais donné de niueau A B S. soit pris quelque point à discretion sur la ligne B *o*, cõme en *d*, & soit menée de *d* vne ligne parallele à A G, cõme *d t*, laquelle coupera B *n*, comme en *x*, soit pris la distance *x d*, & soit porté de B en *p*, puis de *p* soit menée vne ligne parallele à A G, qui coupera B S, comme en R, & soit prise la longueur B R, que l'on transportera du point *d* comme en *t*, & puis de *t* soit menée la ligne *t* B, qui sera le requis. Et partant l'angle A B *t* sera l'angle du talus, ou G B *t*, l'angle de surplomb selon le biais, ou proprement l'angle du lict & de l'areste de deuant du tableau, de quelque porte biaisé en talus ou surplombante.

Et venant à propos, ie demonstreray par la premiere figure de la mesme planche la maniere de descrire les plombs des talus, selon le biais, qui est proprement les arestes des faces, ou testes des pieds droicts. Soit A G, le niueau donné; comme aussi A B S, le biais donné & à niueau; & soit *n* B, le talus donné quarément & au respect du niueau A G, & si l'on desire comme du point C, mener quelque plomb selon le biais, l'on menera C *o* parallele à A G, & coupant la perpendiculaire *o* E, comme en *o*, puis du point B, comme centre soit descrite la portion de cercle *o n*, coupant B *n*. comme en *n*, puis l'on menera *n u* paralleles à A G, & du point C sera menée vne ligne perpendiculaire à A G,

Maniere de faire les plõbs des talus ou surplombs selon les biais.

comme C G H, puis l'on fera G H eſgal à *n u*, puis du point H l'on menera H F parallele au biais donné B S, & coupans





A G commẽ en F, puis du point C l'on mẽnẽra C F, qui ſera le requis, & faiſant B D parallele à iceluy C F, iceluy B D ſera l'areſte de face, ou teſte du pied droit, au reſpect du niueau A B.

Et ſi la ligne *n* B eſtoit vn ſurplomb, il faut faire G *b* eſgal à *n u*, & du point *b* mener *b q*, parallele à B S, & où elle coupera A G comme en *q* mener la ligne C *q*, qui ſera le plomb ſelon le biais de ſurplomb donné, comme auſſi l'areſte de face de pied droit, &c.

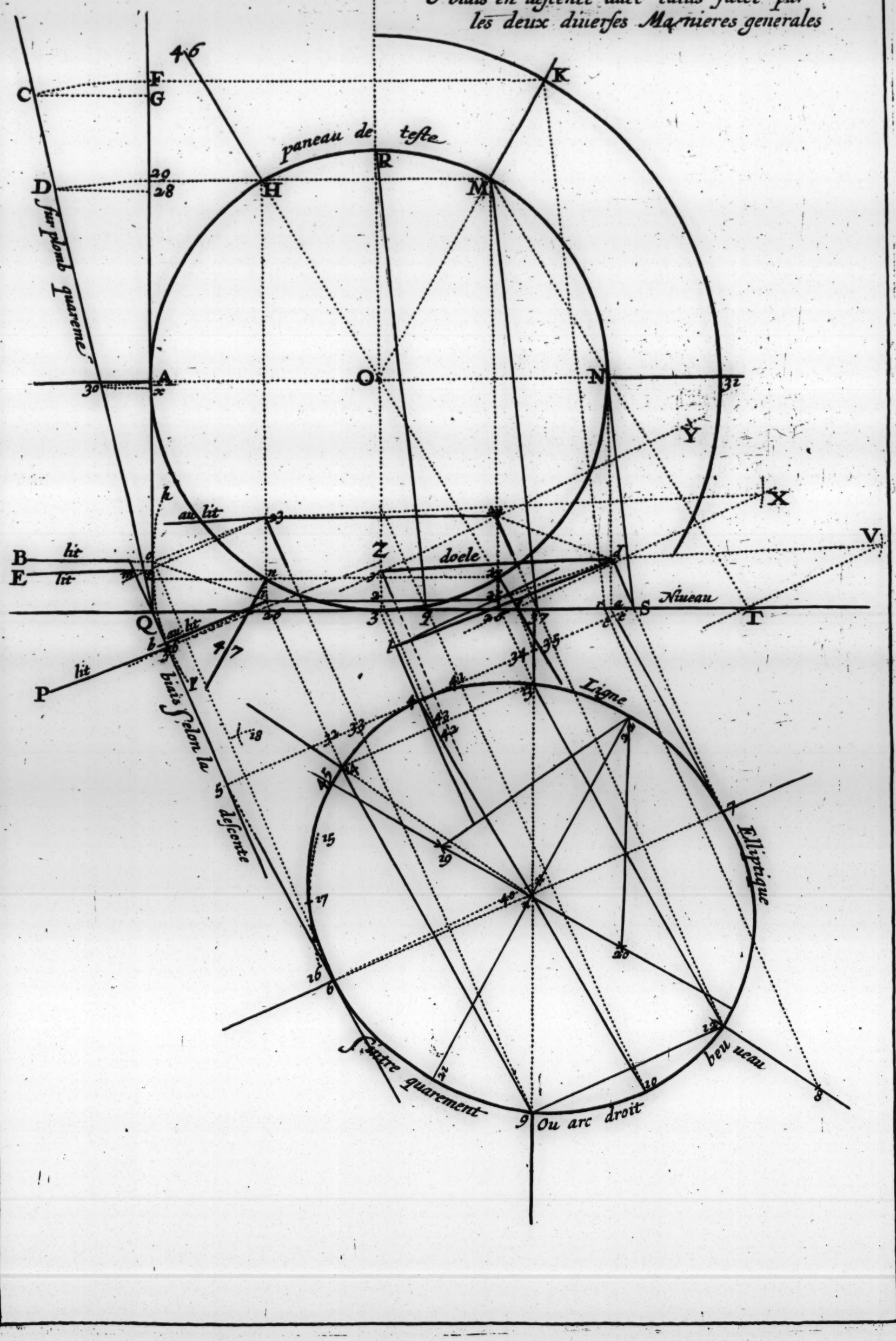
Maniere de reduire la deſcente on

De plus, ſi *n* B eſt talus donné quarément, & B F. eſt deſcente dõnée auſſi quarément, & le tout au reſpect du niueau A G, ſi l'angle F B *n* eſtoit d'eſquierre, icelle décente ne ſeroit qu'vne

ſimple porte biaiſe, & ſi l'angle F B *n* eſtoit gras, icelle deſcente ne ſera qu'vne porte biaiſe ſurplombante. Donc le ſurplomb ſera de ce que l'angle gras ſurpaſſera l'angle deſquierre; & ſi iceluy angle F B *n* eſtoit maigre, alors icelle deſcente ſera vne porte biaiſe en talus; & iceluy angle maigre ſera mis comme en la figure eſt mis A B *n*, & l'angle gras ſera mis comme en G B *n*, & partant l'on rameine la ligne B F en la ligne de niueau A G, qui eſt imaginer la deſcente ou rampe eſtre ramenée au plan de niueau, & par ce moyen s'exempter de la ſubiection de la deſcente ou rampe, ſelon la regle cy deſſus; & n'auoir en ſon lieu qu'vne ſimple porte ſurplombante ou taluente, ainſi qu'il a eſté dit cy deuant, ce qu'eſtant conceu, & par ce moyen du biais cy deuant trouué ſelon la deſcente, l'on operera auec iceluy biais ſur le plan de niueau imaginé, & par le moyen auſſi du talus ou ſurplomb trouué, comme eſt dit cy deſſus. Ce qui eſt vne regle generale, dont ie me ſers pour toutes ouuertures ſur vn mur droit; Et en conſequence de ce que deſſus, ie feray voir vn os en deſcente biaiſe en talus, qui eſt de ma Stereotomie, pratiqué ſelon la traditiue amandée, & par deux 'diuerſes manieres veritablement generales, pour toutes ouuertures ſur faces droites.

rampe à niueau & n'auoir qu'vn ſimple talus ou ſurplomb.

O biais en descente auec talus faict par les deux diuerses Manieres generales

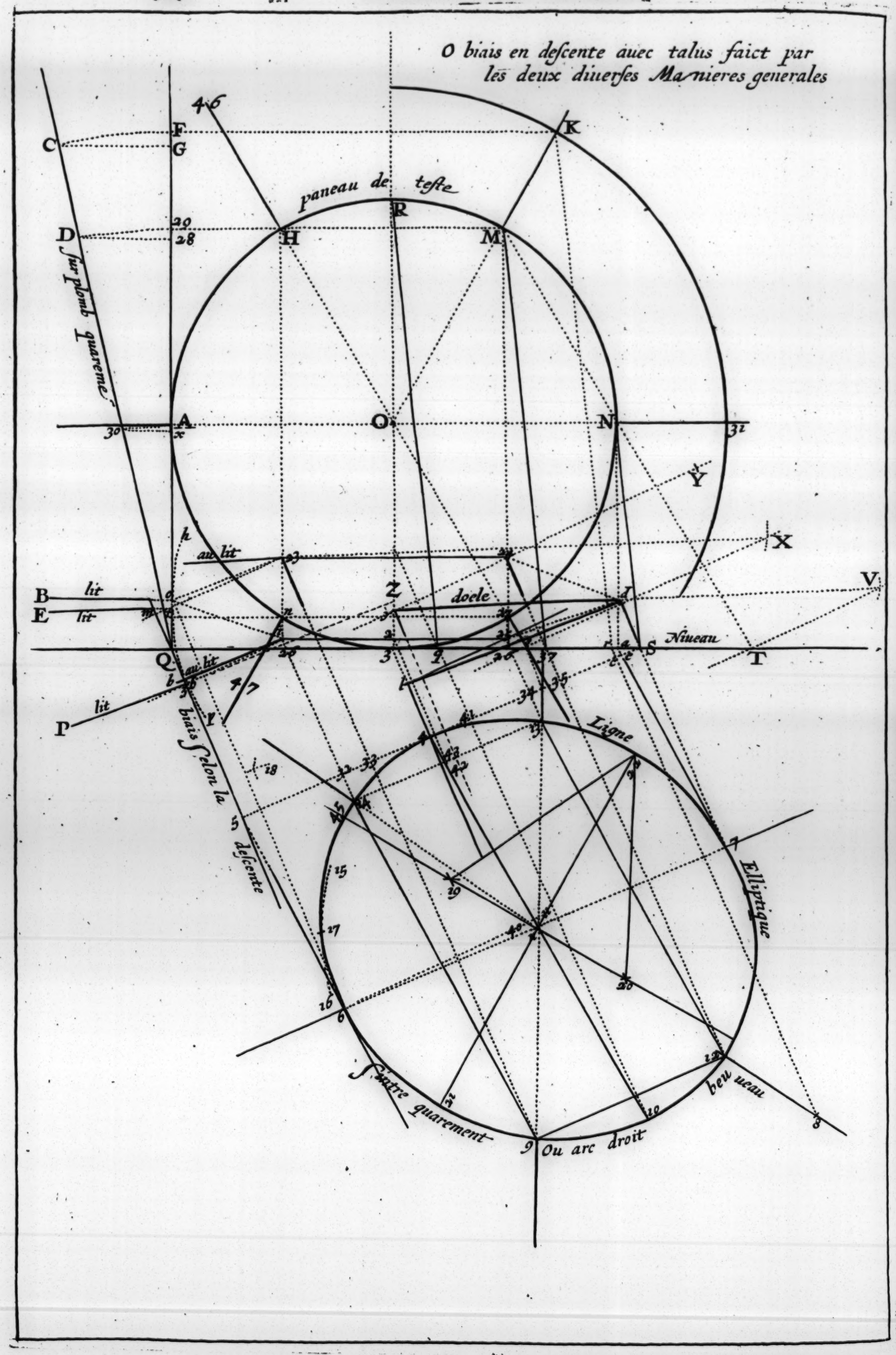


Soit vn pan de mur en talus où l'on deſire d'y faire vn os, lequel os ſoit A 3. N R, marqué comme il paroiſtra à la face d'iceluy mur, Et ſon centre ſoit O, ſoit la ligne de niueau Q, 3. T. touchant l'os en 3. ſoit eſleuée la perpendiculaire au niueau 3. O R, & auſſi ſa parallele F A Q, touchant l'os en A: de plus, ſoit mis l'angle du talus ou ſurplomb, & maigre ou gras en la maniere cy deuant expliquée. Et ſoit poſé eſtre gras comme T Q C, & partant l'os eſt reduit & imaginé eſtre vn biais ſurplombant. De plus & par la maniere cy deuant dite, ſoit T Q 5. biais, ſelon la deſcente & R *q*, plomb ſelon le ſurplomb Q C, & le biais Q 5. ſoit iceluy os departy en telles parties, que deſirerez, comme en H M N 27. *n*. & d'iceux points ſoient tirées les coupes du centre, O, cõme il ſe voit par la figure. Soit menée la parallele à la ligne Q T, comme N A, & M H, & K F, & 27. *n*. iuſques ſur la ligne Q F, comme F 29. A *t*, & d'iceux points ſoient menées les portions de cercle F C & 29. D & A 30. puis du point C D 30. ſoient menées des lignes paralleles à Q T, comme C G, & D 28. & 30 *x*, puis ſoient menez les plombs piquez H *n i* & M 25. & N *a*, ſoit fait *a d*, & Q *o* & 3. Z égal à 30. *x*, & 23 quarément à Q T, comme auſſi 24. quarément à iceluy Q T, ſoit égal à D 28. & ainſi des points *i* & 25.

Demonſtration de deux diuerſes manieres de traict generales pour toutes ouuertures requiſes ſur mur droit appliquées à vn os en deſte biaiſe auec talus par la traditiue amendée.

Maintenant pour auoir le lict ou coupe H 46. & par les deux manieres, conſiderant que N A paſſe au point O, & que partant le point Z eſt le point du plan de ſon ſurplomb, & d'autant que 46. H paſſe au point O. Donc au plan poſant les dérobements ou auances de chacune retombée deſirée, & la diſtance qu'il y a du point à icelle, l'on aura le lict; comme par exemple 23. eſt retombée ou point, au plan reſpondant au point H, & Z; eſt le reſpondant de O. Dõc du point Z deſcriuant vne portion de cercle *h t p* de meſme diſtance, que O H, puis du point 23. mener la ligne 23. *t*, perpendiculaire à Q 5. coupant la portion de cercle *h p*, comme en *t*, puis de Z mener la ligne Z *t* E, alors l'angle 5. *m* E ſera le paneau du lict ou coupe 46. H. & de meſme de la coupe *n*, 47. donc *i* eſt ſon point au plan, menant de *i* vne perpendicule à Q 5. coupant le cercle *h p*, comme en 36. puis l'on mene de Z paſſant en 36. le lict 36. P, partant l'angle P 36. 5. eſt le paneau pour le lict ou coupe *n* 47. & l'on l'auroit encore au reſpect du point de retombée 24. paſſant par 24. vne ligne perpendiculaire à Q 5. puis du point *i* fai-

Raiſonement & maniere de trouuer les paneaux de lict par les deux diuerſes manieres generales.


O biais en descente auec talus faict par les deux diuerses Manieres generales
paneau de teste
sur plomb quarement
biais selon la descente
Ligne
Elliptique
Nyueau
doele
au lit
lit
Ou arc droit
bout veau

re *i* Y égal à *n* M, puis du point Y mener *i b*; alors l'angle 32. *i b*, sera semblable à l'angle 5. 36. P, & ainsi des autres. La secõde maniere pour iceux paneaux se fait par extrados, que beaucoup sçauent, que ie ne demonstre icy pour abreger; & de plus, qui se fait par mesme maniere que les paneaux de doelle que ie vais demonstrer.

Façon de treuuer les paneaux de doelle par les deux diuerses manieres generales.

Pour auoir le paneau de la doelle M N, dont leurs points de retombée sont 24. *d*, soit menée par 24. vne ligne perpendiculaire à Q 5. puis faire *d y* égal à MN, & mener la ligne *y d*, alors 42 *y d* 7. sera le paneau de la doelle M N, qui est l'ãcienne maniere; & partant l'on peut par icelle auoir aussi bien la ligne M K, comme l'on vient d'auoir M N, & par mesme maniere l'on aura la doelle *l d*, mais pour auoir les doelles par vne seconde maniere, comme par exemple la doelle M N, faut conduire M N, iusques sur Q T, & au point T mener vne ligne perpendiculaire à Q 5. puis de *d* faire *d* V égal à N T, puis du point V mener la ligne *d y*, & sera la requise, & ainsi de tous les autres.

Façon de treuuer le cintre quarément par les deux diuerses manieres generales.

Pour trouuer le cintre quarément ou arc droit, soiẽt menées de tous les points de retombée des lignes autant longues que besoin sera, & paralleles à Q 5. comme *o* 6. & *i* 14. & 23. 9. & 3. 4. & 25. 13. & 24. 12. & *d* 7; puis comme du point S, soit menée la ligne S 5. perpendiculaire à Q 5. laquelle ligne seruira de niueau au cintre quarément, ou arc droit 4. 6. 10. 7. comme Q T sert de niueau à l'arc de la face du mur R A 3. N; Maintenant pour auoir la ligne A O N 31. il la faut prendre de Q *x*, & faire 6 & 7. paralleles à 5. S, & de pareil interualle qu'iceluy Q *x*, & partant 6. & 7. sera correspondant à A & N, & *u* milieu de 6. 7. sera correspondant à O, & partant comme en O les coupes M K, & N 31. & 47. *n*, &c. Aussi au cintre quaré les coupes tendront au point *u*. Et pour trouuer les autres points, comme H M, les faut prendre tousiours sur Q F, & faire 9. 33. & 35. 12. égal à Q 28. & ainsi des autres, comme S 8. sera égal à Q G, & ainsi des autres. Donc pour auoir les coupes de l'arc droit, ne faut que mener du centre *u* par 12. 8. & par seconde maniere ayant trouué le point 8. & le point 12. ainsi que cy deuant est dit, faudra mener vne ligne de 12. en 8. & sera la coupe, ainsi des autres. Maintenant les points 14. 13. 7. 12. 9. 6. estant trouuez, il faut descrire vne ligne courbe, qui passe par iceux, laquelle sera elliptique, ce qui sera facile par le moyen de ce qui a esté demonstré cy deuant aux arcs ram-

pans,&d'vne hauteur de ſommité donnée, en cõſiderant la ligne 6.*u*.7. & la hauteur 5.S. & la ligne de rampe 5.6. ou ſa parallele *u* 4.qui touche la hauteur donnée au point 4.ce qui eſt deſia tout trouué plus operant, comme l'vne ou l'autre regle des arcs rampants cy deuant dits l'enſeigne: l'on place le grand diametre 45. 20.l'on trouue auſſi les repaires du ſingliot 19. & 20. enſemble la longueur d'iceluy; puis l'on deſcrit la ligne elliptique 6.10.7. 4. comme on voit par la figure au deuant du Liure; ainſi de tous autres arcs droits.

Cas amandé dans l'ancienne maniere.

Vous remarquerez que l'ancienne maniere dõnoit trois poinrs en chacune doelle, comme 9.10.12.puis par iceux deſcriuoit vne portion de cercle, & ainſi à toutes les doelles. Et partant la ligne elliptique eſtoit deſcrite de pluſieurs & diuerſes portions de cercles, qui faiſoient autant d'angles ou iarets, comme il y auoit de doelles; qui eſt l'vn des cas auquel ladite maniere ancienne auoit beſoin de correction & amendement; Et c'eſtoit de cette façon que le ſieur Deſargues deuoit donner des regles pour les cherches aux fins de creuſer les doelles de toutes ſes portes.

Maintenant ie reprendray la ſuite de l'examen, & diray, que la planche & eſcrit 67. iuſques à 76. ſont employez à demonſtrer par la pretenduë maniere generale dudit ſieur, de conſtruire les traits de pluſieurs portes, tant ſur le coing que dans l'angle, tantoſt en deſcente ou montée ſimple; tantoſt en deſcente & en talus; & operant touſiours ſur des principes faux, cõme ſont ceux que i'ay touchés cy deuant, à ſçauoir ſur des arcs qu'il deſcrit ſur chacune des faces deſdites portes ſans regle, ny raiſonement; où vous remarquerez la lourde faute, que commet ledit ſieur, qui eſt, qu'outre les angles ou iarets & caueures qu'iceux arcs peuuent faire en leur naiſſance, ſur les pieds droits, enſemble leur defaut de proportion, ainſi qu'il a eſté amplement remarqué cy deuant, iceux arcs ſe viennent à couper dans la clef, & font vn angle ou iaret en ladite clef, tout au long du berceau de la porte ou deſcente; ce qui eſt extremement defectueux à la veuë, de voir vn angle ou iaret au milieu, & tout au long du liſe d'vne voute, faiſant le meſme effet de ſes arcs ou voute Gothique, qui ſont compoſées de deux cercles; ce qu'il eſtoit neceſſaire de reformer. Et c'eſtoit en cecy que conſiſtoit le point de la difficulté; & où il auoit matiere ſuffiſante pour exercer ſon raiſonnement; puiſque aucun autheur ne l'auoit conceu iuſques à maintenant, & n'en

Remarque des courbes dudit ſieur qui ſe coupent dans la clef faiſant les voutes defectueuſes.

auoit

auoit point traité ; & non pas de faire les choſes qui eſtoient faites il y auoit ſi long temps, en meilleure forme & auec beaucoup plus d'expedition, ainſi qu'il a eſté cy deuant remarqué, i'entends parler de ſa pretenduë maniere par eſſieu & ſous-eſſieu. Ie dis qu'il eſtoit neceſſaire de ſçauoir la maniere de traçer les arcs des faces des portes ou deſcentes ſur le coing, ou dans l'angle ; en ſorte que l'arc du berceau de la porte n'eût aucun angle ou iaret, mais fuſt fait tout d'vne meſme courbe, & de ſuite, comme les exemples le demonſtreront cy-apres.

Paſſant outre à l'examen, vous verrez que la planche 77. eſt la ſemblable que la planche 27. cy deuant expliquée ; & qui demonſtre ſeulement la ſous-eſſieu & eſſieu, eſtant à coſté de la porte, & operant touſiours ſur pareilles manieres ; là meſme conſequence ſuit des paneaux ; ce que i'ay amplement deduit cy deuant en la planche cy deſſus cotée. Icy ledit ſieur finit les exemples de ſa pretenduë maniere generale, pour toutes ouuertures & portes ſur face droite.

Vous remarquerez quel grand nombre de planches, eſcrits & demonſtrations il a employé, pour faire voir cette maniere qu'il promettoit merueilleuſement generale, & qu'il n'en faloit qu'vn exemple pour toutes ; mais aux effets l'on voit bien le cõtraire ; auſſi eſt il veritable, qu'il y a grande diſtance entre l'effet & la parole, ou promeſſe. Il dira que c'eſtoit pour ſe faire entendre à fonds ; & qu'vn ſeul exemple contient toutes celles-là ; Et l'on luy repartira, qu'il auoit beſoin plus qu'aucun autre de s'entendre ; & qu'il n'a pas eu la veuë aſſez longue, pour voir iuſques à la moitié de la diſtance du fond, ainſi qu'il eſt euident.

Prolixité des figures du Liure dudit ſieur, enſemble le contraire de ſes promeſſes.

Les planches & eſcrits depuis 78. iuſques à 93. ſont employez, tant aux beueaux droits des trompes, qu'à la conſtruction des trompes ſur la face droite, par vne pretenduë ſienne maniere, & vn cintre ſurbaiſſé, eſtant requis, ce qui eſt neceſſaire d'examiner & y reſpondre par ordre.

Les beueaux droits des trompes ſe conſtruiſent comme les beueaux droits des portes, & partant prolixité reïterée.

En premier lieu ie dis, que pour les beueaux droits pour faire le lict des trompes, ils ſe trouuent & conſtruiſent par meſme maniere que les beueaux droits des portes ; & n'eſt autre choſe que trois angles plans eſtant donnés, pour en conſtruire vn angle ſolide, ainſi qu'il a eſté amplement deduit cy deuant, aux planches 46. iuſques à 52.

Pource qui eſt de ſon cintre ſurbaiſſé ; & en vn point donné ;

Faute dudit sieur de n'auoir dõné des manieres pour faire les surbaissemẽs des courbes touchãt vn point donné, ce qui estoit extremement necessaire, cõme estant le fondement surquoy on trauaille.

comme en *g* de la planche 80. il a déja esté dit & redit encore, qu'il n'en donne la maniere ny la regle, pour la construction d'iceux arcs ; mais il les fait à la discretion & conduite de la main de l'ouurier ; comme si aux effets & œuures qui sont grands l'on y pouuoit proceder, ainsi que fait vne plume sur vne petite fueille de papier, & tousiours sans regle ; & partant toute l'operation qu'il execute sur ce faux fondement, reüssira infailliblement auec defaut. * Et ie luy donne aduis qu'il sera demonstré dans ma Section des solides, qu'iceux arcs seront tousiours l'vne des courbes des trois sections coniques, ou autre maniere, par la ionction de deux ellipses.

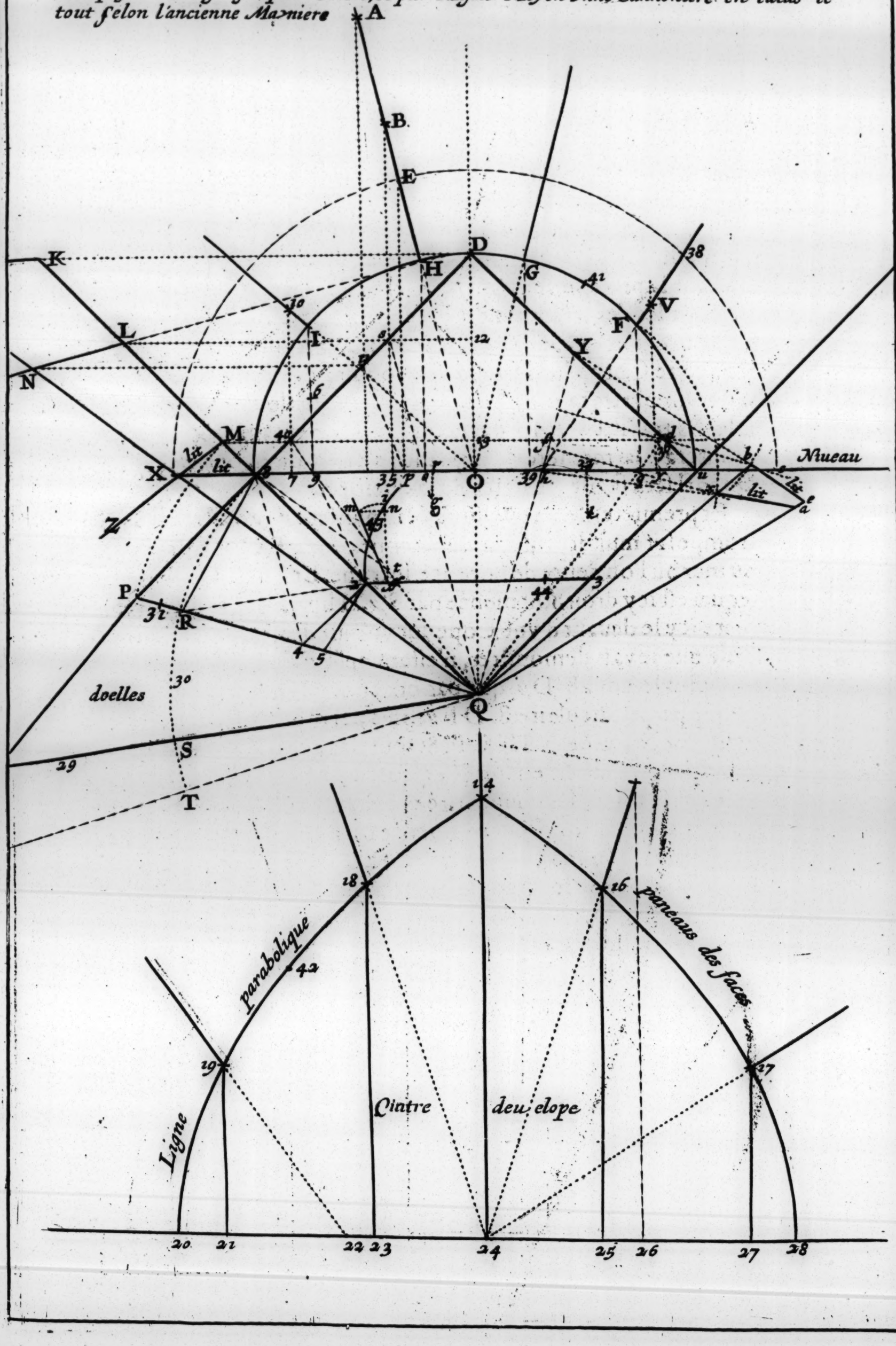
** Que les arcs surbaissez touchant vn point de hauteur dõnée sont tousiours l'vne des trois sections coniques.*

† Pour sa pretenduë maniere de trouuer les paneaux des trompes, par les déguisemens de ses essieux, & en la planche 81. ie vais faire voir qu'elle est de l'ancienne pratique ; & pour déplumer cette Corneille d'Esope, ie produiray les simples manieres d'vne simple trompe sur le mur droit, ou sur le coin, pratiquées selon la traditiue ; mais beaucoup fondamentale, & vous confronteray la maniere de la planche 81. cy dite contre icelle ; & apres il sera facile de iuger de la question.

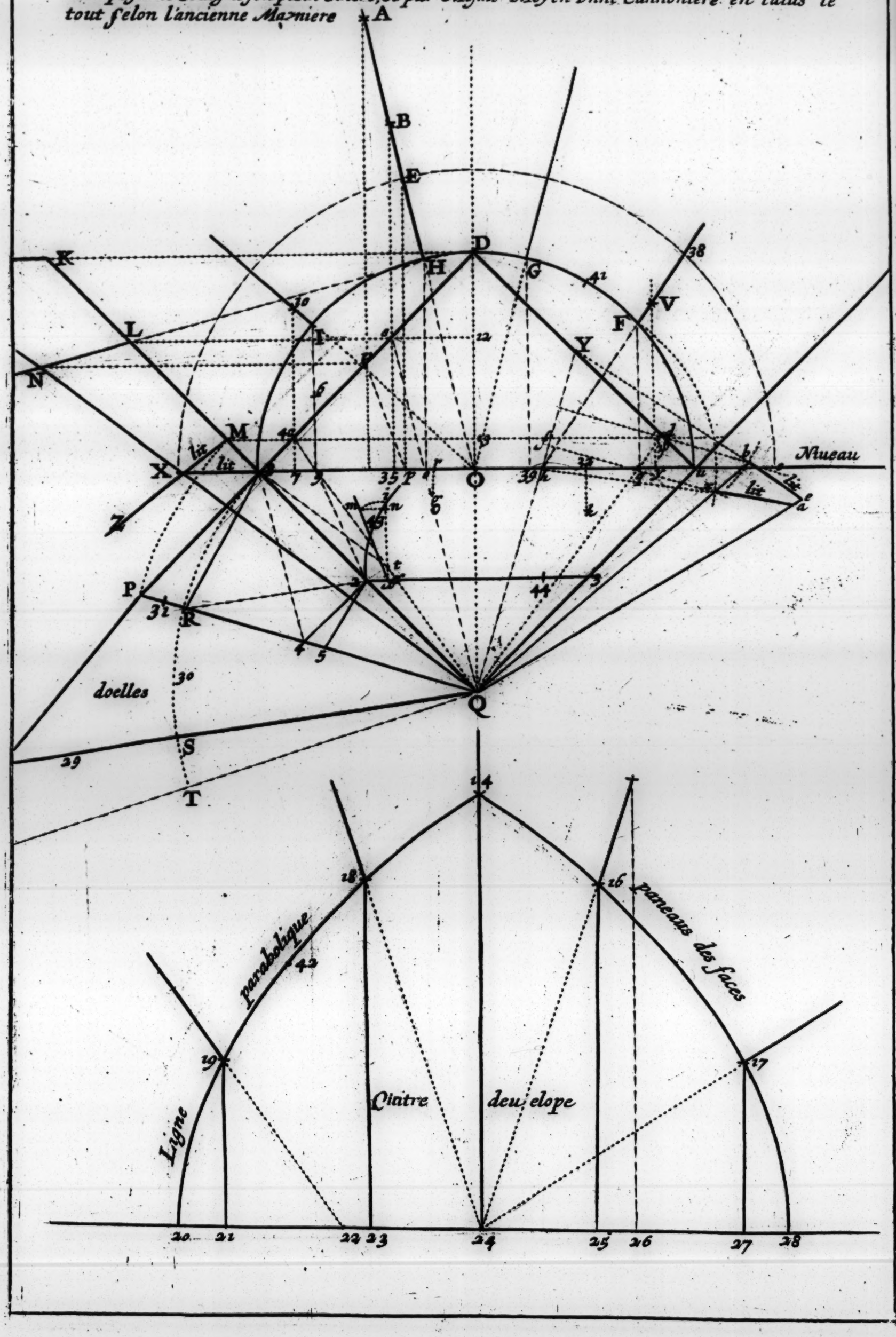
† Que la pretenduë maniere dudit sieur, pour trouuer les paneaux des trompes, est de l'ancienne.

Et premierement ie deduiray la maniere de construire ladite trompe sur mur droit, selon l'ancienne. Soit K Q & Q *u*, l'angle ou mur où l'on desire de faire icelle trõpe. Et soit 8, O *u* son plan, lequel est icy droit ; soit menée par 8. *u*, la ligne X 8. *u c*, qui represente icy le deuant d'vne trõpe sur vn mur droit sans talus, qui est celle que ie vais demonstrer, n'estant besoin maintenant de celle de dessus le coin 8. D *u* que par occasion. Soit fait le cintre de la face perpendiculaire 8. D F *u*, puis party en tant de pieces que desirerez, comme en I H G F, & tout les aplõbs d'iceux menés, & perpendiculairement sur X *c* (qui represente aussi vn plan de niueau) & soient les coupes tirées comme il conuiendra, les vnes en O, les autres en *h*. Pour trouuer le paneau de lict de l'vne d'icelle coupe ; comme par exemple E H, l'on regarde où E H, estant prolongée, coupe le plan de la trompe X *c*, comme en O, puis l'on mene Q O, iusques au plan du deuant de la trompe, comme en D, si elle estoit sur le coin, & en O, estant sur mur droit : de plus l'on mene de Q, passant en *o* vne ligne, iusques en *s*. si c'estoit pour la trompe sur le coin & iusques en *o*, pour la trompe sur mur droit, laquelle Q *s* represente le plan & plomb, que l'areste de la doelle du lict H E, fait sur le plan de niueau X *c*. De plus pour

trompe sur le Coing à son plein Cintre, et par Mesme Moyen Vnne Cannoniere en talus le tout selon l'ancienne Maniere



trompe ſur le Coing à ſon plein Cintre, et par Meſme Moyen Vnne Cannoniere en talus le tout ſelon l'ancienne Maniere



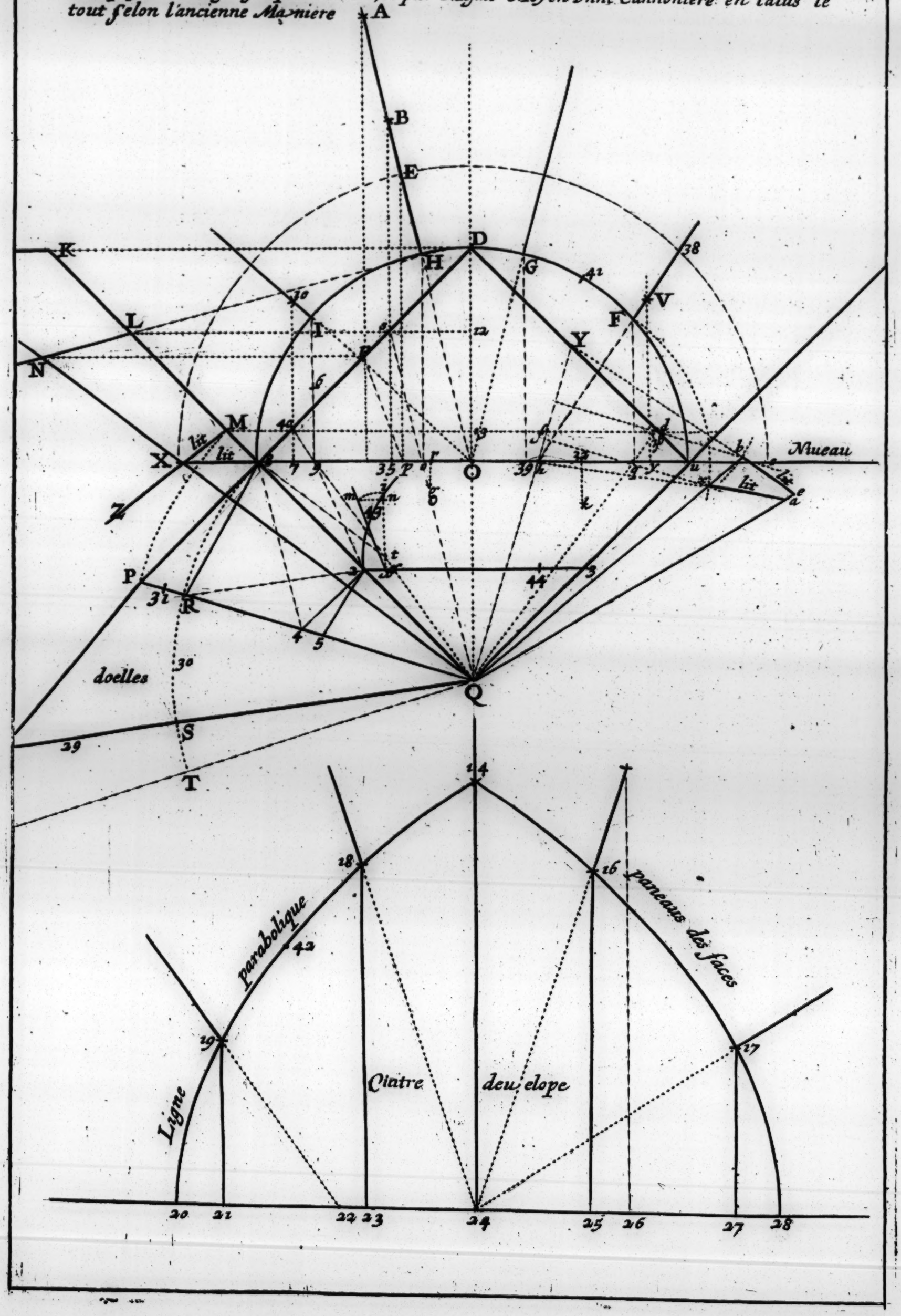
la trompe sur le coin,& au point *s*, l'on passe vne ligne comme 12. L & perpendiculaire à D Q, & l'on fait le semblable pour la trompe sur le mur droit, sçauoir passer en *o* vne ligne comme 8. O,& perpendiculaire à Q O, puis de O l'on prend la distance O H, que l'on porte depuis O,& où elle coupe la ligne O 8.cõme en 8. l'on mene de O la ligne 8. X, & de 8. la ligne 8. Q, & partant l'angle X 8. Q, est le paneau du lict de E H, & en la trompe sur le coin, si l'on prend sur son cintre de faces cette mesme ligne, comme 24.16. & portée en D, coupant 12. L, comme en L, puis de D mener L N,& L Q, alors N L Q, sera paneau de lict pour la coupe E H: l'on procederoit de mesme, si l'on requeroit en la trõpe sur le mur droit cy dite, le paneau d'vne autre coupe, comme F 38.car F 38.continué iusques en *h*, l'on meinera de *h* vne ligne iusques en Q Y, laquelle ligne represente le plõb & plan que l'areste du lict de la doelle F 38.fait sur le plan de niueau X *c*, ainsi qu'il est dit cy deuant. Donc conformément au precepte cy dessus l'on passe par *q* vne ligne perpendiculaire à Q Y, autant grande que besoin est; puis comme a esté demonstré cy dessus, faisant *h x*, esgal à *h* F, faut mener de *h* la ligne *x a* & de *x* la ligne *x* Q, partant Q *x a*. est le paneau de lict F 38.

Donc prenant la peine de confronter & considerer la maniere dudit sieur, en la planche cy dite 81. ie croy que les déguisemens de son essieu ne vous empescheront pas de voir & cognoistre, que c'est la mesme chose que la traditiue cy dessus demonstrée, ce qui est trop palpable. Vous remarquerez aussi que sa trompe cy dite est fausse, la supposant en talus, car il a laissé la liberté en la requisition; & n'a pas dit qu'elle estoit à plomb, ny que cette maniere n'estoit que pour les trompes à plomb.

Que la trompe dudit sieur planche 81. est fausse, la desirant en talus, le prenant au pied de sa proposition.

De mesme suite, quoy que ie ne m'estois proposé de demonstrer entierement la trompe sur le coin, d'autant que cette premiere partie sera vn peu longue, neantmoins tõbant icy à propos, i'en paracheueray la demonstration; & en mesme tẽps feray voir, que la seconde pretenduë maniere vniuerselle dudit sieur, exercée en la planche 105. est de la traditiue; & de plus vous obseruerez d'abord, qu'elle n'est pas vniuerselle, & que si la trompe estoit en talus elle seroit fausse; pourquoy donc nommer cette maniere vniuerselle, puis qu'elle ne l'est pas? & cette faute est reïterée, tant en l'escrit 104. qu'en l'escrit 105. où il n'a fait aucune exception; partant il a creu qu'elle estoit generale; Qu'elle ne

trompe ſur le Coing à ſon plein Cintre, et par Meſme Moyen Vnne Cannoniere en talus le tout ſelon l'ancienne Maniere



soit de la traditiue, il est facile de le verifier.

Soit requis de trouuer le paneau de la coupe F 38. de la trompe sur le coin Q 8. D *u*, l'on procedera par la maniere cy deuant dite, d'auoir le paneau N L Q, mais ayant trouué le paneau X 8. Q de la trompe droite Q 8. *u*, soit qu'elle soit réelle ou supposée; & seulement pour vous en seruir, & que la ligne d'iceluy paneau Q 8. soit prolongée iusques en L, coupant la ligne 12. L, comme en L, alors vous aurez espargné la peine d'auoir pris la longueur L D, sur le cintre de faces, comme 24. 16. ainsi qu'il a esté fait cy-deuant.

Que la trompe dudit sieur planche 105. n'est pas vniuerselle & qu'elle seroit fausse si l'on la desiroit en talus.

Donc ayant tout recemmẽt trouué les paneaux Q *x a* de la coupe F 38. de la trõpe droite Q *u* 8. Et de la trompe sur le coin, Q 8. D *u*, l'on procedera par les raisons cy deuant deduites, sçauoir, menant par *d* vne ligne perpendiculaire Q *h* Y, la où elle coupera la ligne Q *x*, prolongée comme en *b*, alors l'on aura espargné la peine d'aller sur le cintre, & au point 22. qui respond au point Y prendre la distance 22. 19. pour du point Y faire Y *b* égal à icelle 22. 19. puis du point Y l'on tirera la ligne *b e*, l'on trouuera le point *e*, par le point de l'extrados 38. operant par la maniere cy dite, tant pour la trompe sur le mur droit que sur le coin; mais puis que c'est mesme extrados pour les deux trompes, les points Q *a e*, seront en mesme ligne aussi bien que les points Q *x b*.

Demonstration d'vne trompe sur le coing, selon l'ancienne.

Si l'on considere la pretendue maniere generale dudit sieur, demonstrée en la planche cy dite 105. il n'y aura point de sous-essieu qui empesche de recognoistre, que c'est le mesme procedé de l'ancienne maniere cy deduite.

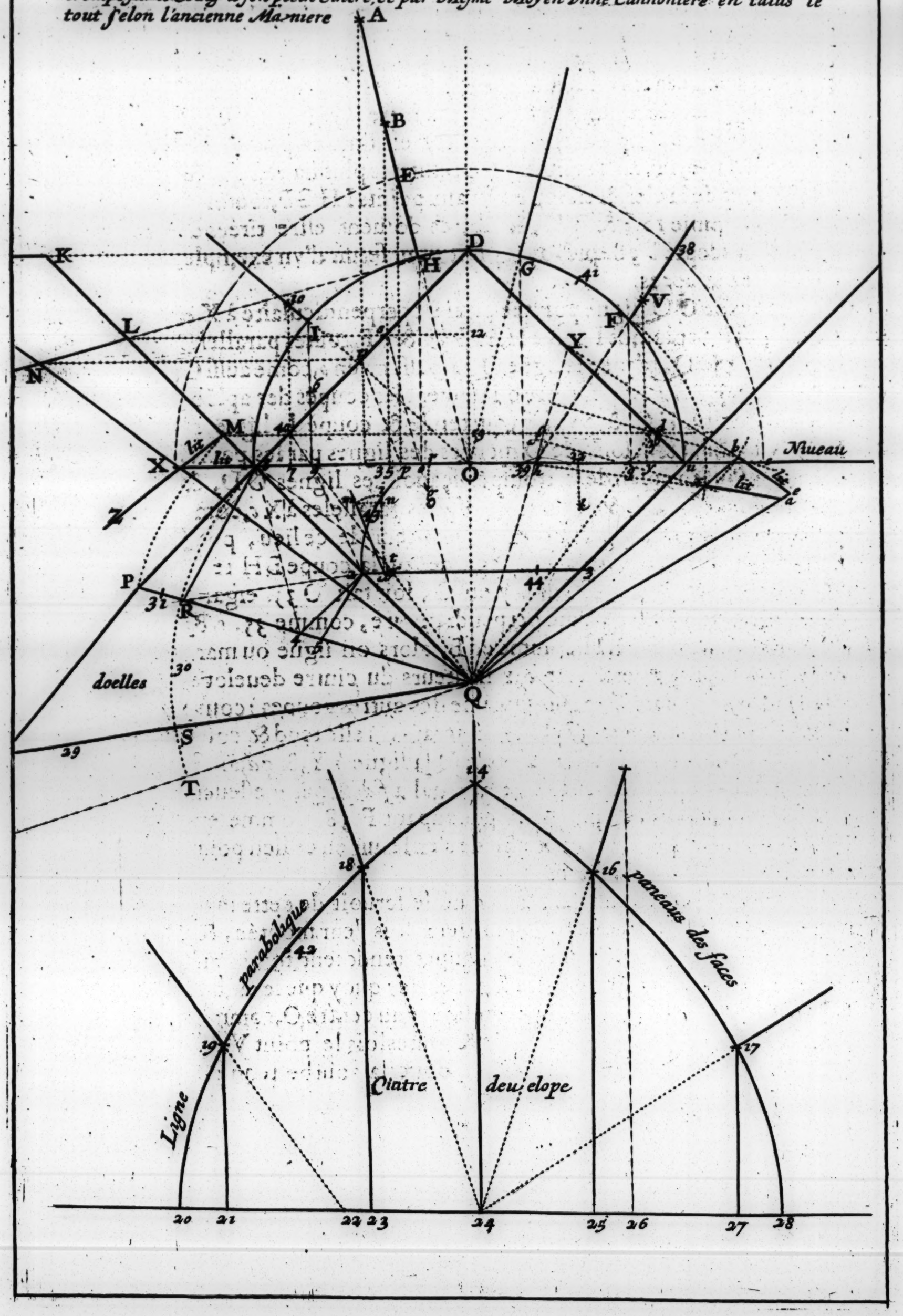
Si ledit sieur vouloit soustenir la negatiue, disant que cette maniere cy deduite n'est de l'ancienne, l'on luy prouuera l'affirmatiue par tous les maistres Massons & appareilleurs cognoissans qui remarqueront d'abord que c'est leur maniere vsitée.

Rapport à preuues contre ledit sieur

Maintenant ayant prouué & demonstré le cy deuant proposé, ie feray vne petite digression pour demonstrer la construction entiere de la presente trompe sur le coin, & pour deux manieres generales selon la traditiue, commentée en quelque point seulement que ie coteray. Et pour commençer de nouueau. Soit K Q *u*, l'angle d'vn mur où l'on desire fabriquer vne trompe appellée sur le coin, comme 8. D *u*, qui est le plan donné d'icelle, & soit 8. D parallele à Q *u*, comme aussi D *u*, parallele à K Q, & les angles 8. D *u*, & 8. Q *u*, soient semblables, sçauoir icy quarez, &

Demonstration de l'ancienne trompe sur le coin par deux diuerses manieres generales qui sont de la traditiue cõmentée

trompe sur le Coing à son plein Cintre, et par Mesme Moyen Vnne Cannoniere en talus le tout selon l'ancienne Maniere



les costez, tant 8. D que D *u*, comme aussi 8. O & O *u*, soient esgaux ; soit menée vne ligne representant le plan de niueau X O *u* & soit descrit le cintre droit 8 H G *u*, qui est icy vn demy cercle, & est pour faire la trompe, ainsi que disent les ouuriers, à deux fois sa montée ; iceluy cintre a icy pour centre le point O, qui est au milieu de 8. *u*, soit iceluy cintre party en tant de pieces qu'il sera necessaire, faisant tousiours les premieres plus hautes que les prochaines de la clef, & est icy party aux points I H G F, toutes les coupes, comme 10. *i* & B H, & autres doiuent estre tirées du point O, excepté F 38. que i'ay icy fait pour seruir d'vn exemple, que verrez cy apres.

Soit menée O Q ligne pointée, qui est perpendiculaire à X *c*, cõme aussi les aplombs I 9. & *o* H, & G 39. & *q* F, toutes paralleles à D Q, soit de Q, menée la ligne Q *o s*, passant en *o*, cõme aussi Q *q d*, passant par *q*, soit consideré où tendent les coupes des aplõbs *o* & *q*, sçauoir E H O, & 38. F *h*, qui tende & coupe la ligne de niueau X *c* en O & *h*, soiẽt de Q menées des lignes par les points O & *h*, iusques au plan de la trompe, sçauoir les lignes O D, & *h* Y. Plus soient par *s*, & menées des lignes paralleles à X *c*, comme *s* 12. & *f d* & pour auoir le point B haulteur de ce lieu, pour le cintre deuelopé des faces, soit regardée où la coupe B H tend & coupe la ligne X *c*, qui est icy en O, soit fait O 35. esgal à 12. *s*, & soit par 35. esleuée vne perpendiculaire, comme 35. *s* B, coupant ladite coupe B H, comme en B, alors on signe ou marque le point B, pour s'en seruir aux hauteurs du cintre deuelopé des faces, & ainsi fait-on pour chacune des autres coupes ; comme par exemple, la coupe 38. F, considerant où elle tend & coupe X *c*, comme en *h*, & par *h*, mener de Q la ligne *h* Y, & *q d*, ainsi qu'il a esté cy deuant dit ; puis faire *h y* esgal à *f d*, & par *y* esleuer la ligne *y* V perpendiculaire à X *c*, & coupant F 38. comme en V, alors l'on marque le point V, afin de s'en seruir en ce lieu pour la hauteur du cintre déuelopé.

Suite de la demonstratiõ de la trompe.

Vous remarquerez que l'ancienne ne se seruoit de cette maniere, qu'aux trompes qui auoient deux fois leur montée ; & ayant leur plein cintre, & dont les coupes tendoient toutes au centre O ; & neantmoins la regle est generale ; quoy que le cintre fust surbaissé ; & que les coupes ne tendissent au centre O, comme la presente F 38. qui tend au point *h*, & toutesfois le point V est treuué par mesme regle que le point B, donc la courbe tend au

Commentatiõ sur l'ancienne.

centre O, ce qui auoit beſoin d'intelligence & explication, pour rendre cette maniere generale.

Conſtruction du cintre deuelopé.

Donc deſdits points trouuez ſur toutes les coupes de la trõpe, & par la maniere generale cy declarée, il en ſera conſtruit le cintre deuelopé des faces, car ayant apporté les diſtances *u d*, ſur & auſſi eſgal à 20.21. & *d* Y eſgal à 21. 22. & ainſi des autres; puis ſoit priſe la hauteur *y* V, & ſoit miſe à 21. 19. & ainſi des autres.

Remarque que l'on n'a que faire d'extrados, ſinon que pour ſeconde maniere.

Et vous remarquerez, qu'en quelque endroit que les coupes, comme B H, & 38. F, &c. auront coupé la ligne de niueau X *c*, comme en O, & *h*, & de Q, par O & *h*, mener les lignes O D & *h* Y, leſdits points D & Y, ſeront les centres des coupes du cintre déuelopé, ainſi que O & *h*, leſont du cintre droit 8. H G *u*, & partant la coupe 19. ſera tirée de 22. & les autres de 24. & ainſi il n'eſt beſoin d'extrados que pour vne ſeconde maniere, pour voir & cognoiſtre ſi l'on a eſté précis.

Et pour faire la ligne 20. 19. 18. 14. l'on a de couſtume de la trouuer par des operations de ligne de milieu, comme en 42. mais i'en deduiray l'erreur en la ſeconde Partie, ſur la trompe ſurbaiſſée & biaiſe, &c.

Pour trouuer les paneaux de doelle de ladite trompe

De plus, pour trouuer les paneaux de doelles, comme de 8. I. l'on a ſon premier lict, qui eſt 8. Q, il n'eſt queſtion que de I, auquel ayant conſideré où il tend, comme en O, puis de Q mener Q O D, puis par 40. bout de la ligne de plan du plomb de I, qui eſt 9. l'on mene vne perpendiculaire à D Q, comme 40. M, puis prenãt ſur le cintre deuelopé 24. 28. ou de face 8. D, la ligne qui reſpond, tant au centre de la coupe 10. I, cõme à ſon plomb, qui partant eſt icy 24. 17. que l'on tranſporte en D M, coupant 40. M, comme en M, qui eſt la longueur reſpondante à I, & Q M, ſera eſgale à P Q, puis mener la ligne 8. P, & égale à 8. I, partant 8. P Q, ſera le paneau dela premiere doelle & ainſi des autres.

Pour trouuer le paneau de lict de ladite trpõe, l'on na beſoin d'extrados pour iceux, que pour ſeconde maniere.

Et pour trouuer le paneau de lict pour la teſte, cõme par exemple 10. I, l'on procedera ſur les manieres generales cy dites, car ayant trouué le point M, & le point D, il ne faut que de D mener la ligne D M Z, alors Q M Z, ſera paneau de lict 10. I, ſeruant pour icelles teſtes, & partant il n'eſt beſoin d'extrados, que pour ſeconde maniere, ce qui ſuffit pour le preſent.

Mais ie vous diray qu'en la planche 92. eſt vne maniere au dire dudit ſieur, *pour eſpargner la place*, laquelle maniere n'eſt autre choſe, que ſi par la traditiue cy deuant deduite l'on exerçoit, cõme

par porfil & ſur vn plan perpendiculaire, & ſitué quarément aux faces,& paſſant au centre de la trompe, ce qui eſt le meſme que l'on exerce ſur le plan de niueau accouſtumé ; & qui paſſe au centre de la trompe.

Que les coupes de la trõpe dudit ſieur faites pour eſpargner s'eſtendent plus ou auſsi loin qu'autrement

Vous remarquerez que ce qu'il dit, que le lict ou coupe des premieres retombées va quelquefois couper bien loin la ligne de niueau, il en arriue autant au lict ou coupes des clefs, d'aller couper ordinairement plus loing la ligne à plomb cy deuant dite, ainſi que le pourrez voir par ſa meſme planche, cy dite, & partant il a mal propoſé.

De plus,la planche 93. eſt la meſme maniere cy dite, excepté qu'au lieu du mur à plomb, il le fait en talus, & d'vn talus requis en vne trompe à face droite, ce que l'on eſt libre de faire, qui eſt le meſme que ſi en l'ancienne maniere l'on exerçoit ſur vn plan rampant d'vne trompe, & paſſant touſiours au centre d'icelle, ce que l'on eſt libre de faire, en faiſant les aplombs ſur le rampant,au lieu de les faire ſur le plan de niueau.

La maniere dudit ſieur de trouuer les paneaux,& de deux lignes ſeules la preparatiõ eſtant faite eſt la maniere de trouuer aux quadrans l'inclination des lignes horaires à l'axe.

La planche 94. eſt auoüée dudit ſieur eſtre de l'ancienne, ce qui fait que ie paſſe aux planches & eſcrits 95. 96. 97. les deux dernieres deſquelles ſont deux manieres de trouuer les paneaux de doelle,ou de lict, auec deux lignes ſeulement, s'entend apres la preparation faite des eſſieux,ſous-eſſieux, & cercles & toutes les choſes requiſes, ce qui eſt le meſme qu'aux biais de l'ancienne maniere, la preparation eſtant faite, comme le biais & aplomb, puis vn cercle, il n'y a plus qu'vne ou deux lignes à tirer pour auoir leſquels des paneaux de doelle ou de lict l'on deſirera ; & cette maniere dudit ſieur, n'eſt autre choſe qu'aux quadrans, trouuer l'inclination que quelque ligne horaire fait auec l'axe, & ſe ſeruant pour abbreuiation des preparations,d'auoir trouué la ſouſtylaire,& l'angle d'icelle à l'axe, ce que ie paſſe briefuement.

Defy dudit ſieur à vn particulier.

Ledit ſieur en ſes eſcrits fueillet 55. parlant à vn particulier,duquel il doute qu'il ne l'entẽde à fonds,dit en ces termes. *Ie luy veux donner dequoy me conuaincre,quand i'aſſeure qu'il ne m'entend à fonds, ſi il trouue auparauant que ie publie vne maniere generale de trouuer,tant le lict que doelle d'vne ſeule ligne ſeulement ; apres la preparation faite, i'auoueray qu'il m'entend à fonds* Ie reſpondrois audit ſieur en deux paroles qui concluroient ſon propoſé;mais ie craindrois que celuy à qui ce defy eſt fait,n'euſt droit de me reprocher, que i'aurois eſté au deuant de ſes lauriers;neantmoins le chemin eſt amplemẽt

La concluſion ny le propoſé du défy n'eſt pas de grand art.

ouuert par les eſclairciſſemens & declarations cy deuant. Et diray pour concluſion, que le propoſé ny la ſolution n'eſt pas de grand art.

Vous remarquerez les manquemẽs des deux planches *96.* & *97.* & en la premiere planche, tant figure d'enhaut, que d'en bas le paneau pour B D, eſt l'angle 2. A *y*, au lieu de 3. A *y*, & en la ſeconde planche, figure d'en bas le paneau de X G, eſt l'angle de ſuplement de K A *x*, à deux angles droits, & non pas l'angle K A *x*, car ledit ſieur opere par dehors l'œuure, ainſi qu'il l'enſeigne dans ſes principes ; ce qui terminera la premiere Partie de l'examen, lequel s'eſt eſtendu plus loin que ie n'eſperois.

SECONDE

SECONDE PARTIE,

ET SVITE DE L'EXAMEN DV LIVRE de la coupe des pierres du sieur Desargues.

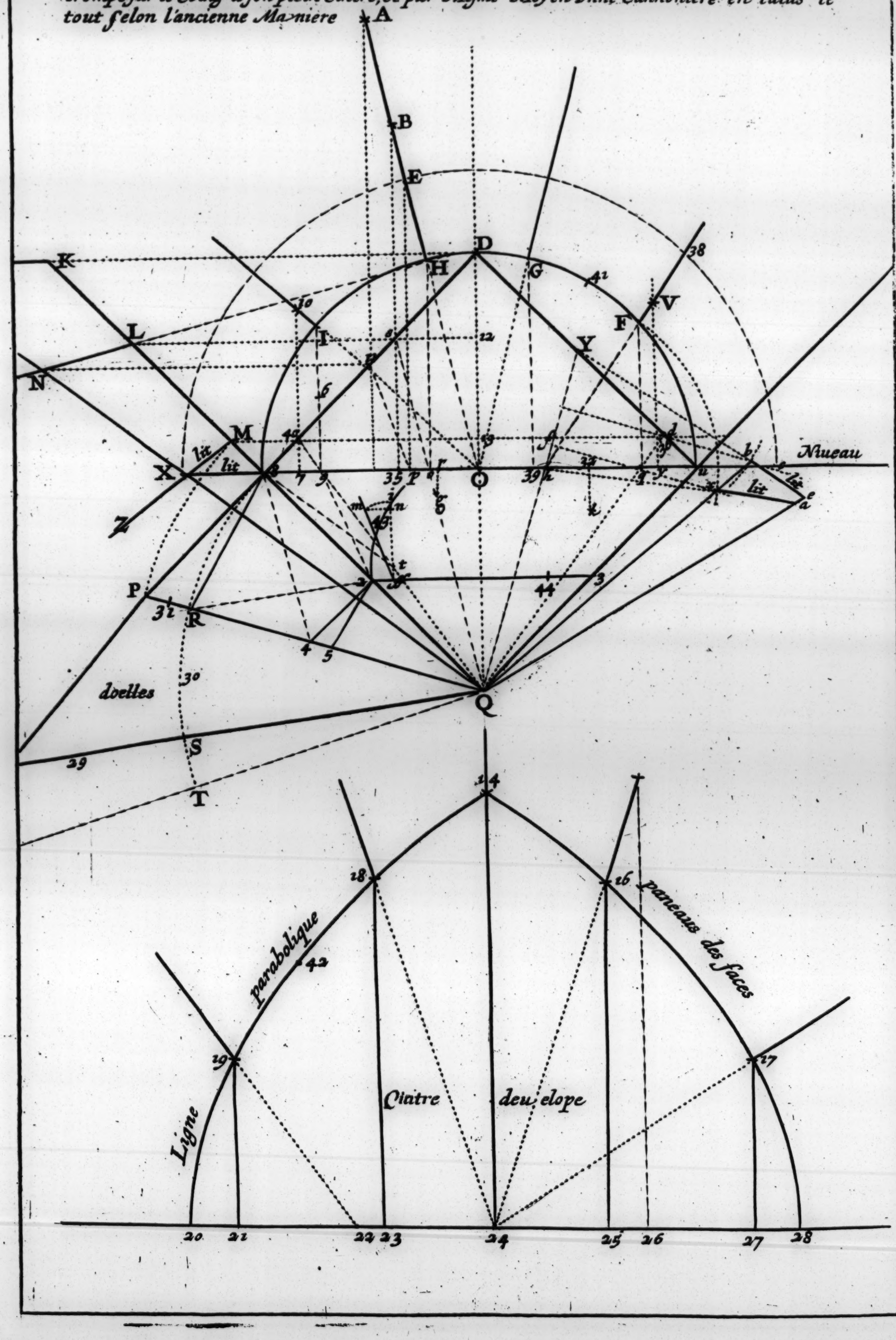
A commencer à la planche 97. & finir où finit ledit Liure.

IE commenceray cette seconde Partie par la remarque que vous ferez auec moy de la figure d'enhaut, de la planche 97. qui est vne autre maniere de faire la preparation, sçauoir comme dit est de la sous-essieu & essieu, &c. Laquelle maniere est absolument fausse, car il dit que l'angle B C N est le beueau de face & chemin (s'entend tousiours l'operation au dehors, & quarémẽt aux faces & niueau, & ainsi que ledit sieur l'entend:) ce procedé auroit lieu si la descente biaise n'auoit point de talus; mais la preparation de la planche 95. qu'il a faite pour ses deux manieres de trouuer paneaux auec deux lignes, la preparation estant faite, icelles preparations sont auec talus; partant il a creu que la figure d'enhault de la planche 97. fust aussi auec talus, puis qu'il la dõne pour faire les mesmes preparations cy dessus dites; ce qui est constant, puis que dans les escrits susdits il n'a mis aucune distinction de proceder, & en ce cas il faloit que l'angle N B C, fust le suplément de l'angle de face & niueau à deux droits, & non pas le faire droit & tousiours continuer la ligne N C, iusques sur la ligne B D.

Faulte à la planche 97. dudit sieur, donnant pour seconde maniere de faire la preparation.

De plus, quand le niueau est auec le chemin, qui est à dire, quand il n'y a point de descente ny rampe, alors la ligne N C, se trouue sur la ligne B N, & A N, sera le demy cercle pour la preparation, &c.

trompe ſur le Coing à ſon plein Cintre, et par Meſme Moyen Vnne Cannoniere en talus le tout ſelon l'ancienne Maniere



Paſſant outre, ie diray que les eſcrits & planches 98. iuſques aux planches 103. ſont employez à diuerſes manieres particulieres de traict pratiquées par ledit ſieur, qui n'eſt autre choſe que ce qui ſe pratique dans l'ancienne; comme ie vais verifier par vne canoniere, & qui conuiendra à l'exemple d'vne qu'il donne à la planche 103.

Que les manieres particulieres dudit ſieur ſont de l'ancienne.

Soit le plan de la canoniere 8. 2. 3. *u*, ſoit 8. *u*, le plan de deuant & 2. 3. le plan de derriere. Soit le cercle 8. D *u*, le cintre de face de deuant; & ſoit 2. 43. telle courbe que deſirerez ou portiõ du cintre de face de derriere, & ſoit 2 *ff m*, talus dõné pour le derriere. Et ſi deſirez que le plan de l'areſte de la doelle, comme icy 9. *t*, tende au point Q, vous executerez mechaniquement la ligne *i ff*, perpendiculaire à 2. 3. en ſorte que coupant 9. Q, comme en *t*, iceluy *t* & *ff* égale *n m*, & s'ẽtend *ff i*, égal à *ff m*. Et ſi vous voulez partir la courbe entiere dudit cintre de face de derriere en égale portion, ou correſpondante aux parties du cintre de face cy deuant dit, & que *i*, fuſt vn point deſdites partitions, ſans qu'il vous importe que le plomb d'areſte de doelle 9. *t*, tende au point Q, faut mener *i ff*, perpendiculaire à 2. 3, & faire *t ff*, eſgal à *m n*, s'entend *m n*, parallele à 2. 3. mainten^ t ſi l'on deſire le paneau de doelle 8. I, l'õ opere par la maniere des trompes, prenant Q 9. & faire 9. 32. ſon égal: puis I 32. ſera eſgal à Q R, & faiſant R 8. eſgal à 8. I alors Q 8. R ſera l'angle des paneaux de doelle. Et pour trouuer le point 4. l'on operera, comme il vient d'eſtre dit, prenant Q *t*, & faire *ff* 44. ſon eſgal, puis 44. *n*. ſera eſgal à Q 4. & 4. 2. eſgalera 2. *i*, & alors R, 8. 4. 2. ſera tout le paneau de doelle de 8. I, & 2. *i*, qui ont pour plan 8. 9. *t* 2. & ainſi de tout autre paneau, tant de doelle que de lict, ce qui eſt purement de l'ancienne maniere.

La canoniere cy dite dudit ſieur, n'eſt en talus, partant icy plus de ſubiection.

Suite de la demonſtratiõ d'vne canoniere ſelon l'ancienne maniere.

Et ſi ledit ſieur diſoit, qu'il ne pratique ny ne deſire que l'on aille chercher le point Q, car aux portes qui ont leurs tableaux paralelles, il n'y aura pas de point Q, il luy ſera repliqué, que ſans ſe ſeruir du point Q, l'on ne laiſſera de trouuer les paneaux requis, & touſiours par meſme maniere de l'ancienne cy deuant dite, comme par exemple prenant 2. 9. & faiſant 9. *r*, ſon eſgal, puis prendre *r* I, eſleuatiõ ou porfil & ſera eſgal à 2. R, & touſiours R 8. eſgal à 8. I, & pour le point 4. ſera pris 8. *t*, que tranſporterez en quelque ligne, comme en 9. *r*, & faiſant 9. 6. eſgal à *ff n*,

alors *r 6.* eſgalera 8.4. & touſiours 2.4. égale à 2.*i*,& ainſi des autres paneaux de doelle ou lict,ayant eſgard de combien eſt la difference de vos points requis, eſleuez ſur le niueau du deuant & derriere; comme par exemple le point *i*, eſt plus eſleué ſur le niueau, que le point 8. de la hauteur *ff i*, auſſi ie me ſuis ſeruy de ce plus pour le porfil. Et partant 8. R 4. 2. fera le paneau de doelle requis,&c. & ſans ſe ſeruir du point Q, Et ſi l'on poſoit la canoniere en deſcente, s'entẽd que 2.3. derriere fuſt plus bas que le niueau,comme de la diſtance *r g*. Et voulant trouuer le porfil, de 9. 2. ayant fait 9. *r*, eſgal à 9. 2. puis *r g*, perpendiculaire à 8. *u*, & *r g*. eſgal à la deſcente cy dite,alors l'on fera 2 R, eſgal à *g* I, & ainſi des autres, ſur le fondement cy dit, qui eſt purement de la traditiue; ainſi que les cognoiſſans le pourront conclurre par le procedé cy deſſus, reconfronté au procedé dudit ſieur.

Erreur en l'vne des pretendues manieres dudit ſieur.

Sera remarqué, qu'il y a erreur en la planche 102. qui eſt, qu'eſtant vne deſcente biaiſe en talus, & les areſtes de face à plomb, choſe qui ne ſe pratique,à cauſe de ſes defectuoſitez, de reduire icelle deſcente ſans talus, s'entend que diminuant la deſcente de ce qu'il y a de talus,alors ce ne ſera plus qu'vne deſcente biaiſe; mais il dit, la reduction eſtant faite par le moyen du triangle rectangle,que l'on operera pour parler ſelon ſon jargon, *par les routes niuelées* ſimplement, qui eſt le biais ſelon le plan de niueau.

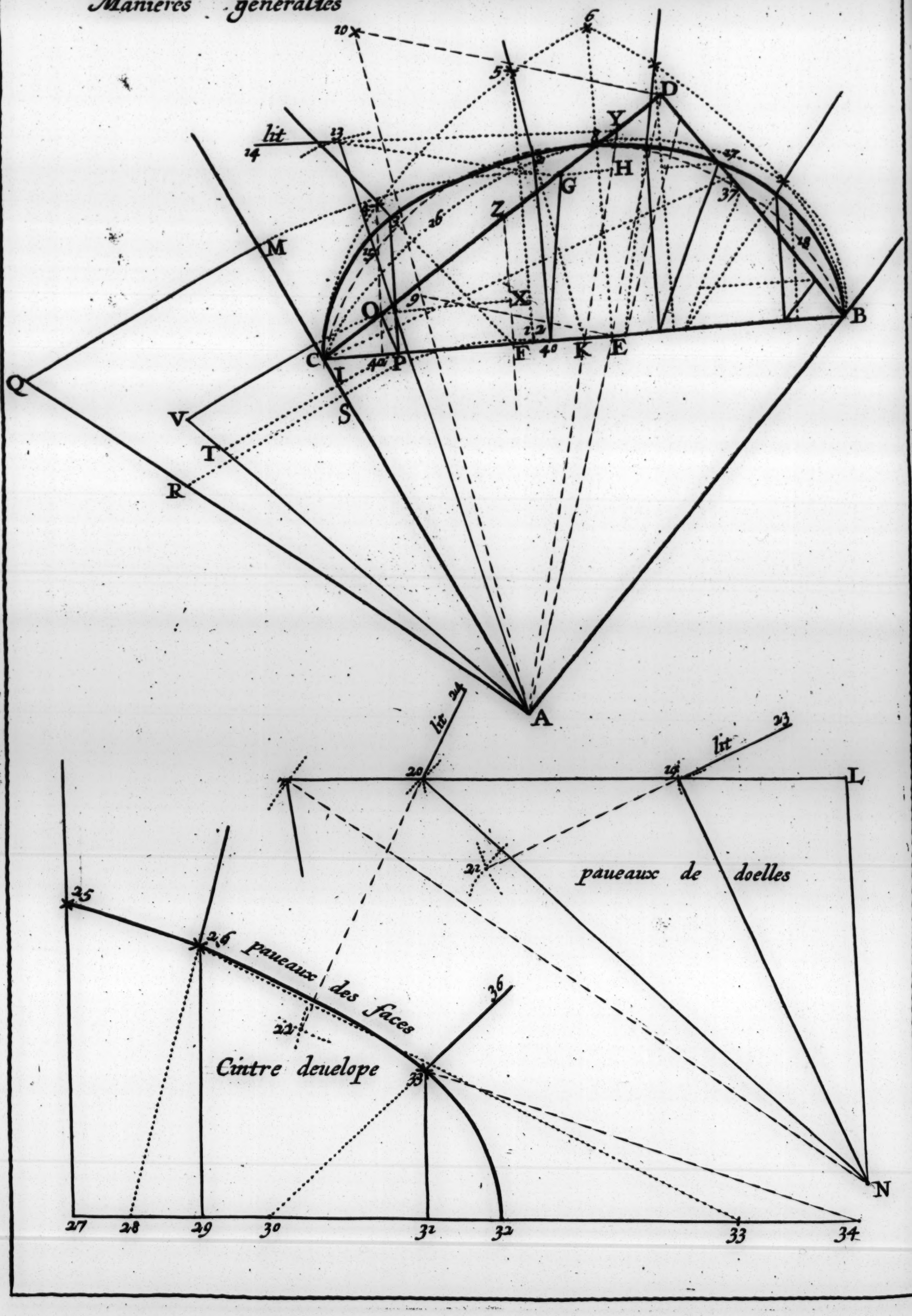
Enſeignement donné audit ſieur.

Ie l'aduiſe qu'il faudra operer par le biais, ſelon le plan perpendiculaire au talus, s'entend iceluy biais reduit pour areſte de face à plomb,& non par le biais, ſelon le plan de niueau ſimplement, ſi l'on veut l'exempter de fauſſeté; & la figure d'embas de ladite planche, ne ſeruira pas d'excuſe, quoy que les meſures reuiennent à ce que ie viens d'enſeigner, car ledit ſieur n'enſeigne pas de les cõſtruire. En la methode dont ie me ſers dans ma Stereotomie, reduiſant les deſcentes biaiſes en talus, en ſimple biais ſurplombant ou taluant, eſtant plus facile aux ouuriers d'effectuer vn biais en talus, qu'vne deſcente de caue biaiſe; mais auparauant ie reduits le biais donné de niueau en biais, ſelon la deſcente par les premieres cognoiſſances, dont i'en ay donné cy deuant vn exemple general en vn os, en deſcente biaiſe & en talus, qui aux intelligens ſuffira pour toute ouuerture requiſe ſur vn mur droit.

Venant

Venant icy à propos, ie donneray encor vn exemple de trompe par deux diuerſes manieres generales, pour toutes trompes à face droite, & perpendiculaire, rampante ſurbaiſſée biaiſe, d'angles inegaux.

Trompe sur le Coing biaise surbaisee Et barlongue dangles Inegaux donc les Coupes Ne Tende En Vng Centre faictes par les deus diuerses Manieres generalles



Soit l'angle d'vne muraille donnée M A B, & soit le plan de la trompe, C D B, & de C, soit menée la ligne C B, que nous prendrons icy pour le niueau. De plus, soit donné l'arc surbaissé C 3. 2. 8. B, & soit iceluy diuisé en tant de parties, que l'on desirera, comme icy en cinq, sçauoir aux points, 3. 2. &c. ainsi qu'il se voit par la figure : soient les coupes menées iusques sur C B, comme 3. F. 2. K & soient menez les aplombs ou perpendicule à la ligne C B, comme 3. P, & 2. 40. plus des points P. & 40. & du point A, soient menées les lignes P O, & 40. G, comme aussi F Z & K Y.

Demonstration d'vne trompe biaise d'angles inegaux & surbaissés, faite par les deux diuerses manieres de la traditiue cõmentée & amandée.

Maintenant pour auoir le cintre de face, soit menée la ligne A D, coupant C B, comme en E, & du point E, soit esleuée la ligne E 6. perpendiculaire à C B, plus soit E 9. & D, 10. perpendiculaire à A D, & soit iceluy E 9. esgal à E 8. plus soit du point A, & par 9. menée la ligne A 9. coupant D. 10. comme en 10. & de plus sur vne ligne comme 27. 34. soit fait 27. 28. esgal à D Y, & 27. 29. esgal à D G, & 27. 30. esgal à D Z, & 27. 31. esgal à D O, & 27. 32. esgal à D C, puis soient esleuées les perpendiculaires à 27. 34. comme 27. 25. & 36. 29. & 31. 33. De plus soit, tant du point G, que O, mené G H, & O X, parallele à C B, puis soit fait K 12. esgal à G H, & F, 42. esgal à O X, puis soit esleué perpendiculairement à C B, 4. 2. 4. & 12. 5. coupant les coupes, comme en 4. & 5. plus sera fait 31. 33. du cintre esgal à 42. 4. & 29. 26. esgal à 12. 5. & 27. 25. esgal à D, 10. & des points 28. & 30. seront menées les coupes 26. & 33. puis par les points 32. & 33. & 26. & 25. sera descrite vne ligne elliptique, qui aura pour grand diametre droit depuis C, iusques ou C D, coupera A B, puis porté depuis 32. & sur 32. 27. & qui se pourra descrire auec le singliot, par la maniere cy deuant declarée aux arcs rampans d'vne hauteur de sommité donnée; mais sans aller chercher ses lignes si lõgues, ie demõstreray dans ma Stereotomie la methode de descrire cette ligne 25. & 26. & 33. 32. sans sortir du champ; comme aussi ie demonstreray la methode de faire les cherches, pour creuser chacune doelle, faite de tant de pieces que l'on voudra, ce qui a esté incognu iusques à present.

Methode de faire ou développer le cintre de face par 2. diuerses manieres

* Pour les doelles des trompes dudit sieur, elles demeureront sans creuser, d'autant qu'il n'en donne les cherches, & n'en parle aucunement dans son Liure, ce qui fait croire qu'elles demeureront imparfaites, & c'est-là où est le nœud de la difficulté, aussi bien que sur la face ou cintre du deuant d'vne trompe, la metho-

* *Faute audit sieur de n'auoir narré ny donné les moyens de creuser les doelles de ses trompes, qui partant demeure imparfaite : cõme aussi les courbes des faces de ses trompes qui sont fausses*

& deffectueuſes.

de de deſcrire conuenablement la courbe 25. & 26. & 32. tant en l'vne ou en l'autre des manieres cy deuant dites.

Seconde maniere pour le cintre de face.

Pour faire le cintre de face par la ſeconde maniere, faut operer par extrados, & ligne de pante ou porfil : comme du point A, & au point G, & 40. mener les portions de cercles G M, & 40. S, puis faire M Q & S R, parallele & perpendicule à M A, & faire S R, eſgal à 40. 2. puis du point A, & par R, mener vne ligne coupant M Q, comme en Q, puis prendre M Q, pour porter au point 29. & ferez 29. 26. eſgal à ladite M Q, & ainſi de tous les autres; & pour trouuer les coupes, comme 33. 36. elles ſe trouueront par extrados, operant par meſme maniere cy dite.

Pour faire les doelles par les 2. manieres, ſoit pris garde aux paroles & non à la preciſion de la planche.

Pour trouuer les doelles, faut faire L N, eſgal à A C, puis prendre l'interualle A O, le portant en 31. 34. puis ferez N, 19. eſgal à 34. 33. & L, 19 eſgal à 32. 33. puis mener 19. N & 19. L, & partant N 19. L. ſera premier paneau de doelle, & ainſi des autres. Et pour ſeconde maniere, l'on la fera par ligne de pente cy deuant expliquée. Et ſera fait 19. N, eſgal à A V, & touſiours 19. L. eſgal à 33. 32. quoy qu'il y ait vne autre maniere de trouuer 19. L, par le cintre droit, ou fondamental, que ie n'explique en cét endroit, pour briefueté.

Pour faire les licts ou teſtes par les 2. diuerſes manieres.

Pour trouuer les paneaux des licts, comme par exemple, pour 3. 4. ſoit fait du point N, la portion de cercle piqué 21. eſgal à A, Z, puis prendre 30. 33. le porter au point 19. faiſant vn point ſur la portion du cercle piqué cy deuant fait, comme en 21. puis du point 21. en paſſant par 19. ſoit mené 19. 23. & N. 19. 23. ſera le paneau de lict, pour la commiſſure 3. 4. & ainſi de tous les autres. Et pour trouuer le paneau de lict, par exemple 2. 5. par ſeconde maniere, outre celle de le trouuer par extrados, que ie n'infereray point icy, d'autant qu'elle ſeroit ſuperfluë, l'on menera par G, la ligne G 13. perpendiculaire à A Y, puis l'on fera A 13. eſgal à N, 20. puis du point Y, paſſant à 13. l'on menera la ligne 13. 14. & A 13. 14. ſera le paneau requis, & eſgal à N 20. & ainſi des autres. Vous ſçaurez auſſi que Y 13. eſt eſgal à 28. 26. & eſt le meſme que l'ancienne demonſtrée cy deuant, en la trompe ſimplement ſur le coin.

Et pour auoir les beueaux des licts, faut les trois angles plans eſtant donnés comme le paneau de doelle, & de lict de face, conſtruire vn angle ſolide par la vingt-troiſiéme de l'vnzieſme d'Euclide; & quoy que par triangles equilateraux, l'on le peut reduire auſſi

auſſi par deux triangles rectangles. I'ay briéuement deduit les deſcriptions cy deſſus, & comme en paſſant, d'autant que dans ma Stereotomie elles ſeront narrées plus au long.

De plus, les planches & eſcrits 104. iuſques à 114. planche derniere du Liure, ſont employées à diuers porfils, tant de porte que de trompe, qui ne ſont autres que la maniere ancienne, ſinon quelques deſguiſemens, & autres choſes qu'il a voulu adiouſter, où il eſt tombé dans l'erreur, ainſi que ie feray voir en la ſuite; neantmoins ledit ſieur donne ſes manieres cy dites en deux diſtributions; l'vne pour eſtre ſa deuziéme maniere vniuerſelle appliquée en la trompe de la planche 105. & quelqu'vn des porfils, comme en la planche 100. qu'il dit eſtre vne ſienne maniere nouuelle, à quoy il eſt facile de reſpondre, & demonſtrer le contraire.

Que les manieres exercées par porfil dudit ſieur ſont de l'ancienne.

Pour faire court, La planche 105. eſt la meſme choſe que la planche 81. & n'eſt point pratiquée par autre maniere, ce qui ſe peut voir à l'œil par les moins cognoiſſans; auſſi ie croy que ledit ſieur ne le pretendra pas, ſinon qu'il pourra dire ſophiſtiquement, qu'il entend parler de la ſeconde teſte, & d'y appliquer le paneau, par le moyen de la premiere teſte; Mais quand i'ay dit cy deſſus, que cette maniere n'eſt point autre que la maniere de la planche 81. c'eſt dire, que tant la premiere que ſeconde teſte, ſe pratiquent ſur meſme fondement & moyen l'vn que l'autre; & que i'ay fait voir eſtre de l'ancienne, en l'examen de la planche cy deſſus dite 81. où vous aurez recours pour obuier aux redites, d'autant que i'y ay amplement ſatisfait.

Que ſa maniere de la premiere & ſeconde teſte ſe pratique ſelon l'ancienne.

Vous remarquerez qu'en la dite planche 105. cette pretenduë maniere vniuerſelle d'ajuſter le paneau à la ſeconde teſte, eſt ſi peu vniuerſelle & generale, que ſans ſortir de la trompe meſme, ſi icelle eſtoit en talus, elle ſeroit fauſſe. S'il replique qu'il a operé en vne trompe à plomb, & non pas à talus; cela n'empeſchera pas que ceux qu'il pretend inſtruire, ne deuineront pas que ſa regle ſoit ſi eſtroitement vniuerſelle, eſtant donnée ſans exception, par ledit ſieur & qualifiée d'vniuerſalité; car il deuoit donc dire vniuerſelle pour toutes trompes à plomb, & alors elle n'auroit point de defaut; auſſi l'ancienne ne la donne que pour les trompes à plomb.

La maniere vniuerſelle de la trompe 105. eſt ſi peu vniuerſelle, que ſi elle eſtoit en talus elle ſeroit fauſſe.

La ſeconde maniere dudit ſieur eſt, dit-il, en l'eſcrit de la planche 104. *Par le moyen des aſsietes niuelées & porfils des voutes d'vne façon plus generale que l'ordinaire de la vieille maniere.*

Que les paroles dudit ſieur ſont fauſſes, de dire que ſa pretendue maniere ſoit plus generale que l'ancienne.

Vous remarquerez que ſes paroles ſont toutes abſolument fauſſes, que ſa pretenduë maniere par les porfils ſoit plus generale que l'ancienne, dautant qu'elle eſt la meſme; à l'exception, que où il l'a voulu plus eſtendre que l'ancienne maniere n'a fait, il y a commis des abſurditez, la prenant generale ainſi qu'il la donne; comme par exemple, ainſi que ie l'ay demonſtré cy deuant, & demonſtreray le cy deſſus dit par ſes figures meſmes, qui ſeront toutes examinées & de rang.

Premierement la planche 104. enſeignè de poſer le plan du parpain ou ſeconde teſte, ſoit droit, oucourbe, & au reſpect de la premiere face, enſemble ſon eſleuation ſoit vn berceau, ou autre. De plus l'eſcrit & planche 106. n'eſt quaſi que meſme choſe, laquelle explique ſa maniere de proceder par porfil, & la donne pour premiere maniere particuliere, mais nouuelle.

Que l'ancienne ne s'employe, pour euiter reſtriction, où ledit ſieur l'applique.

Ie luy dénie qu'elle ſoit nouuelle, n'eſtant que la maniere d'operer par porfils de l'ancienne, excepté que l'ancienne ne l'employe au ſubjet où ledit ſieur l'applique; d'autant que l'employãt à tel ſubjet, icelle regle reçoit telle reſtrictiõ, que ſi les licts du berceau de derriere eſtoient biais au reſpect des licts de doelle de la deſcẽte, icelle regle ſeroit eſchoüée. Il en arriueroit le meſme, ſi icelle deſcente eſtoit ébraſée d'vn coſté, ou ſi elle l'eſtoit des deux. Et voyla pourquoy la traditiue maniere ne ſe ſert que rarement de tels porfils en ce ſubiect, d'autant qu'elle a vne maniere generale pour iceux, qui eſt la troiſieſme eſpece, eſtant la meſme des viz S. Gilles quarrées. Et pour vous expliquer que c'eſt que la planche, tant 106. que 107. c'eſt vne deſcente dont le berceau rencontre vn autre berceau; & directement, s'entend que les licts des doelles du berceau de la deſcente s'enfilent, ou ſont paralleles au lict des doelles du berceau, auquel eſt la rencontre; ſi ledit ſieur ne veut que la planche 106. ſoit de meſme que 107. elle ſe trouuera la meſme que leur ſuiuante 111.

Vous pourrez maintenant examiner & determiner, ſi ce que i'ay dit au precedent eſt vray, ſçachant qu'il opere par porfils, & par le moyen de l'effet du lict, ou areſte de doelle du berceau de la deſcente, fait auec l'areſte ou lict de doelle du berceau rencontré: & vous donneray vn exemple ſur ſa figure meſme de la planche 107. & diray que ſon grand triangle Z *g m*, figure d'enhaut eſt ſemblable aux triangles X *o d*, & que *a* F eſt la ligne de niueau, & M Z, *l* I, X O, luy ſont paralleles, Z *a* & O 4. ſont les plombs du

talus selon le biais, ou aresté des faces du pied droit, Z *m* est porfil de la descente, selon le biais; 7. 5. F, est l'eleuation du berceau, rencontrant le berceau de la descente. Et maintenant vous voyez que cette maniere opere selon les porfils de l'ancienne, en laissant les porfils, comme de la descente Z *m*, & *l i*; où ils se rencontrent, & ne les rameine en vn point; mais la figure de la planche 106. les rameine tous sur vn point de principe, au moins si l'on la veut exempter de faussetés; ce que fait l'ancienne, ainsi comme il se verra en suite, par les diuerses manieres d'exercer porfils.

Que ledit sieur s'explique fort mal, & qu'il y a eu beaucoup de temps a perdre à faire le present examen.

Maintenant vous pouuez iuger & conclure de la mauuaise explication dudit sieur; & de quelle sorte il se fait entendre; veritablement ie ne croy pas, qu'il y ait encor vn homme qui s'explique si mal: pour des Auteurs, ie vous puis asseurer que i'en ay leu assez grande quantité; mais que le pire s'entend & s'explique mieux sans comparaison; aussi iceux Auteurs n'auoient le dessein de cacher le mystere, & s'enrichir des conceptions des autres. Et vous diray, qu'il me fallut resoudre à perdre beaucoup de temps à cet examen, lors que ie deliberay de le faire.

Que tout le reste des porfils dudit sieur sont de mesme maniere exercée sur trois differens points fixes, ce que fait l'ancienne.

Mais passant outre, ie diray que tous les restes de ses porfils exercés sur differentes pieces cy apres cotées, ne sont qu'vne mesme maniere, & seulement pratiquée sur trois differentes places ou points de principe; ce que fait l'ancienne, ainsi que ie le coteray sur chacun.

Premierement, Ez escrits des planches 108. 111. 112. 113. & 114. il cote chacune façon d'icelles planches, pour autre façon de pratiquer les porfils, quoy que (ainsi qu'il est dit cy dessus) ce n'est qu'vne mesme maniere exercée sur trois differentes places, ainsi qu'il se verra en suite. Et pour commencer par sa planche 108. figure d'enhault, que ie deduiray & declareray selon les termes vsitez dans l'ancienne maniere pour plus de facilité. Il est icy question d'vne descente biaise en talus, rachetant le berceau. Il faut entendre que la ligne *a* F, est la ligne de niueau, & P O *c*, est l'ouuerture ou porte, traçée contre le mur en talus, ainsi qu'elle demeurera estant faite: les lignes Z *a*, O 4, & leurs paralleles, sont les plombs contre ledit mur, & selon le biais. Les lignes *a r*, 8 *n*, & leurs paralleles sont lignes biaises selon le niueau, les lignes *c a r*, *b* 9. *r*, & leurs paralleles sont lignes representant le talus selõ le biais ou areste du tableau, s'entend au respect de la ligne *a r*, que l'on prend en ce lieu, pour vne ligne de niueau. Et

Explication des lignes & façon des porfils dudit sieur.

de plus au reſpect d'icelle ligne de niueau, l'on deſcrit la cherche alongée du berceau *r o m*, alongée ſelon le biais de niueau.

Deux fautes remarquées ſur ledit ſieur, l'vne pour n'auoir demonſtré les cherches alongées, l'autre, de n'auoir demonſtré diſtinctement de reduire les biais à niueau, ſelon la deſcente.

Vous remarquerez auant que paſſer plus outre, deux fautes. La premiere, que ledit ſieur n'a enſeigné la maniere de deuëment eſcrire icelle cherche alongée en aucun lieu de ſon liure, & n'en parle aucunemẽt, ce qui eſtoit d'auſſi grande cõſequence que ſon porfil. La ſeconde, qu'il n'a auſſi enſeigné diſtinctement en aucune place de ſon liure, le talus & deſcente eſtant donné, enſemble le biais ſelon la deſcente, de deſcrire *a r*, ligne de biais à niueau, ſi il ne fait vne preparation de ſa ſous-eſſieu, exprés pour la deſcente biaiſe & en talus, ou deſcente donnée, au reſpect du niueau, outre la preparation de la planche 11. vous pouuez iuger à quelle quantité de traict & d'embaras cela conduit.

Suite de l'explication du porfil dudit ſieur.

Mais reuenant à noſtre figure, la ligne *r* F, eſt le biais ou effet, que fait le plan du berceau, au reſpect de *a* F, pris pour le plan du mur de deuant, & le tout de niueau. De plus la ligne *o* O, & ſes paralleles repreſentent la deſcente ſelon le biais, au reſpect de *o d*, porfil de faces ou areſte de tableau, comme cy deuant eſt dit. D'où vous remarquerez, que ledit ſieur en aucun endroit de ſon Liure n'a demonſtré diſtinctement la deſcente eſtant donnée au reſpect des faces, de donner icelle deſcente ſelon le biais, ſinon qu'il la donne par la maniere de donner vn paneau de doelle, ou par la maniere d'vn talus biais, ou deſcente biaiſe, qui ne ſe peut trouuer, qu'il n'ait fait ſa preparation toute entiere d'eſſieux & ſous eſſieux, &c.

Que l'ancienne maniere pratique les meſmes porfils, & en 3. diuers points fixes requis, & qu'elle n'a beſoin d'vne teſte connue, comme le pratique ledit ſieur.

Maintenant il ſe peut facilement cognoiſtre la conſtruction, qui eſt qu'en quelque point que l'on voudra, ou deuant, ou derriere, l'on le rendra fixe, en adiouſtant depuis iceluy les plans des auances des porfils, comme 8. *n*, en *r* 9. & ainſi des autres, où il ne falloit que des points 8. 4. 3. P 2. & par iceux mener des lignes paralleles à *r* F, iuſques en *a r* coupant icelles au point 9. *d e*, &c. & ſeront les points requis. Et le reſte ſe conçoit aſſez pour conſtruire le paneau requis, ayant vne teſte donnée, ſoit celle du berceau, ou celle de face droite, ainſi que les cognoiſſans le peuuent aperçeuoir, & qui recognoiſtrõt d'abord, que l'ancienne maniere pratique telle façon de porfils; mais de plus ſans auoir vne des teſtes dõnées, elle les conſtruit tout enſemble, ce qui eſt beaucoup plus bref; ſi ledit ſieur en vouloit ignorer, il n'auroit qu'a rejetter la veuë ſur la fueil. 54. du Liure de l'art des traits du pere Derand Ieſuite,

Ieſuite, mis au iour deuant le ſien, & qui eſt ſelon l'ancienne maniere pratiquée par les ouuriers,& il verra le point du berceau fixe, ou au moins celuy des faces,ou tous deux,& partant il trouuera la maniere, qui eſt dans ſa figure cy deuant expliquée; il y a ſeulement cette difference, que dans le pere Derand,les auances des porfils ſont données ſelon la deſcente, & ſur icelles tranſportées & en l'autre ſelon le niueau, & ainſi tranſportées ſur iceluy, & touſiours ſelon le biais. Mais vous remarquerez,qu'en celle du pere Derand cy deuant cotée, l'on conſtruit tout enſemble & par meſme regle les deux teſtes, imaginant touſiours vn arc droit ou cintre quarément, ce qui eſt de beaucoup plus expeditif & intelligible, ainſi que les cognoiſſans, & ceux qui effectuent tels ouurages, le pourront teſmoigner.

Sommaire deduction des fautes du Liure de l'art des traits du pere Derande demõſtrée ſelon l'ancienne mis au iour 1643.

Maintenant venant à propos du Liure du pere Derand cy deuant allegué, i'en deduiray ſommairement les manques; Et premierement en la premiere partie de ſon Liure chap. 9. 11.& 12. où il cõmence de deſcrire les deſcẽtes auec biais,&d'vne face ſeulement, ſans expliquer la maniere d'iceluy biais, ſçauoir, s'il eſt ſelon le niueau ou ſelon la deſcente; car il y a difference entre l'vn & l'autre, neantmoins il le faut entendre eſtre donné ſelon le niueau: Au chapitre 12. il commence de demonſtrer les deſcentes biaiſes des deux faces, où il dit. Soit le plan de la deſcente A B, C D,où l'on peut croire manque d'explication,que ce ſoit le plan du biais ſelon la deſcente, & partant iceluy trait ſeroit faux, mais il le faut entendre le plan du biais ſelon le niueau, puis le trait ſera bon. Au chapitre 15. de la ſeconde maniere de porfils,il y a manque; car comme l'on porte les auances des porfils au cintre du porfil, ſur & au long de la deſcente; il faut donc qu'icelle auance ſoit ſelon la deſcente qui ſe trouue, prenant ſur le plan au niueau & auance de quelque retombée,& y deſcrire au deſſous la deſcente de l'vn des points,& de l'autre vne perpendiculaire; & où icelle ligne de la deſcente coupera la perpendiculaite d'iceluy point à l'autre cy deuant dit,ſera l'auance requiſe, pour mettre au long de la deſcente du cintre du porfil, correſpondante à la retombée cy deuant priſe, ce que n'a fait le pere Derand; & partant ſa regle eſt fauſſe; & toutes les deſcentes en ſuite dirigées ſur icelle regle. De plus au chapitre 23.des portes en tour ronde auec talus, il y a erreur à la conſtruction du cintre de face, car 12. E, a plus grande raiſon à 12. 23. que 12. 9. à 9. 24. à cauſe que la courbe

24. 12. eſt plus courbe que la courbe 12. 23. qui ſont ſemblables de baſe ſelon la conſtruction & icelle eſgalité eſt auſſi abſurde; la longueur A 23. eſt auſſi fauſſe. De plus, il y a erreur au tranſport des plombs ſur les porfils. Et ſemblablement à la porte en tour ronde qui la precede, qui a auſſi le meſme defaut cy dit pour le cintre de face, & n'eſt la maniere comme faut operer. Il valoit mieux imaginer la tour ronde, auoir autant de pans qu'il y a de plombs au cintre de faces, ce qui fait que toutes les portes conſtruites, par la maniere cy deuant ſont fauſſes. De plus, chap. 15. de la troiſieſme Partie, qui eſt vne trompe biaiſe ſurbaiſſée & en talus, il y a erreur, & faloit que F G, fuſt égal à B, quarément à C F, & non pas égal à E B, d'autant que le talus F K, eſt donné quarément ſelon l'ordinaire. Il y a auſſi erreur au cintre ſecondaire; outre qu'il eſt à rebours. Il faloit auſſi que 21. 61. fuſt égal à F. 7. & non à F, 8. ainſi des autres. Il faloit auſſi que 55. 61 égalaſt V F, & non V 61. & ainſi des autres; partant toutes les trompes en ſuite d'icelle, & dirigées ſelon cette maniere ſont fauſſes. Et l'excuſe de ſa remarque, fueillet 264. n'eſt pas (ce me ſemble) legitime & receuable, d'autant que le précis eſt neceſſaire & à ſouhaiter, & le faux condemnable & rejetable; il n'y aura pas plus de façons ny d'embaras à le faire, ainſi que ie dis cy deſſus, qu'à le faire faux comme il eſt. Il y a auſſi erreur à toutes les voutes Spheriques, comme cul de four, pendentis, voutes ſur le noyau, & autres faites par paneaux, pour coucher dans les cherches, ou eſcuelles; car iceux paneaux ſont faits pour coucher dans les cherches, faites en long ſeulement & droites en large.

Que l'aduis quatrieſme au pere Dedand au Lecteur eſt reiectable.

Son quatriéme aduis au Lecteur, eſt (à mon aduis) rejettable de dire, *qu'il n'eſt pas neceſſaire d'eſtre précis, & que cela eſt tout à fait inutile, veu que ſans ſe rendre ſi exacte, l'on vient à chef de l'ouurage.*

Maintenant ie retourneray à l'examen dudit ſieur, & vous diray, que l'eſcrit & planche 109. eſt de la meſme maniere que ſa precedente, & ſe pratique de meſme en l'ancienne, pour racheptер les culs de four; ce qui fait que ie paſſeray à la planche 110. qui eſt vne trompe ondée, faite ſelon le porfil precedent, & partant pratiquée ſelon l'ancienne, à l'exception cy apres dite, tant par iceux porfils cy dits, que par les lignes de pante O *h* I *t*, &c. Et vne teſte eſtant dõnée ſelon la maniere dudit ſieur, mais en l'ancienne l'on conſtruit ſans teſte donnée, comme il a eſté remarqué cy deuant.

Vous remarquerez que si la trompe ou porte, ou autre espece, & en ligne courbe estoit en talus, qu'il faudroit à autant de plõbs changer de talus extremément difficiles à trouuer, & dont ledit sieur n'en donne la maniere, ce qui est d'vn grand broüillement & long trauail; ensemble tout le trauail qu'il a falu faire pour les premieres testes, qui ne seruent de rien aux trompes & sont inutiles; plus celuy du beueau de chaque lict, ce qui fait que si l'on faisoit vn proiect de traict, le tout ensemble, selon cette maniere, il y auroit telle confusion de traict, outre le temps perdu à le faire, que l'on auroit force peine de le recognoistre; ce qui est cause que la traditiue n'employe pas ordinairement tels porfils aux trompes, veu qu'elle a vne maniere plus propre & expeditiue pour icelles trompes & est icy l'exception; & ie feray voir par des exemples, dont ie me sers en ma Stereotomie de la traditiue amandée, des manieres expeditiues, au double des cy dessus dudit sieur.

Que la maniere du porfil de trompe dudit sieur est defectueuse.

De plus que l'operation de tout le traict ensemble est en telle quantité, & tellemẽt embrouillée, que les ouuriers n'y recognoistront rien, & que l'ancienne est plus expeditiue du double.

De plus, la figure 111. est pratiquée sur mesme regle & de porfil cy deuant dit; ainsi que vous le pourrez conceuoir par l'éclaircissement que i'ay fait de la figure de la planche 108. & verrez que c'est tousiours la mesme, sinon que l'on a changé de point fixe en celle-cy, & qu'iceluy point est sur les faces de deuant; & ainsi que le pratique la traditiue annotée en la responce de la planche cy dite 108.

L'escrit & planche 112. est la mesme maniere de sa precedente, & est son point fixe, aux faces de deuant & les cherches du cul de four de derriere sont mobiles.

Prolixité dudit sieur, & que l'employ de son essieu est superflu & entieremẽt inutil, & en ce chef, impropremẽt nommé.

La planche 83. s'est escoulée sans examen, lequel i'insereray icy, quoy que le rang soit passé; & diray qu'il ne falloit point faire tant de traicts pour trouuer son essieu; suffisoit de mener la ligne O V, tant & si longue qu'elle coupast L F, en vn point, & d'iceluy mener son essieu passant en A; mais iceluy essieu est superflu & inutil; car selon l'ancienne & sans tant de façons, ayant les lignes de porfil A *g* & A *b*, qui sont données, il ne faut que faire *o u*, esgal à O V, & le paneau requis est fait. Dicy vous iugerez ainsi que deuant l'embaras & superfluité de lignes que ie veux croire estre aussi tost par déguisement, que par ignorance, s'entend seulement à l'addition de nouuelles lignes, neantmoins à telle fin de raison.

Que c'est par ledit

Vous remarquerez en cette planche ce qui est a remarquer à toutes les trompes & portes de son Liure, qu'ayant à trouuer vn

ſieur impropermentnommé eſsieu, la ſectiõ de 2.5. plan ſeulement.

paneau de doelle, comme O V, où icelle ligne ou ſuperficie droite de doelle, coupera la ligne ou plan de niueau L F, il nomme icelle ſection de plans *eſsieu*, & à chacune doelle il y aura diuers eſſieux, & iamais vn eſſieu ou ſection de plan ne ſert ſeulement pour deux doelles, & partant tres-improprement nommé eſſieu; par exemple eſſieu ou axe eſt vne ligne en la Sphere, outre qu'elle eſt au milieu d'icelle, tous les plans des lignes horaires ſe coupent en icelle ligne, & l'on ne nomme plus axe la ligne de ſection que d'autres plans font hors icelle ligne; il en arriue ainſi en la Sphere plane. Mais ledit ſieur croit beaucoup faire, que de broüiller & adiouſter en tous lieux ſon eſſieu, quoy qu'il n'y ſoit qu'en nom; & de là faire croire aux ſimples, que cet eſſieu eſt neceſſaire & accomply, & qu'il fait toutes choſes.

Que les manieres par porſils dudit ſieur, ſont del'ancienne; & aux lieux où il l'a voulu plus eſtendre, que l'ancienne, il y a cõmis des abſurditez.

Reprenant la ſuite des planches, ie diray que l'eſcrit & planche 111. enſemble les planches 112. & 113. ſont pratiquées, l'vne cõme l'autre, ſçauoir mettant le point fixe aux faces de deuant, ou cognuës, & le mobile aux faces ou berceaux & voutes incognuës, & eſt la ſeconde pratique de la maniere d'operer par porſil. La planche 114. & derniere du Liure, admet le point fixe ſur l'arc droit ou quarément, & rend immobiles, tant les faces cognuës que les incognuës, ſi icelles faces ſont biaiſes ou obliques à iceluy arc droit. Et eſt la troiſiéme pratique d'operer par porſils, laquelle maniere de porſils exercée ſur trois differentes places ou points, eſt purement de l'ancienne; comme l'on pourra conclure en reconfrontant les planches déclarées & eſclaircies cy deſſus faites par porſils, aux planches du Liure du Pere Derand cy deuant dit fueillet 54. pour les 2. premieres pratiques, ou pour les trois ſi voulez. Et la planche 48. pour la troiſiéme & derniere pratique; partant tous ſes porſils cy deuant ſont de l'ancienne; & où ledit ſieur l'a voulu eſtendre plus que n'a fait l'ancienne, il y a commis des abſurditez, comme par exemple à la trompe 105. aïnſi que i'ay fait voir cy deuant.

Concluſion de l'examen de la coupe des pierres par 2. remarques, où ſont deduites les fautes du Liure dudit ſieur.

Donc ie concluray par deux remarques. La premiere, qu'il ne deuoit tãt blaſmer l'ancienne & traditiue maniere, puis s'en ſeruir ſous quelques déguiſemens en la plus grand part de ſon Liure, & où il l'a voulu trop eſtendre, y commettre des fautes; & en l'autre partie de ſondit Liure, ſe ſeruir d'vne maniere dont l'on ſe ſert aux quadrãs declinés; laquelle maniere eſt defectueuſe en deux chefs.

Le premier eſtant plus embaraſſé de traict, & partant plus long

longue, les obiets ou eſpeurs de la choſe que l'on deſire faire, ſe trouuent renuerſez & incognus aux ouuriers, qui ont à ſe ſeruir de tels traicts. Le ſecond chef, qu'elle eſt ſi peu generale, que ſans ſortir de la figure, où il l'a appliquée, elle reçoit deux exceptions, ce qui n'arriue à l'ancienne maniere amandée, ainſi que ie l'ay prouué & demonſtré dans l'examen cy deuant.

La ſeconde remarque eſt, qu'il ne deuoit en tout ſon œuure où il y a arc rampant, trauailler ſur choſes fauſſes, i'entend ſur de faux arcs faits & conduits de la main ſeulement, ſans regle ny raiſonnement, dont enſuit que le reſultat eſt defectueux & deſagreable. Il deuoit auſſi donner les cherches quarément, pour creuſer les doelles, tant des portes que des trompes, ce qui a eſté incognu iuſques à preſent; & que ie demonſtreray dans ma Stereotomie pour les trompes, d'autant que notamment toutes les doelles de ſes trompes demeurent à pan, n'ayant dit, ny donné la maniere en aucun lieu de ſon Liure pour les creuſer, ce qui eſt vne marque infaillible de foibleſſe, tant en ce chef que pour les choſes cy deſſus, dont il n'en pouuoit eſperer qu'vne iuſte cenſure, & courir le hazard que ſon œuure ne fuſt ſemblable à cet animal nommé Ephemere, lequel porte ce nom pour la briefueté de ſa vie, d'autant qu'il naiſt, vieillit & meurt en vn meſme iour.

Que ceux qui veulent donner regles des choſes qu'ils ne cognoiſſent que ſuperficiellement tombent dans des erreurs ſubiets à cenſure.

Si ledit ſieur euſt cognu & pratiqué la choſe dont il a voulu parler, il ne fuſt ſans doute tombé dans telles erreurs, la pratique eſtant neceſſaire à ayder & fortifier nos ſens; confirmer, ou infirmer ce que la ſpeculation de noſtre eſprit auroit produit: telle & ſemblable cenſure cy deſſus encourent ceux qui veulent parler des choſes qu'ils n'entendent que ſuperficiellement. Et lors qu'ils s'ingerent d'en donner des regles; ils ne manquent iamais de tomber dans quantité d'erreurs & défaux, qui enfin ſeruent de matiere rifible aux cognoiſſans; & en fin ils ſe deſtruiſent eux meſmes par leurs propres principes.

Fin de l'Examen de la coupe des Pierres.

EST ADIOINT L'EXAMEN DE l'vne des pretenduës manieres vniuerſelles du Sieur Deſargues, touchant la pratique de la Perſpectiue, &c. imprimé en 1636. enſemble vn petit Liuret traitant la meſme matiere, imprimé en 1643. comme auſsi de ſes quadrans, & du moyen de plaçer ſon ſtyle ou axe, inſeré en ſon broüillon de la coupe des pierres, imprimé en 1640.

Que la maniere de perſpectiue dudit ſieur n'eſt autre que celle des anciens, ſinon 2. exceptions defectueuſes.

IL eſt à remarquer que la maniere vniuerſelle du ſieur Deſargues, touchant la Perſpectiue appliquée à la conſtruction d'vne cage, imprimée en *1636.* peut eſtre appliquée à repreſenter toute autre ſorte d'obiets; & n'eſt point d'autre maniere que celle de la plus part de nos anciens, comme du Serlio, de Iean Couſin, de Daniel Barbaro, de Baptiſte Benedict, de Ducercio, du Marolois, &c. ſinon en deux exceptions.

La premiere eſt, que pour conſtruire les racourciſſemens, ils n'ont aproché leur point d'eſloignement dans le tableau; mais cela a pû eſtre aux fins d'en faire mieux entendre les fondemens.

La ſeconde exception eſt, qu'ils ont trauaillé ſur vn Geometral, grand comme leurs ouurages, tant pour les raiſons cy deſſus deduites, que pour ne tomber dans l'erreur où conduit vne commune meſure ou eſchelle, ainſi que l'on peut voir dans le deuis ou petit plan Geometral de la ſuſdite cage, quoy que tourné à la bienſeance des nombres, ce qu'aux effets arriue raremẽt, & neantmoins *g i*, qui eſt nombré $4\frac{1}{4}$ doit eſtre $4\frac{1}{2}$

Que ledit ſieur deuoit faire voir l'origine & fondement de la regle d'aprocher l'eſloignement puis qu'elle eſt ſi facile.

De plus, qui voudra preferer les deuis ou petits plans Geometraux, enſemble vne commune meſure ou eſchelle, à quoy ie ne contreuien pas entierement, eu eſgard à la facilité & briefueté pour les artiſans, & notamment eſtant queſtion d'effectuer en grand, pour leſquels ſes manieres ſont ſeulement faites; cela n'empeſchoit pas que ledit ſieur ne deuſt faire voir les raiſonemens & origines, puis qu'ils ſont ſi faciles, comme il ſe pourra voir cy apres.

Que le ſieur Alleaume

L'on ne ſçauroit nier; (ce que chacun peut voir & ſçauoir) qu'en 1628. il a eſté imprimé vn traicté de Perſpectiue du ſieur

Alleaumē, opērant par des eſchelles, tant d'angles, que d'eſloignemens, & ſans ſortir du champ de l'ouurage. Et pour l'eſchelle des eſloignemens ſeulement, outre l'auoir demonſtrée de deſcrire, & transferer ſur les deux montans du tableau, il l'auoit appliquée ſur vn compas qu'il nommoit de Perſpectiue, que le ſieur de Vaulezard a depuis pratiqué en vn petit Liure de Perſpectiue imprimé en 1631. à l'appendix duquel Liure il a demonſtré la conſtruction dudit compas de Perſpectiue, qui n'eſt autre choſe que la conſtruction de l'eſchelle des eſloignemens, & ſans ſortir du champ de l'ouurage, qui ſe pratique par meſme regle & fondemens, que celle de nos anciens cy deuant citez, ainſi qu'il ſe peut voir à l'exception cy deſſus dite ; ce qui eſt trop clair aux cognoiſſans, qui ne mécognoiſſent pas vne demonſtration pour auoir eſté les lignes changées de place & de nom. Et pour ſatisfaire, ie l'inſereray icy, puis qu'elle eſt l'vne de celles que ie demonſtre en l'Optique 2. tome de mon cours d'Architecture.

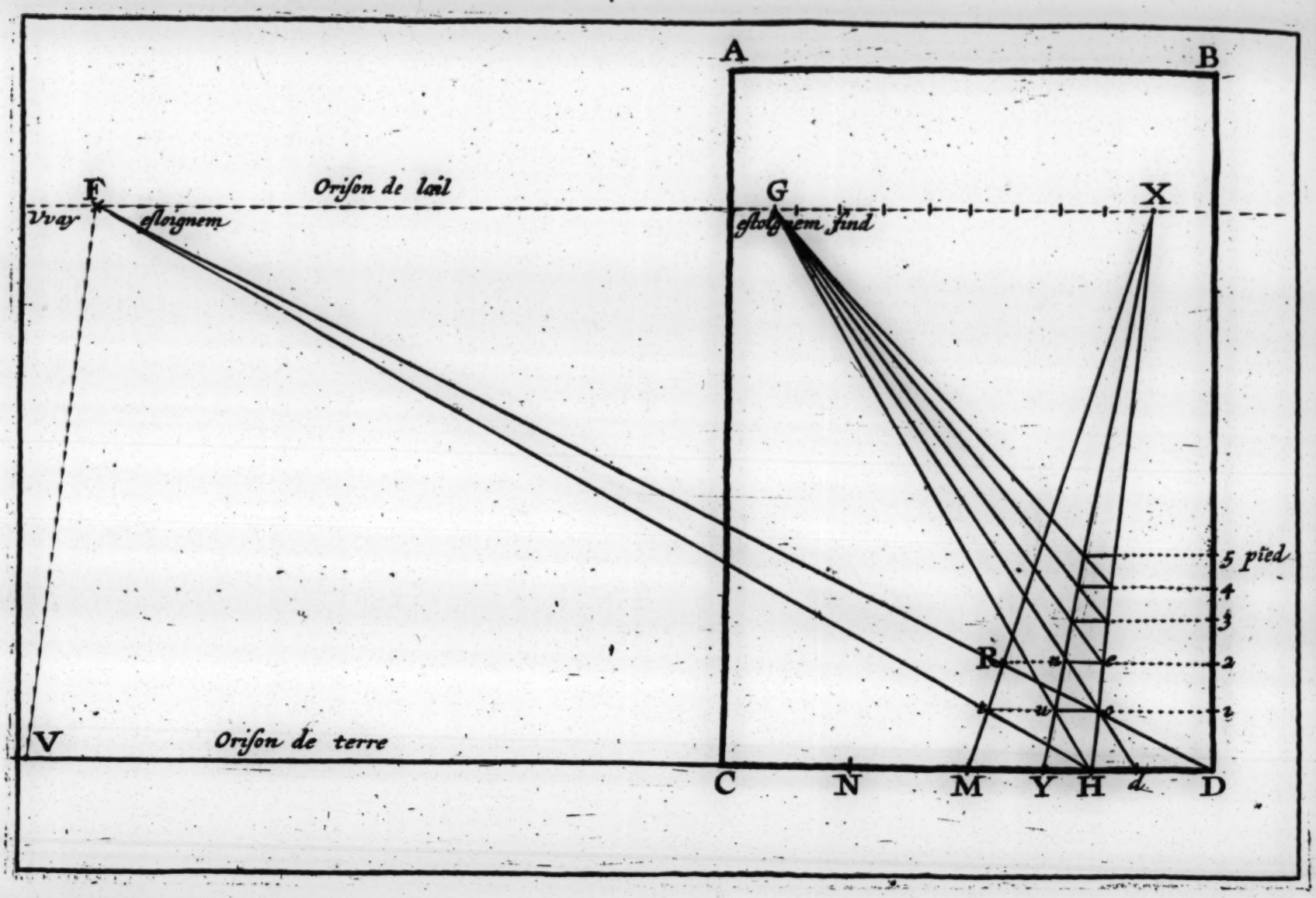
s'eſt ſeruy de la commune meſure de l'eſchelle d'eſloignement pour reduire en perſpectiue ſans ſortir du tableau, qui eſt purement la maniere dudit ſieur.

Soit vn tableau ou tranſparence propoſé A B C D, & ſoit ſur la ligne C D, deſcrit le pied de Roy, qui eſt la commune meſure dont l'on ſe ſert en France, ou autre telle commune meſure qu'il vous plaira, & ſoit C D, quatre pieds comme C N M H D, & ſoit X F, l'eſloignement de l'œil au tableau, qui eſt icy poſé de huicts pieds & demy. La maniere des anciens Autheurs cy deuant citez, eſt de mener du point X, quelque ligne vers C D, comme en H, puis pour auoir ſur la ligne X H, l'eſloignement d'vn pied, ils conduiſent vne ligne de F en D, qui coupera la ligne X H, comme en *o*, qui ſera point d'eſloignement d'vn pied ; plus menant la ligne de X en M, qui coupera la ligne F D, comme en R, qui ſera l'eſloignement de deux pieds, car il y a deux pieds du point M, iuſques en D, mais F H, coupera la ligne X M, en meſme diſtance parallele à C D, que F D, à coupé H X, par les raiſons demonſtrées enſuite ; partant ſi *t a*, eſt parallele à C D, il ne ſera beſoin de mener la ligne F D, du point D, mais du point *o*, & ainſi l'on conſtruira vne eſchelle à l'infiny entre les deux lignes X M, & X H, qui eſt le meſme qu'entre les lignes X H, & X Y.

Conſtruction de l'eſchelle d'eſloignemēt ſelon les anciens.

Maintenant qui n'auroit point de place pour plaçer le point F, qui eſt loin du tableau, & qui voudroit le reſtraindre dans le tableau meſme, l'on le fera par reduction des triangles en meſme

raiſon, c'eſt à dire comme F X, du triangle F X *o* eſt à H D, du triangle H D *o*, ainſi G X, du triangle G X *o*, à H *d*, du triangle



Demonſtratiõ pour approcher tant que voudrez le vray eſloignement, qui par meſme moyen face voir l'originaire rapport des 2. eſchelles, ſçauoir de front & d'eloignement.

H *o d*, lequel G *d*, coupera touſiours X H, au point *o* ; car cõme F X, du triangle F X *o*, à H D, du triangle H D *o*, ainſi X, *o*, à *o* H, coſtez homologues de meſme triangle; car F X, eſtant parallele à C D, les triangles F X *o*, & H D *o*, ſont ſemblables, comme auſſi les triangles G X *o*, & H *o d*, Et partant comme F X, ou G X à H D, ou H *d*, ainſi X *o*, à *o* H. Donc X H, ne pourra par les deux manieres de triangles eſtre diuiſé qu'en *o*, & ainſi de tout autre point à trouuer. Et partant le point G, fait le meſme effet que le point F, Donc dudit point G, y procedant ſelon la maniere accouſtumée, l'on deſcrira vne eſchelle d'eſloignement pour ſe ſeruir mechaniquement en la reduction perſpectiue de toute figure propoſée. Ledit point G, eſt mis à fantaiſie dans le tableau, & com-

& comme il represente le point F, qui est icy posé estre esloigné de X, huict pieds & demy, ainsi sera party G X en 8. $\frac{1}{2}$ & comme H D, est l'vne des parties de F X, aussi sera l'on H *d*, l'vne des parties G X, comme M H esgale H D, aussi Y H doit esgaler H *d*, Et en vn mot, les effets des parties de F X, sur la ligne C D, sont semblables aux effets des parties de G X sur icelle ligne C D, Et partant les points ou sections qui se feront, tant sur la ligne X H, que X Y, estant conduites iusques sur le montan du tableau B D, & perpendiculaires à C D, descriront l'eschelle perspectiue ou d'esloignemens, comme aussi les paralleles à C D, & entre les lignes X M, & X H, sçauoir *t o* & R *e*, & autre, & sur l'eschelle desloignemens, iceux seront les pieds de front seruant † mechaniquement audit tableau. Dicy sort & se voit le rapport & conuenance originaire des deux eschelles de front & d'esloignemens, c'est à dire l'eschelle de la base du tableau C D, & celle de son montan D B.

† *I'appelle mechaniquement, lors que l'on procede par l'instrument d'vne commune mesure.*

De plus, qui considerera la figure du fueillet 5. du traité du sieur Alleaume, cognoistra pour regle generale, que comme le pied de l'œil * iusques à quelque obiet, est à la distance de l'œil au tableau, parallele à la ligne du pied de l'œil, & l'obiet sur mesme plan que le pied de l'œil, ainsi sera la distance du pied de l'œil à l'œil, à vne autre distance qui aura son commencement à l'orison, & sa fin entre iceluy orison & la ligne de terre. Donc estant donnée la distance du pied de l'œil à quelque obiect, comme F X, & H D, ou V D, & la distance de l'œil au tableau, comme F X, ou V H, & la hauteur de l'œil perpendiculaire, ou non, comme X H, ou F V, il est facile de treuuer son quatriesme proportionel X *o*, mais par analogie de proportions, sçauoir, cõme X G & H *d*, à H *d*, ou telle autre ligne qu'il vous plaira mener par le point *o*, qui est icy *d* G, plus proceder aux parties de l'eschelle Perspectiue, comme il est cy deuant enõcé, tant selon nos anciens, que plus recemment par ledit sieur Alleaume, qui a trouué à propos d'operer par vne commune mesure & eschelle, tant d'angle que de Perspectiue, sans sortir du champ de l'ouurage, quant au dernier chef, & le tout pour faciliter l'operation de la pratique. Donc il n'est pas nouueau de voir l'exemple de l'vne des manieres vniuerselles du sieur Desargues, touchant la pratique de Perspectiue, sans employer aucun point, hors du champ de l'ouurage, imprimé comme dit est en 1636.

* *Ie nomme le pied de l'œil quelque point dans la ligne passant par l'œil, parallele au tableau, & ce point se prẽd le plus ordinairement en l'attouchement du plan sur lequel est esleué quelque obiet.*

Demonstratiõ de la construction & rapport des deux eschelles, & sans sortir du champ de l'ouurage, selon le sieur Alleaume qui fait voir que la maniere vniuerselle du sieur Desargues n'est pas nouuelle, mais copiée.

Mais ce qui est risible, est enuiron le bas de l'escrit de la pre- *Matiere risi-*

ſible des propoſitions dudit ſieur touchant la Perſpectiue.

miere fueille d'vn broüillon, proiet pour la coupè des pierres, imprimé, comme a eſté dit cy deuant en 1640. où ledit ſieur ayant loüé ſa pretenduë maniere de Perſpectiue, conclud entre autres mots, *qu'icelle donne la cognoiſſance de ce qui fait repoſer, agir, reſpirer, viure, veiller, dormir, tant en l'illuminé qu'en l'ombre, &c. Ce qui fait paroiſtre frais, meurtry, fort, foible, ſec, tendre, gras, maigre, dur, mol, &c.* Ie m'eſtonne que les amis dudit ſieur & cognoiſſans ne l'ont empeſché, ou diſſuadé de mettre telles paroles au iour, qui tournẽt à ſon grand deſauantage, notamment entre les experts, d'autant que la regle de reduire en Perſpectiue ne demonſtre rien de toutes ces choſes; auſſi n'en a il rien demonſtré.

Que la maniere dudit ſieur pour la pratique du trait, eſt de differente production que la maniere de ſa Perſpectiue, & que toutes les 2. manieres n'ont rien de nouueau que leurs defauts.

De plus, Au commencement de l'eſcrit de ladite fueille dudit broüillon il dit, que *ſa maniere de trait pour la coupe des pierres eſt de la meſme productiõ que la maniere de pratiquer la Perſpectiue*. Vous remarquerez que ſa maniere pretẽduë generale de pratiquer en la coupe des pierres, cõſiſte à plaçer la ſous-eſſieux & cõtre-eſſieux, qui eſt le ſemblable que de plaçer la ſouſtylaire, & l'angle qu'icelle fait auec l'axe du monde, ſur vn quadrant declinant ainſi qu'il eſt demonſtré en l'examen de la coupe des pierres. Vous pouuez conſiderer le rapport qu'a la regle de conſtruire l'eſchelle Perſpectiue cy deuant demonſtrée auec les moyens de plaçer ladite ſouſtylaire, & l'angle qu'icelle fait auec l'axe du monde ſur vn quadrant declinant, & ie m'aſſeure que vous rencontrerez des productions bien contraires; & qui neantmoins n'ont rien de nouueau que leurs fautes, tant eſſentielles qu'accidentelles, & déguiſemens, ainſi qu'il eſt demonſtré dans ce preſent examen.

Qu'il y a manque aux demonſtrations des plãches 114. 115. &c. du petit Liuret de Perſpectiue.

IL ſe voit vn petit liuret de Perſpectiue dudit ſieur Deſargues, addreſſé aux Theoriciens, imprimé en 1643. où il eſt à remarquer, qu'outre les mauuais termes dont il ſe ſert pour s'expliquer, dans l'eſcrit de la planche 112. il y a manques & defauts aux planches 114. 115. &c. & leurs ſemblables; où il n'eſt demonſtré de deſcrire en Perſpectiue, les lignes d'inclination où il met l'eſchelle de Perſpectiue, reſpondant au Geometral, qui peut auoir diuerſes inclinations, quoy que veuës d'vn meſme eſloignement d'œil parallele au tableau, ainſi que ie le vais demonſtrer. De plus, icelles planches cy dites n'ont aucune hypotheſe ny regles de conſtruction, & partant nulles, comme n'ayant aucun fondement de les conſtruire. Mais comme ledit ſieur à la fin d'vne reſponce à cauſes & moyens d'oppoſition, &c. du 16. Decembre,

1642. remet *d'en donner la clef, quand la demonstration de cette grande proposition nommée la Pascale verra le iour. Et que ledit Pascal peut dire, que les quatre premiers Liures d'Apollonius sont ou bien vn cas, ou bien vne consequence immediate de cette grande proposition;* dont i'en laisse la glosse à la liberté du Lecteur.

Que l'abaissement que ledit sieur pretend de faire d'Apollonius, ne releuera pas ses leçõs de tenebres, dont Monsieur de Beaugrand a demonstré les erreurs, en sorte que le public est maintenant priué desdites leçons.

Mais quant à l'égard du sieur Desargues, cét abaissement d'Apollonius ne releue pas *ses leçons de tenebres, ny ses euenemens aux atteintes, que fait vn cone rencontrant vn plan droit,* auquel a suffisamment respondu le sieur de Beaugrand, & demonstré les erreurs en l'année 1639. & imprimé en 1642. en telle sorte que le public depuis ledit temps est priué desdites leçons de tenebres, qui estoient tellement releuées au dire dudit sieur, qu'elles surpassoient de beaucoup les œuures d'Apollonius, ainsi qu'on pourra voir dans la lettre dudit sieur de Beaugrand, imprimée l'année cy dessus.

Ie ne vois pas qu'il falle appeller des œuures plus releuées que celles d'Apollonius, ny mesme celles d'Apollonius, pour demonstrer cette clef cy deuant demonstrée, ny celle qui suit; il ne faut pas estre grand Geometre pour la sçauoir ou conceuoir. Quant à celle des veuës obliques qui suit, elle est vn peu plus difficile; mais ie l'ay icy librement demonstrée, puis qu'elle est de mon Optique, & ie la demonstre ainsi.

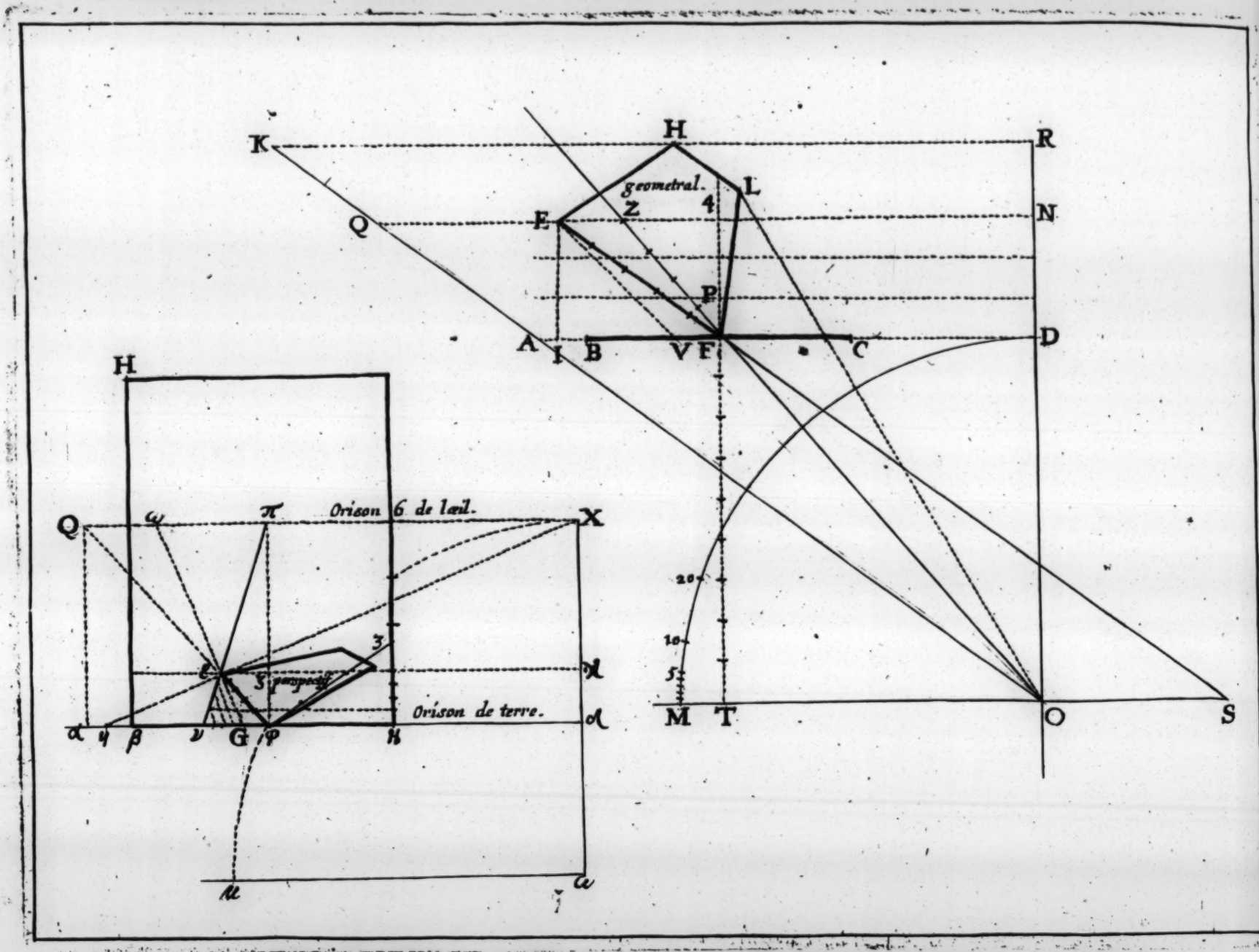
† *Icy ie pose le plan Geometral, grand comme le subiect à representer pour faciliter, n'estant icy la question de reduire au petit pied, qu'vn chacũ peut sçauoir.*

Soit B C, le plan du tableau, & F E H L, plan Geometral, & soit O, le pied de l'œil du spectateur, & soit B D O, angle droit, & soit tant Q N, que K R & M O S, parallele à A D: soit aussi K O, parallele à E F, & soit E F S, ligne droite, comme aussi O F Z. De plus soit β H ϰ, l'eleuation perpendiculaire du tableau. Dõc β ϰ esgale B † C & soit H β perpendiculaire à β ϰ, comme aussi φ ϖ à α Q & ♪ X & α β égale A B, comme aussi ϰ ♪ égale C D, & β φ esgale B F, & soit Q X, l'orison, ou hauteur d'œil; comme aussi β ϰ, la ligne de terre, qui est icy posée en la base du tableau; si ie voulois en cette demonstration chercher l'endroit perpendiculaire au tableau de la ligne de directiõ * O R, ayant à la representer, il faudroit que ie sortisse du tableau, comme au point X, qui sera point visuel, ou si ie desire que l'œil soit tourné vers A, & que la ligne de directiõ soit O K, alors le point visuel sera encor hors le tableau, comme en Q, & la ligne oblique Q A du plan Geometral, sera representée au costé du tableau le long de α Q. D'abondant, si au plan Geometral ie desire que l'œil soit tourné vers F, & que la ligne de direction soit O Z, alors le point visuel sera en π, & la ligne oblique

* *Ie nõme ligne de direction, celle qui passe directement iusques au fond de l'œil, sans faire fraction en la christaline ny dans les autres humeurs que i'ay de-*

monſtré en mon optique,

FZ du plan Geometral, ſera repreſentée au tableau, le long de φ π, & ſi du point O vous tournez l'œil en R, alors au tableau, le



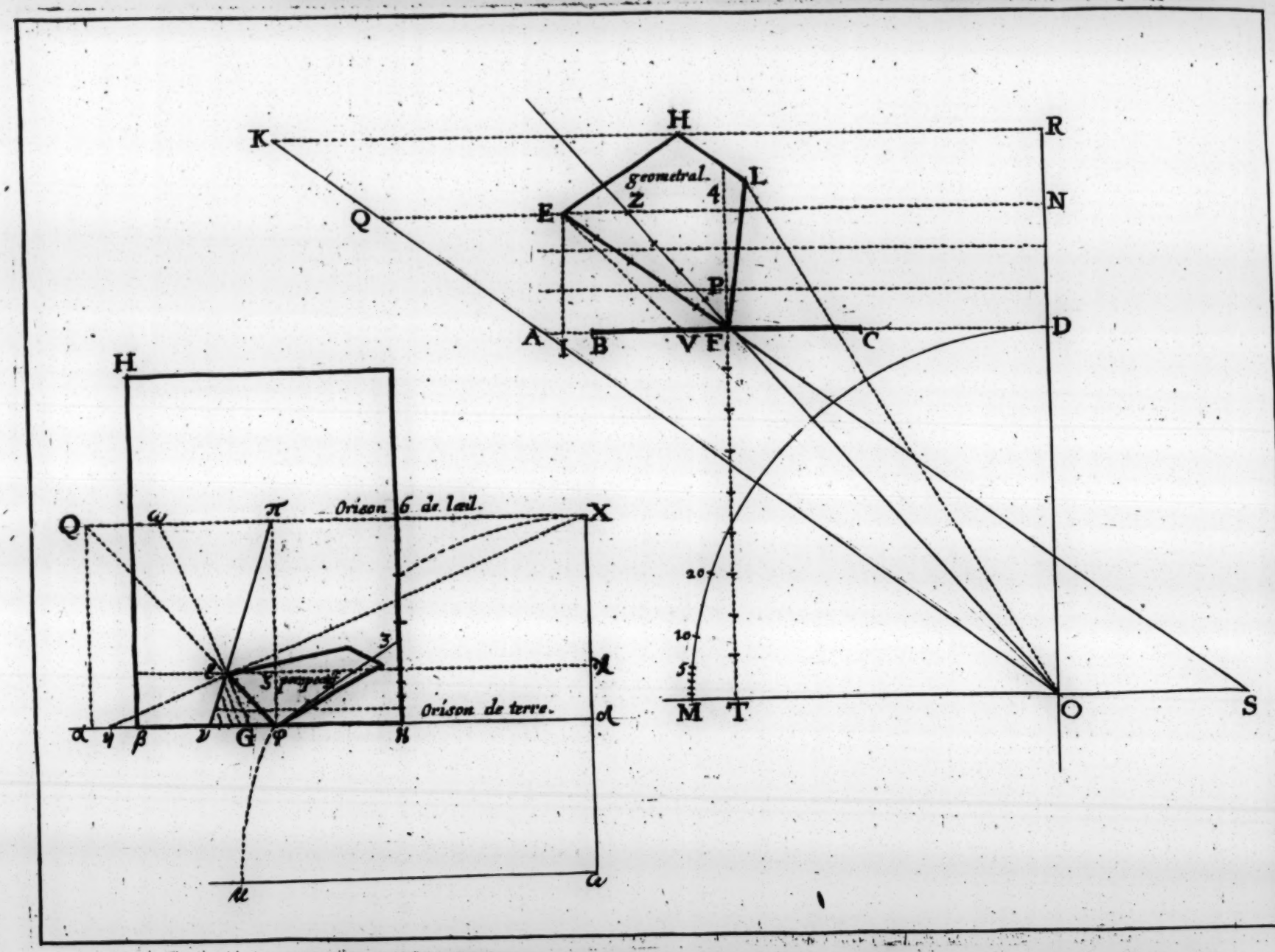
ou celle que l'on dit que l'œil regarde droictement.

point viſuel ſera en X, mais de quelle façon que vous tourniez l'œil, la figure Perſpectiue ε φ, demeurera en meſme forme, excepté quelque difficulté de fraction, que ie demonſtreray dans mon Optique; mais pour trouuer le point E du Geometral poſant O R, la ligne de direction ou regard de l'œil, & ſoit E I, parallele à B C, faut faire au tableau η φ, eſgal à I F, & mener la ligne η X, puis mener la ligne Q φ, qui coupera la ligne η X, comme en ε, alors le point ε du plan Perſpectif, repreſentera le point E du plan Geometral. Et ſoit menée la ligne pointée ε λ, parallele à β κ; car pour trouuer le point ε, il ne faut que faire comme O D, à D N, ſoit X λ, à λ δ ou X ε à ε η, Mais D A, eſt à I F, comme D O à I E, & en leur place Q X à η φ, ainſi O D à D N, & partant auſſi X ε à ε η.

ɛn. Donc tant le point ɛ que la ligne ɛφ est bien representée; d'où il paroist, que si du pied de la station de l'œil, comme O, l'on mene vne ligne comme O Q, parallele à quelque ligne du plan, comme E F, ou icelle ligne coupera le tableau, estant esleuée vne perpendiculaire, comme αQ, si du point Q l'on mene la ligne Perspectiue ɛφ, elle representera la ligne E F, & partant tous les paralleles à E F, seront aussi menez du point Q, &c. De plus, qui voudroit du point O. tourner l'œil vers K, ie dis que le point de perspectiue ɛ sera tousiours en mesme place au tableau, car O D, à D N, & ainsi O A à A Q, partant Xλ à λ ſ; Il est cy deuant dit, que ɛφ, doit estre menée au point Q & partant tousiours, tant le point ɛ que la ligne ɛφ demeurera en sa place. De plus, si l'on vouloit que l'œil fust tourné vers Z, le point ɛ demeurera tousiours en sa place, & ainsi de toute autre obliquité de ligne de direction; car O F à F Z, ainsi O D à D N, & ainsi Xλ à λ ſ. Si l'on fait F V, parallele à Z F, & que γφ soit esgal à V F, & soit menée la ligne πγ, elle coupera la ligne ɛλ, comme en ɛ, car E V, parallele à Z F, ainsi πγ, sera parallele à πφ, & partant l'angle perspectif πγφ, esgaler a l'angle perspectif πφx ou Z F D, du Geometral, & par consequent l'angle perspectif Qφβ esgale l'angle Geometral E F B. Donc le triangle Geometral E V F, égale le triangle perspectif ɛγφ, car leur base γφ, & V F, sont esgales. Et partant le point ɛ ne changera de place, car aussi D F, à V I, ainsi O F, à V E, ou O D, à D N, & aussi πX à ηγ. De plus, si vous voulez trouuer le point ɛ sans sortir du tableau, il n'est question que faire, comme N O à N D, ainsi X ſ à ſλ, puis où coupera la ligne γπ, comme en ɛ, alors le point ɛ du perspectif representera le point E, du plan Geometral; soit que du plan de l'œil O, vous vouliez tourner l'œil en quelque obliquité, qu'il vous plaira, ainsi qu'il vient d'estre demonstré, & le tout, pourueu que l'œil puisse aperçeuoir le tableau, s'entẽd, que l'angle C O B, ne soit pas plus grãd que la moitié d'vn droit, & quelques vns l'ont limité à la moitié d'vn tiers de deux droits, & partant point de regle certaine; mais ie demonstreray cette difficulté dans mõ Optique. Donc ayant treuué le point ɛ & que vous vouliez descrire l'eschelle perspectiue, il ne faut que de quelque point en l'orison du tableau, comme ω, conduire vne ligne passant par ɛ, & finissant comme en G, * & maintenant determiner, laquelle vous desirez qui soit la ligne de direction, & soit F Z qui a de positiõ quatre parties esgales ou telle quantité que

* *Car ω & π sera à γ G, comme πɛ à ɛγ ou Xλ à λſ, ce qui est requis pour construire l'eschelle de Perspectiue.*

voudrés, & ſoit des pieds de Roy, cõmunes meſurés vſitées en Frãce; dont ſi vous diuiſez γ G en quatre parties égales, & ſoit mené



vers ϖ, ils deſcriront le long de la ligne π γ quatre des pieds de l'eſchelle perſpectiue que l'on peut continuer à l'infiny, par la premiere regle demonſtrée. Donc ſi telles ſections de pied ſur la ligne π γ, ſont menées iuſques aux montans du tableau κ 3. & paralelle à β κ', il ſera deſcrit ſur iceux montans l'eſchelle perſpectiue, pour deſcrire tel plan, ou figure que voudrez.

Il eſt à remarquer que de la demonſtration cy deuant faite, l'on en tire les conſequences qui ſuiuent, ſçauoir que deſirant que la ligne F Z, ait de l'inclination ſur le plan Perſpectif, auſſi bien que ſur le plan Geometral, il faudra poſer la ſtation de l'œil en quelque autre endroit, qu'en O, d'autant qu'iceluy O, eſt enfilé de F Z, & ſoit poſé l'autre endroit, comme en T, ou autre tel lieu que de-

sirerez à l'exception cy dite, & que la ligne de direction fust TF 4. ladite ligne TF, est posée en la demonstration, auoir 9. pieds, & d'iceux pieds F 4. en a 3. car si du point 4. l'on mene 4. E, parallele à B C, iceluy 4. E, en la demonstration passe en E, donc si en l'eschelle de perspectiue du tableau faite sur ledit esloignement F T, l'on mene vne ligne passant à trois pieds d'esloignement, comme λ ε, icelle ligne representera la ligne E 4. du Geometral, & si au lieu de 4. F, l'on auoit pris F Z, qui a quatre pieds sur le Geometral, & que sur ladite eschelle d'esloignement cy deuant dite, l'on eust fait la ligne ε λ, à quatre pieds d'esloignement, alors icelle ligne ε λ ne represente la ligne E 4. du Geometral, mais vn pied plus au dela, ce qui fait voir qu'il faut prendre les communes mesures du Geometral sur des lignes paralleles à la ligne de direction, comme E I, ou directes à icelles mesmes, comme F 4. & non sur des lignes obliques, comme F Z, & ainsi des autres.

Que les figures de Perspectiue des planches 114. & leurs suiuantes sont absolument fausses; & qu'il a esté fort improprement addressé aux Theoriciens tant pour ce chef qu'à raison de l'eschelle.

Si l'on considere les figures des planches 114. 115. 116. & 118. où la ligne oblique *e g* au respect de *x y* base du Geometral, reçoit les diuisions des pieds, ou commune mesure de semblable grandeur, que sur la ligne *x y*, neantmoins icelle ligne *e g*, n'est ny parallele ny directe à la ligne de direction de l'œil; aussi ledit sieur ne l'a entendu estre, ny ne le pretend pas, tesmoin les figures de la planche 117. & 118. ou la ligne directe à la ligne de direction de l'œil est representée par la perpendiculaire *a* 90. & la ligne oblique *e g*. figure d'embas de la planche 118. coupe icelle *a* 90. partant *e g* des figures Perspectiues cy deuant dites, n'est ny directe ny parallele à la ligne de direction de l'œil, ce qu'estant constant, il ensuiura par la demonstration cy deuant, que toutes les figures de perspectiue dudit Liuret fabriquées sur telle methode sont absolument fausses. Voila pas vne belle chose adressée aux Theoriciens; & de plus, des manieres operantes par pied, ou commune mesure d'eschelles d'esloignement mechanique faillible, & sans precision requise par les Theoriciens, comme estant incapable de la plus grande partie des proportions.

Que les eschelles d'angles ne sont pas generalement tant necessaires que les eschelles d'éloignemens.

Quant à la planche 117. figure d'enhaut, demonstrant la construction d'vne eschelle d'angles, qui n'est autre chose proposer que de reduire vn demy cercle, ou vne espece de quarré Geometrique en perspectiue, ayant son centre en *a*, excepté que ce quarré Geometrique aura les costez obliques, en sorte qu'ils soient directes au plan de l'œil *h*, pour faire que au tableau les aparances

d'iceux coſtez ſoient perpendiculaires à la baze d'iceſuy tableau, ce qu'vn chacun peut ſçauoir. Ledit ſieur fait eſperer par l'eſcrit du fueillet 2. qu'il donnera quelque autre eſchelle d'angles, plus parfaite que celle cy, ny que celle du ſieur Alleaume; mais il eſt à craindre que ce temps n'arriuera pas ſi toſt. Et pour vous dire mõ ſentiment des eſchelles d'angles, ie ne trouue pas que ce ſoient choſes neceſſaires, d'autant qu'elles ne ſont pas generales, & qu'il faut qu'elles ſe ſeruent de l'eſchelle d'eſloignement, mais l'eſchelle d'eſloignement eſt generale, & n'a que faire d'eſchelle d'angle, partant de beaucoup preferable.

Que le compas de proportion eſt peu vtile en Perſpectiue.

De plus, l'eſcrit & planche 119. & derniere dudit Liuret, demonſtre ce que beaucoup d'autres ont enſeigné cy deuant, qui eſt de trouuer les éloignemens perſpectifs de quelque plan Geometral, par le compas de proportion, qui a cette incommodité, qu'à chaque point que deſirez rendre en perſpectiue, il faut ouurir ou fermer, & faire nouuelle operation, ce qui n'arriue au compas optique: & arriuant des nombres rompus, il faudra recourir au compas commun, pour faire les ruptures en proportion; & outre les incommoditez cy dites, l'on ſera plus de temps à faire l'operation par iceluy compas, que par regle ou eſchelle perſpectiue; & de plus, tel compas n'aura lieu que ſur petits deſſeins & ſubiets de la grandeur d'iceluy compas, & aux effets ou grandes perſpectiues il ſera nul, partant fort peu vtile.

Reſtabliſſement à la rupture du compas Optique contre le propoſé dudit ſieur.

Il eſt à remarquer qu'en l'eſcrit de la fueille 119. ledit ſieur parlant du compas optique, dit ainſi, *Auec vne ligne diuiſée en parties inégales, & conſequemment pour vne ſituation particuliere*, qu'il confirme & reïtere en l'eſcrit du fueillet 2. où il dit, *lequel ne ſçauroit eſtre que pour vne rencontre particuliere*, croyant de là que le compas Optique, qui a ſur ſes branches vne eſchele d'éloignement fait par quelque nombre particulier d'éloignement poſé, qu'iceluy cõpas ne peut faire ny rẽdre en perſpectiue, les obiets propoſez, que touſiours veus du meſme éloignemẽt, dont l'eſchelle de ſes branches a eſté conſtruite; mais par la propoſition cy deuant demonſtrée, il eſt facile de verifier le contraire; & qu'iceluy compas Optique eſt general pour tous éloignemens deſirez, ce que ie demonſtreray ainſi par la figure precedente. Soit O F, éloignemẽt poſé, dont l'échelle Optique ou d'éloignemẽt du cõpas a eſté conſtruite, maintenant ſoit deſiré de reduire en perſpectiue la figure E H L M F, & veuë d'vn plus proche éloignemẽt, que l'échelle du compas cõme de F T,

de FT, il ne faut que prendre les mesures communes ou pieds du plan Geometral, le long de la ligne Z F O, & non pas le long de la ligne 4. F T, & sera cõstruite en Perspectiue ladite figure E H L F, cõme il se voit en ξ. Donc l'vn des éloignemens, cõme la ligne ε λ, ne sera pas plus loing, que si l'on l'auoit fait par vne eschele construite sur l'esloignement desiré F T. De plus, si il estoit requis que ladite figure F E H L, fust veuë d'vn éloignement plus loing que l'éloignement F O, dont l'eschelle Optique dudit compas a esté constrinte, & soit l'éloignement desiré plus loing, comme F S, il faudra operer comme cy deuant est dit, sçauoir prendre touſiours les mesures des pieds au long de la ligne Z F O, & non au long de la ligne E F S, & il ensuiura qu'au plan Perpectif, la ligne d'éloignement ε λ sera touſiours en mesme lieu, & la figure sera seulement veuë plus obliquement, & par d'autres lignes de direction, ce que ledit sieur deuoit excepter, puis il eut eu quelque raison, & non pas dire, *Pour vne situation particuliere, ou pour vne rencontre particuliere*, car il est pour infinies situations & rencontres d'éloignement, ce qui se peut faire encor par autre maniere, rendant le tableau oblique à l'orison, ainsi que les excellens remarqueront par la premiere figure de Perspectiue cy deuant demõstrée: ie demonstreray plus au long dans mon Optique les changemens d'éloignemens, par les deux differentes manieres cy dites; cõme auſſi, qu'estant le triangle du Geometral E F V, reduit au Perspectif ε φ γ, qu'en l'endroit ou γ ε coupera l'orison cõme en π que d'iceluy point π, partiront toutes les lignes qui y pourront construire la figure Perspectiue ξ, & qui representeront toutes les lignes du Geometral paralleles à E V, qui sont lignes des plus commodes, pour reduire en Perspectiue la figure F H L F. Donc ie concluray par la demonstration cy dessus, qu'estant quelque figure proposée à reduire en Perspectiue, & de quelque éloignement, autre que l'éloignement de l'eschelle du compas Optique, elle sera par iceluy compas construite selon l'éloignement proposé, & partant iceluy compas Optique sera general & non particulier; si l'on n'adiouste les exceptions & subiections cy dessus.

Que la maniere dudit ſieur pour traçer quadräs vn ſtyle ou axe eſtant placé, n'eſt vniuerſelle comme il la propoſe.

IL ſe voit à la fin de l'eſcrit d'vn broüillon de la coupe des pierres dudit ſieur imprimé en 1640. *Vne maniere vniuerſelle de traçer, au moyen d'vn ſtyle placé, tous quadrans plats d'heures eſgales au Soleil*, &c. qui outre les mauuais termes de parler & façons defectueuſes de s'expliquer ordinaires audit ſieur, la maniere qu'il dõne n'eſt pas vniuerſelle, & n'eſt qu'vne des manieres ordinaires pour les declinants; d'autant qu'aux quadrants Orientaux ou Occidentaux inclinez ou declinez, la meridienne & equinoctiale, dont il ſe ſert pour deſcrire iceux quadrans, ſont quelquefois paralleles, autrefois elles ne ſe couperont que beaucoup loing, & ordinairement hors la muraille, ſur laquelle on doit deſcrire iceux quadrans, & partant ils demeureront à faire, & encore pis, eſtant iceux Septentrionaux; & les quadrans qui ſeront paralleles à l'equinoctial, n'auront point de ligne equinoctiale, ny de ſix heures, ſelon ſa maniere, & demeureront ſans eſtre faits.

Ie donne pour regle, que le plan orizontal eſt preferable au plan equinoctial, comme le pratique ledit ſieur, en generalité, facilité, & briefueté, operant ſeulement auec la regle, le compas, & le plomb: en generalité, deſcriuant tous les quadrans, tant Orientaux qu'Occidentaux, tant declinez qu'inclinez, enſemble tous les paralleles à l'equinoctial: en facilité, d'autant que l'orizontal eſt plus intelligible aux eſprits communs, & notamment aux declinans & inclinés que l'équinoctial, à cauſe de ſa plus intelligible & facile poſition: & en briefueté, n'ayant pas tant d'operations à faire, qui eſt la methode que ie tiens, & la demonſtreray au long en l'appendix de ma Stereotomie.

Que ſur le plan de l'equinoctial c'eſt improprement nommer vne ligne accidentelle, ligne de 6. heures, & qu'il pouuoit auſsi bien nommer vne ligne de 12. heures.

Quant au ſurplus de la conſtruction par ledit ſieur, comme de tourner l'eſquierre le long du ſtyle ou axe, qui repreſente le plan de l'equinoctial, que la maniere ordinaire trouue auſſi facilement, & neantmoins aucunefois il le nomme la ligne de ſix heures ſimplement, ce qui eſt improprement; d'autant qu'encore que la ligne de ſix heures ſoit ſur iceluy plan, toutes les autres lignes horaires y ſont auſſi. Et que s'il vouloit ſignifier, qu'il auoit deſſein de ſe ſeruir ſur le plan equinoctial d'icelle ligne de ſix heures, il deuoit donc auec autant de raiſon quelquefois nommer iceluy plan la ligne meridiene, dont il ſe ſert en la conſtruction de ſes quadrans, & neantmoins le tout improprement, puis qu'ils

ont leurs appellatiõs & significations particulieres. De plus il deuoit au moins énõcer en quelque sorte s'il desiroit se seruir d'icelle ligne de six heures. La seconde station par sadite esquierre est superfluë: pour le triangle & le cercle descrit de son sommet, puis diuisé en 24. parties esgales, est purement la pratique ordinaire. Donc ie conclus que sa pratique pour traçer quadrans, ayant vn style ou axe placé, n'est pas vniuerselle, comme il le propose, ny des plus faciles & expeditiues, & que ledit sieur a improprement nommé le plan ou ligne sur l'equinoctial, ligne de six heures, & ce pour déguiser ce que sa maniere auoit de commun auec la methode ordinaire.

Que la maniere dudit sieur pour placer son style est fort incommode.

LEdit sieur donne vne maniere vniuerselle de plaçer vn style, selõ l'axe du mõde, sans sçauoir l'eleuation du lieu, ny la declinaison du mur, ce dit-il, *en quelque endroit possible*. Il est à remarquer, qu'outre que la regle est fausse, comme ie le feray voir en suite, cette maniere a de grandes incommoditez, comme qu'il faut auoir des reglets de fer à viue areste, & qu'il faudra les couper de longueur conuenable à chaque occurrence de quadrant, ou en enfonçer quelques vns d'iceux beaucoup auant dans le mur, où l'on desire construire le quadrant. De plus, qu'il faut faire des trous dans le quadrant, qui seront quelquefois bien longs, selon l'obliquité des reglets de fer dressez au Soleil, & capables de gaster toute la surface du quadrant. Il est aussi difficile de seller & arrester auec plastre iceux reglets de fer, regardans directement le Soleil, d'autant qu'il faut du temps à les seller; & neantmoins le Soleil ne donne point de temps, mais chemine tousiours.

Que la maniere pretendue dudit sieur de plaçer le style n'est pas nouuelle, & n'est que l'antithese de celle de Morelic.

Pour le fondement de cette maniere, il n'est autre chose, que supposant que quelque point marqué à vn style, où le bout de quelque verge soit la summité receuant, & par ou passent tous les rais solaires, & dans vn espace de temps que l'on concede la course du Soleil estre circulaire, sera la summité du cone descrit par iceux rais solaires, excepté qu'en l'equinoxe il n'a point de cone, mais vn plan droit, & iceluy bout ou point sera pour lors en l'axe. Morolic en sa description des lignes horaires, demonstre que le Soleil estant hors de l'equinoctial, fait par le moyen d'vn bout de verges, ou point marqué au style ou axe, des rais formant vn cone, semblable au cone du Soleil, qui a pour base quelque cercle pa-

rallele à l'equateur, & la ſummité d'iceluy cone eſt le bout de la verge ou point au ſtyle, & l'axe d'iceluy cone eſt l'axe du monde; & Morolic poſant l'axe du mõde eſtre cognu, enſemble vn point ſur le ſtyle, repreſentãt la ſummité d'iceluy axe, puis par vn ſimple rais de Soleil, il deſcrit le cone ou la ſection, qu'iceux rais ſolaires peuuent faire tout le iour ſur quelque plan, & la maniere dudit ſieur n'eſt que la contre-regle de cecy. Sçauoir, qu'eſtans pluſieurs rais de Soleil, ou portion de cone obſerués ou donnez, enſemble le bout de quelques verges repreſentans la ſummité d'iceluy cone, il eſt facile de trouuer tout le cone, & par conſequent ſon axe, qui repreſentera l'axe du monde. Donc cette maniere dudit ſieur n'eſt pas nouuelle, n'eſtant que l'antitheſe de celle de Morolic.

Que la maniere dudit ſieur de plaçer ſtyle, eſt fauſſe, ce qui a fait que Morolic ne l'a appliquée à plaçer ſtyle, mais ſuiure la maniere vſitée par le pole.

De plus, cette maniere appliquée à plaçer vn ſtyle eſt fauſſe, d'autant qu'elle ne place pas iceluy ſtyle ſelon l'axe du monde; la cauſe eſt que nous ne ſommes pas ſous les poles, ny au centre de la terre, où cette regle auroit lieu ſeulement; mais nous ſommes ſur la ſuperficie qui eſt beaucoup eſloignée du centre; où il eſt queſtion de plaçer le ſtyle; & pour monſtrer qu'il y a erreur grande, c'eſt que le grand cone fait par les rais ſolaires, n'eſt pas iſocele, mais ſcalene, & neantmoins il a touſiours pour baze quelque cercle inſenſible en defaut, & parallele à l'equateur, & faiſant vne ſection en iceluy cone, ou ſon ſemblable qui le rende iſocele, icelle ſection ſera elliptique, & toutesfois ledit ſieur l'a creuë circulaire; & s'en ſert à plaçer ſon ſtyle cõme d'vn cercle, ſans en faire aucun aduertiſſement; la difference qu'il y a d'vne ellipſe à vn cercle apporte plus d'erreur à poſer ſon ſtyle que l'erreur inſenſible de le poſer, ſelon le pole du monde, qui eſt la maniere vſitée; ce qui a fait que Morolic ne s'eſt pas voulu ſeruir de ſa maniere pour poſer ſon ſtyle; mais ſuiure la maniere vſitée cy dite.

Si ledit ſieur dit, que Morolic a donc failly en la deſcription des ſignes du Zodiaque, d'autant qu'il poſe les cones des rais ſolaires eſtre iſocele; Ie diray en faueur de Morolic, que telle deſcription de ſignes faits en vn quadran, ne ſont pas tant, & à beaucoup prez importans, qu'vne fauſſe éleuation. De plus, cette maniere dudit ſieur de poſer ſtyle en pluſieurs poſitions de quadran, n'eſt point en tout certaine, & notamment aux ſaiſons approchans les equinoxes; & il y aura plus de certitude de poſer iceux

ſtyles

styles par la cognoissance de l'eleuation polaire, que par cette maniere.

Ie demonstreray dans l'Appendix de ma Stereotomie vne maniere plus précise, certaine, facile & commode, que celle dudit sieur, pour poser style ou axe, sans aucuns instrumens ordinaires, ny mois, ny esleuation du lieu, mais le plomb, la regle, & le compas, & deux plans, dont l'vn Orizontal, & par trois ou quatre operations de Soleil en vn iour, ce qui confirme de combien le plan Orizontal est preferable à tout autre plan en la construction des quadrans; & tels moyens n'ont esté vsitez iusques à present, pour trouuer esleuation polaire, qui sont neantmoins les plus précis, à cause des fractions de l'air.

Ie n'ay point fait de demonstration du cy dessus, ny mesme en quelques autres pareils endroits de ce Liure. I'ay pretendu parler aux cognoissans (qui auront plus d'esgard à la substance du proposé & conclu, qu'aux punctuelles & estenduës obseruations du demonstré) & non aux simples qui auroient besoin d'instruction entiere sur chacune proposition & conclusion, comme n'estant le subiet de ce Liuret de *l'Examen des Oeuures du sieur Desargues Lyonnois*, par I. Curabelle. 1643.

FIN.

Fautes coulées en l'impression, que le Lecteur excusera, s'il luy plaist, & est aduerty que p. *signifie page*; lg. *signifie ligne, &* li. *signifie lisez.*

Pag.12.lg. 23.niueau de,li.niueau du deuant de,p 16.lg 35.PAE,li.PAF,p.18.lg.4.les beueaux,li. les sauterelles ou fausse équierre,p 18.lg.22.fait & parfait, li. faire & parfaire. p. 21.lg 24 *p s o* li. *p s o* paralelle à *a m*.p 23.lg 32. A *o n* li. A *n*.p.23.lg.33. HIQ. li.HI.p. 24.lg. .XR,li. *x* R p.24.li 3 A *p* esgal à *h* B li.*h* B esgal à A *p*.p.25.lg.4 ferez 24 li ferez 29.p.25 lg.9 ferez 33.26 esgal à 41 19.li ferez 41.19.esgal à 33,26. p. 25.lg.20 42.6.li.42 9.p.25.lg.28. à 97 li à 87. p. 25.lg. 39. de demy ligne, li.auec deux lignes.p 33. lg 37.P 36 5.li.P 36.Q 5.p.35.lg.2.5.36.P.li 5,Q.36. P.p.35.lg.12. 5.8 li.6.8.p.36.lg.2 de rampe, li. oblique p 38.lg 8 confronteray.li.confronterez p.43,lg.33.pour li.par.p.45.lg.9.10.*i*.li. 10.l.p 46 lg 21.de I.li.de I 10. p.49.lg 11.Icelles preparations sont,li.icelle preparation est. p.5 .lg. 8 & 36.li. & 26.p.55.lg.21.4 2.li.42 p.57 lg.12.100.li.104.p.58.lg.34.de l'effect. li. que l'effect p.60 lg. 2.*r o m*. li. *r* O *m*.p 60 lg 5.escrire,li. descrire.p.61.lg.30. & auance li.l'auance p.61.lg.31.te de.li.te selon le biais de,p.64.lg.28.la planche,li.la fueille. p. 71. lg.7.parallele, li.perpendiculaire.

Aux additions, pag.18,ie dis cintre, lis.ie dis sauterelle.p.23.alors L.lis.alors *h* S.p 64.2.5: l.1.

MVSEVM
BRITAN
NICVM

GAUGING EN-TOMIZED:

OR,

An Abbreviation of Solid Geometry, *ſo much as concerns the Buſineſs of* Cask-Gauging; *Wherein you may take a* Cask *in any of the Four Notions following.*

Compiled by *MICHAEL DARY.*

HAving by you a Table of the Squares of Numbers, you may put S = the ſum of the reſpective Tabular Numbers belonging to the Head and Bouldge, D = their difference, and Y = the difference of the Diameters themſelves, L = the length of the Veſſel, and C = the Content thereof.

Prop. 1. If a Cask taken as the middle *Fruſtum* of a *Spheroid* intercepted between two *Planes* parallel, cutting the *Axis* at right Angles: Then,

$:1\frac{1}{2}S+\frac{1}{2}D:\times L=C.$

Prop. 2. If a Cask be taken as the middle *Fruſtum* of a *Parabolick Spindle* intercepted between two *Planes* parallel, cutting the *Axis* at right Angles: Then,

$:1\frac{1}{2}S+\frac{1}{10}D:\times L=C.$

Prop. 3. If a Cask be taken as the middle *Fruſtum* of two *Parabolick Conoids* abutting upon one common Baſe, intercepted between two *Planes* parallel, cutting the *Axis* at right Angles: Then,

$:1\frac{1}{2}S+0\,D:\times L=C.$

Prop. 4. If a Cask be taken as the middle *Fruſtum* of two *Cones* abutting upon one common *Baſe*, intercepted between two *Planes* parallel, cutting the *Axis* at right Angles: Then,

$:1\frac{1}{2}S-\frac{1}{2}YY:\times L=C.$

In all theſe four Equations, you muſt Divide the left ſide of the Equation by 3,82 if you would have the Content in Cubical Inches, by 1077 if you would have *Beer-Gallons*, by 883 if you would have *Wine-Gallons*. But becauſe the Diviſor 883 is a little too much, and 882 ſomething too little, the Diviſor may commodiouſly be done away, thus:

1. Tranſcribe the Dividend one Grade toward the right Hand, and take the third part of it as it there ſtands.
2. This third part, the number tranſcribed, and the Dividend, theſe Three you ſhall add into one Total, cutting off four Decimals, and your Diviſion is effected.

Examples in Wine-Gallons.

Let a Cask be propoſed, whoſe Diameter at the Bouldge is 32 Inches, the Diameter at the Head 22 Inches, and the length of the Veſſel 44 Inches.

Spheroid.	*Parabolick Spindle.*	*Parabolick Conoid.*	*Cone.*
1024	1024	1024	1028
484	484	484	484
1508	1508	1508	1508
754	754	754	754
270	5400	2262	50
2532	23160	44	2212
44	44	9048	44
10128	92640	9048	8848
10128	92640	99528	8848
111408	1019040	99528	97328
111408	1019040	33176	97328
37136	339680	112,7984	32443
126,2624	115,49120		110,3051

This which hath been done by the Table of Squares may be performed more eaſily by a Table of the third part of *Areas* of Circles ready reduced to *Wine-Gallons*: Thus,

Spheroid.	*Parabolick Spindle.*	*Parabolick Conoid.*	*Cone.*
Dia Bung 1,160	1,160	1,160	1,160
Dia. Head 0548	0548	0548	0548
Sum. 1708	1708	1708	1708
$1\frac{1}{2}S$ 854	$\frac{1}{2}S$ 854	854	854
$+\frac{1}{2}$ Diff 306	$+\frac{1}{10}$ Diff 612	2562	565
2868	26232	44	25055
× Long. 44	+ Long. 44	10248	44
11472	104928	10248	100220
11472	104928	112,728	100220
Cont. 126,192.	Cont. 115,4208		110,2420

If any Man object and ſay, that the *Fruſtum* of the *Spheroid* was eaſier done the old way, by taking the Bouldge twice, and the Head once; I confeſs it is ſomewhat the ſhorter way, But that way for the *Spheroid* could not be accommodated to the other three forms which are here laid down, and accommodation was my chief intent for where one Man is of Opinion, that a Cask is a *Fruſtum* of a *Spheroid*, Ten are of a contrary Opinion, and affirm it to be leſs by experience. Now here in this way that I have laid down, you are at liberty to do at pleaſure, for if you imagine a Cask to be ſome Content between a *Spheroid*, and a *Parabolick Spindle*, you may take $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}$, or $\frac{1}{9}$ of the difference, and you have your deſire.

¶ *If a* Cask *be not full (the* Axis *being poſited parallel to the* Horizon*) to find the quantity of Liquor contained in it.*

To do this, you ought to have a Table of *Segments* of a Circle whoſe *Area* is unity, the *Diameter* being Divided into 10000 equal parts, and then this Approximation is the beſt hitherto uſed, which requireth this *Data*; The whole Content of the Cask, the *Diameter* at the Bouldge, and the wet Portion thereof; and the Proportion runs thus;

As the whole *Diameter*, is to its wet Portion:

So is the *Diameter* in the *Table*, (*i. e.*) 10000, to its like Portion: which being ſought in the *Table* of *Segments* abutts you to a *Segment*, by which if you Multiply the whole Content of the Cask, the Product is the Content of the remaining Liquor in the Cask.

LONDON, Printed by *T. Cobbid*, 1669.

CAMBRIDGE: PRINTED BY W. METCALFE.

H:84 5*

The ſumme of what I have heretofore written or ſpoken to you, concerning *the advancement of the Mathematickes*, is this: As long as men want *will*, *wit*, *meanes* or *leiſure* to attend thoſe ſtudies, it is no marvaile if they make no great progreſſe in them. To remedy which, I conceive *theſe meanes* not to be amiſſe.

1. To write a *Conſiliarius Mathematicus*, (ſo I call it) anſwering to theſe 3. queſtions:
   - Q. 1. What *fruit or profit* ariſeth from the ſtudy of *Mathematics?*
   - Q. 2. What *helpes* are there for the attaining this profitable knowledge?
   - Q. 3. What *order* is to be obſerved in uſing theſe helpes?

To this purpoſe it ſhould containe
A plaine and popular *diſcourſe* of *the extent* of the Mathematics, with *the profit* that redounds, firſt to the *Student* himſelfe, and then to the *Country* wherein there are many ſuch grounded Artiſts.
A *Catalog* of Mathematicians and their workes in this order:
1. A Synopſis of all the ſeverall *kindes* of Mathematicall writings, either *extant* in print, or *acceſſible* Manuſcripts in public Libraries, with ſeverall numbers ſet to every kinde.
2. A Chronicall *Catalog* of all *Mathematicians names* that ever were of note, according to the order of the yeares *when they lived*, with the yeare when any of their workes were *firſt printed*.
3. A *Catalog* of the *writings themſelves*, in the order of yeares in which they were *printed* in any language: And this I would contrive thus: Firſt, the yeere of our *Lord*, and then the names of all the Mathematicall bookes printed that yeare in any country or language, after the uſuall manner of Catalogs;
   but 1. *adding* the volume, that is, not onely what fold [4°.8°.&c.] but alſo the number of leaves; that we may eſtimate the bulke of the booke.
   2. *præfixing* before the title, the yeare to which you muſt looke backe to know either when it was written, or when it was laſt before printed, in that or any other language.
   3. *ſetting* in the margent, after the title, 1. the yeare wherein it was the *next time* printed:
      2. the *number of reference* to the *Synopſis* in the firſt page; By which numbers one may preſently runne over all the bookes of one ſort, of this or that particular ſubject.

A *Counſell* directing a ſtudent to the *beſt* bookes in every kinde; In what *order*, and *how* to reade them, What to obſerve, what to beware of in ſome Mathematicaſters, how to *proceed* and keepe all.
A *Parænesis*, *First*, To *all thoſe* who have *meanes* and *leiſure*, and *a wit* not unapt for theſe ſtudies, to *ſet* upon them in regard 1. of their *profitableneſſe* to the ſtudent, and to mankinde, and 2. of that more refined *pleaſure* of hunting out hidden truths, wraſtling with difficult Problemes, and getting the victory; and ſo much the rather, becauſe 3. of the great *facility* that is now in that ſtudy, by reaſon of the many helpes to ſpare much *labour*, *time* and *coſt*, which our forefathers were forced to ſpend. *Secondly*, To all thoſe who have *underſtanding* to eſtimate the worth of theſe ſtudies, and *wealth* wherewith to purchaſe to themſelves laſting honour by the wiſe diſpenſing of it, to take more notice of this ſort of ſtudents, and to encourage them, ſetting apart the choiceſt of them, to perfect the inventions, to which their *Genius* leads them: Eſpecially, To *all Princes* and *Eſtates*, whom it concernes to take a courſe 1. that their dominions may be better furniſhed with this ſort of ſtudents, 2. that the way may be made leſſe laborious and coſtly, 3. that *Mathematica ingenia* may be diſcovered and aſſiſted.

To which end it would be good

To erect a *Public Library*, containing all thoſe bookes, and one inſtrument of every ſort that hath beene invented, with ſuch a revenue, 1. to buy one copy of all thoſe that ſhall be printed yearly in other countries.
and 2. to maintaine a Library keeper of great judgement, to whom it may belong †
1. to peruſe all books of ſuch ſubjects, to be printed within that country, and 1. ſuppreſſe whatſoever is not according to Art; [illegible] be not abuſed, and 2. admoniſh the writers, if they bring nothing but ſtale ſtuffe.
[illegible] to cherish excellent Inventions, and unpartially to commend the inventors to the rewarders.
[illegible] bookes ſo peruſed, ſent into the Library, well bound [illegible] the Author or Book-ſellers charge.
[illegible] to reſolve any ſtudent that enquires about any Book [illegible] whether it be there, or hath beene there already or no, fearing leſt he ſhould *actum agere*, and therefore perhaps ſuppreſſing [illegible] information which he hath light upon, but doubts it is old and to be found in ſome of the bookes of that Library.
[illegible] to receive, &c. all ſuch Manuſcripts as ſhall be brought thither by way of gift, Legacy, or the like.
[illegible] to maintaine correſpondence with ſuch as himſelfe in other countries, to know what is printed there.
[illegible] to take notice of all his countrimen that are fit to be teachers, &c.
[illegible] to keepe a Catalog of all ſuch workmen as are able and fit to be imployed in making Mathematicall inſtruments and repreſentations, working upon wood, magnets, metalls, glaſſe, &c.
[illegible] to give teſtimoniall, after examination, to all ſorts of practitioners, as Pilots, Maſters, Landmeters, Accomptants, &c. of their ſpeculative ability and practicall dexterity, that thoſe that have occaſion to uſe ſuch men, be not abuſed by unable undertakers, to their exceeding great damage.
By the Catalog men might be informed, in that [illegible] of bookes, with which the world is now peſtered, what the *names* are of thoſe bookes that tend to this [illegible] ſort.
In the Library, they might finde the *bookes themſelves* [illegible] them, and if they liked them, know whither to ſend to buy them. Beſides, any, whether forreiners [illegible] might have recourſe thither, and learne what helps that country would afford them in theſe ſtudies.
And this is the beſt courſe that I can thinke on for the making [illegible] *ſuch helpes as we have already.* If men deſire *better* helpes, let them employ *fit* Artiſts
3. To write and publiſh theſe *three new Treatiſes*: †
1. *Pandectæ Mathematicæ*, Comprehending as cleerly, orderly, [illegible]tily, and inventuouſly as may be, whatſoever may be gathered out of all thoſe Mathematicall bookes and inventions that were before us, or that may be inferred as Conſectaries thereon; *citing*, at the end of every period or propoſition, the ancienteſt Author in which it is found, and *branding* all later writers if they be taken ſtealing, or borrowing without acknowledgment, or [which is worſe] expreſly arrogating to themſelves any other mans inventions. This would bring that huge *Library* into farre leſſe roome, to the ſaving of more *labour*, *time* and *coſt*, to all after-ſtudents, than we can yet well imagine. But becauſe this alſo would be too great and cumberſome to carry about us, Let there be compoſed
2. *Comes Mathematicus*, Comprehending in a pocket-booke, [therefore as briefly as may be] the uſefulleſt Tables and the Precepts for their uſe, in ſolving all Problemes, whether purely-Mathematicall, or applied to ſuch practices as mens various occaſions may require.

And laſtly, that in this kinde of Learning alſo, we be no longer tyed to bookes, Let there be compoſed
3. *Mathematicus automatus*, or An inſtruction, ſhewing how any Mathematician that will take the paines, may prepare himſelfe, ſo, as that he may, though he be utterly deſtitute of bookes or inſtruments, reſolve any Mathematicall Probleme as exactly as if he had a complete *Library* by him.

And this is the *Idéa*, which I have long framed to my ſelfe, according to my faſhion, with
ted truth, that the ſureſt way to come to all poſſible excellency *in any thing*, is to propoſe to
we can imagine, then to ſeeke the meanes tending thereto, as rationally as may be, and to
gence; yet, if the *Idéa* prove too high for us, to reſt our ſelves content with *approximation*.

As for this preſent *Idéa*, I am ſo farre from counting it *meerely-impoſſible*, that I ſee not why it might
one man, without any aſſiſtants, provided that he were neither *diſtracted* with cares for his maintenance,
employments.

The excellency of this worke, makes me wiſh mine owne nation the honour of *firſt* undertaking and perfe
And I conceive I have ſome reaſon to hope that it will be ſo. For, though I know few or none that are both
promote deſignes of this nature, yet can I not therefore be perſuaded that this Kingdome is ſo deſtitute of
and Gentry, that there can be found none to countenance and advance *this* part of Learning, even in *this* way,
ſee it poſſible and likely to be effected.

1.2 As for the *Library* and *Catalog*, there can be no doubt but they may eaſily be had, if mony be not wanting. Nor is it unl
that divers of this nation (if they be ſet apart for it) are able to compoſe the other 3. *new Treatiſes*; For though I kno
ſuch, yet I perſuade my ſelfe there may be found amongſt us men able to encounter all the difficulties, and to endure all th
bour, that they muſt needs meet with in the raiſing of ſo great a *Fabricke*. And I the rather beleeve that there are many ſ
becauſe for mine *owne part*, notwithſtanding the want of Counſell and helpes in that ſtudy, and the innumerable diverſions
diſtractions that I have had, I am nevertheleſſe come to ſuch a confidence of my underſtanding the depth of that ſtudy, t
3. were I to pen thoſe *Pandects*, I ſhould lay heavier lawes upon my ſelfe, than I have already mentioned; namely, *Firſt* to
downe ſuch an exact *Method* or deſcription of the proceſſe of Mans reaſon in inventions, that *afterward* it ſhould be impu
meerly to my negligence and diſobedience to my owne lawes, [and not to their inſufficiency] if, from my firſt grounds, ſee
or principles, I did not, in an orderly way, according to that præſcribed Method, deduce, not onely all that ever is to be fo
in our Anteceſſors writings, and whatſoever they may ſeeme to have thought on, but alſo all the Mathematicall inventio
Theoremes, Problemes and Precepts, that it is poſſible for the working wits of our ſucceſſors to light upon, and that in one c
taine, unchanged order, from the firſt ſeeds of Mathematics, to their higheſt and nobleſt applications, as well as to the mean
and moſt ordinary. Not ſetting them down at randon as they come in my head, as thoſe before us have done, ſo that they ſeeme
have light upon their Problemes and the ſolutions of them by chance, not to have found them by one perpetuall, conſtant, in
riable proceſſe of Art. Yet ſuch an Art may men invent, if they accuſtome themſelves, as I have long done, to conſider, not o
the *uſefulneſſe* of mens workes, and the *meaning* and *truth* of their writings, but alſo *how it came to paſſe* that they fell upon ſ
thoughts, and that they propoſed to themſelves ſuch ends, or found out ſuch meanes for them.

Were theſe *Pandects* thus made and finiſhed, I ſuppoſe it is manifeſt; that by their orderly, rationall and uniforme co
pleatneſſe, above all that hath beene hitherto written, they would ſpare after-ſtudents much *labour and time* that is now ſp
in *ſeeking* out of bookes, and *diſorderly reading* them, and *ſtruggling* with their cloudy expreſſions, unapt repreſentations,
ferent Methods, confuſions, tautologies, impertinencies, falſhoods by paralogiſmes and pſeudographemes, uncertainties
cauſe of inſufficient demonſtrations, &c. beſides much *coſt* alſo, now throwne away upon the multitude of bookes, the gr
ter part whereof they had perhaps beene better never to have ſeene.

And it may be ſome would like the Method of that worke ſo well, as to extend it farther, and apply it to other ſtudies
ſpeculation imitating this my warineſſe, that *no falſhood be admitted, and no truth omitted*; and *for practice* enuring themſelv
Any ſubject being propounded, to determine the number of all the Problemes that can be conceived concerning it, and
Probleme being propounded, demonſtratively to ſhew *either all the meanes* of its ſolution, *or the impoſſibility* of it: and if
then whether it be *not yet*, or *not at all* poſſible.

4. Men would eaſily ſee how to contract *theſe Pandects* into *a pocket-booke* for their ordinary uſe. But ſo to lay them up in th
heads, as to need *no booke at all* [according to that *deſideratum* of mine, to have a *Mathematicus αὐτόγραφος*] will perhaps ſee
5. utterly impoſſible to moſt: No man, that ever I heard of, having propoſed ſuch a ſcope to himſelfe before. But perh
they will conceive it more poſſible, if it pleaſe them to ſuſpend their judgements, till they have conſidered *what meanes*
be uſed to fortifie the imagination, to prompt the memory, to regulate our reaſon, and *what effects* may be produced by
uniting of theſe meanes, and the conſtant exerciſing of them.

For mine owne part, the conſideration of the *incomparable excellency, unſtained pleaſure, unvaluable profitablneſſe, and*
doubted poſſibility of this whole deſigne, hath prevailed ſo farre with me, that notwithſtanding all the diſcouragements th
have met withall, I have done more towards it than [illegible] *idéa*.

H-745.

K

6

528.

TABULA

Numerorum *QUADRATORUM* Decies millium, unà cum ipſorum *Lateribus* ab Unitate incipientibus & ordine naturali uſque ad 10000 progredientibus.

Τετράγωνός ἐστιν ἐξ ἀριθμοῦ τινος ἐφ' ἑαυτὸν πολλαπλασιασθέντος, οὗτος γὰρ ὁ ἀριθμὸς καλεῖται Πλευρὰ τοῦ τετραγώνου.

Quadratus *ſit numero aliquo in ſe multiplicato, qui numerus* Latus *quadrati vocatur.*

DIOPHANTUS Lib. 1. Defin. 1.

MVSEVM BRITANNICVM

A TABLE *Of Ten thouſand* SQUARE NUMBERS,

Namely,

Of all the Square Numbers *between* 0 *and* 100 *Millions*;

And of their SIDES *or* ROOTS,

Which are all the whole Numbers between 1 *and Ten thouſand.*

LONDON,

Printed by *Thomas Ratcliffe*, and *Nath. Thompſon*, and are to be ſold by *Moſes Pitt* at the *White Hart* in *Little Britain*, 1672.

Dr Pells Tables

A Table of whole Numbers, and their Squares.

	0..	1..	2..	3..	4..	5..	6..	7..	8..	9..
00	0	1 0000	4 0000	9 0000	16 0000	25 0000	36 0000	49 0000	64 0000	81 0000
01	1	1 0201	4 0401	9 0601	16 0801	25 1001	36 1201	49 1401	64 1601	81 1801
02	4	1 0404	4 0804	9 1204	16 1604	25 2004	36 2404	49 2804	64 3204	81 3604
03	9	1 0609	4 1209	9 1809	16 2409	25 3009	36 3609	49 4209	64 4809	81 5409
04	16	1 0816	4 1616	9 2416	16 3216	25 4016	36 4816	49 5616	64 6416	81 7216
05	25	1 1025	4 2025	9 3025	16 4025	25 5025	36 6025	49 7025	64 8025	81 9025
06	36	1 1236	4 2436	9 3636	16 4836	25 6036	36 7236	49 8436	64 9636	82 0836
07	49	1 1449	4 2849	9 4249	16 5649	25 7049	36 8449	49 9849	65 1249	82 2649
08	64	1 1664	4 3264	9 4864	16 6464	25 8064	36 9664	50 1264	65 2864	82 4464
09	81	1 1881	4 3681	9 5481	16 7281	25 9081	37 0881	50 2681	65 4481	82 6281
10	100	1 2100	4 4100	9 6100	16 8100	26 0100	37 2100	50 4100	65 6100	82 8100
11	121	1 2321	4 4521	9 6721	16 8921	26 1121	37 3321	50 5521	65 7721	82 9921
12	144	1 2544	4 4944	9 7344	16 9744	26 2144	37 4544	50 6944	65 9344	83 1744
13	169	1 2769	4 5369	9 7969	17 0569	26 3169	37 5769	50 8369	66 0969	83 3569
14	196	1 2996	4 5796	9 8596	17 1396	26 4196	37 6996	50 9796	66 2596	83 5396
15	225	1 3225	4 6225	9 9225	17 2225	26 5225	37 8225	51 1225	66 4925	83 7225
16	256	1 3456	4 6656	9 9856	17 3056	26 6256	37 9456	51 2656	66 5856	83 9056
17	289	1 3689	4 7089	10 0489	17 3889	26 7289	38 0689	51 4089	66 7489	84 0889
18	324	1 3924	4 7524	10 1124	17 4724	26 8324	38 1924	51 5524	66 9124	84 2724
19	361	1 4161	4 7961	10 1761	17 5561	26 9361	38 3161	51 6961	67 0761	84 4561
20	400	1 4400	4 8400	10 2400	17 6400	27 0400	38 4400	51 8400	67 2400	84 6400
21	441	1 4641	4 8841	10 3041	17 7241	27 1441	38 5641	51 9841	67 4041	84 8241
22	484	1 4884	4 9284	10 3684	17 8084	27 2484	38 6884	52 1284	67 5684	85 0084
23	529	1 5129	4 9729	10 4329	17 8929	27 3529	38 8129	52 2729	67 7329	85 1929
24	576	1 5376	5 0176	10 4976	17 9776	27 4576	38 9376	52 4176	67 8976	85 3776
25	625	1 5625	5 0625	10 5625	18 0625	27 5625	39 0625	52 5625	68 0625	85 5625
26	676	1 5876	5 1076	10 6276	18 1476	27 6676	39 1876	52 7076	68 2276	85 7476
27	729	1 6129	5 1529	10 6929	18 2329	27 7729	39 3129	52 8529	68 3929	85 9329
28	784	1 6384	5 1984	10 7584	18 3184	27 8784	39 4384	52 9984	68 5584	86 1184
29	841	1 6641	5 2441	10 8241	18 4041	27 9841	39 5641	53 1441	68 7241	86 3041
30	900	1 6900	5 2900	10 8900	18 4900	28 0900	39 6900	53 2900	68 8900	86 4900
31	961	1 7161	5 3361	10 9561	18 5761	28 1961	39 8161	53 4361	69 0561	86 6761
32	1024	1 7424	5 3824	11 0224	18 6624	28 3024	39 9424	53 5824	69 2224	86 8624
33	1089	1 7689	5 4289	11 0889	18 7489	28 4089	40 0689	53 7289	69 3889	87 0489
34	1156	1 7956	5 4756	11 1556	18 8356	28 5156	40 1956	53 8756	69 5556	87 2356
35	1225	1 8225	5 5225	11 2225	18 9225	28 6225	40 3225	54 0225	69 7225	87 4225
36	1296	1 8496	5 5696	11 2896	19 0096	28 7296	40 4496	54 1696	69 8896	87 6096
37	1369	1 8769	5 6169	11 3569	19 0969	28 8369	40 5769	54 3169	70 0569	87 7969
38	1444	1 9044	5 6644	11 4244	19 1844	28 9444	40 7044	54 4644	70 2244	87 9844
39	1521	1 9321	5 7121	11 4921	19 2721	29 0521	40 8321	54 6121	70 3921	88 1721
40	1600	1 9600	5 7600	11 5600	19 3600	29 1600	40 9600	54 7600	70 5600	88 3600
41	1681	1 9881	5 8081	11 6281	19 4481	29 2681	41 0881	54 9081	70 7281	88 5481
42	1764	2 0164	5 8564	11 6964	19 5364	29 3764	41 2164	55 0564	70 8964	88 7364
43	1849	2 0449	5 9049	11 7649	19 6249	29 4849	41 3449	55 2049	71 0649	88 9249
44	1936	2 0776	5 9536	11 8336	19 7136	29 5936	41 4736	55 3536	71 2336	89 1136
45	2025	2 1025	6 0025	11 9025	19 8025	29 7025	41 6025	55 5025	71 4025	89 3025
46	2116	2 1316	6 0516	11 9716	19 8916	29 8116	41 7316	55 6516	71 5716	89 4916
47	2209	2 1609	6 1009	12 0409	19 9809	29 9209	41 8609	55 8009	71 7409	89 6809
48	2304	2 1904	6 1504	12 1104	20 0704	30 0304	41 9904	55 9504	71 9104	89 8704
49	2401	2 2201	6 2001	12 2801	20 1601	30 1401	42 1201	56 1001	72 0801	90 0601

	0..	1..	2..	3..	4..	5..	6..	7..	8..	9..
50	2500	2 2500	6 2500	12 2500	20 2500	30 2500	42 2500	56 2500	72 2500	90 2500
51	2601	2 2801	6 3001	12 3201	20 3401	30 3601	42 3801	56 4001	72 4201	90 4401
52	2704	2 3104	6 3504	12 3904	20 4304	30 4704	42 5104	56 5504	72 5904	90 6304
53	2809	2 3409	6 4009	12 460[illegible]	[illegible]0 5209	30 5809	42 6409	56 7009	72 7609	90 8209
54	2916	2 3716	6 4516	12 [illegible]	[illegible]0 6116	30 6916	42 7716	56 8516	72 9316	91 0116
55	3025	2 4025	6 50[illegible]	12 6[illegible]5	20 7025	30 8025	42 9025	57 0025	73 1025	91 2025
56	3136	2 4336	6 5536	12 6736	20 7936	30 9136	43 0336	57 1536	73 2736	91 3936
57	3249	2 4649	6 6049	12 7449	20 8849	31 0249	43 1649	57 3049	73 4449	91 5849
58	3364	2 4964	6 6564	12 8164	20 9764	31 1364	43 2964	57 4564	73 6164	91 7764
59	3481	2 5281	6 7081	12 8881	21 0681	31 2481	43 4281	57 6081	73 7881	91 9681
60	3600	2 5600	6 7600	12 9600	21 1600	31 3600	43 5600	57 7600	73 9600	92 1600
61	3721	2 5921	6 8121	13 0321	21 2521	31 4721	43 6921	57 9121	74 1[illegible]1	92 3521
62	3844	2 6244	6 8644	13 1044	21 3444	31 5844	43 8244	58 0644	74 3044	92 5444
63	3969	2 6569	6 9169	13 1769	21 4369	31 6969	43 9569	58 2169	74 4769	92 7369
64	4096	2 6896	6 9696	13 2496	21 5296	31 8096	44 0896	58 3696	74 6496	92 9296
65	4225	2 7225	7 0225	13 3225	21 6225	31 9225	44 2225	58 5225	74 8225	93 1225
66	4356	2 7556	7 0756	13 3956	21 7156	32 0356	44 3556	58 6756	74 9956	93 3156
67	4489	2 7889	7 1289	13 4689	21 8089	32 1489	44 4889	58 8289	75 1689	93 5089
68	4624	2 8224	7 1824	13 5424	21 9024	32 2624	44 6224	58 9824	75 3424	93 7024
69	4761	2 8561	7 2361	13 6161	21 9961	32 3761	44 7561	59 1361	75 5161	93 8961
70	4900	2 8900	7 2900	13 6900	22 0900	32 4900	44 8900	59 2900	75 6900	94 0900
71	5041	2 9241	7 3441	13 7641	22 1841	32 6041	45 0241	59 4441	75 8641	94 2841
72	5184	2 9584	7 3984	13 8384	22 2784	32 7184	45 1584	59 5984	76 0384	94 4784
73	5329	2 9929	7 4529	13 9129	22 3729	32 8329	45 2929	59 7529	76 2129	94 6729
74	5476	3 0276	7 5076	13 9876	22 4676	32 9476	45 4276	59 9076	76 3876	94 8676
75	5625	3 0625	7 5625	14 0625	22 5625	33 0625	45 5625	60 0625	76 5625	95 0[illegible]
76	5776	3 0976	7 6176	14 1376	22 6576	33 1776	45 6976	60 2176	76 7376	95 2[illegible]
77	5929	3 1329	7 6729	14 2129	22 7529	33 2[illegible]9	45 8329	60 3729	76 9129	95 4[illegible]
78	6084	3 1684	7 7284	14 2884	22 8484	33 4084	45 9684	60 5284	77 0884	95 6[illegible]
79	6241	3 2041	7 7841	14 3641	22 9441	33 5241	46 1041	60 6841	77 2641	95 8441
80	6400	3 2400	7 8400	14 4400	23 0400	33 6400	46 2400	60 8400	77 4400	96 0400
81	6561	3 2761	7 8961	14 5161	23 1361	33 7561	46 3761	60 9961	77 6161	96 2361
82	6724	3 3124	7 9524	14 5924	23 2324	33 8724	46 5124	61 1524	77 7924	96 4324
83	6889	3 3489	8 0089	14 6689	23 3289	33 9889	46 6489	61 3089	77 9689	96 6289
84	7056	3 3856	8 0656	14 7456	23 4256	34 1056	46 7856	61 4656	78 1456	96 8256
85	7225	3 4225	8 1225	14 8225	23 5225	34 2225	46 9225	61 6225	78 3225	97 0225
86	7396	3 4596	8 1796	14 8996	23 6196	34 3396	47 0596	61 7796	78 4996	97 2196
87	7569	3 4969	8 2369	15 9769	23 7169	34 4569	47 1969	61 9369	78 6769	97 4169
88	7744	3 5344	8 2944	15 0544	23 8144	34 5744	47 3344	62 0944	78 8544	97 6144
89	7921	3 5721	8 3521	15 1321	23 9121	34 6921	47 4721	62 2521	79 0321	97 8121
90	8100	36 100	8 4100	15 2100	24 0100	34 8100	47 6100	62 4100	79 2100	98 0100
91	8281	3 6481	8 4681	15 2881	24 1081	34 9281	47 7481	62 5681	79 3881	98 2081
92	8464	3 6864	8 5264	15 3664	24 2064	35 0464	47 8864	62 7264	79 5664	98 4064
93	8649	3 7249	8 5849	15 4449	24 3049	35 1649	48 0249	62 8849	79 7449	98 6049
94	8836	3 7636	8 6436	15 5236	24 4036	35 2836	48 1636	63 0436	79 9236	98 8036
95	9025	3 8025	8 7025	15 6025	24 5025	35 4025	48 3025	63 2025	80 1025	99 0025
96	9216	3 8416	8 7616	15 6816	24 6016	35 5216	48 4416	63 3616	80 2816	99 2016
97	9409	3 8809	8 8209	15 7609	24 7009	35 6409	48 5809	63 5209	80 4609	99 4009
98	9604	3 9204	8 8804	15 8404	24 8004	35 7604	48 7204	63 6804	80 6404	99 6004
99	9801	3 9601	8 9401	15 9201	24 9001	35 8801	48 8601	63 8401	80 8201	99 8001

A Table of whole Numbers, and their Squares.

	10..	11..	12..	13..	14..	15..	16..	17..
00	100 0000	121 0000	144 0000	169 0000	196 0000	225 0000	256 0000	289 0000
01	100 2001	121 2201	144 2401	169 2601	196 2801	225 3001	256 3201	289 3401
02	100 4004	121 4404	144 4804	169 5204	196 5604	225 6004	256 6404	289 6804
03	100 6009	121 6609	144 7209	169 7809	196 8409	[illegible] 009	256 9609	290 0209
04	100 8016	121 8816	144 9616	170 0416	197 1216	[illegible] 6	257 2816	290 3616
05	101 0025	122 1025	145 2025	170 3025	197 4025	[illegible]	257 6025	290 7025
06	101 2036	122 3236	145 4436	170 5636	197 6836	[illegible] 6	257 9236	291 0436
07	101 4049	122 5449	145 6849	170 8249	197 9649	227 1049	258 2449	291 3849
08	101 6064	122 7664	145 9264	171 0864	198 2464	227 4064	258 5664	291 7264
09	101 8081	122 9881	146 1681	171 3481	198 5281	227 7081	258 8881	292 0681
10	102 0100	123 2100	146 4100	171 6100	198 8100	228 0100	259 2100	292 4100
11	102 2121	123 4321	146 6521	171 8721	199 0921	228 3121	259 5321	292 7521
12	102 4144	123 6544	146 8944	172 1344	199 3744	228 6144	259 8544	293 0944
13	102 6169	123 8769	147 1369	172 3969	199 6569	228 9169	260 1769	293 4369
14	102 8196	124 0996	147 3796	172 6596	199 9396	229 2196	260 4996	293 7796
15	103 0225	124 3225	147 6225	172 9225	200 2225	229 5225	260 8225	294 1225
16	103 2256	124 5456	147 8656	173 1856	200 5056	229 8256	261 1456	294 4656
17	103 4289	124 7689	148 1089	173 4489	200 7889	230 1289	261 4689	294 8089
18	103 6324	124 9924	148 3524	173 7124	201 0724	230 4324	261 7924	295 1524
19	103 8361	125 2161	148 5961	173 9761	201 3561	230 7361	262 1161	295 4961
20	104 0400	125 4400	148 8400	174 2400	201 6400	231 0400	262 4400	295 8400
21	104 2441	125 6641	149 0841	174 5041	201 9241	231 3441	262 7641	296 1841
22	104 4484	125 8884	149 3284	174 7684	202 2084	231 6484	263 0884	296 5284
23	104 6529	126 1129	149 5729	175 0329	202 4929	231 9529	263 4129	296 8729
24	104 8576	126 3376	149 8176	175 2976	202 7776	232 2576	263 7376	297 2176
25	105 0625	126 5625	150 0625	175 5625	203 0625	232 5625	264 0625	297 5625
26	105 2676	126 7876	150 3076	175 8276	203 3476	232 8676	264 3876	297 9076
27	105 4729	127 0129	150 5529	176 0929	203 6329	233 1729	264 7129	298 2529
28	105 6784	127 2384	150 7984	176 3584	203 9184	233 4784	265 0384	298 5984
29	105 8841	127 4641	151 0441	176 0241	204 2041	233 7841	265 3641	298 9441
30	106 0900	127 6900	151 2900	176 8900	204 4900	234 0900	265 6900	299 2900
31	106 2961	127 9161	151 5361	177 1561	204 7761	234 3961	266 0161	299 6361
32	106 5024	128 1424	151 7824	177 4224	205 0624	234 7024	266 3424	299 9824
33	106 7089	128 3689	152 0289	177 6889	205 3489	235 0089	266 6689	300 3289
34	106 9156	128 5956	152 2756	177 9556	205 6356	235 3156	266 9956	300 6756
35	107 1225	128 8225	152 5225	178 2225	205 9225	235 6225	267 3225	301 0225
36	107 3296	129 0496	152 7696	178 4896	206 2096	235 9296	267 6496	301 3696
37	107 5369	129 2769	153 0169	178 7569	206 4969	236 2369	267 9769	301 7169
38	107 7444	129 5044	153 2644	179 0244	206 7844	236 5444	268 3044	302 0644
39	107 9521	129 7321	153 5121	179 2921	207 0721	236 8521	268 6321	302 4121
40	108 1600	129 9600	153 7600	179 5600	207 3600	237 1600	268 9600	302 7600
41	108 3681	130 1881	154 0081	179 8281	207 6481	237 4681	269 2881	303 1081
42	108 5764	130 4164	154 2564	180 0964	207 9364	237 7764	269 6164	303 4564
43	108 7849	130 6449	154 5049	180 3649	208 2249	238 0849	269 9449	303 8049
44	108 9936	130 8736	154 7536	180 6336	208 5136	238 3936	270 2736	304 1536
45	109 2025	131 1025	155 0025	180 9025	208 8025	238 7025	270 6025	304 5025
46	109 4116	131 3316	155 2516	181 1716	209 0916	239 0116	270 9316	304 8516
47	109 6209	131 5609	155 5009	181 4409	209 3809	239 3209	271 2609	305 2009
48	109 8304	131 7904	155 7504	181 7104	209 6704	239 6304	271 5904	305 5504
49	110 0401	132 0201	156 0001	181 9801	209 9601	239 9401	271 9201	305 9001

	10..	11..	12..	13..	14..	15..	16..	17..
50	110 2500	132 2500	156 2500	182 2500	210 2500	240 2500	272 2500	306 2500
51	110 4601	132 4801	156 5001	182 5201	210 5401	240 5601	272 5801	306 6001
52	110 6704	132 7104	156 7504	182 7904	210 8304	240 8704	272 9104	306 9504
53	110 8809	132 9409	157 0009	183 0609	211 1209	241 1809	273 2409	307 3009
54	111 0916	133 1716	157 2516	183 3316	211 4116	241 4916	273 5716	307 6516
55	111 3025	133 4025	157 5025	183 6025	211 7025	241 8025	273 9025	308 0025
56	111 5136	133 6336	157 7736	183 8736	211 9936	242 1136	274 2336	308 3536
57	111 7249	133 8649	158 0149	104 1449	212 2849	242 4249	274 5649	308 7040
58	111 9364	134 0964	158 2464	184 4164	212 5764	242 7364	274 8964	309 0564
59	112 1481	134 3281	158 5881	184 6881	212 8681	243 0481	275 2281	309 4081
60	112 3600	134 5600	158 7600	184 9600	213 1600	243 3600	275 5600	309 7600
61	112 5721	134 7921	159 0121	185 2321	213 4521	243 6721	275 8921	310 1121
62	112 7844	135 0244	159 2644	185 5044	213 7444	243 9844	276 2244	310 4644
63	112 9969	135 2569	159 5169	185 7769	214 0369	244 2969	276 5569	310 8169
64	113 2096	135 4896	159 7696	186 0496	214 3296	244 6096	276 8896	311 1696
65	113 4225	135 7225	160 0225	186 3225	214 6225	244 9225	277 2225	311 5225
66	113 6356	135 9556	160 2756	186 5956	214 9156	245 2356	277 5556	311 8756
67	113 8489	136 1889	160 5289	186 8689	215 2089	245 5489	277 8889	312 2289
68	114 0624	136 4224	160 7824	187 1424	215 5024	245 8624	278 2224	312 5824
69	114 2761	136 6561	161 0361	187 4161	215 7961	246 1761	278 5561	312 9361
70	114 4900	136 8900	161 2900	187 6900	216 0900	246 4900	278 8900	313 2900
71	114 7041	137 1241	161 5441	187 9641	216 3841	246 8041	279 2241	313 6441
72	114 9184	137 3584	161 7984	188 2384	216 6784	247 1184	279 5584	313 9984
73	115 1329	137 5929	162 0529	188 5129	216 9729	247 4329	279 8929	314 3529
74	115 3476	137 8276	162 3076	188 7876	217 2676	247 7476	280 2276	314 7076
75	115 5625	138 0625	162 5625	189 0625	217 5625	248 0625	280 5625	315 0625
76	115 7776	138 2976	162 8176	189 3376	217 8576	248 3776	280 8976	315 4176
77	115 9929	138 5329	163 0729	189 6129	218 1529	248 6929	281 2329	315 7729
78	116 2084	138 7684	163 3284	189 8884	218 4484	249 0084	281 5684	316 1284
79	116 4241	139 0041	163 5841	190 1641	218 7441	249 3241	281 9041	316 4841
80	116 6400	139 2400	163 8400	190 4400	219 0400	249 6400	282 2400	316 8400
81	116 8561	139 4761	164 0961	190 7161	219 3361	249 9561	282 5761	317 1961
82	117 0724	139 7124	164 3524	190 9924	219 6324	250 2724	282 9124	317 5524
83	117 2889	139 9489	164 6089	191 2689	219 9289	250 5889	283 2489	317 9089
84	117 5056	140 1856	164 8656	191 5456	220 2256	250 9056	283 5856	318 2656
85	117 7225	140 4225	165 1225	191 8125	220 5225	251 2225	283 9225	318 6225
86	117 9396	140 6596	165 3796	192 0996	220 8196	251 5396	284 2596	318 9796
87	118 1569	140 8969	165 6369	192 3769	221 1169	251 8569	284 5969	319 3369
88	118 3744	141 1344	165 8944	192 6544	221 4144	252 1744	284 9344	319 6944
89	118 5921	141 3721	166 1521	192 9321	221 7121	252 4921	285 2721	320 0521
90	118 8100	141 6100	166 4100	193 2100	222 0100	252 8100	285 6100	320 4100
91	119 0281	141 8481	166 6681	193 4881	222 3081	213 1281	285 9481	320 7681
92	119 2464	142 0864	166 9264	193 7664	222 6064	253 4464	286 2864	321 1264
93	119 4649	142 3249	167 1849	194 0449	222 9049	253 7649	286 6249	321 4849
94	119 6836	142 5636	167 4436	194 3236	223 2036	254 0836	286 9636	321 8436
95	119 9025	142 8025	167 7025	194 6025	223 5025	254 4025	287 3025	322 2025
96	120 1216	143 0416	167 9616	194 8816	223 8016	254 7216	287 6416	322 5616
97	120 3409	143 2809	168 2209	185 1609	224 1009	255 0409	287 9809	322 9209
98	120 5604	143 5204	168 4804	195 4404	224 4004	255 3604	288 3204	323 2804
99	120 7801	143 7601	168 7401	195 7201	224 7001	255 6801	288 6601	323 6401

A Table of whole Numbers, and their Squares.

	18..	19..	20..	21..	22..	23..	24..	25..
00	324 0000	361 0000	400 0000	441 0000	484 0000	529 0000	576 0000	625 0000
01	324 3601	361 3801	400 4001	441 4201	484 4401	529 4601	576 4801	625 5001
02	324 7204	361 7604	400 8004	441 8404	484 8804	529 9204	576 9604	626 0004
03	325 0809	362 1409	401 2009	442 2609	485 3209	530 3809	577 4409	626 5009
04	325 4416	362 5216	401 6016	442 6816	485 7616	530 8416	577 9216	627 0016
05	325 8025	362 9025	402 0025	443 1025	486 2025	531 3025	578 4025	627 5025
06	326 1636	363 2836	402 4036	443 5236	486 6436	531 7636	578 8836	628 0036
07	326 5249	363 6649	402 8049	443 9449	487 0849	532 2249	579 3649	628 5049
08	326 8864	364 0464	403 2064	444 3664	487 5264	532 6864	579 8464	629 0064
09	327 2481	364 4281	403 6081	444 7881	487 9681	533 1481	580 3281	629 5081
10	327 6100	364 8100	404 0100	445 2100	488 4100	533 6100	580 8100	630 0100
11	327 9721	365 1921	404 4121	445 6321	488 8521	534 0721	581 2921	630 5121
12	328 3344	365 5744	404 8144	446 0544	489 2944	534 5344	581 7744	631 0144
13	328 6969	365 9569	405 2169	446 4769	489 7369	534 9969	582 2569	631 5169
14	329 0596	366 3396	405 6196	446 8996	490 1796	535 4596	582 7396	632 0196
15	329 4225	366 7225	406 0225	447 3225	490 6225	535 9225	583 2225	632 5225
16	329 7856	367 1056	406 4256	447 7456	491 0656	536 3856	583 7056	633 0256
17	330 1489	367 4889	406 8289	448 1689	491 5089	536 8489	584 1889	633 5289
18	330 5124	367 8724	407 2324	448 5924	491 9524	537 3124	584 6724	634 0324
19	330 8761	368 2561	407 6361	449 0161	492 3961	537 7761	585 1561	634 5361
20	331 2400	368 6400	408 0400	449 4400	492 8400	538 2400	585 6400	635 0400
21	331 6041	369 0241	408 4441	449 8641	493 2841	538 7041	586 1241	635 5441
22	331 9684	369 4084	408 8484	450 2884	493 7284	539 1684	586 6084	636 0484
23	332 3329	369 7929	409 2529	450 7129	494 1729	539 6329	587 0929	636 5529
24	332 6976	370 1776	409 6576	451 1376	494 6176	540 0976	587 5776	637 0576
25	333 0625	370 5625	410 0625	451 5625	495 0625	540 5625	588 0625	637 5625
26	333 4276	370 9476	410 4676	451 9876	495 5076	541 0276	588 5476	638 0676
27	333 7929	371 3329	410 8729	452 4129	495 9529	541 4929	589 0329	638 5729
28	334 1584	371 7184	411 2784	452 8384	496 3984	541 9584	589 5184	639 0784
29	334 5241	372 1041	411 6841	453 2641	496 8441	542 4241	590 0041	639 5841
30	334 8900	372 4900	412 0900	453 6900	497 2900	542 8900	590 4900	640 0900
31	335 2561	372 8761	412 4961	454 1161	497 7361	543 3561	590 9761	640 5961
32	335 6224	373 2624	412 9024	454 5424	498 1824	543 8224	591 4624	641 1024
33	335 9889	373 6489	413 3089	454 9689	498 6289	544 2889	591 9489	641 6089
34	336 3556	374 0356	413 7156	455 3956	499 0756	544 7556	592 4356	642 1156
35	336 7225	374 4225	414 1225	455 8225	499 5225	545 2225	592 9225	642 6225
36	337 0896	374 8096	414 5296	456 2496	499 9696	545 6896	593 4096	643 1296
37	337 4569	375 1969	414 9369	456 6769	500 4169	546 1569	593 8969	643 6369
38	337 8244	375 5844	415 3444	457 1044	500 8644	546 6244	594 3844	644 1444
39	338 1921	375 9721	415 7521	457 5321	501 3121	547 0921	594 8721	644 6521
40	338 5600	376 3600	416 1600	457 9600	501 7600	547 5600	595 3600	645 1600
41	338 9281	376 7481	416 5681	458 3881	502 2081	548 0281	595 8481	645 6681
42	339 2964	377 1364	416 9764	458 8164	502 6564	548 4964	596 3364	646 1764
43	339 6649	377 5249	417 3849	459 2449	503 1049	548 9649	596 8249	646 6849
44	340 0336	377 9136	417 7936	459 6736	503 5536	549 4336	597 3136	647 1936
45	340 4025	378 3025	418 2025	460 1025	504 0025	549 9025	597 8025	647 7025
46	340 7716	378 6916	418 6116	460 5316	504 4516	550 3716	598 2916	648 2116
47	341 1409	379 0809	419 0209	460 9609	504 9009	550 8409	598 7809	648 7209
48	341 5104	379 4704	419 4304	461 3904	505 3504	551 3104	599 2704	649 2304
49	341 8801	379 8601	419 8401	461 8201	505 8001	551 7801	599 7601	649 7401

	18..	19..	20..	21..	22..	23..	24..	25..
50	342 2500	380 2500	420 2500	462 2500	506 2500	552 2500	600 2500	650 2500
51	342 6201	380 6401	420 6601	462 6801	506 7001	552 7201	600 7401	650 7601
52	342 9904	381 0304	421 0704	463 1104	507 1504	553 1904	601 2304	651 2704
53	343 3609	381 4209	421 4809	463 5409	507 6009	553 6609	601 7209	651 7809
54	343 7316	381 8116	421 8916	463 9716	508 0516	554 1316	602 2116	652 2916
55	344 1025	382 2025	422 3025	464 4025	508 5025	554 6025	602 7025	652 8025
56	344 4736	382 5936	422 7136	464 8336	508 9536	555 0736	603 1936	653 3136
57	344 8449	382 9849	423 1249	465 2649	509 4049	555 5449	603 6849	653 8249
58	345 2164	383 3764	423 5364	465 6964	509 8564	556 0164	604 1764	654 3364
59	345 5881	383 7681	423 9481	466 1281	510 3081	556 4881	604 6681	654 8481
60	345 9600	384 1600	424 3600	466 5600	510 7600	556 9600	605 1600	655 3600
61	346 3321	384 5521	424 7721	466 9921	511 2121	557 4321	605 6521	655 8721
62	346 7044	384 9444	425 1844	467 4244	511 6644	557 9044	606 1444	656 3844
63	347 0769	385 3369	425 5969	467 8569	512 1169	558 3769	606 6369	656 8969
64	347 4496	385 7296	426 0096	468 2896	512 5696	558 8496	607 1296	657 4096
65	347 8225	386 1225	426 4225	468 7225	513 0225	559 3225	607 6225	657 9225
66	348 1956	386 5156	426 8356	469 1556	513 4756	559 7956	608 1156	658 4356
67	348 5689	386 9089	427 2489	469 5889	513 9289	560 2689	608 6089	658 9489
68	348 9424	387 3024	427 6624	470 0224	514 3824	560 7424	609 1024	659 4624
69	349 3161	387 6961	428 0761	470 4561	514 8361	561 2161	609 5961	659 9761
70	349 6900	388 0900	428 4900	470 8900	515 2900	561 6900	610 0900	660 4900
71	350 0641	388 4841	428 9041	471 3241	515 7441	562 1641	610 5841	661 0041
72	350 4384	388 8784	429 3184	471 7584	516 1984	562 6384	611 0784	661 5184
73	350 8129	389 2729	429 7329	472 1929	516 6529	563 1129	611 5729	662 0329
74	351 1876	389 6676	430 1476	472 6276	517 1076	563 5876	612 0676	662 5476
75	351 5625	390 0625	430 5625	473 0625	517 5625	564 0625	612 5625	663 0625
76	351 9376	390 4576	430 9776	473 4976	518 0176	564 5376	613 0576	663 5776
77	352 3129	390 8529	431 3929	473 9329	518 4729	565 0129	613 5529	664 0929
78	352 6884	391 2484	431 8084	474 3684	518 9284	565 4884	614 0484	664 6084
79	353 0641	391 6441	432 2241	474 8041	519 3841	565 9641	614 5441	665 1241
80	353 4400	392 0400	432 6400	475 2400	519 8400	566 4400	615 0400	665 6400
81	353 8161	392 4361	433 0561	475 6761	520 2961	566 9161	615 5361	666 1561
82	354 1924	392 8324	433 4724	476 1124	520 7524	567 3924	616 0324	666 6724
83	354 5689	393 2289	433 8889	476 5489	521 2089	567 8689	616 5289	667 1889
84	354 9456	393 6256	434 3056	476 9856	521 6656	568 3456	617 0256	667 7056
85	355 3225	394 0225	434 7225	477 4225	522 1225	568 8225	617 5225	668 2225
86	355 6996	394 4196	435 1396	477 8596	522 5796	569 2996	618 0196	668 7396
87	356 0769	394 8169	435 5569	478 2969	523 0369	569 7769	618 5169	669 2569
88	356 4544	395 2144	435 9744	478 7344	523 4944	570 2544	619 0144	669 7744
89	356 8321	395 6121	436 3921	479 1721	523 9521	570 7321	619 5121	670 2921
90	357 2100	396 0100	436 8100	479 6100	524 4100	571 2100	620 0100	670 8100
91	357 5881	396 4081	437 2281	480 0481	524 8681	571 6881	620 5081	671 3281
92	357 9664	396 8064	437 6464	480 4864	525 3264	572 1664	621 0064	671 8464
93	358 3449	397 2049	438 0649	480 9249	525 7849	572 6449	621 5049	672 3649
94	358 7236	397 6036	438 4836	481 3636	526 2436	573 1236	622 0036	672 8836
95	359 1025	398 0025	438 9025	481 8025	526 7025	573 6025	622 5025	673 4025
96	359 4816	398 4016	439 3216	482 2416	527 1616	574 0816	623 0016	673 9216
97	359 8609	398 8009	439 7409	482 6809	527 6209	574 5609	623 5009	674 4409
98	360 2404	399 2004	440 1604	483 1204	528 0804	575 0404	624 0004	674 9604
99	360 6201	399 6001	440 5801	483 5601	528 5401	575 5201	624 5001	675 4801

A Table of whole Numbers, and their Squares.

	26..	27..	28..	29..	30..	31..	32..
00	676 0000	729 0000	784 0000	841 0000	900 0000	961 0000	1024 0000
01	676 5201	729 5401	784 5601	841 5801	900 6001	961 6201	1024 6401
02	677 0404	730 0804	785 1204	842 1604	901 2004	962 2404	1025 2804
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15	683 8225	737 1225	792 4225	849 7225	909 0225	970 3225	1033 6225
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17	684 8689	738 2089	793 5489	850 8889	910 2289	971 5689	1034 9089
18	685 3924	738 7524	794 1124	851 4724	910 8324	972 1924	1035 5524
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23	688 0129	741 4729	796 9329	854 3929	913 8529	975 3129	1038 7729
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27	690 1129	743 6529	799 1929	856 7329	916 2729	977 8129	1041 3529
28	690 6384	744 1984	799 7584	857 3184	916 8784	978 4384	1041 9984
29	691 1641	744 7441	800 3241	857 9041	917 4841	979 0641	1042 6441
30	691 6900	745 2900	800 8900	858 4900	918 0900	979 6900	1043 2900
31	692 2161	745 8361	801 4561	859 0761	918 6961	980 3161	1043 9361
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33	693 2689	746 9289	802 5889	860 2489	919 9089	981 5689	1045 2289
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62	708 6244	762 8644	819 1044	877 3444	937 5844	999 8244	1064 0644
63	709 1569	763 4169	819 6769	877 9369	938 1969	1000 4569	1064 7169
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67	711 2889	765 6289	821 9689	880 3089	940 6489	1002 9889	1067 3289
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69	712 3561	766 7361	823 1161	881 4961	941 8761	1004 2561	1068 6361
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85	720 9225	775 6225	832 3225	891 0225	951 7225	1014 4225	1079 1225
86	721 4596	776 1796	832 8996	891 6196	952 3396	1015 0596	1079 7796
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89	723 0721	777 8521	834 6321	893 4121	954 1921	1016 9721	1081 7521
90	723 6100	778 4100	835 2100	894 0100	954 8100	1017 6100	1082 4100
91	724 1481	778 9681	835 7881	894 6081	955 4281	1018 2481	1083 0681
92	724 6864	779 5264	836 3664	895 2064	956 0464	1018 8864	1083 7264
93	725 2249	780 0849	836 9449	895 8049	956 6649	1019 5249	1084 3849
94	725 7636	780 6436	837 5236	896 4036	957 2836	1020 1636	1085 0436
95	726 3025	781 2025	838 1025	897 0025	957 9025	1020 8025	1085 7025
96	726 8416	781 7616	838 6816	897 6016	958 5216	1021 4416	1086 3616
97	727 3809	782 3209	839 2609	898 2009	959 1409	1022 0809	1087 0209
98	727 9204	782 8804	839 8404	898 8004	959 7604	1022 7204	1087 6804
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37	1113 5569	1181 2969	1251 0369	1322 7769	1396 5169	1472 2569	1549 9969
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67	1133 6689	1202 0089	1272 3489	1344 6889	1419 0289	1495 3689	1573 7089
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81	1143 1161	1211 7361	1282 3561	1354 9761	1429 5961	1506 2161	1584 8361
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86	1146 4996	1215 2196	1285 9396	1358 6596	1433 3796	1510 0996	1588 8196
87	1147 1769	1215 9169	1286 6569	1359 3969	1434 1369	1510 8769	1589 6169
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22	1617 6484	1699 0884	1782 5284	1867 9684	1955 4084	2044 8484	2136 2884
23	1618 4529	1699 9129	1783 3729	1868 8329	1956 2929	2045 7529	2137 2129
24	1619 2576	1700 7376	1784 2176	1869 6976	1957 1776	2046 6576	2138 1376
25	1620 0625	1701 5625	1785 0625	1870 5625	1958 0625	2047 5625	2139 0625
26	1620 8676	1702 3876	1785 9076	1871 4276	1958 9476	2048 4676	2139 9876
27	1621 6729	1703 2129	1786 7529	1872 2929	1959 8329	2049 3729	2140 9129
28	1622 4784	1704 0384	1787 5984	1873 1584	1960 7184	2050 2784	2141 8384
29	1623 2841	1704 8641	1788 4441	1874 0241	1961 6041	2051 1841	2142 7641
30	1624 0900	1705 6900	1789 2900	1874 8900	1962 4900	2052 0900	2143 6900
31	1624 8961	1706 5161	1790 1361	1875 7561	1963 3761	2052 9961	2144 6161
32	1625 7024	1707 3424	1790 9824	1876 6224	1964 2624	2053 9024	2145 5424
33	1626 5089	1708 1689	1791 8289	1877 4889	1965 1489	2054 8089	2146 4689
34	1627 3156	1708 9956	1792 6756	1878 3556	1966 0356	2055 7156	2147 3956
35	1628 1225	1709 8225	1793 5225	1879 2225	1966 9225	2056 6225	2148 3225
36	1628 9296	1710 6496	1794 3696	1880 0896	1967 8096	2057 5296	2149 2496
37	1629 7369	1711 4769	1795 2169	1880 9569	1968 6969	2058 4369	2150 1769
38	1630 5444	1712 3044	1796 0644	1881 8244	1969 5844	2059 3444	2151 1044
39	1631 3521	1713 1321	1796 9121	1882 6921	1970 4721	2060 2521	2152 0321
40	1632 1600	1713 9600	1797 7600	1883 5600	1971 3600	2061 1600	2152 9600
41	1632 9681	1714 7881	1798 6081	1884 4281	1972 2481	2062 0681	2153 8881
42	1633 7764	1715 6164	1799 4564	1885 2964	1973 1364	2062 9764	2154 8164
43	1634 5849	1716 4449	1800 3049	1886 1649	1974 0249	2063 8849	2155 7449
44	1635 3936	1717 2736	1801 1536	1887 0336	1974 9136	2064 7936	2156 6736
45	1636 2025	1718 1025	1802 0025	1887 9025	1975 8025	2065 7025	2157 6025
46	1637 0116	1718 9316	1802 8516	1888 7716	1976 6916	2066 6116	2158 5316
47	1637 8209	1719 7609	1803 7009	1889 6409	1977 5809	2067 5209	2159 4609
48	1638 6304	1720 5904	1804 5504	1890 5104	1978 4704	2068 4304	2160 3904
49	1639 4401	1721 4201	1805 4001	1891 3801	1979 3601	2069 3401	2161 3201

	40..	41..	42..	43..	44..	45..	46..
50	1640 2500	1722 2500	1806 2500	1892 2500	1980 2500	2070 2500	2162 2500
51	1641 0601	1723 0801	1807 1001	1893 1201	1981 1401	2071 1601	2163 1801
52	1641 8704	1723 9104	1807 9504	1893 9904	1982 0304	2072 0704	2164 1104
53	1642 6809	1724 7409	1808 8009	1894 8609	1982 9209	2072 9809	2165 0409
54	1643 4916	1725 5716	1809 6516	1895 7316	1983 8116	2073 8916	2165 9716
55	1644 3025	1726 4025	1810 5025	1896 6025	1984 7025	2074 8025	2166 9025
56	1645 1136	1727 2336	1811 3536	1897 4736	1985 5936	2075 7136	2167 8336
57	1645 9249	1728 0649	1812 2049	1898 3449	1986 4849	2076 6249	2168 7649
58	1646 7364	1728 8964	1813 0564	1899 2164	1987 3764	2077 5364	2169 6964
59	1647 5481	1729 7281	1813 9081	1900 0881	1988 2681	2078 4481	2170 6281
60	1648 3600	1730 5600	1814 7600	1900 9600	1989 1600	2079 3600	2171 5600
61	1649 1721	1731 3921	1815 6121	1901 8321	1990 0521	2080 2721	2172 4921
62	1649 9844	1732 2244	1816 4644	1902 7044	1990 9444	2081 1844	2173 4244
63	1650 7969	1733 0569	1817 3169	1903 5769	1991 8369	2082 0969	2174 3569
64	1651 6096	1733 8896	1818 1696	1904 4496	1992 7296	2083 0096	2175 2896
65	1652 4225	1734 7225	1819 0225	1905 3225	1993 6225	2083 9225	2176 2225
66	1653 2356	1735 5556	1819 8756	1906 1956	1994 5156	2084 8356	2177 1556
67	1654 0489	1736 3889	1820 7289	1907 0689	1995 4089	2085 7489	2178 0889
68	1654 8624	1737 2224	1821 5824	1907 9424	1996 3024	2086 6624	2179 0224
69	1655 6761	1738 0561	1822 4361	1908 8161	1997 1961	2087 5761	2179 9561
70	1656 4900	1738 8900	1823 2900	1909 6900	1998 0900	2088 4900	2180 8900
71	1657 3041	1739 7241	1824 1441	1910 5641	1998 9841	2089 4041	2181 8241
72	1658 1184	1740 5584	1824 9984	1911 4384	1999 8784	2090 3184	2182 7584
73	1658 9329	1741 3929	1825 8529	1912 3129	2000 7729	2091 2329	2183 6929
74	1659 7476	1742 2276	1826 7076	1913 1876	2001 6676	2092 1476	2184 6276
75	1660 5625	1743 0625	1827 5625	1914 0625	2002 5625	2093 0625	2185 5625
76	1661 3776	1743 8976	1828 4176	1914 9376	2003 4576	2093 9776	2186 4976
77	1662 1929	1744 7329	1829 2729	1915 8129	2004 3529	2094 8929	2187 4329
78	1663 0084	1745 5684	1830 1284	1916 6884	2005 2484	2095 8084	2188 3684
79	1663 8241	1746 4041	1830 9841	1917 5641	2006 1441	2096 7241	2189 3041
80	1664 6400	1747 2400	1831 8400	1918 4400	2007 0400	2097 6400	2190 2400
81	1665 4561	1748 0761	1832 6961	1919 3161	2007 9361	2098 5561	2191 1761
82	1666 2724	1748 9124	1833 5524	1920 1924	2008 8324	2099 4724	2192 1124
83	1667 0889	1749 7489	1834 4089	1921 0689	2009 7289	2100 3889	2193 0489
84	1667 9056	1750 5856	1835 2656	1921 9456	2010 6256	2101 3056	2193 9856
85	1668 7225	1751 4225	1836 1225	1922 8225	2011 5225	2102 2225	2194 9225
86	1669 5396	1752 2596	1836 9796	1923 6996	2012 4196	2103 1396	2195 8596
87	1670 3569	1753 0969	1837 8369	1924 5769	2013 3169	2104 0569	2196 7969
88	1671 1744	1753 9344	1838 6944	1925 4544	2014 2144	2104 9744	2197 7344
89	1671 9921	1754 7721	1839 5521	1926 3321	2015 1121	2105 8921	2198 6721
90	1672 8100	1755 6100	1840 4100	1927 2100	2016 0100	2106 8100	2199 6100
91	1673 6281	1756 4481	1841 2681	1928 0881	2016 9081	2107 7281	2200 5481
92	1674 4464	1757 2864	1842 1264	1928 9664	2017 8064	2108 6464	2201 4864
93	1675 2649	1758 1249	1842 9849	1929 8449	2018 7049	2109 5649	2202 4249
94	1676 0836	1758 9636	1843 8436	1930 7230	2019 6036	2110 4836	2203 3636
95	1676 9025	1759 8025	1844 7025	1931 6025	2020 5025	2111 4025	2204 3025
96	1677 7216	1760 6416	1845 5616	1932 4816	2021 4016	2112 3216	2205 2416
97	1678 5409	1761 4809	1846 4209	1933 3609	2022 3009	2113 2409	2206 1809
98	1679 3604	1762 3204	1847 2804	1934 2404	2023 2004	2114 1604	2207 1204
99	1680 1801	1763 1601	1848 1401	1935 1201	2024 1001	2115 0801	2208 0601

A Table of whole Numbers, and their Squares.

	47..	48..	49..	50..	51..	52..	53..
00	2209 0000	2304 0000	2401 0000	2500 0000	2601 0000	2704 0000	2809 0000
01	2209 9401	2304 9601	2401 9801	2501 0001	2602 0201	2705 0401	2810 0601
02	2210 8804	2305 9204	2402 9604	2502 0004	2603 0404	2706 0804	2811 1204
03	2211 8209	2306 8809	2403 9409	2503 0009	2604 0609	2707 1209	2812 1809
04	2212 7616	2307 8416	2404 9216	2504 0016	2605 0816	2708 1616	2813 2416
05	2213 7025	2308 8025	2405 9025	2505 0025	2606 1025	2709 2025	2814 3025
06	2214 6436	2309 7636	2406 8836	2506 0036	2607 1236	2710 2436	2815 3636
07	2215 6849	2310 7249	2407 8649	2507 0049	2608 1449	2711 2849	2816 4249
08	2216 5264	2311 6864	2408 8464	2508 0064	2609 1664	2712 3264	2817 4864
09	2217 4681	2312 6481	2409 8281	2509 0081	2610 1881	2713 3681	2818 5481
10	2218 4100	2313 6100	2410 8100	2510 0100	2611 2100	2714 4100	2819 6100
11	2219 3521	2314 5721	2411 7921	2511 0121	2612 2321	2715 4521	2820 6721
12	2220 2944	2315 5344	2412 7744	2512 0144	2613 2544	2716 4944	2821 7344
13	2221 2369	2316 4969	2413 7569	2513 0169	2614 2769	2717 5369	2822 7969
14	2222 1796	2317 4596	2414 7396	2514 0196	2615 2996	2718 5796	2823 8596
15	2223 1225	2318 4225	2415 7225	2515 0225	2616 3225	2719 6225	2824 9225
16	2224 0656	2319 3856	2416 7056	2516 0256	2617 3456	2720 6656	2825 9856
17	2225 0089	2320 3489	2417 6889	2517 0289	2618 3689	2721 7089	2827 0489
18	2225 9524	2321 3124	2418 6724	2518 0324	2619 3924	2722 7524	2828 1124
19	2226 8961	2322 2761	2419 6561	2519 0361	2620 4161	2723 7961	2829 1761
20	2227 8400	2323 2400	2420 6400	2520 0400	2621 4400	2724 8400	2830 2400
21	2228 7841	2324 2041	2421 6241	2521 0441	2622 4641	2725 8841	2831 3041
22	2229 7284	2325 1684	2422 6084	2522 0484	2623 4884	2726 9284	2832 3684
23	2230 6729	2326 1329	2423 5929	2523 0529	2624 5129	2727 9729	2833 4329
24	2231 6176	2327 0976	2424 5776	2524 0576	2625 5376	2729 0176	2834 4976
25	2232 5625	2328 0625	2425 5625	2525 0625	2626 5625	2730 0625	2835 5625
26	2233 5076	2329 0276	2426 5476	2526 0676	2627 5876	2731 1076	2836 6276
27	2234 4529	2329 9929	2427 5329	2527 0729	2628 6129	2732 1529	2837 6929
28	2235 3984	2330 9584	2428 5184	2528 0784	2629 6384	2733 1984	2838 7584
29	2236 3441	2331 9241	2429 5041	2529 0841	2630 6641	2734 2441	2839 8241
30	2237 2900	2332 8900	2430 4900	2530 0900	2631 6900	2735 2900	2840 8900
31	2238 2361	2333 8561	2431 4761	2531 0961	2632 7161	2736 3361	2841 9561
32	2239 1824	2334 8224	2432 4624	2532 1024	2633 7424	2737 3824	2843 0224
33	2240 1289	2335 7889	2433 4489	2533 1089	2634 7689	2738 4289	2844 0889
34	2241 0756	2336 7556	2434 4356	2534 1156	2635 7956	2739 4756	2845 1556
35	2242 0225	2337 7225	2435 4225	2535 1225	2636 8225	2740 5225	2846 2225
36	2242 9696	2338 6896	2436 4096	2536 1296	2637 8496	2741 5696	2847 2896
37	2243 9169	2339 6569	2437 3969	2537 1369	2638 8769	2742 6169	2848 3569
38	2244 8644	2340 6244	2438 3844	2538 1444	2639 9044	2743 6644	2849 4244
39	2245 8121	2341 5921	2439 3721	2539 1521	2640 9321	2744 7121	2850 4921
40	2246 7600	2342 5600	2440 3600	2540 1600	2641 9600	2745 7600	2851 5600
41	2247 7081	2343 5281	2441 3481	2541 1681	2642 9881	2746 8081	2852 6281
42	2248 6564	2344 4964	2442 3364	2542 1764	2644 0164	2747 8564	2853 6964
43	2249 6049	2345 4649	2443 3249	2543 1849	2645 0449	2748 9049	2854 7649
44	2250 5536	2346 4336	2444 3136	2544 1936	2646 0736	2749 9536	2855 8336
45	2251 5025	2347 4025	2445 3025	2545 2025	2647 1025	2751 0025	2856 9025
46	2252 4516	2348 3716	2446 2916	2546 2116	2648 1316	2752 0516	2857 9716
47	2253 4009	2349 3409	2447 2809	2547 2209	2649 1609	2753 1009	2859 0409
48	2254 3504	2350 3104	2448 2704	2548 2304	2650 1904	2754 1504	2860 1104
49	2255 3001	2351 2801	2449 2601	2549 2401	2651 2201	2755 2001	2861 1801

	47..	48..	49..	50..	51..	52..	53..
50	2256 2500	2352 2500	2450 2500	2550 2500	2652 2500	2756 2500	2862 2500
51	2257 2001	2353 2201	2451 2401	2551 2601	2653 2801	2757 3001	2863 3201
52	2258 1504	2354 1964	2452 2304	2552 2704	2654 3104	2758 3504	2864 3904
53	2259 1009	2355 1609	2453 2209	2553 2809	2655 3409	2759 4009	2865 4609
54	2260 0516	2356 1316	2454 2116	2554 2916	2656 3716	2760 4516	2866 5316
55	2261 0025	2357 1025	2455 2025	2555 3025	2657 4025	2761 5025	2867 6025
56	2261 9536	2358 0736	2456 1936	2556 3136	2658 4336	2762 5536	2868 6736
57	2262 9049	2359 0449	2457 1849	2557 3249	2659 4649	2763 6049	2869 7449
58	2263 8564	2360 0164	2458 1764	2558 3364	2660 4964	2764 6564	2870 8164
59	2264 8081	2360 9881	2459 1681	2559 3481	2661 5281	2765 7081	2871 8881
60	2265 7600	2361 9600	2460 1600	2560 3600	2662 5600	2766 7600	2872 9600
61	2266 7121	2362 9321	2461 1521	2561 3721	2663 5921	2767 8121	2874 0321
62	2267 6644	2363 9044	2462 1444	2562 3844	2664 6244	2768 8644	2875 1044
63	2268 6169	2364 8769	2463 1369	2563 3969	2665 6569	2769 9169	2876 1769
64	2269 5696	2365 8496	2464 1296	2564 4096	2666 6896	2770 9696	2877 2496
65	2270 5225	2366 8225	2465 1225	2565 4225	2667 7225	2772 0225	2878 3225
66	2271 4756	2367 7956	2466 1156	2566 4356	2668 7556	2773 0756	2879 3956
67	2272 4289	2368 7689	2467 1089	2567 4489	2669 7889	2774 1289	2880 4689
68	2273 3824	2369 7424	2468 1024	2568 4624	2670 8224	2775 1824	2881 5424
69	2274 3361	2370 7161	2469 0961	2569 4761	2671 8561	2776 2361	2882 6161
70	2275 2900	2371 6900	2470 0900	2570 4900	2672 8900	2777 2900	2883 6900
71	2276 2441	2372 6641	2471 0841	2571 5041	2673 9241	2778 3441	2884 7641
72	2277 1984	2373 6384	2472 0784	2572 5184	2674 9584	2779 3984	2885 8384
73	2278 1529	2374 6129	2473 0729	2573 5329	2675 9929	2780 4529	2886 9129
74	2279 1076	2375 5876	2474 0676	2574 5476	2677 0276	2781 5076	2887 9876
75	2280 0625	2376 5625	2475 0625	2575 5625	2678 0625	2782 5625	2889 0625
76	2281 0176	2377 5376	2476 0576	2576 5776	2679 0976	2783 6176	2890 1376
77	2281 9729	2378 5129	2477 0529	2577 5929	2680 1329	2784 6729	2891 2129
78	2282 9284	2379 4884	2478 0484	2578 6084	2681 1684	2785 7284	2892 2884
79	2283 8841	2380 4641	2479 0441	2579 6241	2682 2041	2786 7841	2893 3641
80	2284 8400	2381 4400	2480 0400	2580 6400	2683 2400	2787 8400	2894 4400
81	2285 7961	2382 4161	2481 0361	2581 6561	2684 2761	2788 8961	2895 5161
82	2286 7524	2383 3924	2482 0324	2582 6724	2685 3124	2789 9524	2896 5924
83	2287 7089	2384 3689	2483 0289	2583 6889	2686 3489	2791 0089	2897 6689
84	2288 6656	2385 3456	2484 0256	2584 7056	2687 3856	2792 0656	2898 7456
85	2289 6225	2386 3225	2485 0225	2585 7225	2688 4225	2793 1225	2899 8225
86	2290 5796	2387 2996	2486 0196	2586 7396	2689 4596	2794 1796	2900 8996
87	2291 5369	2388 2769	2487 0169	2587 7569	2690 4969	2795 2369	2901 9769
88	2292 4944	2389 2544	2488 0144	2588 7744	2691 5344	2796 2944	2903 0544
89	2293 4521	2390 2321	2489 0121	2589 7921	2692 5721	2797 3521	2904 1321
90	2294 4100	2391 2100	2490 0100	2590 8100	2693 6100	2798 4100	2905 2100
91	2295 3681	2392 1881	2491 0081	2591 8281	2694 6481	2799 4681	2906 2881
92	2296 3264	2393 1664	2492 0064	2592 8464	2695 6864	2800 5264	2907 3664
93	2297 2849	2394 1449	2493 0049	2593 8649	2696 7249	2801 5849	2908 4449
94	2298 2436	2395 1236	2494 0036	2594 8836	2697 7636	2802 6436	2909 5236
95	2299 2025	2396 1025	2495 0025	2595 9025	2698 8025	2803 7025	2910 6025
96	2300 1616	2397 0816	2496 0016	2596 9216	2699 8416	2804 7616	2911 6816
97	2301 1209	2398 0609	2497 0009	2597 9409	2700 8809	2805 8209	2912 7609
98	2302 0804	2399 0404	2498 0004	2598 9604	2701 9204	2806 8804	2913 8404
99	2303 0401	2400 0201	2499 0001	2599 9801	2702 9601	2807 9401	2914 9201

	54..	55..	56..	57..	58..	59..	60..
00	2916 0000	3025 0000	3136 0000	3249 0000	3364 0000	3481 0000	3600 0000
01	2917 0801	3026 1001	3137 1201	3250 1401	3365 1601	3482 1801	3601 2001
02	2918 1604	3027 2004	3138 2404	3251 2804	3366 3204	3483 3604	3602 4004
03	2919 2409	3028 3009	3139 3609	3252 4209	3367 4809	3484 5409	3603 6009
04	2920 3216	3029 4016	3140 4816	3253 5616	3368 6416	3485 7216	3604 8016
05	2921 4025	3030 5025	3141 6025	3254 7025	3369 8025	3486 9025	3606 0025
06	2922 4836	3031 6036	3142 7236	3255 8436	3370 9636	3488 0836	3607 2036
07	2923 5649	3032 7049	3143 8449	3256 9849	3372 1249	3489 2649	3608 4049
08	2924 6464	3033 8064	3144 9664	3258 1264	3373 2864	3490 4464	3609 6064
09	2925 7281	3034 9081	3146 0881	3259 2681	3374 4481	3491 6281	3610 8081
10	2926 8100	3036 0100	3147 2100	3260 4100	3375 6100	3492 8100	3612 0100
11	2927 8921	3037 1121	3148 3321	3261 5521	3376 7721	3493 9921	3613 2121
12	2928 9744	3038 2144	3149 4544	3262 6944	3377 9344	3495 1744	3614 4144
13	2930 0569	3039 3169	3150 5769	3263 8369	3379 0969	3496 3569	3615 6169
14	2931 1396	3040 4196	3151 6996	3264 9796	3380 2596	3497 5396	3616 8196
15	2932 2225	3041 5225	3152 8225	3266 1225	3381 4225	3498 7225	3618 0225
16	2933 3056	3042 6256	3153 9456	3267 2656	3382 5856	3499 9056	3619 2256
17	2934 3889	3043 7289	3155 0689	3268 4089	3383 7489	3501 0889	3620 4289
18	2935 4724	3044 8324	3156 1924	3269 5524	3384 9124	3502 2724	3621 6324
19	2936 5561	3045 9361	3157 3161	3270 6961	3386 0761	3503 4561	3622 8361
20	2937 6400	3047 0400	3158 4400	3271 8400	3387 2400	3504 6400	3624 0400
21	2938 7241	3048 1441	3159 5641	3272 9841	3388 4041	3505 8241	3625 2441
22	2939 8084	3049 2484	3160 6884	3274 1284	3389 5684	3507 0084	3626 4484
23	2940 8929	3050 3529	3161 8129	3275 2729	3390 7329	3508 1929	3627 6529
24	2941 9776	3051 4576	3162 9376	3276 4176	3391 8976	3509 3776	3628 8576
25	2943 0625	3052 5625	3164 0625	3277 5625	3393 0625	3510 5625	3630 0625
26	2944 1476	3053 6676	3165 1876	3278 7076	3394 2276	3511 7476	3631 2676
27	2945 2329	3054 7729	3166 3129	3279 8529	3395 3929	3512 9329	3632 4729
28	2946 3184	3055 8784	3167 4384	3280 9984	3396 5584	3514 1184	3633 6784
29	2947 4041	3056 9841	3168 5641	3282 1441	3397 7241	3515 3041	3634 8841
30	2948 4900	3058 0900	3169 6900	3283 2900	3398 8900	3516 4900	3636 0900
31	2949 5761	3059 1961	3170 8161	3284 4361	3400 0561	3517 6761	3637 2961
32	2950 6624	3060 3024	3171 9424	3285 5824	3401 2224	3518 8624	3638 5024
33	2951 7489	3061 4089	3173 0689	3286 7289	3402 3889	3520 0489	3639 7089
34	2952 8356	3062 5156	3174 1956	3287 8756	3403 5556	3521 2356	3640 9156
35	2953 9225	3063 6225	3175 3225	3289 0225	3404 7225	3522 4225	3642 1225
36	2955 0096	3064 7296	3176 4496	3290 1696	3405 8896	3523 6096	3643 3296
37	2956 0969	3065 8369	3177 5769	3291 3169	3407 0569	3524 7969	3644 5369
38	2957 1844	3066 9444	3178 7044	3292 4644	3408 2244	3525 9844	3645 7444
39	2958 2721	3068 0521	3179 8321	3293 6121	3409 3921	3527 1721	3646 9521
40	2959 3600	3069 1600	3180 9600	3294 7600	3410 5600	3528 3600	3648 1600
41	2960 4481	3070 2681	3182 0881	3295 9081	3411 7281	3529 5481	3649 3681
42	2961 5364	3071 3764	3183 2164	3297 0564	3412 8964	3530 7364	3650 5764
43	2962 6249	3072 4849	3184 3449	3298 2049	3414 0649	3531 9249	3651 7849
44	2963 7136	3073 5936	3185 4736	3299 3536	3415 2336	3533 1136	3652 9936
45	2964 8025	3074 7025	3186 6025	3300 5025	3416 4025	3534 3025	3654 2025
46	2965 8916	3075 8116	3187 7316	3301 6516	3417 5716	3535 4916	3655 4116
47	2966 9809	3076 9209	3188 8609	3302 8009	3418 7409	3536 6809	3656 6209
48	2968 0704	3078 0304	3189 9904	3303 9504	3419 9104	3537 8704	3657 8304
49	2969 1601	3079 1401	3191 1201	3305 1001	3421 0801	3539 0601	3659 0401

	54..	55..	56..	57..	58..	59..	60..
50	2970 2500	3080 2500	3192 2500	3306 2500	3422 2500	3540 2500	3660 2500
51	2971 3401	3081 3601	3193 3801	3307 4001	3423 4201	3541 4401	3661 4601
52	2972 4304	3082 4704	3194 5104	3308 5504	3424 5904	3542 6304	3662 6704
53	2973 5209	3083 5809	3195 6409	3309 7009	3425 7609	3543 8209	3663 8809
54	2974 6116	3084 6916	3196 7716	3310 8516	3426 9316	3545 0116	3665 0916
55	2975 7025	3085 8025	3197 9025	3312 0025	3428 1025	3546 2025	3666 3025
56	2976 7936	3086 9136	3199 0336	3313 1536	3429 2736	3547 3936	3667 5136
57	2977 8849	3088 0249	3200 1649	3314 3049	3430 4449	3548 5849	3668 7249
58	2978 9764	3089 1364	3201 2964	3315 4564	3431 6164	3549 7764	3669 9364
59	2980 0681	3090 2481	3202 4281	3316 6081	3432 7881	3550 9681	3671 1481
60	2981 1600	3091 3600	3203 5600	3317 7600	3433 9600	3552 1600	3672 3600
61	[illegible] 2521	3092 4721	3204 6921	3318 9121	3435 1321	3553 3521	3673 5721
62	[illegible] 3444	3093 5844	3205 8244	3320 0644	3436 3044	3554 5444	3674 7844
63	2984 4369	3094 6969	3206 9569	3321 2169	3437 4769	3555 7369	3675 9969
64	2985 5296	3095 8096	3208 0896	3322 3696	3438 6496	3556 9296	3677 2096
65	2986 6225	3096 9225	3209 2225	3323 5225	3439 8225	3558 1225	3678 4225
66	2987 7156	3098 0356	3210 3556	3324 6756	3440 9956	3559 3156	3679 6356
67	2988 8089	3099 1489	3211 4889	3325 8289	3442 1689	3560 5089	3680 8489
68	2989 9024	3100 2624	3212 6224	3326 9824	3443 3424	3561 7024	3682 0624
69	2990 9961	3101 3761	3213 7561	3328 1361	3444 5161	3562 8961	3683 2761
70	2992 0900	3102 4900	3214 8900	3329 2900	3445 6900	3564 0900	3684 4900
71	2993 1841	3103 6041	3216 0241	3330 4441	3446 8641	3565 2841	3685 7041
72	2994 2784	3104 7184	3217 1584	3331 5984	3448 0384	3566 4784	3686 9184
73	2995 3729	3105 8329	3218 2929	3332 7529	3449 2129	3567 6729	3688 1329
74	2996 4676	3106 9476	3219 4276	3333 9076	3450 3876	3568 8676	3689 3476
75	2997 5625	3108 0625	3220 5625	3335 0625	3451 5625	3570 0625	3690 5625
76	2998 6576	3109 1776	3221 6976	3336 2176	3452 7376	3571 2576	3691 7776
77	2999 7529	3110 2929	3222 8329	3337 3729	3453 9129	3572 4529	3692 9929
78	3000 8484	3111 4084	3223 9684	3338 5284	3455 0884	3573 6484	3694 2084
79	3001 9441	3112 5241	3225 1041	3339 6841	3456 2641	3574 8441	3695 4241
80	3003 0400	3113 6400	3226 2400	3340 8400	3457 4400	3576 0400	3696 6400
81	3004 1361	3114 7561	3227 3761	3341 9961	3458 6161	3577 2361	3697 8561
82	3005 2324	3115 8724	3228 5124	3343 1524	3459 7924	3578 4324	3699 0724
83	3006 3289	3116 9889	3229 6489	3344 3089	3460 9689	3579 6289	3700 2889
84	3007 4256	3118 1056	3230 7856	3345 4656	3462 1456	3580 8256	3701 5056
85	3008 5225	3119 2225	3231 9225	3346 6225	3463 3225	3582 0225	3702 7225
86	3009 6196	3120 3396	3233 0596	3347 7796	3464 4996	3583 2196	3703 9396
87	3010 7169	3121 4569	3234 1969	3348 9369	3465 6769	3584 4169	3705 1569
88	3011 8144	3122 5744	3235 3344	3350 0944	3466 8544	3585 6144	3706 3744
89	3012 9121	3123 6921	3236 4721	3351 2521	3468 0321	3586 8121	3707 5921
90	3014 0100	3124 8100	3237 6100	3352 4100	3469 2100	3588 0100	3708 8100
91	3015 1081	3125 9281	3238 7481	3353 5681	3470 3881	3589 2081	3710 0281
92	3016 2064	3127 0464	3239 8864	3354 7264	3471 5664	3590 4064	3711 2464
93	3017 3049	3128 1649	3241 0249	3355 8849	3472 7449	3591 6049	3712 4649
94	3018 4036	3129 2836	3242 1636	3357 0436	3473 9236	3592 8036	3713 6836
95	3019 5025	3130 4025	3243 3025	3358 2025	3475 1025	3594 0025	3714 9025
96	3020 6016	3131 5216	3244 4416	3359 3616	3476 2816	3595 2016	3716 1216
97	3021 7009	3132 6409	3245 5809	3360 5209	3477 4609	3596 4009	3717 3409
98	3022 8004	3133 7604	3246 7204	3361 6804	3478 6404	3597 6004	3718 5604
99	3023 9001	3134 8801	3247 8601	3362 8401	3479 8201	3598 8001	3719 7801

 A Table of whole Numbers, and their Squares.

	61..	62..	63..	64..	65..	66.	67..
00	3721 0000	3844 0000	3969 0000	4096 0000	4225 0000	4356 0000	4489 0000
01	3722 2201	3845 2401	3970 2601	4097 2801	4226 3001	4357 3201	4490 3401
02	3723 4404	3846 4804	3971 5204	4098 5604	4227 6004	4358 6404	4491 6804
03	3724 6609	3847 7209	3972 7809	4099 8409	4228 9009	4359 9609	4493 0209
04	3725 8816	3848 9616	3974 0416	4101 1216	4230 2016	4361 2816	4494 3616
05	3727 1025	3850 2025	3975 3025	4102 4025	4231 5025	4362 6025	4495 7025
06	3728 3236	3851 4436	3976 5636	4103 6836	4232 8036	4363 9236	4497 0436
07	3729 5449	3852 6849	3977 8249	4104 9649	4234 1049	4365 2449	4498 3849
08	3730 7664	3853 9264	3979 0864	4106 2464	4235 4064	4366 5664	4499 7264
09	3731 9881	3855 1681	3980 3481	4107 5281	4236 7081	4367 8881	4501 0681
10	3733 2100	3856 4100	3981 6100	4108 8100	4238 0100	4369 2100	4502 4100
11	3734 4321	3857 6521	3982 8721	4110 0921	4239 3121	4370 5321	4503 7521
12	3735 6544	3858 8944	3984 1344	4111 3744	4240 6144	4371 8544	4505 0944
13	3736 8769	3860 1369	3985 3969	4112 6569	4241 9169	4373 1769	4506 4369
14	3738 0996	3861 3796	3986 6596	4113 9396	4243 2196	4374 4996	4507 7796
15	3739 3225	3862 6225	3987 9225	4115 2225	4244 5225	4375 8225	4509 1225
16	3740 5456	3863 8656	3989 1856	4116 5056	4245 8256	4377 1456	4510 4656
17	3741 7689	3865 1089	3990 4489	4117 7889	4247 1289	4378 4689	4511 8089
18	3742 9924	3866 3524	3991 7124	4119 0724	4248 4324	4379 7924	4513 1524
19	3744 2161	3867 5961	3992 9761	4120 3561	4249 7361	4381 1161	4514 4961
20	3745 4400	3868 8400	3994 2400	4121 6400	4251 0400	4382 4400	4515 8400
21	3746 6641	3870 0841	3995 5041	4122 9241	4252 3441	4383 7641	4517 1841
22	3747 8884	3871 3284	3996 7684	4124 2084	4253 6484	4385 0884	4518 5284
23	3749 1129	3872 5729	3998 0329	4125 4929	4254 9529	4386 4129	4519 8729
24	3750 3376	3873 8176	3999 2976	4126 7776	4256 2576	4387 7376	4521 2176
25	3751 5625	3875 0625	4000 5625	4128 0625	4257 5625	4389 0625	4522 5625
26	3752 7876	3876 3076	4001 8276	4129 3476	4258 8676	4390 3876	4523 9076
27	3754 0129	3877 5529	4003 0929	4130 6329	4260 1729	4391 7129	4525 2529
28	3755 2384	3878 7984	4004 3584	4131 9184	4261 4784	4393 0384	4526 5984
29	3756 4641	3880 0441	4005 6241	4133 2041	4262 7841	4394 3641	4527 9441
30	3757 6900	3881 2900	4006 8900	4134 4900	4264 0900	4395 6900	4529 2900
31	3758 9161	3882 5361	4008 1561	4135 7761	4265 3961	4397 0161	4530 6361
32	3760 1424	3883 7824	4009 4224	4137 0624	4266 7024	4398 3424	4531 9824
33	3761 3689	3885 0289	4010 6889	4138 3489	4268 0089	4399 6689	4533 3289
34	3762 5956	3886 2756	4011 9556	4139 6356	4269 3156	4400 9956	4534 6756
35	3763 8225	3887 5225	4013 2225	4140 9225	4270 6225	4402 3225	4536 0225
36	3765 0496	3888 7696	4014 4896	4142 2096	4271 9296	4403 6496	4537 3696
37	3766 2769	3890 0169	4015 7569	4143 4969	4273 2369	4404 9769	4538 7169
38	3767 5044	3891 2644	4017 0244	4144 7844	4274 5444	4406 3044	4540 0644
39	3768 7321	3892 5121	4018 2921	4146 0721	4275 8521	4407 6321	4541 4121
40	3769 9600	3893 7600	4019 5600	4147 3600	4277 1600	4408 9600	4542 7600
41	3771 1881	3895 0081	4020 8281	4148 6481	4278 4681	4410 2881	4544 [illegible]
42	3772 4164	3896 2564	4022 0964	4149 9364	4279 7764	4411 6164	4545 [illegible]
43	3773 6449	3897 5049	4023 3649	4151 2249	4281 0849	4412 9449	4546 8049
44	3774 8736	3898 7536	4024 6336	4152 5136	4282 3936	4414 2736	4548 1536
45	3776 1025	3900 0025	4025 9025	4153 8025	4283 7025	4415 6025	4549 5025
46	3777 3316	3901 2516	4027 1716	4155 0916	4285 0116	4416 9316	4550 8516
47	3778 5609	3902 5009	4028 4409	4156 3809	4286 3209	4418 2609	4552 2009
48	3779 7904	3903 7504	4029 7104	4157 6704	4287 6304	4419 5904	4553 5504
49	3781 0201	3905 0001	4030 9801	4158 9601	4288 9401	4420 9201	4554 9001

	61..	62..	63..	64..	65..	66..	67..
50	3782 2500	3906 2500	4032 2500	4160 2500	4290 2500	4422 2500	4556 2500
51	3783 4801	3907 5001	4033 5201	4161 5401	4291 5601	4423 5801	4557 6001
52	3784 7104	3908 7504	4034 7904	4162 8304	4292 8704	4424 9104	4558 9504
53	3785 9409	3910 0009	4036 0609	4164 1209	4294 1809	4426 2409	4560 3009
54	3787 1716	3911 2516	4037 3316	4165 4116	4295 4916	4427 5716	4561 6516
55	3788 4025	3912 5025	4038 6025	4166 7025	4296 8025	4428 9025	4563 0025
56	3789 6336	3913 7536	4039 8736	4167 9936	4298 1136	4430 2336	4564 3536
57	3790 8649	3915 0049	4041 1449	4169 2849	4299 4249	4431 5649	4565 7049
58	3792 0964	3916 2564	4042 4164	4170 5764	4300 7364	4432 8964	4567 0564
59	3793 3281	3917 5081	4043 6881	4171 8681	4302 0481	4434 2281	4568 4081
60	3794 5600	3918 7600	4044 9600	4173 1600	4303 3600	4435 5600	4569 7600
61	3795 7921	3920 0121	4046 2321	4174 4521	4304 6721	4436 8921	4571 1121
62	3797 0244	3921 2644	4047 5044	4175 7444	4305 9844	4438 2244	4572 4644
63	3798 2569	3922 5169	4048 7769	4177 0369	4307 2969	4439 5569	4573 8169
64	3799 4896	3923 7696	4050 0496	4178 3296	4308 6096	4440 8896	4575 1696
65	3800 7225	3925 0225	4051 3225	4179 6225	4309 9225	4442 2225	4576 5225
66	3801 9556	3926 2756	4052 5956	4180 9156	4311 2356	4443 5556	4577 8756
67	3803 1889	3927 5289	4053 8689	4182 2089	4312 5489	4444 8889	4579 2289
68	3804 4224	3928 7824	4055 1424	4183 5024	4313 8624	4446 2224	4580 5824
69	3805 6561	3930 0361	4056 4161	4184 7961	4315 1761	4447 5561	4581 9361
70	3806 8900	3931 2900	4057 6900	4186 0900	4316 4900	4448 8900	4583 2900
71	3808 1241	3932 5441	4058 9641	4187 3841	4317 8041	4450 2241	4584 6441
72	3809 3584	3933 7984	4060 2384	4188 6784	4319 1184	4451 5584	4585 9984
73	3810 5929	3935 0529	4061 5129	4189 9729	4320 4329	4452 8929	4587 3529
74	3811 8276	3936 3076	4062 7876	4191 2676	4321 7476	4454 2276	4588 7076
75	3813 0625	3937 5625	4064 0625	4192 5625	4323 0625	4455 5625	4590 0625
76	3814 2976	3938 8176	4065 3376	4193 8576	4324 3776	4456 8976	4591 4176
77	3815 5329	3940 0729	4066 6129	4195 1529	4325 6929	4458 2329	4592 7729
78	3816 7684	3941 3284	4067 8884	4196 4484	4327 0084	4459 5684	4594 1284
79	3818 0041	3942 5841	4069 1641	4197 7441	4328 3241	4460 9041	4595 4841
80	3819 2400	3943 8400	4070 4400	4199 0400	4329 6400	4462 2400	4596 8400
81	3820 4761	3945 0961	4071 7161	4200 3361	4330 9561	4463 5761	4598 1961
82	3821 7124	3946 3524	4072 9924	4201 6324	4332 2724	4464 9124	4599 5524
83	3822 9489	3947 6089	4074 2689	4202 9289	4333 5889	4466 2489	4600 9089
84	3824 1856	3948 8656	4075 5456	4204 2256	4334 9056	4467 5856	4602 2656
85	3825 4225	3950 1225	4076 8225	4205 5225	4336 2225	4468 9225	4603 6225
86	3826 6596	3951 3796	4078 0996	4206 8196	4337 5396	4470 2596	4604 9796
87	3827 8969	3952 6369	4079 3769	4208 1169	4338 8569	4471 5969	4606 3369
88	3829 1344	3953 8944	4080 6544	4209 4144	4340 1744	4472 9344	4607 6944
89	3830 3721	3955 1521	4081 9321	4210 7121	4341 4921	4474 2721	4609 0521
90	3831 6100	3956 4100	4083 2100	4212 0100	4342 8100	4475 6100	4610 4100
91	3832 8481	3957 6681	4084 4881	4213 3081	4344 1281	4476 9481	4611 7681
92	3834 0864	3958 9264	4085 7664	4214 6064	4345 4464	4478 2864	4613 1264
93	3835 3249	3960 1849	4087 0449	4215 9049	4346 7649	4479 6249	4614 4849
94	3836 5636	3961 4436	4088 3236	4217 2036	4348 0836	4480 9636	4615 8436
95	3837 8025	3962 7025	4089 6025	4218 5025	4349 4025	4482 3025	4617 2025
96	3839 0416	3963 9616	4090 8816	4219 8016	4350 7216	4483 6416	4618 5616
97	3840 2809	3965 2209	4092 1609	4221 1009	4352 0409	4484 9809	4619 9209
98	3841 5204	3966 4804	4093 4404	4222 4004	4353 3604	4486 3204	4621 2804
99	3842 7601	3967 7401	4094 7201	4223 7001	4354 6801	4487 6601	4622 6401

A Table of whole Numbers, and their Squares.

	68..	69..	70..	71..	72..	73..	74..
00	4624 0000	4761 0000	4900 0000	5041 0000	5184 0000	5329 0000	5476 0000
01	4625 3601	4762 3801	4901 4001	5042 4201	5185 4401	5330 4601	5477 4801
02	4626 7204	4763 7604	4902 8004	5043 8404	5186 8804	5331 9204	5478 9604
03	4628 0809	4765 1409	4904 2009	5045 2609	5188 3209	5333 3809	5480 4409
04	4629 4416	4766 5216	4905 6016	5046 6816	5189 7616	5334 8416	5481 9216
05	4630 8025	4767 9025	4907 0025	5048 1025	5191 2025	5336 3025	5483 4025
06	4632 1636	4769 2836	4908 4036	5049 5236	5192 6436	5337 7636	5484 8836
07	4633 5249	4770 6649	4909 8049	5050 9449	5194 0849	5339 2249	5486 3649
08	4634 8864	4772 0464	4911 2064	5052 3664	5195 5264	5340 6864	5487 8464
09	4636 2481	4773 4281	4912 6081	5053 7881	5196 9681	5342 1481	5489 3281
10	4637 6100	4774 8100	4914 0100	5055 2100	5198 4100	5343 6100	5490 8100
11	4638 9721	4776 1921	4915 4121	5056 6321	5199 8521	5345 0721	5492 2921
12	4640 3344	4777 5744	4916 8144	5058 0544	5201 2944	5346 5344	5493 7744
13	4641 6969	4778 9569	4918 2169	5059 4769	5202 7369	5347 9969	5495 2569
14	4643 0596	4780 3396	4919 6196	5060 8996	5204 1796	5349 4596	5496 7396
15	4644 4225	4781 7225	4921 0225	5062 3225	5205 6225	5350 9225	5498 2225
16	4645 7856	4783 1056	4922 4256	5063 7456	5207 0656	5352 3856	5499 7056
17	4647 1489	4784 4889	4923 8289	5065 1689	5208 5089	5353 8489	5501 1889
18	4648 5124	4785 8724	4925 2324	5066 5924	5209 9524	5355 3124	5502 6724
19	4649 8761	4787 2561	4926 6361	5068 0161	5211 3961	5356 7761	5504 1561
20	4651 2400	4788 6400	4928 0400	5069 4400	5212 8400	5358 2400	5505 6400
21	4652 6041	4790 0241	4929 4441	5070 8641	5214 2841	5359 7041	5507 1241
22	4653 9684	4791 4084	4930 8484	5072 2884	5215 7284	5361 1684	5508 6084
23	4655 3329	4792 7929	4932 2529	5073 7129	5217 1729	5362 6329	5510 0929
24	4656 6976	4794 1776	4933 6576	5075 1376	5218 6176	5364 0976	5511 5776
25	4658 0625	4795 5625	4935 0625	5076 5625	5220 0625	5365 5625	5513 0625
26	4659 4276	4796 9476	4936 4676	5077 9876	5221 5076	5367 0276	5514 5476
27	4660 7929	4798 3329	4937 8729	5079 4129	5222 9529	5368 4929	5516 0329
28	4662 1584	4799 7184	4939 2784	5080 8384	5224 3984	5369 9584	5517 5184
29	4663 5241	4801 1041	4940 6841	5082 2641	5225 8441	5371 4241	5519 0041
30	4664 8900	4802 4900	4942 0900	5083 6900	5227 2900	5372 8900	5520 4900
31	4666 2561	4803 8761	4943 4961	5085 1161	5228 7361	5374 3561	5521 9761
32	4667 6224	4805 2624	4944 9024	5086 5424	5230 1824	5375 8224	5523 4624
33	4668 9889	4806 6489	4946 3089	5087 9689	5231 6289	5377 2889	5524 9489
34	4670 3556	4808 0356	4947 7156	5089 3956	5233 0756	5378 7556	5526 4356
35	4671 7225	4809 4225	4949 1225	5090 8225	5234 5225	5380 2225	5527 9225
36	4673 0896	4810 8096	4950 5296	5092 2496	5235 9696	5381 6896	5529 4096
37	4674 4569	4812 1969	4951 9369	5093 6769	5237 4169	5383 1569	5530 8969
38	4675 8244	4813 5844	4953 3444	5095 1044	5238 8644	5384 6244	5532 3844
39	4677 1921	4814 9721	4954 7521	5096 5321	5240 3121	5386 0921	5533 8721
40	4678 5600	4816 3600	4956 1600	5097 9600	5241 7600	5387 5600	5535 3600
41	4679 9281	4817 7481	4957 5681	5099 3881	5243 2081	5389 0281	5536 8481
42	4681 2964	4819 1364	4958 9764	5100 8164	5244 6564	5390 4964	5538 3364
43	4682 6649	4820 5249	4960 3849	5102 2449	5246 1049	5391 9649	5539 8249
44	4684 0336	4821 9136	4961 7936	5103 6736	5247 5536	5393 4336	5541 3136
45	4685 4025	4823 3025	4963 2025	5105 1025	5249 0025	5394 9025	5542 8025
46	4686 7716	4824 6916	4964 6116	5106 5316	5250 4516	5396 3716	5544 2916
47	4688 1409	4826 0809	4966 0209	5107 9609	5251 9009	5397 8409	5545 7809
48	4689 5104	4827 4704	4967 4304	5109 3904	5253 3504	5399 3104	5547 2704
49	4690 8801	4828 8601	4968 8401	5110 8201	5254 8001	5400 7801	5548 7601

	68..		69..		70..		71..		72..		73..		74..	
50	4692	2500	4830	2500	4970	2500	5112	2500	5256	2500	5402	2500	5550	2500
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52	4694	9904	4833	0304	4973	0704	5115	1104	5259	1504	5405	1904	5553	2304
53	4696	3609	4834	4209	4974	4809	5116	5409	5260	6009	5406	6609	5554	7209
54	4697	7316	4835	8116	4975	8916	5117	9716	5262	0516	5408	1316	5556	2116
55	4699	1025	4837	2025	4977	3025	5119	4025	5263	5025	5409	6025	5557	7025
56	4700	4736	4838	5936	4978	7136	5120	8336	5264	9536	5411	0736	5559	1936
57	4701	8449	4839	9849	4980	1249	5122	2649	5266	4049	5412	5449	5560	6849
58	4703	2164	4841	3764	4981	5364	5123	6964	5267	8564	5414	0164	5562	1764
59	4704	5881	4842	7681	4982	9481	5125	1281	5269	3081	5415	4881	5563	6681
60	4705	9600	4844	1600	4984	3600	5126	5600	5270	7600	5416	9600	5565	1600
61	4707	3321	4845	5521	4985	7721	5127	9921	5272	2121	5418	4321	5566	6521
62	4708	7044	4846	9444	4987	1844	5129	4244	5273	6644	5419	9044	5568	1444
63	4710	0769	4848	3369	4988	5969	5130	8569	5275	1169	5421	3769	5569	6369
64	4711	4496	4849	7296	4990	0096	5132	2896	5276	5696	5422	8496	5571	1296
65	4712	8225	4851	1225	4991	4225	5133	7225	5278	0225	5424	3225	5572	6225
66	4714	1956	4852	5156	4992	8356	5135	1556	5279	4756	5425	7956	5574	1156
67	4715	5689	4853	9089	4994	2489	5136	5889	5280	9289	5427	2689	5575	6089
68	4716	9424	4855	3024	4995	6624	5138	0224	5282	3824	5428	7424	5577	1024
69	4718	3161	4856	6961	4997	0761	5139	4561	5283	8361	5430	2161	5578	5961
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80	4733	4400	4872	0400	5012	6400	5155	2400	5299	8400	5446	4400	5595	0400
81	4734	8161	4873	4361	5014	0561	5156	6761	5301	2961	5447	9161	5596	5361
82	4736	1924	4874	8324	5015	4724	5158	1124	5302	7524	5449	3924	5598	0324
83	4737	5689	4876	2289	5016	8889	5159	5489	5304	2089	5450	8689	5599	5289
84	4738	9456	4877	6256	5018	3056	5160	9856	5305	6656	5452	3456	5601	0256
85	4740	3225	4879	0225	5019	7225	5162	4225	5307	1225	5453	8225	5602	5225
86	4741	6996	4880	4196	5021	1396	5163	8596	5308	5796	5455	2996	5604	0196
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92	4749	9664	4888	8064	5029	6464	5172	4864	5317	3264	5464	1664	5613	0064
93	4751	3449	4890	2049	5031	0649	5173	9249	5318	7849	5465	6449	5614	5049
94	4752	7236	4891	6036	5032	4836	5175	3636	5320	2436	5467	1236	5616	0036
95	4754	1025	4893	0025	5033	9025	5176	8025	5321	7025	5468	6025	5617	5025
96	4755	4816	4894	4016	5035	3216	5178	2416	5323	1616	5470	0816	5619	0016
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98	4758	2404	4897	2004	5038	1604	5181	1204	5326	0804	5473	0404	5622	0004
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01	5626 5001	5477 5201	5930 5401	6085 5601	6242 5801	6401 6001	6562 6201
02	5628 0004	5779 0404	5932 0804	6087 1204	6244 1604	6403 2004	6564 2404
03	5629 5009	5780 5609	5933 6209	6088 6809	6245 7409	6404 8009	6565 3609
04	5631 0016	5782 0816	5935 1616	6090 2416	6247 3216	6406 4016	6567 4816
05	5632 5025	5783 6025	5936 7025	6091 8025	6248 9025	6408 0025	6569 1025
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07	5635 5049	5786 6449	5939 7849	6094 9249	6252 0649	6411 2049	6572 3449
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15	5647 5225	5798 8225	5952 1225	6107 4225	6264 7225	6424 0225	6585 3225
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17	5650 5289	5801 8689	5955 2089	6110 5489	6267 8889	6427 2289	6588 5689
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19	5653 5361	5804 9161	5958 2961	6113 6761	6271 0561	6430 4361	6591 8161
20	5655 0400	5806 4400	5959 8400	6115 2400	6272 6400	6432 0400	6593 4400
21	5656 5441	5807 9641	5961 3841	6116 8041	6274 2241	6433 6441	6595 0641
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23	5659 5529	5811 0129	5964 4729	6119 9329	6277 3929	6436 8529	6598 3129
24	5661 0576	5812 5376	5966 0176	6121 4976	6278 9776	6438 4576	6599 9376
25	5662 5625	5814 0625	5967 5625	6123 0625	6280 5625	6440 0625	6601 5625
26	5664 0676	5815 5876	5969 1076	6124 6276	6282 1476	6441 6676	6603 1876
27	5665 5729	5817 1129	5970 6529	6126 1929	6283 7329	6443 2729	6604 8129
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37	5680 6369	5832 3769	5986 1169	6141 8569	6299 5969	6459 3369	6621 0769
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42	5688 1764	5840 0164	5993 8564	6149 6964	6307 5364	6467 3764	6629 2164
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45	5692 7025	5844 6025	5998 5025	6154 4025	6312 3025	6472 2025	6634 1025
46	5694 2116	5846 1316	5900 0516	6155 9716	6313 8916	6473 8116	6635 7316
47	5695 7209	5847 6609	5901 6009	6157 5409	6315 4809	6475 4209	6637 3609
48	5697 2304	5849 1904	5903 1504	6159 1104	6317 0704	6477 0304	6638 9904
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53	5704 7809	5856 8409	6010 9009	6166 9609	6325 0209	6485 0809	6647 1409
54	5706 2916	5858 3716	6012 4516	6168 5316	6326 6116	6486 6916	6648 7716
55	5707 8025	5859 9025	6014 0025	6170 1025	6328 2025	6488 3025	6650 4025
56	5709 3136	5861 4336	6015 5536	6171 6736	6329 7936	6489 9136	6652 0336
57	5710 8249	5862 9649	6017 1049	6173 2449	6331 3849	6491 5249	6653 6649
58	5712 3364	5864 4964	6018 6564	6174 8164	6332 9764	6493 1364	6655 2964
59	5713 8481	5866 0281	6020 2081	6176 3881	6334 5681	6494 7481	6656 9281
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61	5716 8721	5899 0921	6023 3121	6179 5321	6337 7521	6497 9721	6660 1921
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63	5719 8969	5872 1569	6026 4169	6182 6769	6340 9369	6501 1969	6663 4569
64	5721 4096	5873 6896	6027 9696	6184 2496	6342 5296	6502 8096	6665 0896
65	5722 9225	5875 2225	0029 5225	6185 8225	6344 1225	6504 4225	6666 7225
66	5724 4356	5876 7556	6031 0756	6187 3956	6345 7156	6506 0356	6668 3556
67	5725 9489	5878 2889	6032 6289	6188 9689	6347 3089	6507 6489	6669 9889
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73	5735 0329	5887 4929	6041 9529	6198 4129	6356 8729	6517 3329	6679 7929
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75	5738 0625	5890 5625	6045 0625	6201 5625	6360 0625	6520 5625	6683 0625
76	5739 5776	5892 0976	6046 6176	6203 1376	6361 6576	6522 1776	6684 6976
77	5741 0929	5893 6329	6048 1729	6204 7129	6363 2529	6523 7929	6686 3329
78	5742 6084	5895 1684	6049 7284	6206 2884	6364 8484	6525 4084	6687 9684
79	5744 1241	5896 7041	6051 2841	6207 8641	6366 4441	6527 0241	6689 6041
80	5745 6400	5898 2400	6052 8400	6209 4400	6368 0400	6528 6400	6691 2400
81	5747 1561	5899 7761	6054 3961	6211 0161	6369 6361	6530 2561	6692 8761
82	5748 6724	5901 3124	6055 9524	6212 5924	6371 2324	6531 8724	6694 5124
83	5750 1889	5902 8489	6057 5089	6214 1689	6372 8289	6533 4889	6696 1489
84	5751 7056	5904 3856	6059 0656	6215 7456	6374 4256	6535 1056	6697 7856
85	5753 2225	5905 9225	6060 6225	6217 3225	6376 0225	6536 7225	6699 4225
86	5754 7396	5907 4596	6062 1796	6218 8996	6377 6196	6538 3396	6701 0596
87	5756 2569	5908 9969	6063 7369	6220 4769	6379 2169	6539 9569	6702 6969
88	5757 7744	5910 5344	6065 2944	6222 0544	6380 8144	6541 5744	6704 3344
89	5759 2921	5912 0721	6066 8521	6223 6321	6382 4121	6543 1921	6705 9721
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91	5762 3281	5915 1481	6069 9681	6226 7881	6385 6081	6546 4281	6709 2481
92	5763 8464	5916 6864	6071 5264	6228 3664	6387 2064	6548 0464	6710 8864
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95	5768 4025	5921 3025	6076 2025	6233 1025	6392 0025	6552 9025	6715 8025
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97	5771 4409	5924 3809	6079 3209	6236 2609	6395 2009	6556 1409	6719 0809
98	5772 9604	5925 9204	6080 8804	6237 8404	6396 8004	6557 7604	6720 7204
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03	6728 9209	6893 9809	7061 0409	7230 1009	7401 1609	7574 2209	7749 2809
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05	6732 2025	6897 3025	7064 4025	7233 5025	7404 6025	7577 7025	7752 8025
06	6733 8436	6898 9636	7066 0836	7235 2036	7406 3236	7579 4436	7754 5636
07	6735 4849	6900 6149	7067 7649	7236 9049	7408 0449	7581 1849	7756 3249
08	6737 1264	6902 2864	7069 4464	7238 6064	7409 7664	7582 9264	7758 0864
09	6738 7681	6903 9481	7071 1281	7240 3081	7411 4881	7584 6681	7759 8481
10	6740 4100	6905 6100	7072 8100	7242 0100	7413 2100	7586 4100	7761 6100
11	6742 0521	6907 2721	7074 4921	7243 7121	7414 9321	7588 1521	7763 3721
12	6743 6944	6908 9344	7076 1744	7245 4144	7416 6544	7589 8944	7765 1344
13	6745 3369	6910 5969	7077 8569	7247 1169	7418 3769	7591 6369	7766 8969
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15	6748 6225	6913 9225	7081 2225	7250 5225	7421 8225	7595 1225	7770 4225
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17	6751 9089	6917 2489	7084 5889	7253 9289	7425 2689	7598 6089	7773 9489
18	6753 5524	6918 9124	7086 2724	7255 6324	7426 9924	7600 3524	7775 7124
19	6755 1961	6920 5761	7087 9561	7257 3361	7428 7161	7602 0961	7777 4761
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21	6758 4841	6923 9041	7091 3241	7260 7441	7432 1641	7605 5841	7781 0041
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23	6761 7729	6927 2329	7094 6929	7264 1529	7435 6129	7609 0729	7784 5329
24	6763 4176	6928 8976	7096 3776	7265 8576	7437 3376	7610 8176	7786 2976
25	6765 0625	6930 5625	7098 0625	7267 5625	7439 0625	7612 5625	7788 0625
26	6766 7076	6932 2276	7099 7476	7269 2676	7440 7876	7614 3076	7789 8276
27	6768 3529	6933 8929	7101 4329	7270 9729	7442 5129	7616 0529	7791 5929
28	6769 9984	6935 5584	7103 1184	7272 6784	7444 2384	7617 7984	7793 3584
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30	6773 2900	6938 8900	7106 4900	7276 0900	7447 6900	7621 2900	7796 8900
31	6774 9361	6940 5561	7108 1761	7277 7961	7449 4161	7623 0361	7798 6561
32	6776 5824	6942 2224	7109 8624	7279 5024	7451 1424	7624 7824	7800 4224
33	6778 2289	6943 8889	7111 5489	7281 2089	7452 8689	7626 5289	7802 1889
34	6779 8756	6945 5556	7113 2356	7282 9156	7454 5956	7628 2756	7803 9556
35	6781 5225	6947 2225	7114 9225	7284 6225	7456 3225	7630 0225	7805 7225
36	6783 1696	6948 8896	7116 6096	7286 3296	7458 0496	7631 7696	7807 4896
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39	6788 1121	6953 8921	7121 6721	7291 4521	7463 2321	7637 0121	7812 7921
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41	6791 4081	6957 2281	7125 0481	7294 8681	7466 6881	7640 5081	7816 3281
42	6793 0564	6958 8964	7126 7364	7296 5764	7468 4164	7642 2564	7818 0964
43	6794 7049	6960 5649	7128 4249	7298 2849	7470 1449	7644 0049	7819 8649
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46	6799 6516	6965 5716	7133 4916	7303 4116	7475 3316	7649 2516	7825 1716
47	6801 3009	6967 2409	7135 1809	7305 1209	7477 0609	7651 0009	7826 9409
48	6802 9504	6968 9104	7136 8704	7306 8304	7478 7904	7652 7504	7828 7104
49	6804 6001	6970 5801	7138 5601	7308 5401	7480 5201	7654 5001	7830 4801

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51	6807 9001	6973 9201	7141 9401	7311 9601	7483 9801	7658 0001	7834 0201
52	6809 5504	6975 5904	7143 6304	7313 6704	7485 7104	7659 7504	7835 7904
53	6811 2009	6977 2609	7145 3209	7315 3809	7487 4409	7661 5009	7837 5609
54	6812 8516	6978 9316	7147 0116	7317 0916	7789 1716	7663 2516	7839 3316
55	6814 5025	6980 6025	7148 7025	7318 8025	7490 9025	7665 0025	7841 1025
56	6816 1536	6982 2736	7150 3936	7320 5136	7492 6336	7666 7536	7842 8736
57	6817 8049	6983 9449	7152 0849	7322 2249	7494 3649	7668 5049	7844 6449
58	6819 4564	6985 6164	7153 7764	7323 9364	7496 0964	7670 2564	7846 4164
59	6821 1081	6987 2881	7155 4681	7325 6481	7497 8281	7672 0081	7848 1881
60	6822 7600	6988 9600	7157 1600	7327 3600	7499 5600	7673 7600	7849 9600
61	6824 4121	6990 6321	7158 8521	7329 0721	7501 2921	7675 5121	7851 7321
62	6826 0644	6992 3044	7160 5444	7330 7844	7503 0244	7677 2644	7853 5044
63	6827 7169	6993 9769	7162 2369	7332 4969	7504 7569	7679 0169	7855 2769
64	6829 3696	6995 6496	7163 9296	7334 2096	7506 4896	7680 7696	7857 0496
65	6831 0225	6997 3225	7165 6225	7335 9225	7508 2225	7682 5225	7858 8225
66	6832 6756	6998 9956	7167 3156	7337 6356	7509 9556	7684 2756	7860 5956
67	6834 3289	7000 6689	7169 0089	7339 3489	7511 6889	7686 0289	7862 3689
68	6835 9824	7002 3424	7170 7024	7341 0624	7513 4224	7687 7824	7864 1424
69	6837 6361	7004 0161	7172 3961	7342 7761	7515 1561	7689 5361	7865 9161
70	6839 2900	7005 6900	7174 0900	7344 4900	7516 8900	7691 2900	7867 6900
71	6840 9441	7007 3641	7175 7841	7346 2041	7518 6241	7693 0441	7869 4641
72	6842 5984	7009 0384	7177 4784	7347 9184	7520 3584	7694 7984	7871 2384
73	6844 2529	7010 7129	7179 1729	7349 6329	7522 0929	7696 5529	7873 0129
74	6845 9076	7012 3876	7180 8676	7351 3476	7523 8276	7698 3076	7874 7876
75	6847 5625	7014 0625	7183 5625	7353 0625	7525 5625	7700 0625	7876 5625
76	6849 2176	7015 7376	7184 2576	7354 7776	7527 2976	7701 8176	7878 3376
77	6850 8729	7017 4129	7185 9529	7356 4929	7529 0329	7703 5729	7880 1129
78	6852 5284	7019 0884	7187 6484	7358 2084	7530 7684	7705 3284	7881 8884
79	6854 1841	4020 7641	7189 3441	7359 9241	7532 5041	7707 0841	7883 6641
80	6855 8400	7022 4400	7191 0400	7361 6400	7534 2400	7708 8400	7885 4400
81	6857 4961	7024 1161	7192 7361	7363 3561	7535 9761	7710 5961	7887 2161
82	6859 1524	7025 7924	7194 4324	7365 0724	7537 7124	7712 3524	7888 9924
83	6860 8089	7027 4689	7196 1289	7366 7889	7539 4489	7714 1089	7890 7689
84	6862 4656	7029 1456	7197 8256	7368 5056	7541 1856	7715 8656	7892 5456
85	6864 1225	7030 8225	7199 5225	7370 2225	7542 9225	7717 6225	7894 3225
86	6865 7796	7032 4996	7201 2196	7371 9396	7544 6596	7719 3796	7896 0996
87	6867 4369	7034 1769	7202 9169	7373 6569	7546 3969	7721 1369	7897 8769
88	6869 0944	7035 8544	7204 6144	7375 3744	7548 1344	7722 8944	7899 6544
89	6870 7521	3037 5321	7206 3121	7377 0921	7549 8421	7724 6521	7901 4321
90	6872 4100	7039 2100	7208 0100	7378 8100	7551 6100	17726 4100	7903 2100
91	6874 0681	7070 8881	7209 7081	7380 5281	7553 3481	7728 1681	7904 9881
92	6875 7264	7042 5664	7211 4064	7382 2464	7555 0864	7729 9264	7906 7664
93	6877 3849	7044 2449	7213 1049	7383 9649	7556 8249	7731 6849	7908 5449
94	6879 0436	7045 9236	7214 8036	7385 6836	7558 5636	7733 4436	7910 3236
95	6880 7025	7045 6025	7216 5025	7387 4025	7560 3025	7735 2025	7912 1025
96	6882 3616	7049 2816	7218 2016	7389 1216	7562 0416	7736 9616	7913 8816
97	6884 0209	7050 9609	7219 9009	7390 8409	7563 7809	7738 7209	7915 6609
98	6885 6804	7052 6404	7221 6004	7392 5604	7565 5204	7740 4804	7917 4404
99	6887 3401	7054 3201	7223 3001	7394 2801	7567 2601	7742 2401	7919 2201

	89..	90..	91..	92..	93..	94..	95..
00	7921 0000	8100 0000	8281 0000	8464 0000	8649 0000	8836 0000	9025 0000
01	7922 7801	8101 8001	8282 8201	8465 8401	8650 8601	8837 8801	9026 9001
02	7924 5604	8103 6004	8284 6404	8467 6804	8652 7204	8839 7604	9028 8004
03	7926 3409	8105 4009	8286 4609	8469 5209	8654 5809	8841 6409	9030 7009
04	7928 1216	8107 2016	8288 2816	8471 3616	8656 4416	8843 5216	9032 6016
05	7929 9025	8109 0025	8290 1025	8473 2025	8658 3025	8845 4025	9034 5025
06	7931 6836	8110 8036	8291 9236	8475 0436	8660 1636	8847 2836	9036 4036
07	7933 4649	8112 6049	8293 7449	8476 8849	8662 0249	8849 1649	9038 3049
08	7935 2464	8114 4064	8295 5664	8478 7264	8663 8864	8851 0464	9040 2064
09	7937 0281	8116 2081	8297 3881	8480 5681	8665 7481	8852 9281	9042 1081
10	7938 8100	8118 0100	8299 2100	8482 4100	8667 6100	8854 8100	9044 0100
11	7940 5921	8119 8121	8301 0321	8484 2521	8669 4721	8856 6921	9045 9121
12	7942 3744	8121 6144	8302 8544	8486 0944	8671 3344	8858 5744	9047 8144
13	7944 1569	8123 4169	8304 6769	8487 9369	8673 1969	8860 4569	9049 7169
14	7945 9396	8125 2196	8306 4996	8489 7796	8675 0596	8862 3396	9051 6196
15	7947 7225	8127 0225	8308 3225	8491 6225	8676 9225	8864 2225	9053 5225
16	7949 5056	8128 8256	8310 1456	8493 4656	8678 7856	8866 1056	9055 4256
17	7951 2889	8130 6289	8311 9689	8495 3089	8680 6489	8867 9889	9057 3289
18	7953 0724	8132 4324	8313 7924	8497 1524	8682 5124	8869 8724	9059 2324
19	7954 8561	8134 2361	8315 6161	8498 9961	8684 3761	8871 7561	9061 1361
20	7956 6400	8136 0400	8317 4400	8500 8400	8686 2400	8873 6400	9063 0400
21	7958 4241	8137 8441	8319 2641	8502 6841	8688 1041	8875 5241	9064 9441
22	7960 2084	8139 6484	8321 0884	8504 5284	8689 9684	8877 4084	9066 8484
23	7961 9929	8141 4529	8322 9129	8506 3729	8691 8329	8879 2929	9068 7529
24	7963 7776	8143 2576	8324 7376	8508 2176	8693 6976	8881 1776	9070 6576
25	7965 5625	8145 0625	8326 5625	8510 0625	8695 5625	8883 0625	9072 5625
26	7967 3476	8146 8676	8328 3876	8511 9076	8697 4276	8884 9476	9074 4676
27	7969 1329	8148 6729	8330 2129	8513 7529	8699 2929	8886 8329	9076 3729
28	7970 9184	8150 4784	8332 0384	8515 5984	8701 1584	8888 7184	9078 2784
29	7972 7041	8152 2841	8333 8641	8517 4441	8703 0241	8890 6041	9080 1841
30	7974 4900	8154 0900	8335 6900	8519 2900	8704 8900	8892 4900	9082 0900
31	7976 2761	8155 8961	8337 5161	8521 1361	8706 7561	8894 3761	9083 9961
32	7978 0624	8157 7024	8339 3424	8522 9824	8708 6224	8896 2624	9085 9024
33	7979 8489	8159 5089	8341 1689	8524 8289	8710 4889	8898 1489	9087 8089
34	7981 6356	8161 3156	8342 9956	8526 6756	8712 3556	8900 0356	9089 7156
35	7983 4225	8163 1225	8344 8225	8528 5225	8714 2225	8901 9225	9091 6225
36	7985 2096	8164 9296	8346 6496	8530 3696	8716 0896	8903 8096	9093 5296
37	7986 9969	8166 7369	8348 4769	8532 2169	8717 9569	8905 6969	9095 4369
38	7988 7844	8168 5444	8350 3044	8534 0644	8719 8244	8907 5844	9097 3444
39	7990 5721	8170 3521	8352 1321	8535 9121	8721 6921	8909 4721	9099 2521
40	7992 3600	8172 1600	8353 9600	8537 7600	8723 5600	8911 3600	9101 1600
41	7994 1481	8173 9681	8355 7881	8539 6081	8725 4281	8913 2481	9103 0681
42	7995 9364	8175 7764	8357 6164	8541 4564	8727 2964	8915 1364	9104 9764
43	7997 7249	8177 5849	8359 4449	8543 3049	8729 1649	8917 0249	9106 8849
44	7999 5136	8179 3936	8361 2736	8545 1536	8731 0336	8918 9136	9108 7936
45	8001 3025	8181 2025	8363 1025	8547 0025	8732 9025	8920 8025	9110 7025
46	8003 0916	8183 0116	8364 9316	8548 8516	8734 7716	8922 6916	9112 6116
47	8004 8809	8184 8209	8366 7609	8550 7009	8736 6409	8924 5809	9114 5209
48	8006 6704	8186 6304	8368 5904	8552 5504	8738 5104	8926 4704	9116 4304
49	8008 4601	8188 4401	8370 4201	8554 4001	8740 3801	8928 3601	9118 3401

	89..	90..	91..	92..	93..	94..	95..
50	8010 2500	8190 2500	8372 2500	8556 2500	8742 2500	8930 2500	9120 2500
51	8012 0401	8192 0601	8374 0801	8558 1001	8744 1201	8932 1401	9122 1601
52	8013 8304	8193 8704	8375 9104	8559 9504	8745 9904	8934 0304	9124 0704
53	8015 6209	8195 6809	8377 7409	8561 8009	8747 8609	8935 9209	9125 9809
54	8017 4116	8197 4916	8379 5716	8563 6516	8749 7316	8937 8116	9127 8916
55	8019 2025	8199 3025	8381 4025	8565 5025	8751 6025	8939 7025	9129 8025
56	8020 9936	8201 1136	8383 2336	8567 3536	8753 4736	8941 5936	9131 7136
57	8022 7849	8202 9249	8385 0649	8569 2049	8755 3449	8943 4849	9133 6249
58	8024 5764	8204 7364	8386 8964	8571 0564	8757 2164	8945 3764	9135 5364
59	8026 3681	8206 5481	8388 7281	8572 9081	8759 0881	8947 2681	9137 4481
60	8028 1600	8208 3600	8390 5600	8574 7600	8760 9600	8949 1600	9139 3600
61	8029 9521	8210 1721	8392 3921	8576 6121	8762 8321	8951 0521	9141 2721
62	8031 7444	8211 9844	8394 2244	8578 4644	8764 7044	8952 9444	9143 1844
63	8033 5369	8213 7969	8396 0569	8580 3169	8766 5769	8954 8369	9145 0969
64	8035 3296	8215 6096	8397 8896	8582 1696	8768 4496	8956 7296	9147 0096
65	8037 1225	8217 4225	8399 7225	8584 0225	8770 3225	8958 6225	9148 9225
66	8038 9156	8219 2356	8401 5556	8585 8756	8772 1956	8960 5156	9150 8356
67	8040 7089	8221 0489	8403 3889	8587 7289	8774 0689	8962 4089	9152 7489
68	8042 5024	8222 8624	8405 2224	8589 5824	8775 9424	8964 3024	9154 6624
69	8044 2961	8224 6761	8407 0561	8591 4361	8777 8161	8966 1961	9156 5761
70	8046 0900	8226 4900	8408 8900	8593 2900	8779 6900	8968 0900	9158 4900
71	8047 8841	8228 3041	8410 7241	8595 1441	8781 5641	8969 9841	9160 4041
72	8049 6784	8230 1184	8412 5584	8596 9984	8783 4384	8971 8784	9162 3184
73	8051 4729	8231 9329	8414 3929	8598 8529	8785 3129	8973 7729	9164 2329
74	8053 2676	8233 7476	8416 2276	8600 7076	8787 1876	8975 6676	9166 1476
75	8055 0625	8235 5625	8418 0625	8602 5625	8789 0625	8977 5625	9168 0625
76	8056 8576	8237 3776	8419 8976	8604 4176	8790 9376	8979 4576	9169 9776
77	8058 6529	8239 1929	8421 7329	8606 2729	8792 8129	8981 3529	9171 8929
78	8060 4484	8241 0084	8423 5684	8608 1284	8794 6884	8983 2484	9173 8084
79	8062 2441	8242 8241	8425 4041	8609 9841	8796 5641	8985 1441	9175 7241
80	8064 0400	8244 6400	8427 2400	8611 8400	8798 4400	8987 0400	9177 6400
81	8065 8361	8246 4561	8429 0761	8613 6961	8800 3161	8988 9361	9179 5561
82	8067 6324	8248 2724	8430 9124	8615 5524	8802 1924	8990 8324	9181 4724
83	8069 4289	8250 0889	8432 7489	8617 4089	8804 0689	8992 7289	9183 3889
84	8071 2256	8251 9056	8434 5856	8619 2656	8805 9456	8994 6256	9185 3056
85	8073 0225	8253 7225	8436 4225	8621 1225	8807 8225	8996 5225	9187 2225
86	8074 8196	8255 5396	8438 2596	8622 9796	8809 6996	8998 4196	9189 1396
87	8076 6169	8257 3569	8440 0969	8624 8369	8811 5769	9000 3169	9191 0569
88	8078 4144	8259 1744	8441 9344	8626 6944	8813 4544	9002 2144	9192 9744
89	8080 2121	8260 9921	8443 7721	8628 5521	8815 3321	9004 1121	9194 8921
90	8082 0100	8262 8100	8445 6100	8630 4100	8817 2100	9006 0100	9196 8100
91	8083 8081	8264 6281	8447 4481	8632 2681	8819 0881	9007 9081	9198 7281
92	8085 6064	8266 4464	8449 2864	8634 1264	8820 9664	9009 8064	9200 6464
93	8087 4049	8268 2649	8451 1249	8635 9849	8822 8449	9011 7049	9202 5649
94	8089 2036	8270 0836	8452 9636	8637 8436	8824 7236	9013 6036	9204 4836
95	8091 0025	8271 9025	8454 8025	8639 7025	8826 6025	9015 5025	9206 4025
96	8092 8016	8273 7216	8456 6416	8641 5616	8828 4816	9017 4016	9208 3216
97	8094 6009	8275 5409	8458 4809	8643 4209	8830 3609	9019 3009	9211 0409
98	8096 4004	8277 3604	8460 3204	8645 2804	8832 2404	9021 2004	9211 2604
99	8098 2001	8279 1801	8462 1601	8647 1401	8834 1201	9023 1001	9211 4801

	96..	97..	98..	99..
00	9216 0000	9409 0000	9604 0000	9801 0000
01	9217 9201	9410 9401	9605 9601	9802 9801
02	9219 8404	9412 8804	9607 9204	9804 9604
03	9221 7609	9414 8209	9609 8809	9806 9409
04	9223 6816	9416 7616	9611 8416	9808 9216
05	9225 6025	9418 7025	9613 8025	9810 9025
06	9227 5236	9420 6436	9615 7636	9812 8836
07	9229 4449	9422 5849	9617 7249	9814 8649
08	9231 3664	9424 5264	9619 6864	9816 8464
09	9233 2881	9426 4681	9621 6481	9818 8281
10	9235 2100	9428 4100	9623 6100	9820 8100
11	9237 1321	9430 3521	9625 5721	9822 7921
12	9239 0544	9432 2944	9627 5344	9824 7744
13	9240 9769	9434 2369	9629 4969	9826 7569
14	9242 8996	9436 1796	9631 4596	9828 7396
15	9244 8225	9438 1225	9633 4225	9830 7225
16	9246 7456	9440 0656	9635 3856	9832 7056
17	9248 6689	9442 0089	9637 3489	9834 6889
18	9250 5924	9443 9524	9639 3124	9836 6724
19	9252 5161	9445 8961	9641 2761	9838 6561
20	9254 4400	9447 8400	9643 2400	9840 6400
21	9256 3641	9449 7841	9645 2041	9842 6241
22	9258 2884	9451 7284	9647 1684	9844 6084
23	9260 2129	9453 6729	9649 1329	9846 5929
24	9262 1376	9455 6176	9651 0976	9848 5776
25	9264 0625	9457 5625	9653 0625	9850 5625
26	9265 9876	9459 5076	9655 0276	9852 5476
27	9267 9129	9461 4529	9656 9929	9854 5329
28	9269 8384	9463 3984	9658 9584	9856 5184
29	9271 7641	9465 3441	9660 9241	9858 5041
30	9273 6900	9467 2900	9662 8900	9860 4900
31	9275 6161	9469 2361	9664 8561	9862 4761
32	9277 5424	9471 1824	9666 8224	9864 4624
33	9279 4689	9473 1289	9668 7889	9866 4489
34	9281 3956	9475 0756	9670 7556	9868 4356
35	9283 3225	9477 0225	9672 7225	9870 4225
36	9285 2496	9478 9696	9674 6896	9872 4096
37	9287 1769	9480 9169	9676 6569	9874 3969
38	9289 1044	9482 8644	9678 6244	9876 3844
39	9291 0321	9484 8121	9680 5921	9878 3721
40	9292 9600	9486 7600	9682 5600	9880 3600
41	9294 8881	9488 7081	9684 5281	9882 3481
42	9296 8164	9490 6564	9686 4964	9884 3364
43	9298 7449	9492 6049	9688 4649	9886 3249
44	9300 6736	9494 5536	9690 4336	9888 3136
45	9302 6025	9496 5025	9692 4025	9890 3025
46	9304 5316	9498 4516	9694 3716	9892 2916
47	9306 4609	9500 4009	9696 3409	9894 2809
48	9308 3904	9502 3504	9698 3104	9896 2704
49	9310 3201	9504 3001	9700 2801	9898 2601

In this Table, To find the Square *of any Whole Number less than* Ten Thousand.

2. *To find the* Side *of any Square Number less than* 100 *Millions*

Imitate these following Examples.

I desire the Square of 9738; That is, of 9700 and 38. (For so every Side in this Table is parted into two Portions.) A Column bearing 97.. on the head of it, is found only in this 28. Page, and another in the 29. In the Margin of this 28. pag. I find 38. A right Line running from this 38, crosseth the column marked 97.. at the number 94828644. This 94828644 is the Square of 9738.

But the Square of 9756 is not found in this pag. because 56 is more than 49. In the margin of pag. 29, I find 56. A right line, running from this 56, sheweth you the number 95179536, in the column on whose top stands 97.. This 95179536 is the Square of 9756.

Thus in p. 24, you see 67551961 in the right Line running from the Marg. 19, to the column whose Vertical is 82.. Then say, 67551961 is the Square of 8219. And so I find 70829056 to be the Square of 8416. In pag. 9. you have 7080921 the Square of 2661. And in pag. 2, you have 675684 the Square of 822. As in pag. 3, you find 70756 the Square of 266.

That *First opening*, in Pag. 2. & Pag. 3, contains *all Square Numbers less than a Million*, and *all Sides less than* 1000. And the *first Column* of those two Pages [2 and 3] hath the Square of every Number less than 100; That is to say, It hath all square numbers less than 10000. Wherfore the *Square* number 62821476 being *given*, and its *Side* being *sought*, you may cut off the two last Figures [76] and may find all the rest of the Figures in that *First Opening*; thus,

Pag. 2. 62 is the Square of 7 and more.

Pag. 3. 6282 is the Square of 79 †

Pag. 3. 628214. is the Square of 792 †

This sends you to seek a Column, whose head bears 79 which is in *Pag.* 22. and 23. In *Pag.* 22. you may find the entire number 62821476. to be the Square of 7926.

	96..	97..	98..	99..
50	9312 2500	9506 2500	9702 2500	9900 2500
51	9314 1801	9508 2001	9704 2201	9902 2401
52	9316 1104	9510 1504	9706 1904	9904 2304
53	9318 0409	9512 1009	9708 1609	9906 2209
54	9319 9716	9514 0516	9710 1316	9908 2116
55	9321 9025	9516 0025	9712 1025	9910 2025
56	9323 8336	9517 9536	9714 0736	9912 1936
57	9325 7649	9519 9049	9716 0449	9914 1849
58	9327 6964	9521 8564	9718 0164	9916 1764
59	9329 6281	9523 8081	9719 9881	9918 1681
60	9331 5600	9525 7600	9721 9600	9920 1600
61	9333 4921	9527 7121	9723 9321	9922 1521
62	9335 4244	9529 6644	9725 9044	9924 1444
63	9337 3569	9531 6169	9727 8769	9926 1369
64	9339 2896	9533 5696	9729 8496	9928 1296
65	9341 2225	9535 5225	9731 8225	9930 1225
66	9343 1556	9537 4756	9733 7956	9932 1156
67	9345 0889	9539 4289	9735 7689	9934 1089
68	9347 0224	9541 3824	9737 7424	9936 1024
69	9348 9561	9543 3361	9739 7161	9938 0961
70	9350 8900	9545 2900	9741 6900	9940 0900
71	9352 8241	9547 2441	9743 6641	9942 0841
72	9354 7584	9549 1984	9745 6384	9944 0784
73	9356 6929	9551 1529	9747 6129	9946 0729
74	9358 6276	9553 1076	9749 5876	9948 0676
75	9360 5625	9555 0625	9751 5625	9950 0625
76	9362 4976	9557 0176	9753 5376	9952 0576
77	9364 4329	9558 9729	9755 5129	9954 0529
73	9366 3684	9560 9284	9757 4884	9956 0484
79	9368 3041	9562 8841	9759 4641	9958 0441
80	9370 2400	9564 8400	9761 4400	9960 0400
81	9372 1761	9566 7961	9763 4161	9962 0361
82	9374 1124	9568 7524	9765 3924	9964 0324
83	9376 0489	9570 7089	9767 3689	9966 0289
84	9377 9856	9572 6656	9769 3456	9968 0256
85	9379 9225	9574 6225	9771 3225	9970 0225
86	9381 8596	9576 5796	9773 2796	9972 0196
87	9383 7969	9578 5369	9775 2769	9974 0169
88	9385 7344	9580 4944	9777 2544	9976 0144
89	9387 6721	9582 4521	9779 2321	9978 0121
90	9389 6100	9584 4100	9781 2100	9980 0100
91	9391 5481	9586 3681	9783 1881	9982 0081
92	9393 4864	9588 3264	9785 1664	9984 0064
93	9395 4249	9590 2849	9787 1449	9986 0049
94	9397 3636	9592 2436	9789 1236	9988 0036
95	9399 3025	9594 2025	9791 1025	9990 0025
96	9401 2416	9596 1616	6793 0816	9992 0016
97	9403 1809	9598 1209	9795 0609	9994 0009
98	9405 1204	9600 0804	9797 0404	9996 0004
99	9407 0601	9602 0401	9799 0201	9998 0001

This table of Sides and Squares may be continued by Addition of all the Odd numbers greater than 19997. Thus.

When the difference of two Sides is an Unite, The difference of their Squares is the ſum of thoſe Sides.

Sides.	*Squares.*
9999	99980001
	19999
10000	100000000
	20001
10001	100020001
	20003
10002	100040004
&c.	&c.

If it be *continued* to the Square of 100000, it may be printed like the foregoing Table. Every page may have ſix columns; then 300 pages will hold 90000 Squares. And ſo the whole Table of 100 thouſand Squares may end in pag. 329. *A Book* of 168 leaves is not an inch thick.

But *without* ſuch a *Book*, the numbers in theſe 29 pages, may be made uſe of.

1. *To find the Square of any whole number leſs than* 100 *thouſand. Thus,*

A	2 9947	9 4706
B	2 9940	9 4700
C	5 9887	18 9406
D	7	6
E	41 9209	113 6436
F	89640 3600	89 6809 0000
G	89682 2809	89 6922 6436

A	The given Number: Of 5 figures
D	The fifth or laſt figure of A.
B	Remainder, after ſubſtract. D from A.
C	The Sum of A and B
E	C multiplyed by D
F	The Square of B. 00 after a Tabular
G	The Sum of E and F. The Square of A.

2. *To find the Side of any Square number, of more than* 8 *figures.*

From the end of the given number, Cut off as many pairs of figures as may leave 7 or 8 figures. From theſe ſubſtract as great a Tabular number as you can. Note the 4 figures of the Side of that Tabular ſquare, *&c.*

As	2 9 9 4 7	9 4 7 0 6
	8 9682 2809	89 6922 6436
	8 9640 36	89 6809 00
	41 9209	113 6436
	5 9887	18 9406
	41 9209	113 6436
	0	0

Terminationes ſive Poſteriores notæ Quadratorum Numerorum.

Omnis *Quadrati* numeri Nota *poſtrema* eſt 0, 1, 4, 5, 6, vel 9.
Poſteriores binas, ternas, quaternas exhibent hæc tria Cribra Synoptica.

22 *Endings of Square numbers in 2 figures.*										
0										0
0	2	4	6	8						1
0	2	4	6	8						4
	2									5
	1	3	5	7	9					6
0	2	4	6	8	1					9

159 *Endings of Square numbers in 3 figures.*										
0.1	4.5.6	9								00
0	2	4	6	8	1					01
0.1.2.3.4.5.6.7.8.9										04
0	2	4	6	8						09
0.1.2.3.4.5.6	7.8	9								16
1	3	5	7	9						21
0	1.2.3.4.5	6.7	8.9							24
0	2	6								25
1	3	5	7	9						29
0.1.2.3.4.5.6.7.8.9										36
0	2	4	6	8						41
0.1.2.3.4	5.6.7.8.9									44
0	2	4	6	8						49
0.1.2.3.4.5.6.7.8.9										56
1	3	5	7	9						61
0.1.2.3.4.5.6.7.8.9										64
1	3	5	7	9						69
0.1.2.3.4.5.6	7.8.9									76
0	2	4	6	8						81
0.1.2.3.4.5.6.7.8.9										84
0	2	4	6	8						89
0.1.2.3.4.5.6.7	8.9									96

1044 Endings of Square Numbers in 4 Figures.

0										000
0.1.2.3.4.5.6.7.8.9										001
0	2	4	6	8						004
0.1.2.3.4.5.6.7.8.9										009
0	2	4	6	8						016
1	3	5	7	9						024
0.1.2.3.4.5.6.7.8.9										025
0	2	4	6	8						036
0.1.2.3.4.5.6.7.8.9										041
1	3	5	7	9						044
0.1.2.3.4.5.6.7.8.9										049
1	3	5	7	9						056
0	2	4	6	8						064
1	3	5	7	9						076
0.1.2.3.4.5.6.7.8.9										081
0	2	4	6	8						084
0.1.2.3.4.5.6.7.8.9										089
0	2	4	6	8						096
0	2	4	6	8						100
1	3	5	7	9						104
0	2	4	6	8						116
0.1.2.3.4.5.6.7.8.9										121
1	3	5	7	9						124
0.1.2.3.4.5.6.7.8.9										129
1	3	5	7	9						136
0	2	4	6	8						144
1	3	5	7	9						156
0.1.2.3.4.5.6.7.8.9										161
0	2	4	6	8						164
0.1.2.3.4.5.6.7.8.9										169
0	2	4	6	8						176
1	3	5	7	9						184
0	2	4	6	8						196
0.1.2.3.4.5.6.7.8.9										201
1	3	5	7	9						204
0.1.2.3.4.5.6.7.8.9										209
1	3	5	7	9						216
0	2	4	6	8						224
0.1.2.3.4.5.6.7.8.9										225
1	3	5	7	9						236
0.1.2.3.4.5.6.7.8.9										241
0	2	4	6	8						244
0.1.2.3.4.5.6.7.8.9										249
0	2	4	6	8						256
1	3	5	7	9						264
0	2	4	6	8						276
0.1.2.3.4.5.6.7.8.9										281
1	3	5	7	9						284
0.1.2.3.4.5.6.7.8.9										289
1	3	5	7	9						296
0	2	4	6	8						304
1	3	5	7	9						316
0.1.2.3.4.5.6.7.8.9										321
0	2	4	6	8						324
0.1.2.3.4.5.6.7.8.9										329
0	2	4	6	8						336
1	3	5	7	9						344
0	2	4	6	8						356
0.1.2.3.4.5.6.7.8.9										361
1	3	5	7	9						364
0.1.2.3.4.5.6.7.8.9										369
1	3	5	7	9						376
0	2	4	6	8						384
1	3	5	7	9						396
0	2	4	6	8						400
0.1.2.3.4.5.6.7.8.9										401
0	2	4	6	8						404
0.1.2.3.4.5.6.7.8.9										409
0	2	4	6	8						416
1	3	5	7	9						424
0	2	4	6	8						436
0.1.2.3.4.5.6.7.8.9										441
1	3	5	7	9						444
0.1.2.3.4.5.6.7.8.9										449
1	3	5	7	9						456
0	2	4	6	8						464
1	3	5	7	9						476
0.1.2.3.4.5.6.7.8.9										481
0	2	4	6	8						484
0.1.2.3.4.5.6.7.8.9										489
0	2	4	6	8						496
2										500
1	3	5	7	9						504
0	2	4	9	8						516
0.1.2.3.4.5.6.7.8.9										521
1	3	5	7	9						524
0.1.2.3.4.5.6.7.8.9										529
1	3	5	7	9						536
0	2	4	6	8						544
1	3	5	7	9						556
0.1.2.3.4.5	6.7.8.9									561
0	2	4	6	8						564
0.1.2.3.4.5.6.7.8.9										569
0	2	4	6	8						576
1	3	5	7	9						584
0	2	4	6	8						596
1	3	5	7	9						600
0.1.2.3.4.5.6.7.8.9										601
1	3	5	7	9						604
0.1.2.3.4.5.6.7.8.9										609
1	3	5	7	9						616
0	2	4	6	8						624
0	5									625
1	3	5	7	9						636
0.1.2.3.4.5.6.7.8.9										641
0	2	4	6	8						644
0.1.2.3.4.5.6.7.8.9										649
0	2	4	6	8						656
1	3	5	7	9						664
0	2	4	6	8						676
0.1.2.3.4.5.6.7.8.9										681
1	3	5	7	9						684
0.1.2.3.4.5.6.7.8.9										689
1	3	5	7	9						696
0	2	4	6	8						704
1	3	5	7	9						716
0.1.2.3.4.5.6.7.8.9										721
0	2	4	6	8						724
0.1.2.3.4.5.6.7.8.9										729
0	2	4	6	8						736
1	3	5	7	9						744
0	2	4	6	8						756
0.1.2.3.4.5.6.7.8.9										761
1	3	5	7	9						764
0.1.2.3.4.5.6.7.8.9										769
1	3	5	7	9						776
0	2	4	6	8						784
1	3	5	7	9						796
0.1.2.3.4.5.6.7.8.9										801
0	2	4	6	8						804
0.1.2.3.4.5.6.7.8.9										809
0	2	4	6	8						816
1	3	5	7	9						824
0	2	4	6	8						836
0.1.2.3.4.5.6.7.8.9										841
1	3	5	7	9						844
0.1.2.3.4.5.6.7.8.9										849
1	3	5	7	9						856
0	2	4	6	8						864
1	3	5	7	9						876
0.1.2.3.4.5.6.7.8.9										881
0	2	4	6	8						884
0.1.2.3.4.5.6.7.8.9										889
0	2	4	6	8						896
0	2	4	6	8						900
1	3	5	7	9						904
0	2	4	6	8						916
0.1.2.3.4.5.6.7.8.9										921
1	3	5	7	9						924
0.1.2.3.4.5.6.7.8.9										929
1	3	5	7	9						936
0	2	4	6	8						944
1	3	5	7	9						956
0.1.2.3.4.5.6.7.8.9										961
0	2	4	6	8						964
0.1.2.3.4.5.6.7.8.9										969
0	2	4	6	8						976
1	3	5	7	9						984
0	2	4	6	8						996

This Synopſis may ſeem too ſhort. See them all at large in the next leaf.

In this Table to find the Square of any whole Number Less than Ten Thousand.

2 To find the Side of any Square Number Less than 100 Million

Imediately the following Example.

I desire the Square of 9604 which as you see in this Table is parted into two portions 4 Columns wide 97 on the Left hand, is found only in the 28th. page, and one line more 29th. in the Margin of this 28th. page I find 38. a Right line running from this 38, crosseth the Column 97 at the number 92416.

This 9402 & 14. is the Square of 36.

But if the Number 9604 be not found in any page Column 4 Figures more than 49. in the Margin of page 29, I find 36 a Right line running thence this 36, underneath upon the Number 97 in 36, in the Column standing 97.

Square of 36.

Thus in 24 pages of this Book are placed in the Margin 24 Right lines running from the Margin of the 1st. page to the 24th. And the Number 67551961 is the Square of 8219. And to find the Square of 20056 the Square of the Root is 141. and the Square of 2001 is 44. and the Square of 2002 is 44. and the Square of 206.

2 This Table contains all Square Numbers under 10000. And the first Column of each page (2 and 3 Numbers) Square of any number less than 100 and the Number 9604 is less than

Pag. 2. 62 is the square of 7 and more

Pag. 3. 6282 is the square of 79 +

Pag. 3. 62814 is the square of 792 +

This Lemma we have a Column in the next head board 79 in the
is in page 22. & 23. In pag. 22. you may find the Entire Num
62821476. to be the square of 7926.

Every Square Number endeth in 0. 1. 4. 5. 6 or 9. None endeth in 2. 3. 7 or 8.
Every Square number hath one of theſe two and twenty endings
00. 01, 04. 09, 15, 21. 24. 25. 29. 36. 41. 44. 49. 56. 61. 64. 69. 76. 81. 84. 89. 96.
Every Square number hath one of theſe 159 Endings.

000	044	100	161	216	276	329	396	444	500	561	609	664	724	784	844	900	961
001	049	104	164	224	281	336	400	449	504	564	616	676	729	796	849	904	964
004	056	116	169	225	284	344	401	456	516	569	624	681	736	801	856	916	969
009	064	121	176	236	289	356	404	464	521	576	625	684	744	804	864	921	976
016	076	124	184	241	296	361	409	476	524	584	636	689	756	809	876	924	984
024	081	129	196	244	304	364	416	481	529	596	641	696	761	816	881	929	996
025	084	136	249	249	316	369	424	484	536	600	644	704	764	824	884	936	
036	089	144	256	256	321	376	436	489	544	601	649	716	769	836	889	944	
041	096	156	264	264	324	384	441	496	556	604	656	721	776	841	896	956	

Every Square number hath one of theſe 1044 Endings.

0000	0304	0641	0969	1296	1641	2001	2321	2644	2976	3316	3649	4004	4324
0001	0321	0644	0976	1316	1649	2004	2324	2649	2996	3321	3664	4009	4329
0004	0324	0649	0996	1321	1664	2009	2329	2656	3001	3329	3681	4016	4336
0009	0329	0656	1001	1329	1681	2016	2336	2676	3009	3344	3684	4025	4356
0016	0336	0676	1009	1344	1684	2025	2356	2681	3024	3361	3689	4036	4361
0025	0356	0681	1024	1361	1689	2036	2361	2689	3025	3364	3696	4041	4369
0036	0361	0689	1025	1364	1696	2041	2369	2704	3041	3369	3716	4049	4384
0041	0369	0704	1041	1369	1716	2049	2384	2721	3044	3376	3721	4064	4400
0049	0384	0721	1044	1376	1721	2064	2400	2724	3049	3396	3729	4081	4401
0064	0400	0724	1049	1396	1729	2081	2401	2729	3056	3401	3744	4084	4404
0081	0401	0729	1056	1401	1744	2084	2404	2736	3076	3409	3761	4089	4409
0084	0404	0736	1076	1409	1761	2089	2409	2756	3081	3424	3764	4096	4416
0089	0409	0756	1081	1424	1764	2096	2416	2761	3089	3441	3769	4100	4436
0096	0416	0761	1089	1441	1769	2100	2436	2769	3104	3444	3776	4116	4441
0100	0436	0769	1104	1444	1776	2116	2441	2784	3121	3449	3796	4121	4449
0116	0441	0784	1121	1449	1796	2121	2449	2801	3124	3456	3801	4129	4464
0121	0449	0801	1124	1456	1801	2129	2464	2804	3129	3476	3809	4144	4481
0129	0464	0804	1129	1476	1809	2144	2481	2809	3136	3481	3824	4161	4484
0144	0481	0809	1136	1481	1824	2161	2484	2816	3156	3489	3841	4164	4489
0161	0484	0816	1156	1489	1841	2164	2489	2836	3161	3504	3844	4169	4496
0164	0489	0836	1161	1504	1844	2169	2496	2841	3169	3521	3849	4176	4516
0169	0496	0841	1169	1521	1849	2176	2500	2849	3184	3524	3856	4196	4521
0176	0516	0849	1184	1524	1856	2196	2516	2864	3201	3529	3876	4201	4529
0196	0521	0864	1201	1529	1876	2201	2521	2881	3204	3536	3881	4209	4544
0201	0529	0881	1204	1536	1881	2209	2529	2884	3209	3556	3889	4224	4561
0209	0544	0884	1209	1556	1889	2224	2544	2889	3216	3561	3904	4225	4564
0224	0561	0889	1216	1561	1904	2225	2561	2896	3225	3569	3921	4241	4569
0225	0564	0896	1225	1569	1921	2241	2564	2900	3236	3584	3924	4244	4576
0241	0569	0900	1236	1584	1924	2244	2569	2916	3241	3600	3929	4249	4596
0244	0576	0916	1241	1600	1929	2249	2576	2921	3249	3601	3936	4256	4601
0249	0596	0921	1249	1601	1936	2256	2596	2929	3264	3604	3956	4276	4609
0256	0601	0929	1264	1604	1956	2276	2601	2944	3281	3609	3961	4281	4624
0276	0609	0944	1281	1609	1961	2281	2609	2961	3284	3616	3969	4289	4641
0281	0624	0961	1284	1616	1969	2289	2624	2964	3289	3636	3984	4304	4644
0289	0625	0964	1289	1636	1984	2304	2641	2969	3296	3641	4001	4321	4649
													4656

The 41 Last Notes of Square numbers.

4656	5044	5441	5809	6196	6569	6964	7344	7729	8116	8489	8889	9264	9649
4676	5049	5444	5824	6201	6576	6969	7351	7744	8121	8496	8896	9281	9664
4681	5056	5449	5841	6209	6596	6976	7361	7761	8129	8516	8900	9284	9681
4689	5076	5456	5844	6224	6601	6996	7369	7764	8144	8521	8916	9289	9684
4704	5081	5476	5849	6225	6609	7001	7376	7769	8161	8529	8921	9296	9689
4721	5089	5481	5856	6241	6524	7009	7396	7716	8164	8544	8929	9316	9696
4724	5104	5489	5876	6244	6541	7024	7401	7796	8169	8561	8944	9321	9716
4729	5121	5504	5881	6249	6544	7025	7409	7801	8176	8564	8961	9329	9721
4736	5124	5521	5889	6256	6649	7041	7424	7309	8196	8569	8964	9344	9729
4756	5129	5524	5904	6276	6556	7041	7441	7824	8201	8576	8969	9361	9744
4761	5136	5529	5921	6281	6576	7049	7444	7841	8209	8596	8976	9364	9761
4769	5156	5536	5924	6289	6681	7056	7449	7344	8224	8601	8996	9369	9764
4784	5161	5556	5929	6304	6689	7076	7456	7849	8225	8609	9001	9376	9769
4801	5169	5561	5936	6321	6704	7081	7476	7356	8241	8624	9009	9396	9776
4804	5184	5569	5956	6324	6721	7089	7481	7376	8241	8641	9024	9401	9796
4809	5201	5584	5961	6329	6724	7104	7489	7381	8249	8644	9025	9409	9801
4816	5204	5600	5969	6336	6729	7121	7501	7389	8256	8649	9041	9424	9809
4836	5209	5601	5984	6356	6736	7124	7521	7904	8276	8656	9044	9441	9824
4841	5216	5604	6001	6361	6756	7129	7524	7921	8281	8676	9049	9444	9841
4849	5225	5609	6004	6369	6761	7136	7529	7924	8289	8681	9056	9449	9844
4864	5236	5616	6009	6384	6769	7156	7536	7929	8304	8689	9076	9456	9849
4881	5241	5625	6016	6400	6784	7161	7556	7936	8321	8704	9081	9476	9856
4884	5249	5636	6025	6401	6801	7169	7561	7956	8324	8721	9089	9481	9876
4889	5264	5641	6036	6404	6804	7184	7569	7961	8329	8724	9104	9489	9881
4896	5281	5649	6041	6409	6809	7201	7584	7969	8336	8729	9121	9504	9889
4900	5284	5664	6049	6416	6816	7204	7600	7984	8356	8736	9124	9521	9904
4916	5289	5681	6064	6436	6836	7209	7601	8001	8361	8756	9129	9524	9921
4921	5296	5684	6081	6441	6841	7216	7604	8004	8369	8761	9136	9529	9924
4929	5316	5689	6084	6449	6849	7225	7609	8009	8384	8769	9156	9536	9929
4944	5321	5696	6089	6464	6864	7236	7616	8016	8400	8784	9161	9556	9936
4961	5329	5716	6096	6481	6881	7241	7636	8025	8401	8801	9169	9561	9956
4964	5344	5721	6100	6484	6884	7249	7641	8036	8404	8804	9184	9569	9961
4969	5361	5729	6116	6489	6889	7264	7649	8041	8409	8809	9201	9584	9969
4976	5364	5744	6121	6496	6896	7281	7664	8049	8416	8816	9204	9600	9984
4996	5369	5761	6129	6516	6900	7284	7681	8064	8436	8836	9209	9601	
5001	5376	5764	6144	6521	6916	7289	7684	8081	8441	8841	9216	9604	
5009	5396	5769	6161	6529	6921	7296	7689	8084	8449	8849	9225	9609	
5024	5301	5776	6164	6544	6929	7316	7696	8089	8464	8864	9236	9616	
5025	5309	5796	6169	6561	6944	7321	7716	8096	8481	8881	9241	9636	
5041	5324	5801	6176	6564	6961	7329	7721	8100	8484	8884	9249	9641	

How long soever a Number be, if its ending be *not found* in its proper place in *Pag.* 31 or 32, *it is no Square*. You have 4 numbers, of which *you* tell me nothing, but that one of them ends in 2, another in 35, another in 789, and the last ends in 6796. *Pag.* 31. 32. tell me, that no square number can end so. I therefore tell you, that none of your 4 numbers is a Square. But of these 8968 14656. 89683 5044. 89684 5441. 89693 05809. 89694 16196. 89694 26569, I finde the 4 last figures in the uppermost line of this page, and yet I must not say that any of them is a true Square. *Page* 29 makes me see they are not.

Knowing the two, three or four last *figures of any Square number, to exhibit as many of the last figures of its Root*, is a New Question: To which, the just Answers are manifold and not obvious, [illegible] when it shall be desired.

A Tale of a TUB, or The Greenwich PROBLEM.

GREENWICH, *the Twenty sixth of* July, 1673.

THe Dimensions of Mr. *Peter Cooper* his third Round Tun, taken in Inches by *N. G.* and *M. D.*

Data
- The Diameters at the top 119 and 120.5; the Geometrical Mean 119.24 = A A.
- The Diameters at the bottom 121.5 and 127; the Geometrical Mean 124.21 = B B.
- The Perpendicular from the top to the bottom 46.5 = P P.
- After these Dimensions were taken in the dry Tun, there was (upon the Fifth of *December*, 1674.) water 2:1:5 two Barrels, one Firkin, five Gallons, put in to replete the Horizon; and then at the Extremities of the Diameter C C, on the Surface of the water, there was taken wet the Depths C D = 2 and C D = 2.

Ques. The Frustum of Air A A C C A?

Solu. The Frustum of Air A A C C A = 50:3:7

But the Frustum A A C C A found by Art = 50:3:7 and the Frustum C C B B C measured by water = 2:1:5 is the whole Content of this Tun in Beer Measure = 53:1:3 not 54:0:3.

The Dimensions of the said Tun as they were taken in Inches the Fifth of *December*, 1674.

Data
- The Diameters at the top 117.3 and 120.5; the Geometrical Mean 118.88 = A A.
- The Diameters at the bottom 125.5 and 126.75; the Geometrical Mean 126.12 = B B.
- The Perpendicular from the top to the bottom 46.4 = P P.
- After these Dimensions were taken in the dry Tun, there was water 2:1:5 put in to replete the Horizon; and then at the Extremities of the Diameter C C, on the Surface of the water, there was taken wet the Depths C D = 2 and C D = 2.

Ques. The Frustum of Air A A C C A?

Solu. The Frustum of Air A A C C A = 51:1:6.

But the Frustum A A C C A found by Art = 51:1:6 and the Frustum C C B B C measured by water = 2:1:5 is the whole Content of this Tun in Beer Measure = 53:3:2 not 54:1:5.

Object. There were several other Depths as well as C D, taken in the Perimeter of the bottom of this Tun, and therefore this way of Casting up this Tun cannot be true.

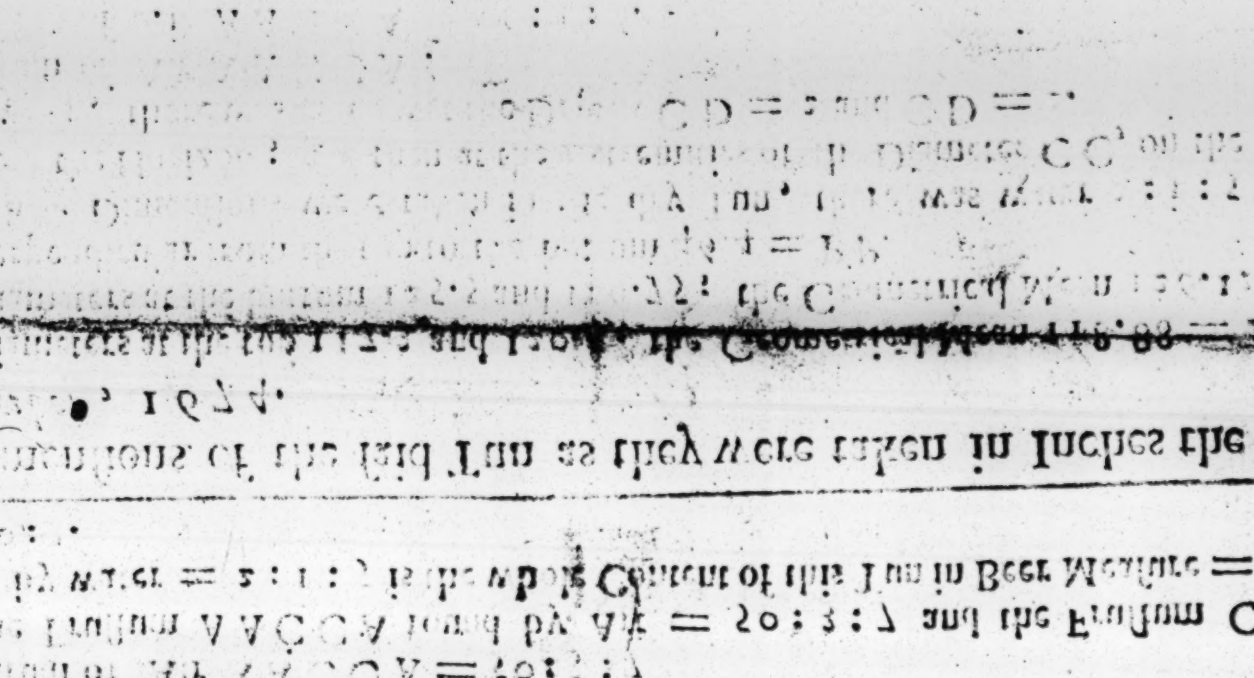
Answ. Although there were several Depths, as well as C D, taken in the Perimeter of the bottom of this Tun, yet there was no other pair of Depths diametrically opposite and equal, but these C D and C D; and if the Perimeter of the bottom (*viz.* the Perimeter at the Croes, I do not mean the whole Surface of the bottom) be granted to be in one and the same Plane, I do aver, that this way of Casting up of this Tun is true to any way or every way of Casting of it up.

Object. But the Croes of this Tun are of a Snaky Form or Figure, and consequently not in one and the same Plane.

Answ. Then there is no way afforded for any Analogy between the Diameters taken on the bottom (*viz.* the Perimeter at the Croes, I do not mean the whole Surface of the bottom) and those on the Surface of the water: So then, if the Croes be not granted to be in one and the same Plane, there is no way afforded to get the Diameters on the Surface of the water, but by taking of them actually; which all Practical *Gangers* judge very uncertain and impracticable.

The reason that the Casting of *December* 5. 1674. exceeds that of *July* 26. 1673. is, for that the Diameters of the bottom are not Rectangular Conjugates, and I judge may be said to be an insufficient *Data*, because *July* 26. shews a longer and a shorter upon the same bottom.

By *MICHAEL DARY.*



A TABLE OF LOGARITHMS, or The Geometrick KROBLEM.

An *Excellent* TABLE *for the finding the* Periferies *or* Circumferences *of all Elleipſes or* Ovals, *ſo near the Truth as any Mechanical Practice can require, Calculated with great Diligence and Care by Sir* JONAS MOORE, *and not done before.*

Axis.	Periferies.	Diff.	Axis.	Periferies.	Diff.
1	2.0000		50	2.4218	
		-12			124
1	2.0012	16	51	2.4342	125
2	2.0028	20	52	2.4467	127
3	2.0048	26	53	2.4594	129
4	2.0072	28	54	2.4723	129
5	2.0100		55	2.4852	131
		-33			
6	2.0133	37	56	2.4983	131
7	2.0170	43	57	2.5114	131
8	2.0213	48	58	2.5245	132
9	2.0261	53	59	2.5377	133
10	2.0314		60	2.5510	134
		-56			
11	2.0370	62	61	2.5644	135
12	2.0432	64	62	2.5779	136
13	2.0496	68	63	2.5915	137
14	2.0564	70	64	2.6052	137
15	2.0634		65	2.6189	138
		-74			
16	2.0708	76	66	2.6327	138
17	2.0784	78	67	2.6465	139
18	2.0862	80	68	2.6604	140
19	2.0942	82	69	2.6744	140
20	2.1024		70	2.6884	141
		-82			
21	2.1106	86	71	2.7025	141
22	2.1192	89	72	2.7166	143
23	2.1281	92	73	2.7309	144
24	2.1373	94	74	2.7453	146
25	2.1467		75	2.7599	146
		-94			
26	2.1561	97	76	2.7745	146
27	2.1658	98	77	2.7891	147
28	2.1756	100	78	2.8038	148
29	2.1856	100	79	2.8186	148
30	2.1956	101	80	2.8334	148
31	2.2057	103	81	2.8482	148
32	2.2160	104	82	2.8630	149
33	2.2264	104	83	2.8779	150
34	2.2368	106	84	2.8929	150
35	2.2474	107	85	2.9080	151
36	2.2581	111	86	2.9231	151
37	2.2692	111	87	2.9382	152
38	2.2803	112	88	2.9534	152
39	2.2915	113	89	2.9686	153
40	2.3028	114	90	2.9839	154
41	2.3142	114	91	2.9993	154
42	2.3256	115	92	3.0147	155
43	2.3371	117	93	3.0302	156
44	2.3488	119	94	3.0458	156
45	2.3607	119	95	3.0614	157
46	2.3726	122	96	3.0771	157
47	2.3848	122	97	3.0928	158
48	2.3970	124	98	3.1086	158
49	2.4094	124	99	3.1244	158
50	2.4218		⊙	3.1402	

EXAMPLE I.

WHere the longer *Axis* of the Elleipſis is 1, and the ſhorter .78; Becauſe the Table is made for ſuch Elleipſis, enter with .78, the Perifery of that Elleipſis will be 2.8038.

EXAMPLE II.

The longer *Axis* 1, the ſhorter .4382, I enter with .43, gives 2.3371: Then to find the part anſwering to .71, ſay, If 100 give .117; what ſhall .71 give? Anſw. .83, which added to 2.3371, gives 2.3454 for the Perifery deſired.

EXAMPLE III.

Where the longer *Axis* is 388, the ſhorter 280, firſt ſay, 388 : 280 :: 1.000 : Anſw. .721, ſeek in the Table for .72, it gives 2.7166; then ſay, 1.0000 : 2.7166 :: 388 : Anſw. 1054.06, which is the Circumference deſired.

EXAMPLE IV.

The longer Diameter 32.54, the ſhorter 18.64; ſay; 32.54 : 18.64 :: 1.000 ; 572; to which in the Table anſwers 2.5114, and the part proportional for 2 is 26, which makes the whole 2.5140; then 1.000 : 2.5140 :: 32.54 : 81.805 the Perifery required. The Area or Superficies of an Elleipſis is eaſily got by this Rule. As the longer Diameter, is to the ſhorter: So is the Circle of the longer Diameter, to the Elleipſis.

I have made above 45000 *Arithmetical Operations* for this *Table*, and am now well pleaſed it is finiſhed. Some perhaps may find ſhorter ways, as I believed I had my ſelf, 'till adviſed otherwiſe by the truly *Honourable the Lord BRUNCKER.* I therefore purſued the Rules given by me, in that *Contemplation of the Elleipſis* Printed in my *Arithmetick*, taking 100 Elleipſis betwixt that which falls upon the *Diameter* equal in this caſe to 2.0000 the firſt in the Table, and the greateſt which is the Circle 3.1402 the laſt.

SOLI DEO GLORIA.

EXAMPLE I.

EXAMPLE II.

EXAMPLE III.

EXAMPLE IV.

EXAMPLE V.

EXAMPLE VI.

A Brief (but full)

ACCOUNT

OF THE

DOCTRINE

OF

Trigonometry,

BOTH

PLAIN and SPHERICAL.

BY

JOHN CASWELL, M. A.

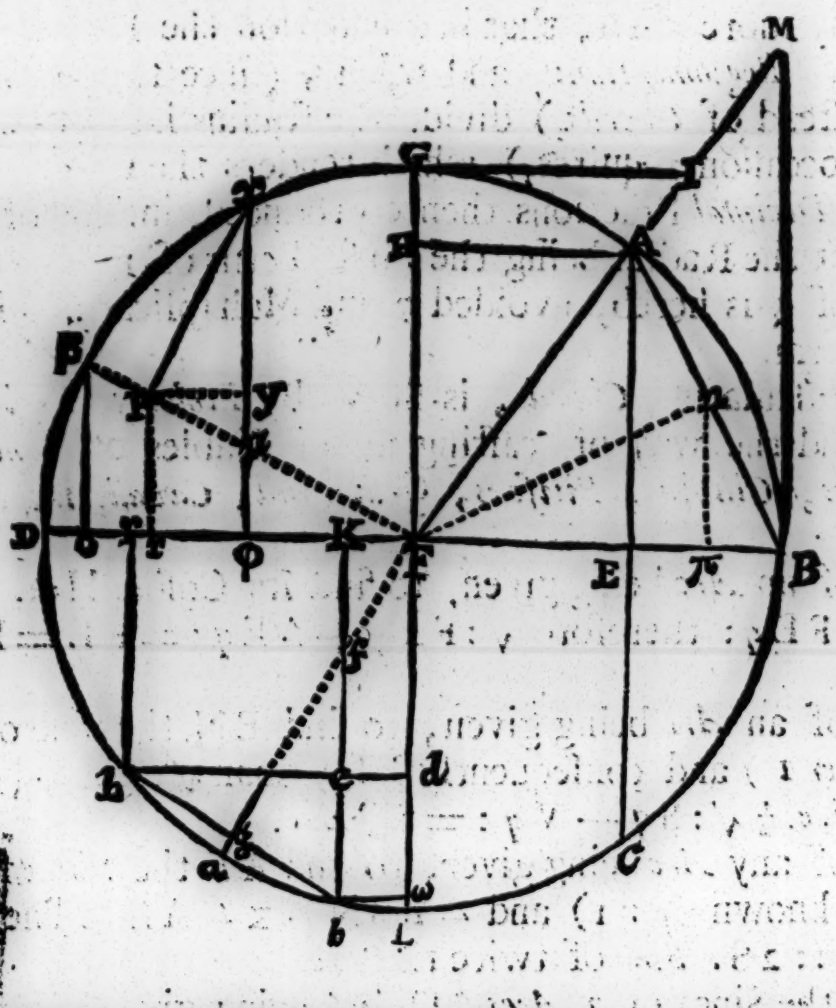
LONDON:

Printed by *John Playford*, for *Richard Davis*, Bookseller, in the University of OXFORD, M. DC. LXXXV.

PLAIN
TRIGONOMETRY.

IN this Treatiſe beſide the Symbols, R. S. Σ. ſ. σ. T. τ. V. v. for the *Radius*, *Sine*, *Co-ſine*, *Secant*, *Co-ſecant*, *Tangent*, *Co-tangent*, *Verſed* Sine of the *Arch* and its *Complement*,) and thoſe commonly received, (+ — × = > <,) the Author makes uſe of theſe alſo.

∴. Equiangular.
th. therefore
Δ. Triangle.
⊥. Perpendicular.
∥. Parallel.
∠∠. Angle. Angles.
∟. Right Angle.
Σcr. Sum of the Legs.
Xcr. Difference of the Legs.
Σ∠∠. Sum of the Two Angles.
X∠∠. Difference of the Two Angles.
c. h. (in the Column under *Giv.*) Leg. Hypothenuſe.



A *Chord* or *Subtenſe*, is a Right Line, joining the extremities of an *Arc*; as AC is the *Chord* of the Arcs ABC, ADC.

A *Right Sine* which is alſo ſingly called a *Sine*, is a Right Line drawn from one end of an *Arc*, perpendicular to the *Diameter* drawn through the other end: or it is half the *Chord* of twice the *Arc*. AE is the *Right Sine* of the *Arcs* AB, AD. The *Radius* or *Sine* of 90 degrees, is called the *whole Sine*, and is the greateſt of all *Sines*; for the *Sine* of an *Arc* greater than a *Quadrant*, is leſs than the *Radius*.

A *Verſed Sine*, called by the Ancients a *Dart* (*ſagitta*) is the Segment of the *Radius*, between the *Arc* and its *Right Sine*. EB is the *Verſed Sine* of the *Arc* AB, and ED of the *Arc* AD.

The *Secant* of an *Arc*, is a Right Line, drawn from the Center through one end of the *Arc*, till it meet with the *Tangent*, *i. e.* a Right Line touching the Circle at the neareſt end of that Diameter which cuts the other end of the *Arc*. F M is the *Secant*, and B M the *Tangent* of the *Arc* A B, or of A D.

The difference of an *Arc* from a Quadrant, whether it be greater or leſs, is call'd the *Complement* of that *Arc*. G A is the *Complement* of the *Arcs* A B, A D; and H A is the *Sine* of that *Complement*; G I the *Tangent* of the *Complement*; F I the *Secant* of the *Complement*: Or as the *Engliſh* uſe to call them, H A is the *Co-ſine* of the *Arc* A B; G I the *Co-tangent*; F I the *Co-ſecant*.

The difference of an *Arc* from a Semicircle, is called its *Supplement*.

That part of the Radius, which is betwixt the Center and its *Right Sine*, is equal to the *Co-ſine*. F E = H A; and F O is equal to the *Co-ſine* of the *Arc* D β.

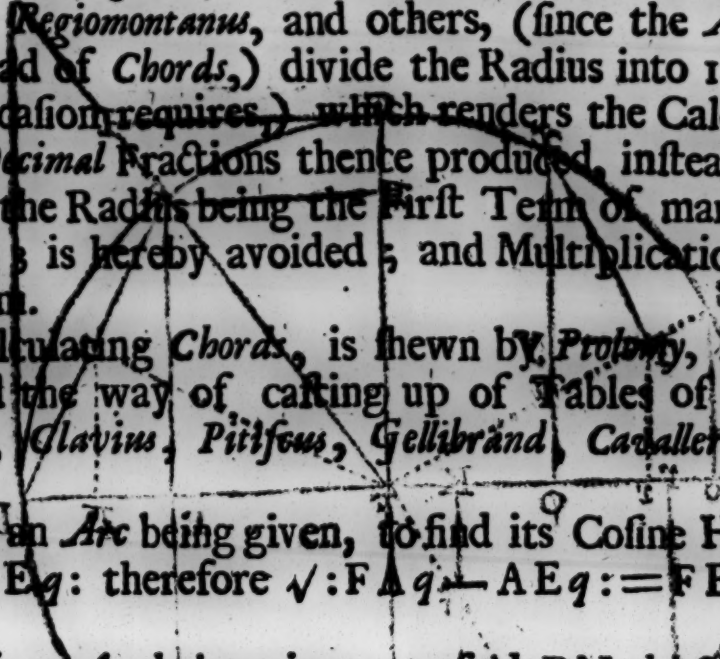
If an *Arc* be greater or leſſer than a Quadrant; the Sum or difference, accordingly, of the Radius and Co-ſine is equal to the *Verſed Sine*. F D + H A = E D. and F B — H A = E B.

Of making the Tables of Natural Sines, Tangents, *and* Secants.

IN a Triangle are 6 parts, *i. e.* 3 Sides, and 3 Angles; of theſe 6, any 3 being given, except the 3 Angles of a Plain Triangle, the 3 other parts may be thereby found; if, ſuppoſing the *Radius* divided into any number of equal parts, we know how many of thoſe parts, are in the *Chord*, *Sine*, *Tangent* or *Secant* of any *Arc* propoſed.

Ptolomy, with the *Ancients*, divided the *Diameter* into 120 parts, which number was choſen by reaſon of its many Aliquot parts. But becauſe in this diviſion, many *Chords* had Fractions annext, and many were Surd Roots, which created much trouble in Calculation; therefore later Mathematicians have divided the Diameter into many more parts, that in calculation the Reſidual Fractions may be ſafely neglected. *Regiomontanus*, and others, (ſince the *Arabians* brought in the uſe of *Sines* inſtead of *Chords*,) divide the Radius into 10000, *&c.* (adding as many Ciphers as occaſion requires,) which renders the Calculation much more eaſy, by means of *Decimal* Fractions thence produced, inſtead of *Sexageſimal* and *Vulgar*, and for that the Radius being the Firſt Term of many Proportions, Diviſion in the Rule of 3 is hereby avoided; and Multiplication, in caſe it be the Second or Third Term.

The Method of calculating *Chords*, is ſhewn by *Ptolomy*, *Regiomontanus*, *Rheticus*, *Copernicus*, and the way of caſting up of Tables of *Sines* and *Tangents* is ſhewn by *Rheticus*, *Clavius*, *Pitiſcus*, *Gellibrand*, *Cavalerius* and *Snellius*: as follows.



1. A E the *Sine* of an *Arc* being given, to find its Coſine H A.

F A q = A E q + F E q: therefore $\surd$: F A q — A E q :: = F E = H A. or $\surd$: R q — S q :: = Σ.

2. A E the Sine of an *Arc* being given, to find B N the Sine of half the *Arc*.

F E is known (*per* 1) and conſequently E B; then $\surd$: A E q + E B q :: = A B; and $\frac{1}{2}$ A B = B N. *i. e.* $\frac{1}{2}\surd$: S q — V q :: = S $\frac{1}{2}$ *Arc*.

3. B N the *Sine* of any *Arc* being given, to find A E the *Sine* of twice the *Arc*. F N the *Co-ſine* is known (*per* 1) and $\triangle$ F B N is $\sim$ $\triangle$ A B E. Therefore F B. F N :: A B. A E. *i. e.* R. Σ :: 2 S. *Sine* of twice the *Arc*.

4. β O and x P, the Sines of 2 *Arcs* β D, x β being given, to find $x\varphi$ the Sine of the *Sum* of the *Arcs*.

The *Co-ſines* F O, F P, are known (by 1) but F β. F P :: β O. P r; and $\triangle y$ P $x \sim$ ($\triangle$ P U x, $\sim$ $\triangle\varphi$ U F, $\sim$) $\triangle$ O β F. Therefore F β. F O :: x P. xy; then xy + P r = $x\varphi$.

5. Suppoſe the *Arc* L u = 30°, and ab = ah, and $h\omega$ the Sine of b L, and b K its Co-ſine; alſo bd the Sine of b L, and br its Co-ſine; draw F a cutting bh in g. Then $4ghq$ = bhq = beq + ehq; Therefore, $3ghq$ = beq; and $gh \times \surd 3$ = be; and $h\omega$ + $gh \times \surd 3$ = bd. *i. e.* the Sine of an *Arc* leſs than 30 Degrees, adding $\surd 3$ × Sine of the defect, makes the Sine of an *Arc*, as much exceeding 30°. Therefore, if the Sines of all *Arcs* leſs than 30°, be known; the reſt as far as 60°, may be had by one Addition, and a Multiplication into $\surd 3$.

6. $\triangle gh$

6. $\Delta ghf \simeq \Delta heb$, but $\angle hfg = gFL = 30°$; therefore $\angle hbe = 30°$. And ſuppoſing a Circle on the Center g deſcrib'd through beb; be the Chord of 60° will be $= bg$ the Radius. Therefore $rb + bg = Kh$: *i.e.* the Sine of any Arc b D leſs than 60°, adding the Sine of the Defect bn, makes Kh the Sine of an Arc ſo much exceeding 60°. Conſequently the Sines of all Arts leſs than 60° being known, the reſt to 90° may be found by one Addition. For inſtance, the Sine of $58° + S2° = S62°$.

The Radius is equal to the Chord of 60°, and $\frac{1}{2}$ R = S 30°, then (by 2*d*) are known the Sines of the halves 15°. 7°, 30'. 3°, 45'. 1°, 52', 30". 56', 15". 28', 7", 30"'. 14', 3", 45"'. 7', 1", 52"', 30"". 3', 30", 56"', 15"". 1', 45", 28"', 7"", 30""'. 52", 44"', 3"", 45""'. So that by 12 Diviſions, we come to Sines which have the ſame ſenſible proportion as their Arcs; for the laſt Sine ſave one, is double of the laſt Sine to all ſenſe, as one Arc is double of the other. But $1800 \times 1' = 30° = 2048 \times 52", 44"', 3"", 45""'$. Therefore 1800. 2048:: Arc 52", 44"', 3"", 45""'. Arc 1' :: S 52", 44"', 3"", 45""'. S 1'.

And thus by a continued Biſection of Arcs, the Sine of one Minute being had (by 3*d*) is found the Sine of 2', then (by 4*th*) the Sine of 3', and ſo to 30°, then (by 5*th*) to 60°, and (by 6*th*) the reſt to 90°.

Gellibrand and *Vieta* find the Sines by the *Analyſis* of *Angular Sections*; inſtead of which *Pitiſcus* and others uſe the *Rule* of *falſe*.

Having the Sines, we may find the *Tangents* and *Secants* by the following Proportions.

FE. FB :: EA. BM, and FE. FB :: FA. FM. and BM. BF :: FG. GI. *i.e.* Σ. R::S.T, and Σ. R :: R. $\int$. and T. R :: R. τ.

LEMMA. I.

EB. BN :: AB. FB :: ($\frac{1}{2}$ AB) BN. $\frac{1}{2}$ FB. *i. e.* V.S $\frac{1}{2}$ Arc :: S $\frac{1}{2}$ Arc. $\frac{1}{2}$ R, and therefore $\frac{1}{2}$ VR = S*q* $\frac{1}{2}$ Arc.

LEMMA II.

$\frac{1}{2}$ R *v* = Σ *q* $\frac{1}{2}$ Arc. For draw $n\pi \perp$ FB. th: Fπ = FE + (Eπ) $\frac{1}{2}$ EB = $\frac{1}{2}$ ED. but F$\pi \times$ FB = FN*q*.

LEMMA III.

The *Tangents* of 2 Arcs A, B, are reciprocally proportional to their *Co-tangents*. For T, A. R :: R. τ, A. and T, B. R :: R. τ, B. Therefore T, A $\times \tau$, A = R $\times$ R = T, B $\times \tau$ B; Therefore T, A. T, B :: τ, B. τ A.

LEMMA. IV.

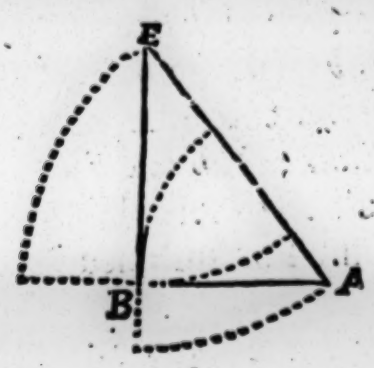
The Co-ſines of 2 Arcs A, B, are reciprocally proportional to their Secants. For Σ, A. R :: R. $\int$, A. And Σ, B. R :: R. $\int$, B. Therefore Σ, A. Σ, B :: $\int$, B. $\int$, A.

Plain and Spheric Trigonometry, are uſually reſolved into 4 fundamental Theorems, call'd Axioms.

The First AXIOM.

In a Right-angled Triangle, if one Leg of the Right Angle be made the Radius of a Circle; the Hypothenuſe will be the Secant of the Adjacent Angle, and the other Leg will be the Tangent of that Angle. But if the Hypothenuſe be Radius, the 2 Legs will be Sines of the oppoſite Angles; as is manifeſt by the following Figure.

In the following Proportions, I ſuppoſe that 2 Lines being eſtimated in parts of any meaſure for example in parts of the Table are proportional to themſelves



ſelves reckon'd aecording to any other meaſure; ſo A B reckon'd as Radius of parts 100000, is to B E Tangent of the Angle A 30°, of Tabular parts 57735 :: ſo the ſame A B of 213 feet to B E 123 feet almoſt.

Note alſo, that becauſe few Books have Tables of Logarithmic Secants, I have declined their uſe for the moſt part, which had I admitted, I might eaſily have varied the following proportions both of Plain and Spheric Trigonometry many other ways, as *Clavius* has done. But I have regarded the giving not ſo much a multitude, as of one good Solution to each caſe; and ſuch I count that proportion to be, which has the Radius in the firſt place, for which end I have given an inſtance or Two of the Secants.

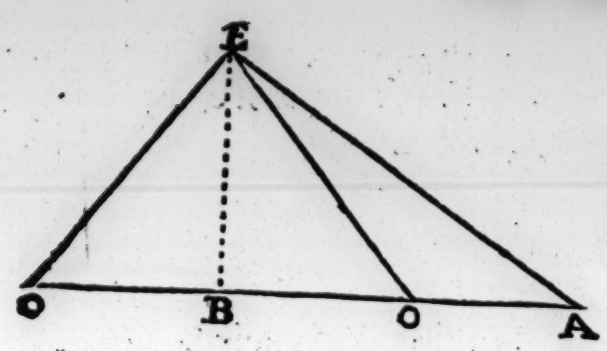
In a Right-angled Triangle, one Acute Angle is the Reſidue of the other Acute to 90°, conſequently one being known, the other is known: And in an Oblique-Angled Triangle, Two Angles being known, the Third is alſo known, as being the Supplement or Reſidue of the Sum of the other 2 to 180°.

The Seven Caſes of Right Angled Triangles.

See the laſt Figure.

Given	Requir'd	Proportions	giv.	req.
AB. *LL*	BE	R. AB :: T, A. BE	*c. LL*	*c*
AB. *LL*	AE	S, E. AB :: R. AE or, R. AB :: *ſ*, A. AE	*c. LL*	*h*
AB. AE	*LL*	AE. R :: AB. S, E	*c. h.*	*LL*
AB. AE	BE	AE. R :: AB. S, E then R. T, A :: AB. BE. or √ : AE + AB : × : AE — AB : = BE	*c. h.*	*c*
AB. BE	*LL*	AB. BE :: R. T, A.	*cc*	*LL*
AB. BE	AE	AB. BE :: R. T, A, then S, A. R :: BE. AE.	*cc*	*h*
AE. *LL*	AB	R. S, E :: AE. AB.	*h. LL*	*c*

The Second A X I O M.



In any Triangle O E A, the Sides are proportional to the *Sines* of the oppoſite Angles. For O E. R :: B E. S, O, and A E. R :: B E. S A. Therefore O E. A E :: S, A. S, O.

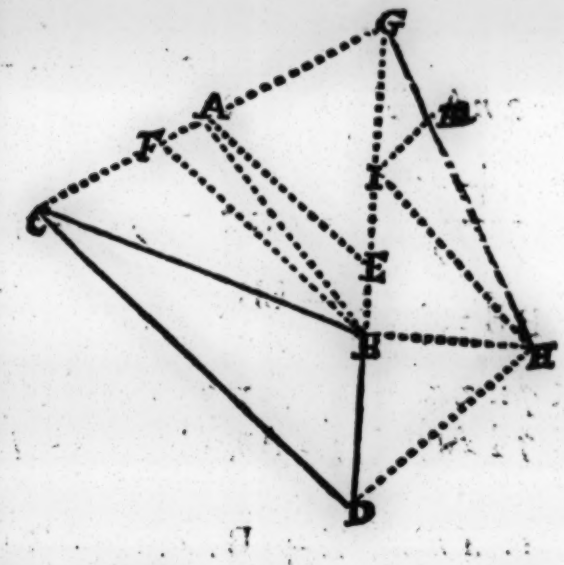
The Six Caſes of Oblique-angled Triangles.

OE. AE. A.	O	OE. AE :: S, A. S, O.	2 *l. Lop*	*Lop*
		Note here, that whereas any Arc has the ſame *Sine*, *Tangent*, and *Secant*, with its *Supplement*; the Angle O is ſo ambiguous: *i. e.* Whether 'tis Acute or Obtuſe, cannot be diſcerned, from the Three things here given, therefore its kind muſt be found from ſome other Circumſtance of the Queſtion.		
OE. AE. A	AO	OE. AE :: S, A. S, O. S. A. OE :: S, E. AO. Whence the Angle E will be alſo known, then	2 *L l. op*	*l*
A. O. AE	OE	S, O. AE :: S, A. OE or	2 *L. l op* 3 *L. l*	*l. op* *l*

The

The Third AXIOM.

The Sum of the Legs of an Angle DBC, is to the difference of the Legs:: as the Tangent of half the ſum of the oppoſite Angles, is to the Tangent of half their difference *Dem.* producing DB, take BG=BC, and and divide DG equally in E, and GC in A. Therefore BA is ⊥ GC, and AE || CD, and ∠ABC = ½GBC = ½Z∠∠*op.* Draw BF || DC; therefore ∠FBC = altern ∠BCD. And if from ∠ABC the ½ Sum of the Angles BCD, BDC, you take the Leſſer CBF, there will remain ∠ABF the ½ difference of the ſame BCD, BDC; and if from the ½ Sum of the Legs ED, you take the leſſer Leg BD, the Reſidue EB will be the ½ difference of the Legs. But putting AB Radius, AC is the Tangent of ∠ABC, and AF is Tangent of ∠ABF; therefore Z*cr.* X*cr* :: (½ Z*cr.* ½ X*cr* :: ED. EB :: AC. AF::) T, ½Z∠∠*op.* T, ½X∠∠*op.*



B.BC. BD. *i.e.* L.2 l*opr.*	C.D.	BC + BD. BC — BD :: T, ½Z∠∠*op.* T, ½X∠∠*op.* But ½Z + ½X = greater ∠D, and ½Z — ½X = leſſer ∠C.
BC. BD. B *i.e.* L.2 l*opr.*	CD	Find the Angles C, D by the laſt, then S, D. BC :: S, B. CD, or S, C. BD :: S, B. CD.

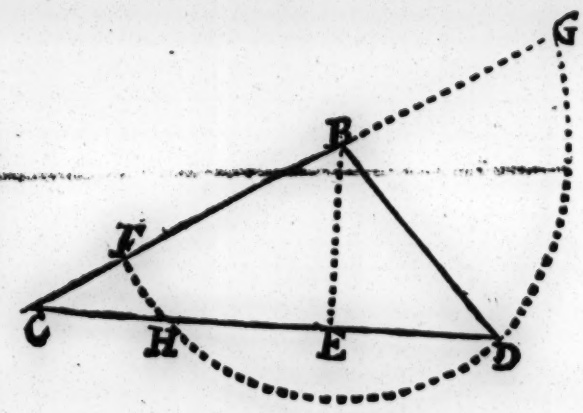
Having an Angle CBD, and the Logarithms of the Legs CB, BD, to find the other 2 Angles, which is a frequent caſe in Aſtronomy. *See the laſt Figure.*

The leſſer Leg BD, is to the greater BC:: as the Radius to the Tangent of an Arc, from which taking 45 Degrees, as the Radius to the Tangent of the remaining Arc :: ſo the Tangent of the ½ Sum of C, D, to the Tangent of their ½ difference.

Dem. Draw BH ⊥ and = BD = BI, and IM ⊥ IH. then BH. BG :: R. T, ∠BHG, whence taking ∠BHI, R. T, ∠IHG :: IH. IM :: DH. IM :: DG. IG :: [by 3*d* Axiom] T, ½Z∠∠*op.* T, ½X∠∠*op.*

The Fourth AXIOM.

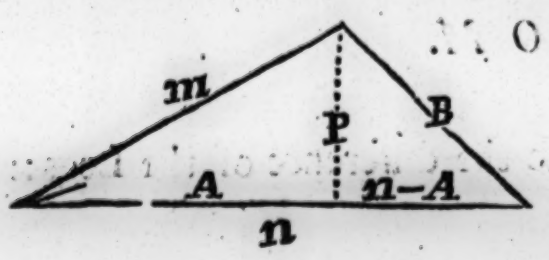
Having drawn a Perpendicular from an Angle to its oppoſite Baſe, the Baſe CD will be to the Sum of the Legs CG :: difference of the Legs CF, to the difference of the Segments of the Baſe CH.



3 Sides BC. BD. CD,	3∠	CD. CB + BD :: CB — BD. CH. then ½CD + ½CH = CE. and ½CD — ½CH = ED. and then CB. R :: CE. ∠, C, and BD. R :: ED. ∠, D.

This

This laſt caſe may be better ſolved by Four other Theorems. For the analyſis and demonſtration whereof, ſuppoſe m, n, the legs of the Angle required; B its Baſe, A and N ∽ A, the Segments of the Baſe made by P, a Perpendicular let fall from another Angle, whether it fall within or without the Triangle $z = m + n$, $x = m - n$, $\mathfrak{Z} = \frac{1}{2}$ Sum of m, n, B.



Then is $mm - AA = PP = BB - nn - AA + 2nA$. Therefore $\frac{mm + nn - BB}{2n} = A$; therefore R. ∡ of the Angle :: m. $\frac{mm + nn - BB}{2n}$ :: $2mn$. $mm + nn - BB$. Therefore $2mn$. ($2mn \pm mm \pm nn \mp BB =$) $zz - BB$. $BB - xx$ :: R . R $\pm$ ∑ = v . V :: $\frac{1}{2}$ R R . $\frac{1}{2}$ R v = Σq . $\frac{1}{2}$ RV = Sq . $\frac{1}{2}$ Ang. Therefore $4mn$. R q :: $zz - BB$. ∑ q $\frac{1}{2}$ Ang :: $BB - xx$. Sq $\frac{1}{2}$ Ang. and $zz - BB$. $BB - xx$ :: ∑ q $\frac{1}{2}$ Ang. Sq $\frac{1}{2}$ Ang. :: R q. T q $\frac{1}{2}$ Ang. :: τq. R q.

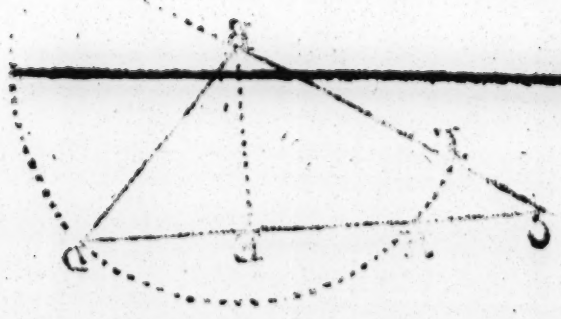
1. $4mn$. $Z + B : x : Z - B ::$ R q . ∑ q $\frac{1}{2}$ Ang. or mn . $\mathfrak{Z} : x :: \mathfrak{Z} - B ::$ R q . ∑ q . $\frac{1}{2}$ Ang.

2. $4mn$. $B + x : x : B - X ::$ R q . Sq $\frac{1}{2}$ Ang. or mn . $\mathfrak{Z} - m : x : \mathfrak{Z} - n ::$ R q . Sq $\frac{1}{2}$ Ang.

3. $Z + B : x : Z - B$. $B + X : x : B - X ::$ R q . T q $\frac{1}{2}$ Ang. or $m : x : \mathfrak{Z} - n ::$ R q . T q $\frac{1}{2}$ Ang.	$\mathfrak{Z} : x : \mathfrak{Z} - B$. $\mathfrak{Z} -$
4. $B + X : x : B - X$. $Z + B : x : Z - B ::$ R q . τq $\frac{1}{2}$ Ang. or $x : \mathfrak{Z} - B ::$ R q . τq $\frac{1}{2}$ Ang.	$\mathfrak{Z} - m : x : \mathfrak{Z} - n$. $\mathfrak{Z} :$

Becauſe $zz - xx = 4mn$, if you have an Angle given, with its Baſe, and the Sum or difference of its Legs, you will have by theſe Theorems the Square of the Difference or Sum, and ſo both the Sum and Difference, and conſequently the very Legs.

If inſtead of Tables, you would work with a Sector, or Gunters line, or other proportional inſtrument; the firſt Theorem $2mn$. $BB - xx ::$ R . V, is to be reſolv'd into 2 Proportions. 2 m. $B + x :: B - x$. G . then n . G :: R . V. or into theſe, $B + x$. $2n ::$ m . H . then, H . $B - x ::$ R . V.



Having drawn a Perpendicular from an Angle to its oppoſite baſe, the Baſe CD will be to the Sum of the Legs as CG :: difference of the Legs CF, to the difference of the Segments of the Baſe CE.

3 Sides	CD . CB + BD :: CB − BD . CB. then $\frac{1}{2}$CD + $\frac{1}{2}$CH = CE. and $\frac{1}{2}$CD − $\frac{1}{2}$CH = ED. and then CB. R :: CE. $\sin$ C, and BD. R :: ED. $\sin$ D.

SPHE-

This

SPHERIC
TRIGONOMETRY.

A Spheric Triangle is that which is contained between the Arcs of 3 great Circles of the Sphere.

A Spheric Angle is the same with the mutual aperture or inclination of the planes of those 2 Circles which constitute the Angle.

Affections of Spheric Triangles.

1. When a Circle falls on another Circle, the Sum of the 2 Angles made thereby is $= 2 \sqcup$.

2. When a Circle crosses a Circle, the Vertical Angles made thereby are mutually equal.

3. The greater Angle is oppos'd to the greater side.

4. An isosceles Triangle has its 2 Angles at the Base mutually equal, and on the contrary, if a Triangle has 2 Angles equal, it has 2 sid[illegible] qual.

5. Two Triangles mutually equilateral, are also Equi[illegible]ular one to the other.

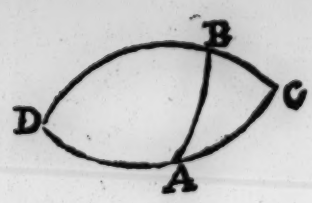
These 5 proprieties, with the 2 next are common to plain Triangles, and have a like demonstration.

6. If there be 2 Triangles, and in each, one Angle and the 2 Sides including respectively equal; or if one Side and the 2 Angles adjacent be severally equal; then the 2 Triangles are equal; for if laid one upon another, they will agree.

7. Two Sides of a Triangle are bigger than one: For the Arc of a great Circle is the shortest distance betwixt 2 points on the Surface of a Sphere; as a Streight Line is betwixt 2 points in a plain.

8. All great circles cut each other into 2 equal parts; for their common Section is a Diameter of the Sphere, and consequently the 2 Sections of the Peripherys of 2 great Circles are at a Semicircles distance.

Hence it follows that every Side of a Spheric Triangle, is less than a Semicircle. DB is less than the Semicircle DC.



9. The Opposite Angles at the Sections of 2 Circles are equal, $\angle D = \angle C$, for the same Planes constitute both Angles.

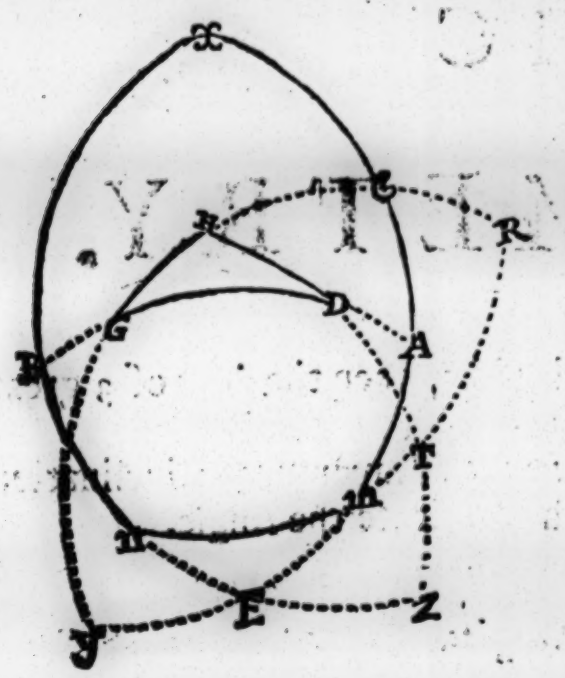
10. In any Spheric Triangle, if the Sum of the Legs of an Angle be $> = <$ (greater, equal, less than a) Semicircle, the intern Angle at the Base is (accordingly) $> = <$ outward opposite, and consequently the Sum of the 2 intern Angles at the Base is $> = < 2 \sqcup$. *Dem.* If $DB + BA > = < DC$, then BA is $> = < BC$, and therefore $\angle (C) D > = < \angle BAC$, and $\angle D + \angle DAB > = < (\angle BAC + DAB =) 2 \sqcup$.

11. Coroll. In an Isosceles Triangle, if one of the Equal Legs is $> = <$ Quadrant, the Angle at the Base is $> = < \sqcup$.

12. The Sum of the 3 Sides of a Triangle, is less than a Circle; for $BA < BC + AC$. *th.* $DB + DA + BA < DBC + DAC$.

13. If from the point of an Angle as a Pole, you describe a great Circle, or which is the same, if you describe a Circle at the distance of 90 Degrees from the point, the Arc of this Circle intercepted between the Legs of the Angle, is the measure of the Angle.

14. The poles of the Sides of any Triangle G H D, conſtitute another Triangle *n x m*, which we may call ſupplemental to the △ G H D; for the Supplements of the Angles and Sides of △ *n x m*, are equal to the Sides and Angles of the Triangle G H D.



Dem. From the points G H D as Poles, deſcribe 3 great Circles, *x* A Y, R T *mn*, *x* B *n* Z; then is Y *m* = Quadrant = A *x*; becauſe *m* is the Pole of H G Y, and *x* or E the Pole of G A, therefore *m x* = A Y = Supplement of C A = ∠ H G D. and Z *n* Quadrant = B X, therefore *n x* = B Z = *Suppl.* ∠ H D G. and *n* T = Quadrant = *m* R. Therefore *n m* = T R = Supplement of ∠ D H G.

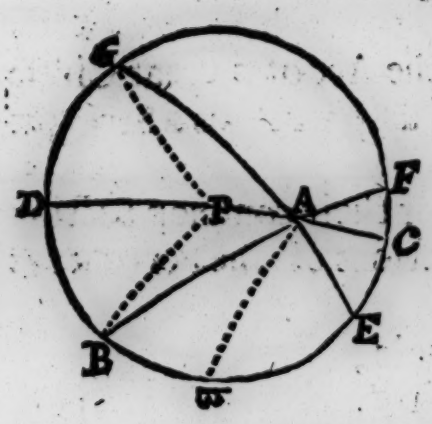
Note that the Triangle *n* E *m* conſtituted between the 3 next poles, has its 3 Sides and Angles = Angles and Sides of △ G H D, ſave that the greateſt Side *n m* is the Supplement of the greateſt ∠ H, and ∠ E of the Side G D.

15. Any Angle of a Triangle with the difference of the other 2, is < 2 ⌟. For *x n* < *x m* + *m n*. *i.e.* 2 ⌟ − D < 2 ⌟ − G + 2 ⌟ − H. Therefore G + H − D < 2 ⌟.

16. If 2 Triangles are mutually æquiangular, they are alſo mutually equilateral; for becauſe they are Equiangular, their Supplemental Triangles are equilalateral (by 14*th*) ⌐. B therefore Equiangular (by 5*th*) and therefore the propoſed Triangles are Eq. lateral (by 14*th*.)

17. The 3 Angles of every Triangle are > 2 ⌟, and < 6 ⌟. For *n x* + *x m* + *n m* < 4 ⌟ (by the 12*th*,) *i.e.* 6 ⌟ − D − G − H < 4 ⌟. *i.e.* 2 ⌟ < D + G + H. Secondly, The Sum of the Intern Angles, is leſs than the Sum of the Intern and Extern both, which in all, make but 6 ⌟.

18. Of ſeveral Arcs of great Circles falling from the ſame point of the Spheres Surface on another Circle, the greateſt is that which paſſes through the Pole of the Circle, and the next to this, is greater than that which is farther off. For ſuppoſe P the Pole of the Circle C *ϖ* D, and *ϖ* the Pole of D P C; then is A D > A B > A E > A C; and the Arc B *ϖ* C > B P > B D.



19. A great Circle paſſing through the Poles of another great Circle cuts it at Right Angles; and on the contrary, if it cut it at Right Angles, it paſſes through its Poles. ∠ P B D = ⌟ = P G D = P D B = *ϖ* A C.

20. In an Oblique-angled Triangle, if the Angles at the Baſe are like or of the ſame kind, *i.e.* both Acute, or both Obtuſe; the Perpendicular falls within the Triangle, and the Quadrantal Arc without: But if they be unlike, the Perpendicular falls without, and the Quadrant within. For △ E A F has ∠∠ E, F Acute, and the Perpendicular A C falls within, and the Quadrant A *ϖ* without. Alſo △ B A G has ∠∠ B, G Obtuſe, and the Perpendicular A D within and the Quadrant A *ϖ* without. But the △ B A E has ∠∠ B E of different kinds, and the Perpendicular A C without, and the Quadrant A *ϖ* within.

Moreover by the ſame Figure 'tis manifeſt, how the Ambiguities of Right Angled Triangles may be ſolved, *viz.*

SOLUTIONS.

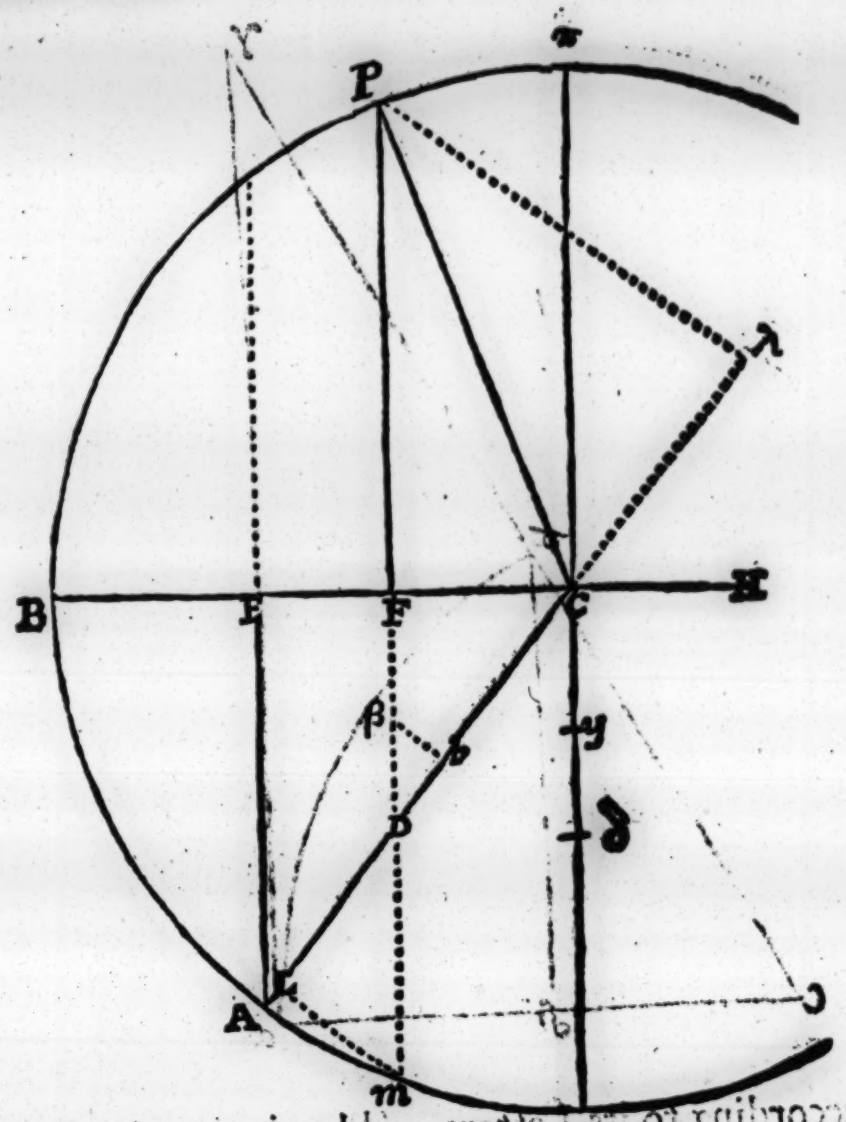
1. The Legs of the Right Angle are of the ſame kind with the oppoſite Angles. So in the △ B D A, becauſe D A > Quadrant D P, the ∠ D B A > ⌟ D B P. And in △ B C A, becauſe A C < Quadrant P C, ∠ C B A < ⌟ C B P.

2. If the Legs, (and conſequently the Angles) are of the ſame or different kind; the Hypothenuſe is accordingly < > Quadrant. So in the Triangles E D A, E C A, the Hypothenuſe A E is < Quadrant, but in △ B D A, the Hypothenuſe A B is > Quadrant B P.

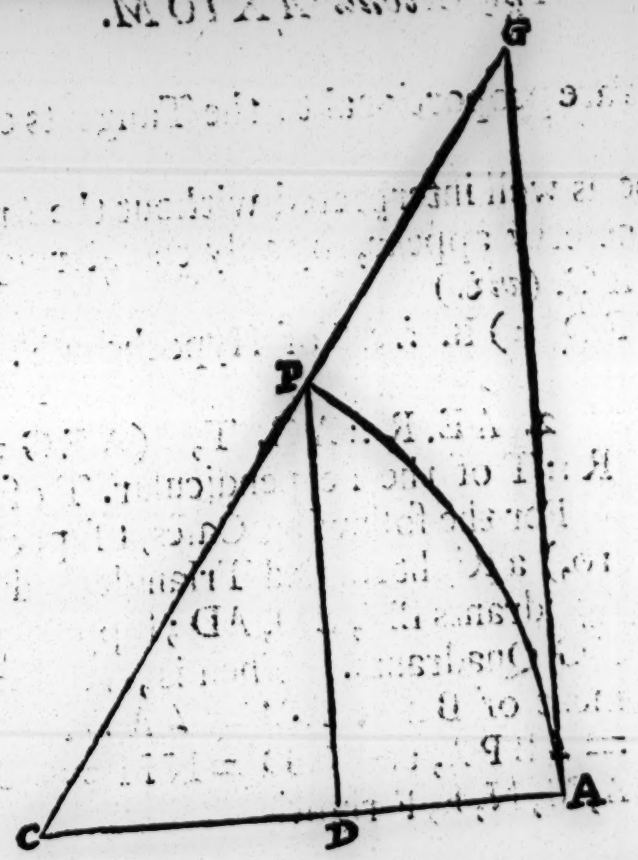
3. If

3. If the Hypothenuse is < > Quadrant, either Leg with its adjacent Angle, is accordingly of the ſame or different kind, as follows from the laſt.

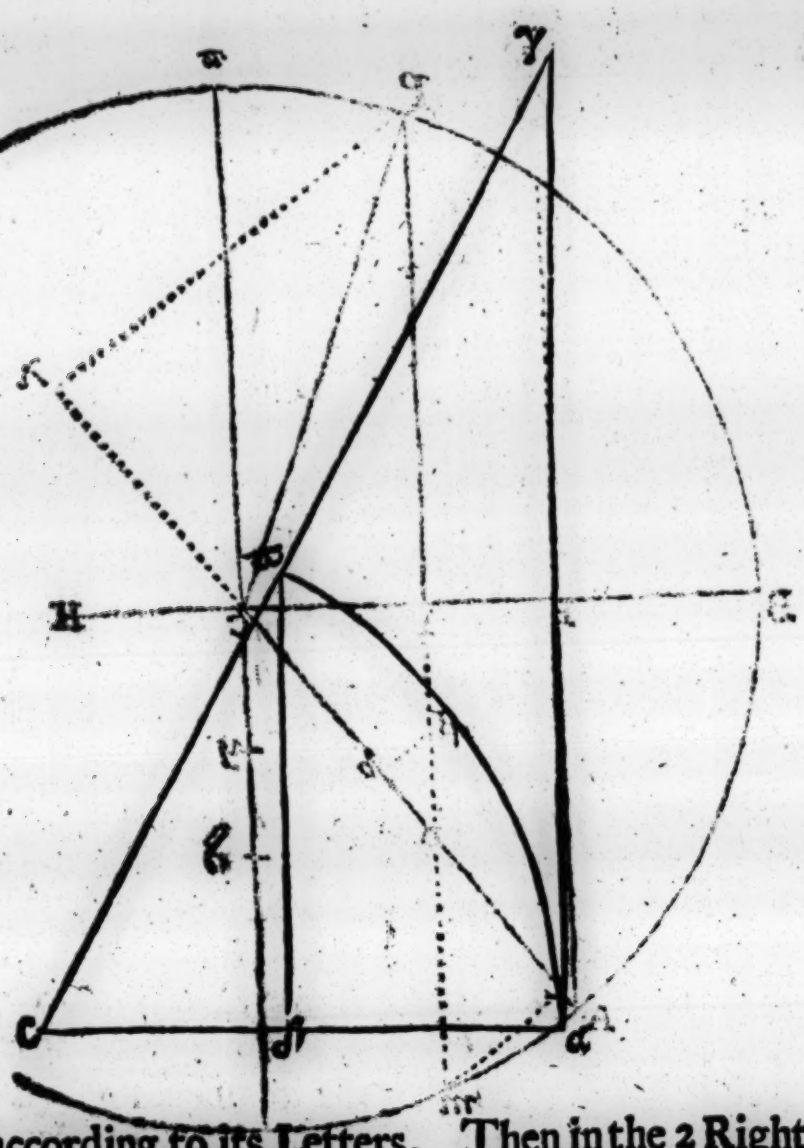
For the viewing the Sines, Coſines and other Right Lines of Arcs which are not viſible in a common Sphere: I uſe the Arcs of 3 great Circles of Card Paſt-board,



put together as in an Armillar Sphere. As ſuppoſe 2 Arcs BP, BA, and that BPH the Plane of the greater Arc were turn'd round BH, till that a Right Line falling from P Perpendicular to the Plane BAH, may fall on ſome point of the Line CA, ſuppoſe on D; for in that Poſition PAB will be a Spheric Triangle Right Angled at A, and BP the Hypothenuſe, BA the Baſe, PA the Perpendicular Arc. And ſuppoſe PA (*Fig.* 2*d pag.* 9.) = PA of the Triangle, and fitted according



to its Letters therein; and draw AE, PF ⊥ BC; ſo AE, PF, PD, will be Sines of the Arcs BA, BP, PA, and their Co-ſines will will be EC, FC, DC. Theſe things being done or conceiv'd, the 2 firſt Axioms of Spheric Trigonometry, will preſently appear,



fitted in the Solid according to its Letters, Then in the 2 Right Angled Spheric Triangles P B A, π B α, having the same Acute Angle B at the Base,

The First AXIOM.

The Sines of the Hypothenuses are Proportional to the Sines of the Perpendiculars.

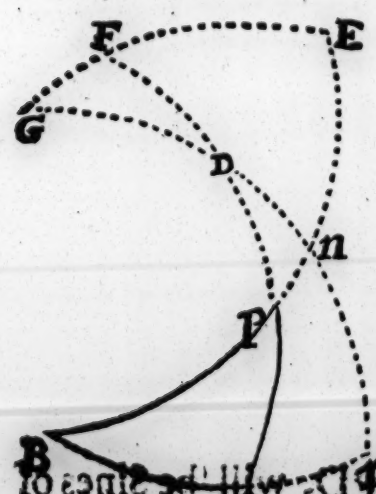
The Second AXIOM.

The Sines of the Bases are proportional to the Tangents of the Perpendiculars.

AE. AG :: α C. α γ.

These 2 Axioms may be as well interpreted without the Arc π α, and their congruity with Plain Trigonometry appear, by only considering the plain Right-angled Triangles F D P, E A G. (*viz.*)

1. PF. R :: PD. S ∠ (P F D =) B. *i. e.* S of Hypothenuse. R :: S of Perpendicular. S of ∠ at Base



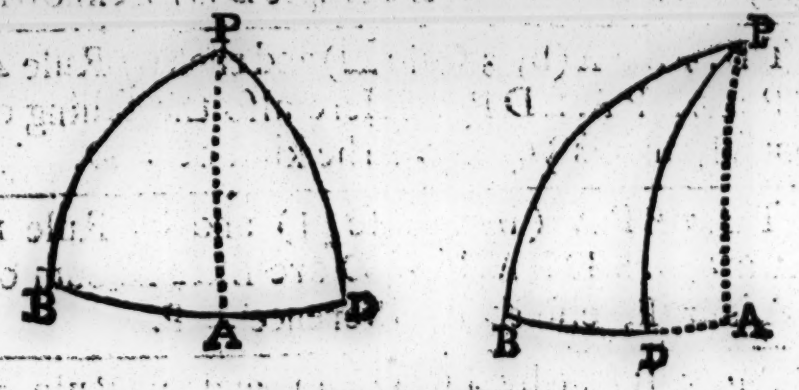
2. AE. R :: AG, T, ∠ (A E G =) B. *i. e.* S of Base. R :: T of the Perpendicular. T of ∠ at the Base.

For the following Cases, I suppose B A P (*Fig. 2d pag.* 10,) a Right Angled Triangle, and its Sides produc'd to Quadrants B N, B M, A D; suppose also, P E, P F, N G and E G Quadrants. Then is, N E = B P, and Complement of B A = A M = ∠ A D M, and F E = ∠ F P E = ∠ B P A, and G D = N M = ∠ B. and the Angles at A, M, N, E, F right.

Given

Given	Req.	The Proportions for the 16 Cases of Right Angled Triangles, with the Solutions of their Ambiguities.	giv.	req.	nu.
BA. PA	BP	S. Diff. S. AM :: S. DP. SPN (by 1 *Ax.*) *i. e.* R. ∑BA :: ∑PA. ∑BP. | *Sol.* 2.	*cc*	*h*	1
BA. PA	B	S. BM. S. BA :: T. MN. TPA (by 2*d Ax.*) *i. e.* R. SBA (:: T, B. T, PA) :: (by *Lem.* 3*d*) τPA. τB | *Sol.* 1.	*cc*	*L*	2
BP. P.	B	SPE. SPN :: TEF. TND. (by 2*d Ax.*) *i. e.* R. ∑BP :: TP. τB :: by 3*d Lem.* T, B. τP | *Sol.* 1.	*h. L*	*L*	3
BP. PA	P	S. GE. SGF :: TEN. TFD. *i. e.* R. ∑P :: TBP. TPA. or R. ∑B :: TBP. TBA. | *Sol.* 3.	*h. L*	*L adj*	4
BP. P.	BA	R. S BP :: SP. SBA. (by *Ax.* 1) *Sol.* 1.	*h. L*	*c. op*	5
PA. P.	BP	SGE. SGF :: TEN. TFD :: τFD. τEN. *i. e.* R. ∑P :: τPA. τBP | *Sol.* 3.	*c. L adj*	*h*	6
PA. P.	B	SPF. SPD :: SFE. SDN. *i. e.* R. ∑PA :: SP. ∑B. | *Sol.* 1.	*c. L adj*	*L*	7
BA. B.	PA	SBM. SBA :: TMN. TPA. *i. e.* R. SBA :: TB. TPA. | *Sol.* 1.	*c. L adj*	*c*	8
PA. B.	BA	R. τB :: (TB. R ::) TPA. SBA (by *Ax* 2) | *Ambig.*	*c. L op*	*c*	9
PA. B.	BP	S B. SPA :: R. S BP (*Ax* 1.) *Ambig.*	*c. L op*	*h*	10
PA. B.	P	SPD. SPF :: SDN. SFE. *i. e.* ∑PA. R :: ∑B. SP | *Ambig.*	*c. L op*	*L*	11
PA. BP	P	R. τBP :: (TBP. R :: (by 4*th* Case) TPA. ∑P | *Sol* 3.	*c. h.*	*L adj*	12
PA. BP	B	SBP. R :: SPA. SB | *Sol.* 1.	*c. h.*	*L op*	13
PA. BP	BA	∑PA. R :: ∑BP. ∑BA (by 1*st* Case) | *Sol.* 2.	*c. h.*	*c*	14
B. P	BP	R. τP :: (τP. R :: (by 3*d* Case) τB. ∑BP | *Sol.* 2.	*LL*	*h*	15
B. P	PA	SP. R :: ∑B. ∑PA (by 11 Case) | *Sol.* 1.	*LL*	*c*	16

In Oblique-Angled Triangles, having let fall a perpendicular to make two Right-Angled Triangles.



RULE I.

The Co-sines of the Angles at the Base are proportional to the Sines of the Angles at the Vertex. For (by 7 Case of ⌐ *tr.*)

∑B. S ∠BPA :: (∑PA. R ::) ∑D. S ∠DPA.

RULE II.

The Co-sines of the Sides are Proportional to the Co-sines of the Bases. For by 1 C

∑BA. ∑BP :: (R. ∑PA ::) ∑DA. ∑DP.

RULE

RULE III.

The Sines of the Baſes are reciprocally proportional to the Tangents of the Angles at the Baſe. For (by 2*d Ax.*)

SBA. R:: TPA. TB. and SDA. R :: TPA. TD. *th.* SBA. SDA :: TD. TB.

RULE IV.

The Tangents of the Sides are reciprocally proportional to the Co-ſines of the Angles at the top. For (by 4 Caſe of ⌟ *tr.*)

TBP. TPA :: R. ΣBPA. and TDP. TPA :: R. ΣDPA. *th.* TBP. TDP :: ΣDPA. ΣBPA.

The Third AXIOM.

In any Triangle, the Sines of the Sides are proportional to the Sines of the Oppoſite Angles. For (by 1*ſt Ax.*)

SBP. R :: SPA. SB. and SDP. R :: SPA. SD. *th.* SBP. SDP :: SD. SB.

Given	Req.	The Proportions for the 12 Caſes of Oblique-angled Triangles.	giv.	req.	nu.
BP. PD. B	D	SPD. SB :: SBP. SD | *Ambig.*	2 *l.* ∠ *op*	∠ *op*	1
BP. B. D	PD	SD. SBP :: SB. SPD | *Ambig.*	2 ∠ *l. op*	*l op*	2

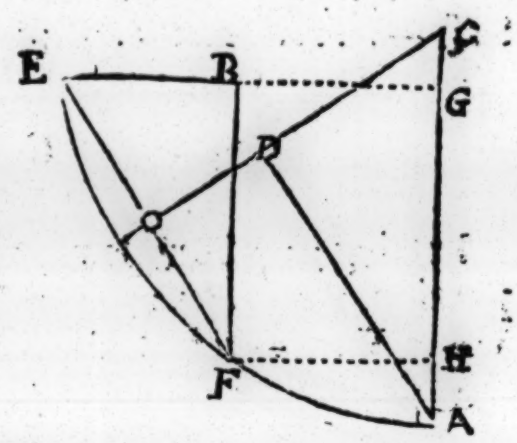
The 8 following Caſes are reſolv'd by letting fall from the extream of a given Side, a Perpendicular oppoſite to a Given Angle. And you muſt obſerve the Addition or Subtraction both of the Segments of the Baſe, and Angles at the top, according as the Perpendicular falls within or without the Triangle.

See the two laſt Figures.

BP. PD. B	BD	R. Σ B :: TBP. TBA, by 4 Caſe ⌟. And then by Rule 2*d*. ΣBP. Σ BA :: Σ DP. Σ DA. | But here 'tis doubtful whether the Perpendicular falls within or without the Triangle, unleſs the kind of the Angle D is foreknown.	2 *l.* ∠ *op*	*l*	3
BP. PD. B	P	R. ΣBP :: TB. τBPA (by 3 Caſe ⌟) and then by Rule 4. TDP. TBP :: ΣBPA. ΣDPA. | Here alſo the falling of PA is doubtful, unleſs you know the kind of ∠ D.	2 *l.* ∠ *op*	∠ *opp*	4
BP. B. D	P	R. ΣBP :: TB. τBPA (by 3 Caſe ⌟) and by Rule 1. ΣB. SBPA :: ΣD. SDPA | If B, D are like, the Sum of BPA, DPA is = P, elſe their difference = P.	2 ∠ *l. op*	∠	5
BP. B. D	BD	R. ΣB :: TBP. TBA (by 4 Caſe ⌟) and by Rule 3. TD. TB :: SBA. SDA. | If B is like D, the Sum of BA, AD is = BD, elſe the difference.	2 ∠ *l. op*	*l. opp*	6
B. P. BP	D	R. ΣBP :: TB. τBPA. (by 3 Caſe ⌟) then by Rule 1. SBPA. SDPA :: ΣB. ΣD. | If BPA is greater than BPD; and alſo B $\begin{matrix}\text{Acute}\\\text{Obtuſe}\end{matrix}$, D is $\begin{matrix}\text{Obtuſe}\\\text{Acute}\end{matrix}$. But if BPA is > BPD and B $\begin{matrix}\text{Acute}\\\text{Obtuſe}\end{matrix}$, D is $\begin{matrix}\text{Acute}\\\text{Obtuſe}\end{matrix}$.	2 ∠ *l opp*	*l*	7
B. P. BP	DP	R. ΣBP :: TB. τ BPA. (by 3 Caſe ⌟.) then by Rule 4. ΣDPA. ΣBPA :: TBP. TDP | If DPA is like, unlike B: DP is leſs greater than a Quadrant.	2 ∠ *l opp*	*l. op*	8

BP.BD.B	DP	R. Σ B :: T B P. T B A (by 4 Case ⊥.) then by Rule 2. Σ B A. Σ B P :: Σ D A. Σ D P | If D A is like, unlike (P A) ∠ B: then P D > < 90°.	2 *l.* Σ *opr*	*l*	9
BP.BD.B	D	R. Σ B :: T B P. T B A (by 4 Case ⊥.) then by Rule 3. S D A. S B A :: T B. T D | If B A <=> B D: D will be like, unlike B.	2 *l.* Δ *opr*	*L*	10

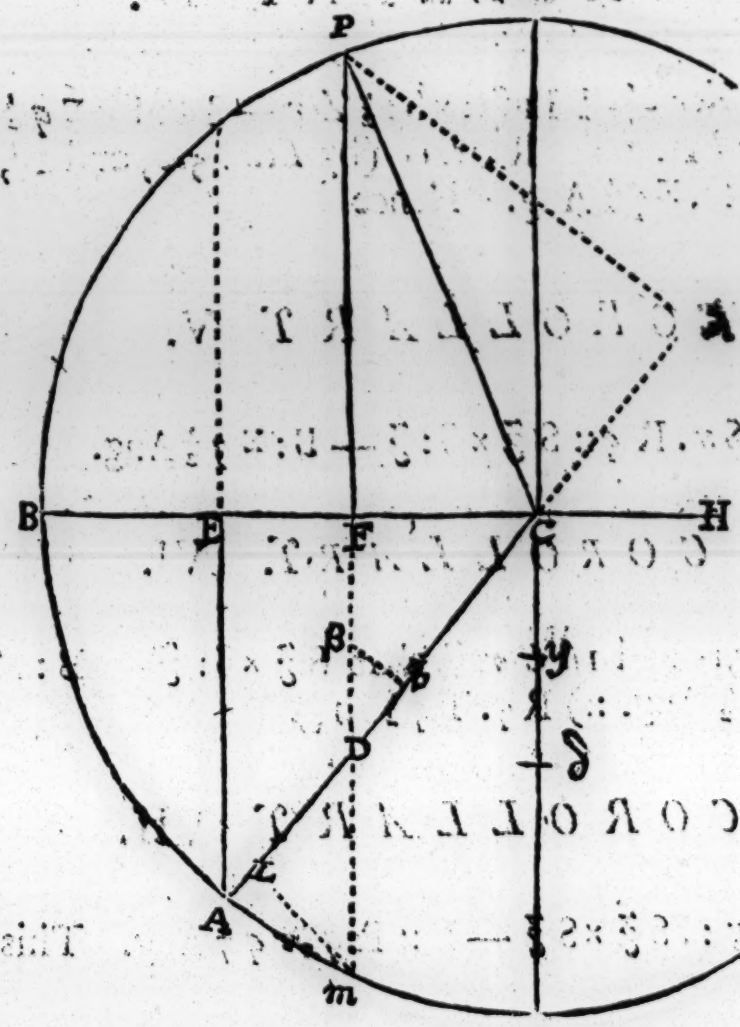
LEMMA V.



The difference of the Verſed Sines of 2 Arcs × ½ R = S ½ Sum of the Arcs Multiplied by the Sine of ½ difference of the Arcs. For ſuppoſe A F, A E, the 2 Arcs, the difference of the Verſed Sines is A G — A H = E B, and S ½ X Arcs = F O, and S ½ Z Arcs = A D. But Δ A C D ≈ Δ E F B. *th*: A C. A D :: F E. F B. *th*: ½ A C × F B = A D × F O.

The Fourth AXIOM.

A Rectangle or Product of the Sines of the Legs. R q :: difference of the Verſed Sines of the Baſe and of the difference of the Legs, to the Verſed Sine of the Vertical Angle. *Dem.* Reſume the foreſaid Circles of Paſtboard, and ſuppoſe there B the Angle requir'd, B A, B P (= B M) its Legs, P A the Baſe to be any



very Oblique not Perpendicular to the Plane B A as before. Let fall P β ⊥ β *m*, and *b y* ⊥ C *a*, (and P *b* ⊥ *b* A, and *m* L ⊥ *r* A. *th*: β *b* ⊥ *r* A, and *b* L = *b* A — L A = M Baſe — V diff: *er*. But A E. A C :: *b* L. *m* . (and *m* F. *a r* :: β *m*. *y a*. *th*: Multiplying the Correſpondent Terms of both Proportions; A E × M F. A C × C H :: (*b* L × β *m*. β *m* × *y a* ::) *b* L. *y a*.

Caſe

Case II. The 3 Sides of any Spheric Triangle, being given to find an Angle.
▯ Sines *cr*. R q :: S: $\frac{1}{2}$ Base + $\frac{1}{2}$ diff. *cr*. × S: $\frac{1}{2}$ Base − $\frac{1}{2}$ diff. *cr*. S q $\frac{1}{2}$ Angle.
Dem. A E × m F. R q:: (by 4*th Ax.*) b L. $y\alpha$ :: b L × $\frac{1}{2}$ R. $y\alpha$ × $\frac{1}{2}$ R. *i. e.* (by *Lemma* 5*th* and 1*st.*) :: S: $\frac{1}{2}$ Base + $\frac{1}{2}$ diff *cr*: S: $\frac{1}{2}$ Base − $\frac{1}{2}$ diff *cr*. S q $\frac{1}{2}$ Angle.

COROLLARY I.

Suppose m, n the Legs of the Angle, B the Base, $Z = m + n$, $x = m - n$. $\frac{Z}{2}$ = $\frac{1}{2}$ Sum of m, n, B. *th.* ▯ Sine *cr*. R q :: S: $\frac{Z}{2} - m$: × S: $\frac{Z}{2} - n$, S q $\frac{1}{2}$ Angle.
Vlacq. prescribes this following, which comes to the same. S m. S $\frac{Z}{2} - m$ :: S: $\frac{Z}{2} - n$. A. and S n. R :: A. Q. then R × Q = S q $\frac{1}{2}$ Angle.

COROLLARY II.

Putting μ = Complement of m *Gunter* resolves the 11*th* Case by this single Proportion. S m. σ n :: Σ B υ S: μ + n : V of the Angle. For S n) R R (= σ n. and V B υ V: $m - n$ = Σ B υ Σ: $m - n$: = Σ B υ S: μ + n.

COROLLARY III.

Being a Proposition like the 4*th* Axiom. ▯ Sines *cr*. R q :: V Z *cr*. − V Base. v Angle. For resuming the Circles of the Pastboard, reduce the Plane B P to the same Plane with B A, so that they may make one Circle, and let fall P λ ⊥ A C. Then $b\lambda$ = λ A. − B A: *i. e.* V Z *cr*. − V Base. But A E. A C :: $b\chi$. β P. and m F. R :: β P. v Ang. *th*: A E × m F. R q :: ($b\lambda$ × β P: β P × v ::) $b\lambda$. v. Hence follows

COROLLARY IV.

▯ Sine *cr*. R q :: S: $\frac{1}{2}$ Z *cr* + $\frac{1}{2}$ Base: × S: $\frac{1}{2}$ Z *cr* − $\frac{1}{2}$ Base: Σ q $\frac{1}{2}$ Ang. For A E × m F. R q :: $b\lambda$. v :: $b\lambda$ × $\frac{1}{2}$ R. v × $\frac{1}{2}$ R. *i. e.* (by *Lem.* 5*th*, and 2*d*,) :: S: $\frac{1}{2}$ Z *cr*. + $\frac{1}{2}$ Base: × : S $\frac{1}{2}$ Z *cr* − $\frac{1}{2}$ Base. Σ q $\frac{1}{2}$ Ang. Hence

COROLLARY V.

S m × S n. R q:: S $\frac{Z}{2}$ × S: $\frac{Z}{2}$ − B: Σ q $\frac{1}{2}$ Ang.

COROLLARY. VI.

It follows from the 5*th* and 1*st Coroll.* that S $\frac{Z}{2}$ × S: $\frac{Z}{2}$ − B :. S: $\frac{Z}{2}$ − m: × S: $\frac{Z}{2}$ − n: :: (Σ q $\frac{1}{2}$ Ang. S q $\frac{1}{2}$ Ang. ::) R q. T q $\frac{1}{2}$ Ang.

COROLLARY VII.

S: $\frac{Z}{2} - m$ × S: $\frac{Z}{2} - n$:. S $\frac{Z}{2}$ × S $\frac{Z}{2}$ − B :: R q : T q $\frac{1}{2}$ Ang. This follows from the last *Corollary.*

COROLLARY VIII.

V Z − V X. Diameter :: V B − V X. V $\mathcal{L}$. or :: V Z − V B. v $\mathcal{L}$. Which is the practise of *Foster* with his Line of Versed Sines. *Dem.* (by 4*th Ax.*) V B − V X. V $\mathcal{L}$ :: S m × S n. R q:: (by 3*d Cor.*) V Z − V B. v $\mathcal{L}$:: Sum of the 1*st* and 5*th* Terms. Sum of 2*d* and 6*th*. *i. e.* :: V Z − V X. Diameter.

COROL-

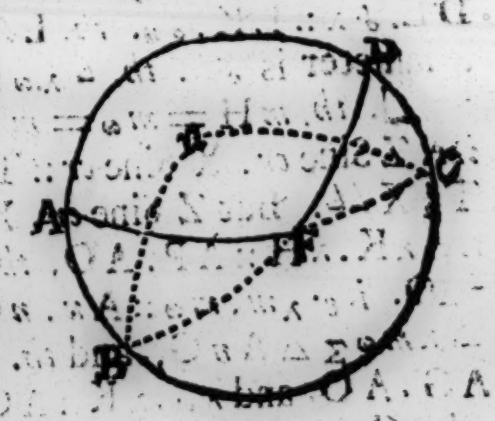
COROLLART. IX.

(Following from 8 *Cor*,) $\frac{\Sigma Z \backsim \Sigma X}{2} \cdot R :: \Sigma B \backsim \Sigma X . V L$. or. $\frac{\Sigma Z \backsim S : \mu + n .}{2}$ $R :: \Sigma B \backsim S \mu + n . V L$. Which is a Theorem of frequent use with *Kepler*.

12 Case. The 3 Angles being given, to find a Side.

The Angles adjacent to the Side requir'd, call Legs, and the Angle opposite call the Base, then work as in the 11*th* Case. For such is the operation in the Supplemental Triangle, whose Angles and Sides are equal to the Supplements of the Sides and Angles of the Triangle propos'd. But Arcs and their Supplements, have the same Sines and Tangents.

In a Triangle that is Right Angled or Quadrantal, the 2 parts which are adjacent to the Right Angle or Quadrant, together with the Complements of the other 3, are called by *Neper* the 5 Circular parts. And if the 3 parts which enter the Question, (*viz.* 2 are given, and one required) have no interruption, now though a Right Angle or Quadrant come between, 'tis not counted an interruption; that part which is betwixt the other 2, is call'd the middle part, and then the other 2 are call'd Extreams adjacent or conjunct. But if there be an interruption, that part which is separate from the other 2, is called the middle part, and the other 2 Extreams opposite or disjunct. This being premised, *Neper* after a diligent view of the Solutions of all the Cases of Right Angled and Quadrantal Triangles has observed, that they all agree in one or 2 Propositions. *viz.* That Rad. × S. of the middle part is = Rectangle or Product made of the Tangents of the Extreams conjunct, or to a Rectangle made of the Co-sines of the Extreams disjunct. This Proposition invented by *Neper* purely for the ease of Memory, has been since applyed in all its Cases by *Gellybrand*, *Cavallerius*, *Orsinus*, *Oughtred*, *Norwood*, *Vlacq*, *Ward* and *Wing*.

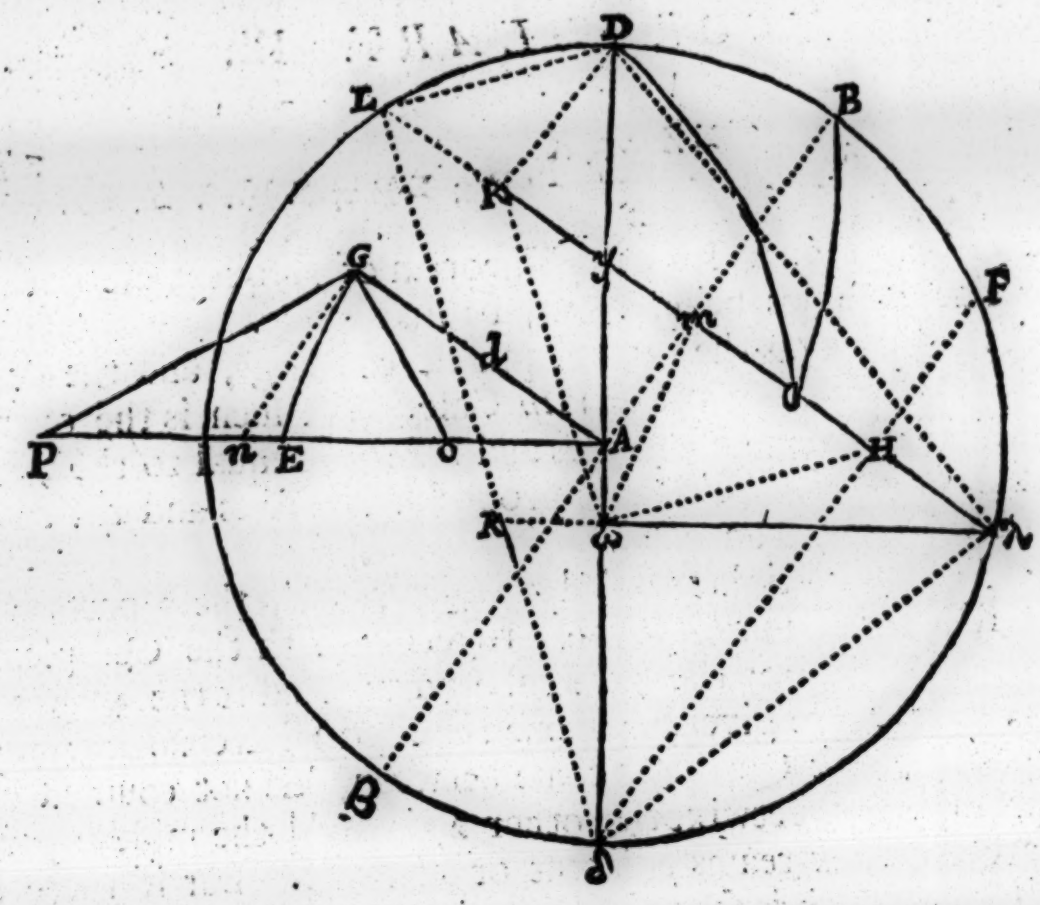


In any Spheric Triangle DAF, 2 Right Angles, are to the excess of the 3 Angles above 2 ⌋, as (G) the Surface of the Sphere, to (4 △) 4 times the Surface of the Triangle, *Dem.* Continue the Sides to Semicircles, continue also FB, FC to Semicircles meeting again in *m. th.* △BmC = △MFD. But 4 ⌋. ∠DAF + ∠ADF + ∠AFD :: G. to the parts of G which are between the 3 pairs of Semicircles AFC ADC, DAB DFB, BBE FCE; which parts are manifestly = G + 2 △. *th.* 2 ⌋ ∠∠∠A. G. G + 4 △. or 2 ⌋. ∠∠∠ — 2 ⌋ :: G. 4 △. Suppose ϖ the Periphery of a great Circle, and E an Arc thereof = ∠∠ — 2 ⌋. *th.* $\frac{1}{2}\varpi$. E :: G. 4 △. *th.* 2 ϖ △ = EG = 2 R ϖ E. *th.* △ = RE.

The last Proposition is I think due to *Cavallerius*, but the next following was invented by *Neper*, and is celebrated not less for its subtilty, than usefulness in resolving the last tenth Case, without letting fall a Perpendicular, or any Ambiguity.

In any Spheric Triangle DCB, 2 Sides DB, BC, and ∠ B included being given, to find the other Angles DC, by 2 proportions. (*viz.*)

$S \frac{1}{2} Z\, cr. S \frac{1}{2} X\, cr :: \tau \frac{1}{2}$ Vertical Angle. $T \frac{1}{2} X \angle\angle$. and $\Sigma \frac{1}{2} Z\, cr. \Sigma \frac{1}{2} X\, cr :: \tau \frac{1}{2}$ Vert. ∠. $T \frac{1}{2} Z \angle\angle$.



Dem. Suppoſe A E G Poles of the Sides D B, D C, B C. *th.* the Arc A E = ∠ D, Arc E G = ∠ C, Arc A G = 180° — ∠ B. Suppoſe the Arc E O = Arc E G = Arc E P. Then if the points G A O E P be Stereographically projected, the Right Line A E will be = T ½ ∠ D, and A G = τ ½ B, and A O = T ½ X ∠∠, and A P = T ½ Z ∠∠; and O G P will be a Semicircle deſcribed from its Pole E through G, whoſe Center ſuppoſe *n*. Then take B λ = B C = B L, and draw the Diameter B *m* A β, and D K ‖ B *m* ‖ δ H. *th.* D B = δ β, and *m* K = *m* H, and F λ = D L. Draw λ ω ⊥ D δ, then △ λ δ *y* ≅ △ D L *y*, and D L K ≅ △ λ δ ω. *th.* L *y*. δ *y* :: D L. δ λ :: L K. δ ω. *th.* L δ ‖ K ω: But the Points δ λ H ω are in a Circle, whoſe Diameter is δ λ. *th.* ∠ λ ω H = (∠ λ δ H = ∠ L δ D =) K ω *y*. *th.* ∠ K ω H = *y* ω λ = ⌟. *th.* *m* H = *m* ω = *m* K. Alſo λ K. L K :: (ω λ . ω R ::) T ∠ D δ λ. T ∠ D δ L. *i. e.* Z Sine *cr*. X Sine *cr* :: T ½ Z *cr* : T ½ X *cr*. *th.* Z Sine ∠∠. X Sine ∠∠ :: T ½ Z ∠∠. T ½ X ∠∠. But Z Sine *cr*. X Sine *cr* :: (Z Sine ∠∠. X Sine ∠∠ ::) T ½ Z ∠∠. T ½ X ∠∠. *i. e.* λ K. λ H :: A P. A O. *th.* ½ λ K + ½ λ H. ½ λ K — ½ λ H :: ½ A P + ½ A O. ½ A P — ½ A O. *i. e.* λ *m*. *m* ω :: A *n*. *n* G. But alſo Angle *m* λ ω ≅ ∠ G A *n*. *th.* (by 7.6 *Encl.*) △ λ *m* ω ≅ △ A *n* G, and *th.* △ λ H ω ≅ △ A O G, and △ λ K ω ≅ △ A P G. *th.* λ ω. λ H :: A G. A O. and λ ω. λ K :: A G. A P. But D λ. λ ω :: D L (L K =) λ H. and δ λ. λ ω :: δ L. (L H =) λ K. *th.* D λ. D L :: (λ ω. λ H ::) A G. A O. and δ λ. δ L :: (λ ω. λ K ::) A G, A P. *i. e.* S ½ Z *cr*. S ½ X *cr* :: τ ½ Vert. ∠, T ½ X ∠∠. and Σ ½ Z *cr*. Σ ½ X *cr* :: τ ½ Vert. ∠. T ½ Z ∠∠.

COROLLARIES.

In any Spheric Triangle A E G.

1. T ½ Baſe, T ½ Z *cr* :: T ½ X *cr*. T ½ X of the Segments of the Baſe A G made by a Perpendicular Arc falling thereon from E.

A G. A O :: A P. A *d*.

2. T ½ Baſe T ½ Z *cr* :: Σ ½ Z ∠∠. Σ ½ X ∠∠. A G. A P :: δ λ. δ L. For ∠ E A G = Arc D B, and ∠ A G E = B C.

3. T ½ Baſe T ½ X *cr* :: S ½ Z ∠∠. S ½ X ∠∠.

A G. A O :: D λ. D L.

4. If ∠ E G A be ſuppoſed Right, then A P × A O = A G *q*. *i. e.* In a Right Angled Spheric Triangle, E G A. T: ½ Hypot. + ½ Perpend. × T: ½ Hypot. — ½ Perpend. = T *q* ½ B aſe.

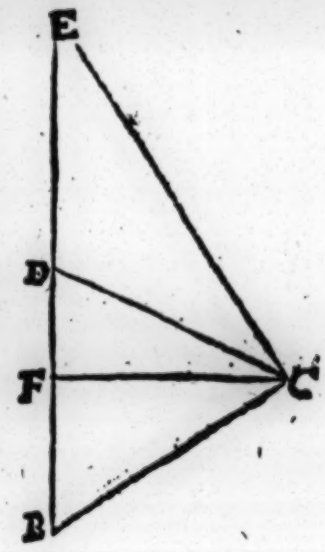
I ſhall

I shall here add 3 Theorems, serving to find by Calculation the Diameters or Centers of the Circles of the Sphere, in the Stereographic projection, or for the more easy making and proving the Tables of natural Tangents and Secants.

Draw CF ⊥ BE, and CD = DE = DB. *th.*

1. ∠DCE = ½FDC = ½ Complement of FCD; but FD + CD = FE. *i.e.* T of an Arc + S Arc = T Arc + ½ Co-arc = T: 45° + ½ Arc.

2. ∠DCF + 2FCB = ∠B + FCB = ⌟ = DCF + 2DCE. *th.* FCB = DCE = ½ Compl. FCD, and BCE = ⌟. But DF + FB = CD. *i.e.* T Arc + T ½ Co-arc = ∫ Arc. For example T 60° + T 15° = ∫60° = 2 Rad. and *th.* 2 T 60° + T 15° = T 60° + 2 R = T 60° + ∫ 60° = (by 1*st* Theorem) T 75°.



3. $\frac{FE - FB}{2} = FD$. *i.e.* $\frac{\tau \text{ Arc} - T \text{ Arc}}{2} = \tau$ 2 Arc. and $\frac{FB + FE}{2} = CD$. *i.e.* $\frac{\tau \text{ Arc} + T \text{ Arc}}{2} = \sigma$ 2 Arc.

FINIS.

ERRATA.

Pag. 2. l. 44. for Sq — read Sq +. l. 54. for 4gbq = read 4gbq (4cbq) = p. 5. l. 18. for EB × read EB ::